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ECONOMETRICS AND ARMS RACES: A CRITICAL REVIEW AND SOME EXTENSIONS *

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ABSTRACT

Recent years have witnessed an increasing use of econometric analysis in arms race studies. The present article reviews this past and the potential future contribution of econometrics to arms race research and concludes that so far the main impact of the new approach has been on efforts at parameter estimation. Therefore, a number of problems typically arising in the application of econometric techniques to the estimation of arms race models from empirical time series data are discussed, and the need for a thorough investigation of the small-sample properties of the results of common procedures is demonstrated. Finally, the considerable potential of econometrics for refining current arms race models is illustrated in the context of asymmetric response and distributed lag hypotheses.

1. Introduction

Empirical analysis of theoretically inspired arms race models dates back to Richardson's work on the outbreak of World War I. Despite theoretical elegance, Richardson's studies – which culminated in a posthumous book-length publication in 1960 – suffer from crude methodology which caused several questionable or erroneous inferences [1]. The same goes for some of the first quantitative arms race studies (e.g. Smoker 1963a, 1963b) which followed Rapoport's (1957) attempt to bring Richardson's work to the attention of a wider audience.

Modern methodology did not make its way into the field of arms

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race research until recently so that we are only now witnessing the beginning of systematic empirical evaluation of arms race models. This process should finally yield some insights on whether and when such models do in fact explain observed reality or merely offer heuristic insights. At the heart of this current revolution of arms race research is the borrowing of econometric techniques. It is therefore useful to ask at the outset how econometrics has so far been utilized in arms race studies, what are the typical problems posed by its application to the field, and whether it offers additional benefits which might be exploited.

2. The Econometric Approach to Arms Race Research

In a recent review (Luterbacher, 1975) of the "state of the art" of arms race modeling and research, three contributions are claimed for the econometric perspective:

1. Analysis of arms races in terms of rational decision-making subject to a goal function and to cost constraints.
2. Apprehension of the discrete character of decision-making by using difference equations rather than differential equations.
3. Application of advanced methodological techniques for estimating model parameters.

The second issue is a purely formal one and will not be considered in this article. Since the estimation of models is at the core of econometrics, there can be no doubt about its methodological impact on arms race studies. It is questionable, however, whether the notion of an arms race as an interaction of rationally designed strategies constrained by the scarcity of resources can be attributed to the econometric perspective.

Assuming 1971 as the year that saw the first significant contribution of the econometric approach to the study of arms race phenomena it is obvious that by that time the analysis of goal-directed behavior in armament competitions already had an impressive history. 1971 is suggested by the publication of Lambelet's (1971) article on the Middle East and of Mihalka's (1971) thesis. Prior to that date a number of studies had appeared, among them those by Ash (1951), Burns (1959), Hoag (1961), Kent (1963), McGuire (1965), Intriligator (1964, 1968a, 1968b), and Deagle (1967), that all departed from Richardson's black-box approach to the internal structure of the opponents in an arms race by treating their policies as deliberate and rational strategies derived from more or less sophisticated goals. The intellectual origins of most of these studies can be traced to operations research, decision- and game-theory, and cost-utility analysis. Most recent contributions in the same vein are the articles by Brito (1972, 1974) and Intriligator (1975). Thus, there was a wealth of theoretical literature on strategic models of arms races upon which econometric analysis could draw when it finally made its way into the field.

Similarly, the most convincing departure from Richardson's simplistic treatment of cost and supply constraints was published years ahead of the first econometric study of arms race behavior. Those who have summarized the literature (e.g. Busch, 1970, Chatterjee, 1971, or Rattinger, 1975) agree that Caspary's (1967) model comprises the most elaborate view of the inhibiting effects of economic burdens on armament races. This model, however, is not inspired by the econometric approach. Those recent contributions to the field that have used econometric techniques, on the other hand, have consistently proceeded with models involving cost constraints in much cruder fashion than proposed by Caspary. As Luterbacher has pointed out this is largely due to the nonlinear character of Caspary's model which poses a serious identification problem.

All this suggests that so far only the third point quoted at the outset describes the genuine innovation of the econometric perspective in arms race research. This statement does not belittle the importance of econometrics but serves to focus attention where it belongs, i.e. on the methodological issues raised by attempts to estimate arms race models. Such problems could be recognized right from the beginning. Lambelet (1971), e.g., obtained impressive fits for his models of the Middle East arms race up to 1965 but many model parameters were not significant due to multicollinearity. Luterbacher has listed some further problems typically arising in the econometric analysis of arms races in his recent review article. The present article will now turn to a few of these as well as to some other issues that so far have received little attention in arms race studies.

3. Some Problems of Estimating Arms Race Models

Luterbacher (1975) names three problems complicating the estimation of parameters of arms race models:

1. Substantial considerations, e.g. on the effect of cost constraints, might lead researchers to use nonlinear models. Most standard procedures of econometrics, however, are geared to linear formulations.
2. Substantial arguments, again, suggest that in the long run more

complex models than those currently in vogue will be introduced. Approximating models to reality by including additional variables, however, will add free parameters and thus produce a serious degrees of freedom problem -- especially with the short duration of some arms races.

3. There might be a chance that established techniques cannot yield reliable estimates of the parameters of endogenously unstable relationships which are often alleged to be characteristic of arms races.

These three issues will now be discussed together with available solutions. We will then turn to some difficulties with autocorrelated residuals which deserve as much attention as the problems mentioned by Luterbacher.

ESTIMATION OF NONLINEAR MODELS

Talking about nonlinear models, one has to remember the distinction between models that are nonlinear with respect to variables but linear in the parameters that are to be estimated and models nonlinear both in variables and parameters. Models of the first group are also called "intrinsically linear" whereas those of the second type are designated as "intrinsically nonlinear" (Kmenta, 1971).

The basic feature of intrinsically linear models is that they can be converted into linear models by an appropriate transformation of variables. They therefore present no particular estimation problems. A simple example is

$$y_t = ax^k_{t-i} + e_t \tag{1}$$

where the error term e_t is randomly distributed around zero with constant variance. Regardless of k there is a transformation $z_{t-i} = x^k_{t-i}$ which allows (1) to be rewritten as

$$y_t = az_{t-i} + e_t \tag{2}$$

which is linear.

Models with interaction terms serve as another example of intrinsically linear models. Interaction terms are the simplest way of formalizing the notion that the change in the dependent variable corresponding to a given change in an independent variable x_1 depends on the level of another independent variable x_2.

$$y_t = ax_{1,t-i}x_{2,t-j} + e_t \tag{3}$$

If we write $z_{t-i} = x_{1,t-i}\, x_{2,t-j}$ we again have the linear model (2). After a series of such transformations, e.g., Richardson's nonlinear submissiveness model poses no unusual obstacles to estimation.

As a third case of an intrinsically linear model [2] consider the multiplicative model which is analogous to Lambelet's model of the Middle East arms race:

$$y_t = a x_{1,t-i}^{b_1} \dots x_{k,t-j}^{b_k}\, e_t \tag{4}$$

This model is linear in the logarithms of the variables and can be rewritten as

$$y'_t = a' + b_1 x'_{1,t-i} + \dots + b_k x'_{k,t-j} + e_t \tag{5}$$

where $y'_t = n(y_t)$ etc. There are many other nonlinear models which can be similarly transformed into linear equations, as e.g. exponential or hyperbolic functions.

For intrinsically nonlinear models things are more complicated. Imagine, e.g., that in (4) the disturbance has been entered additively or consider Caspary's (1967) nonlinear arms race model. In those cases there is no transformation leading to a linear model as nonlinearity pertains to parameters. A general solution to the estimation problem is available, however, which also applies to such models.

It is necessary, as a first step, to set up the likelihood function for the model to be estimated. Then, its partial derivatives with respect to all unknown parameters are taken and set equal to zero. The problem of this maximum likelihood approach then is to solve the resulting system of normal equations for the unknown parameters. Generally there will be no straightforward analytical solutions available whenever these equations are nonlinear in parameters and iterative numerical procedures must be employed. Such programs usually start from "intelligent guesses" of the numerical values of the parameters, directly compute the likelihood function, and then vary those values in order to maximize the likelihood function [3]. These procedures thus output maximum likelihood estimates of model parameters.

A final point concerning nonlinear models should briefly be made. Usually such models receive a substantial rationale, i.e. it is assumed a priori that certain factors govern the behavior of arms racing nations *and* that the functional form of their influence is a nonlinear one (e.g. Luterbacher, 1975, p. 212). Assumptions of this kind ignore the possibility to treat the specification of a model as testable. In view of the practical difficulties with the estimation of intrinsically nonlinear models it seems worthwhile to test for linearity of the relationship between dependent variables and potential explanatory variables before employing nonlinear models by available tests for linearity (Kmenta, 1971, chapter 11).

SHORT TIME SERIES VERSUS COMPLEX MODELING

If we measure the complexity of an arms race model by its number of variables it is obvious that the complexity of modeling is constrained by the duration of the race a particular model is to be applied to. With limited-information methods there is generally no difficulty in estimating a model when the overall number of predetermined variables exceeds the number of observations as long as this is not the case for any single equation (Theil, 1973). But the issue then is to determine whether estimation results do support the specification of the model. The reader is reminded that if we now briefly discuss inferences from short arms races we are really addressing only one facet of a complex problem. On the one hand one might argue that arms race data are population and not sample data, and on the the other hand it is conceivable that highly significant results will not be reproducible across cases, e.g. due to systematic measurement error. Because of space limitations there can be no attempt here to reconcile these considerations with the econometric perspective.

The goodness-of-fit of a multiple regression equation – as measured by R^2 – is going to go up whenever we reduce the number of observations and/or increase the number of variables. It is therefore unwise to be overwhelmed by R^2s that look impressive in bivariate analysis as such "good fits" have to be expected whenever there is a serious problem with degrees of freedom. If we have, e.g., six observations but as many as four explanatory variables only R^2s above .99 will be significant at the .05-level. Goodnes-of-fit by itself thus is a weak criterion for accepting hypotheses or not and should be augmented by F-ratios and their significance levels. This is especially relevant whenever we compare models with different numbers of explanatory variables.

Usually it should also prove useful to consider parameter estimates and their standard errors because in many cases we will previously have set up hypotheses on their approximate numerical values or on some parameters being significantly distinct from each other or from zero, etc. [4]. As long as one has at least a few degrees of freedom left it should be possible to perform tests like these and to reject or modify one's model accordingly. The main difficulty in applying arms race models to races of short duration therefore is not the availability of appropriate methodology but rather this necessity to maintain a minimum number of degrees of freedom.

This prerequisite can only be met by fitting extremely parsimonious models. The opportunity for more complex modeling is thus tied to the occurrence of armament races of longer duration so that often complexity will have to be sacrificed for testability. This poses a dilemma if one or more variables are known to have a substantial impact on the behavior or arms racing opponents but have to be excluded from the model for the sake of simplicity. The choice then presents itself between knowingly committing a specification error in order to be able to estimate the resulting model or correctly to specify a model which cannot be estimated. Fortunately, under certain (testable) assumptions (Kmenta, 1971, chapter 10) the detrimental effects of omitting a relevant explanatory variable are limited and lead to conservative inferences so that the former option does not have to be self-defeating.

ESTIMATION IN THE UNSTABLE CASE

Most challenging of the three problems raised in Luterbacher's review is the third. Consider as given a set of data on the armament policies of two antagonists over time and imagine that precise knowledge on the behavioral laws governing this interaction and on their parameters is available. If we now remove all exogenous variables from the system – i.e. those variables which do not appear as dependent variables – it becomes possible to apply criteria for equilibria as well as stability to the resulting closed system. Assume this closed system to be an unstable one. According to Luterbacher (1975: 215) least squares methods will then not produce reliable parameter estimates from the data. More precisely, he thinks that least squares estimates will be biased downward toward stability.

Before confronting this assertion with some results from the literature a few remarks on its implications for arms race research seem in order. Looking for stability or instability of arms races would not be worthwhile if there was in fact a systematic bias as one would often produce erroneous conclusions in favor of stability based on biased parameter estimates of *truly* unstable systems. But also if one does not think analyses of stability to be among the most relevant contributions of arms race studies the alleged bias could lead to serious difficulties, particularly in attempts to forecast.

Viewed in a somewhat broader and skeptical perspective, however, the problem of endogenously unstable systems loses some of its importance. If we realize that in their studies of arms races researchers are frequently forced to apply almost ridiculously oversimplified models to short series of unreliable data it should be obvious that even without this additional problem there will usually be reason enough to expect biased and unreliable parameter estimates. More often than not models will be ridden by specification errors, measurement error will be non-random, and the assumptions of the linear regression model will be more or less severely violated. Under such circumstances one should be content if estimation results allow some crude conclusions at least on the relative impact of a number of variables on arms race behavior. According to this view, confidence in the exact numerical values of parameter estimates would be inappropriate anyway and regardless of stability.

But let us now assume a world where all these standard problems of applied regression analysis do not exist and our only worry is the estimation of explosive systems. To begin with, consider the simple first-order autoregressive process

$$y_t = ay_{t-1} + e_t \tag{6}$$

where e_t is a normally distributed random error term with mean zero and finite standard deviation. The least squares (and also maximum likelihood) estimate of a, $\hat{a}$, is

$$\hat{a} = \left(\sum_{t=1}^{T} y_t y_{t-1}\right) \Big/ \left(\sum_{t=1}^{T} y_{t-1}^2\right) \tag{7}$$

The properties of $\hat{a}$ can be analyzed in four situations. First, its asymptotic (large-sample) properties can be confronted with its properties in small samples. Second, the stable case with $|a| < 1$ has to be distinguished from the unstable or explosive case $|a| > 1$. The circular case $|a| = 1$ will not be considered here [5].

Anderson (1959) has shown that with increasing sample size $\hat{a}$ is an unbiased and consistent estimate of a and tends to be normally distributed around a with finite variance if the successive errors e_t are distributed independently and identically. These results have been generalized for the stable vector case (Anderson, 1971, chapter 5-5) which is of particular concern for arms race analysis. This implies that in large samples stable autoregressive systems can be estimated by the usual procedures.

A number of authors have investigated the asymptotic properties of estimates of non-stationary autoregressive systems [6]. They all face the difficulty described by Luterbacher, i.e. that for $|a| > 1$ the variance of y_t as well as y_t itself grow towards infinity with large t even though the variance of e_t has been assumed constant in selecting $\hat{a}$ as an estimate. Nevertheless, all explorations of the asymptotic properties of $\hat{a}$ reported in the literature seem to contradict his conjecture that this assumption might produce a stability bias in $\hat{a}$. Studies by Rubin (1950), White (1958, 1959), Anderson (1959), and Rao (1961) all allow the conclusion that $\hat{a}$ will be an asymptotically unbiased, consistent, and efficient estimate of a if the errors e_t are independently normally distributed and if $|a| > 1$. Most of these results have also been shown to hold for unstable autoregressive processes of arbitrary order (Rao, 1961). Anderson (1959) has generalized his analysis for the unstable vector case, and he concludes that the usual procedures are applicable. All these results have been obtained, however, assuming a zero constant term.

For arms race research with its short series of data, proofs on asymptotic properties of estimates offer little consolation. This becomes evident if we now turn to stable autoregressive processes in small samples. The relevant body of literature is very restricted. For sample sizes of three and four Hurwicz (1950) has proved that $\hat{a}$ has a downward bias if $0 \leqslant a \leqslant 1$, and he has also analytically derived the size of the bias for various other small sample sizes. He finds downward bias to be strongest when a tends toward zero, i.e. when the process is maximally damped. If a approaches unity the limit of the ratio $\hat{a}/a$ also is one regardless of sample size. The largest bias ($\hat{a}/a = .73$) occurs with samples of size four and a approaching zero. With growing samples greater than four $\hat{a}/a$ monotonically increases toward unity for all values of a. These results suggest that the bias in $\hat{a}$ is negligible even in small samples if the true value of $|a|$ is close to one. The smaller $|a|$, however, the more observations we need for $\hat{a}$ to be an approximately unbiased estimate of a [7]. Unfortunately, analogous investigations so far have not been reported for higher-order autoregressive processes and/or the autoregressive vector case.

To the best of my knowledge, no analyses exist of the problems arising in efforts to estimate explosive autoregressive systems from few observations. As the objective of this section is only to survey the literature for available results no attempts will be made to overcome this deficiency. A series of Monte Carlo experiments over a range of models, sample sizes and values of a appears as the most promising strategy to generate the required information on the existence of bias in small-sample estimates. Considering Hurwicz's results for the stable case it seems possible that the bias conjectured by Luterbacher for the asymptotic case might in fact instead be present in estimates from small samples describing explosive processes.

AUTOCORRELATED RESIDUALS IN AUTOREGRESSIVE MODELS

The errors e_t in (6) have been assumed to be randomly and independently distributed. This assumption will frequently be violated in empirical applications. For non-autoregressive models, however, autocorrelation of errors will not lead to biased or inconsistent least squares estimates, not even from small samples (Kmenta, 1971, chapter 8-2).

For autoregressive models like (6) and therefore for most arms race models things are different. The combination of lagged endogenous variables and serially correlated errors generates biased and inconsistent least squares estimates in small and large samples. Even with an infinite number of observations ordinary least squares estimates will not converge to the true parameter values (Theil, 1971, chapter 8-7). This can be shown for (6) by assuming that the errors are themselves governed by a first-order autoregressive process

$$e_t = pe_{t-1} + u_t \tag{8}$$

where u_t is randomly distributed with zero mean and finite standard deviation. (6) can then be written as:

$$y_t = (a + p)y_{t-1} - pay_{t-2} + u_t \tag{9}$$

Since the explanatory variable y_{t-2} is excluded when (6) is estimated, the least squares estimate $\hat{a}$ from (6) can be shown to be for large samples (Hibbs, 1974):

$$\lim \hat{a} = (a + p)/(1 + pa) \tag{10}$$

This estimate contains a bias which tends to

$$\hat{a} - a = p(1 - a^2)/(1 + pa) \tag{11}$$

It has been similarly demonstrated that the least squares estimate $\hat{p}$ of the first-order autocorrelation coefficient derived from observed residuals from (6) also contains a bias which is equal to (11) in magnitude but opposite in sign (Malinvaud, 1970, chapter 15-5). This implies that an empirical estimate $\hat{p}$ will not enable us to correct for bias in $\hat{a}$. This example is a very extreme one as in (6) there are no exogenous variables at all. Malinvaud (1970) has shown analytically and by Monte Carlo experiments that least squares bias in the estimation of autoregressive processes with autocorrelated disturbances is greatly reduced by the presence of exogenous variables.

There remains the problem of arriving at consistent estimates of the parameters of an autoregressive model whenever these parameters and/or the process generating its disturbances are unknown. If there are sound a priori reasons for assuming a specific interdependence structure of the disturbances the parameters of both the model and the error structure can be estimated by ordinary least squares from an equation analogous to (9). Generally, however, such prior knowledge is unavailable and other techniques have to be used which investigate error structures and estimate model parameters at the same time [8]. Unfortunately the small-sample properties of available methods are either unsatisfactory or as yet unknown. Together with the foregoing section this suggests that the exploration of the properties of estimation procedures for stationary and explosive autoregressive processes with systematically interacting errors in very small samples by means of experimental analyses deserves highest priority if the way is to be paved for a large-scale application of econometric estimation methods in arms race research.

4. The Potential of Econometrics for Modeling Arms Races

As said earlier, the benefit of econometrics for the study of arms races so far has been in the borrowing of its estimation methods. It might now be useful to demonstrate that its contribution does not have to be confined to estimation but that it offers a considerable potential for the construction and refinement of models. This will be illustrated here in the context of asymmetric response and distributed lag models.

ASYMMETRIC RESPONSE MODELS

Models of arms races have generally assumed the effects of changes in explanatory variables upon dependent variables to be symmetric. This assumption is easily illustrated for Richardson's classical two-nation model. Substituting first differences for derivatives and allowing for variable lags in the response to the enemy's arms we write this model as

$$x_t = ax_{t-1} + cy_{t-i} + g \qquad (12)$$

$$y_t = by_{t-1} + dx_{t-j} + h \qquad (13)$$

It is obvious from (12) that the absolute effects on x of a unit increase and a unit decrease of y are identical.

This implication of the classical model can be challenged on substantial grounds, and some interpretations of the current East-West arms competition can serve as an example. Many political and military leaders of the Western Alliance maintain – whether rightly or not does not matter here – that the Soviet Union and her allies have, during the last decade or so, greatly intensified their defense efforts while the miliary preparations of the West have slackened or even decreased in the same period. If these *observations were correct* the Richardsonian model would be unable to explain this confrontation as it predicts restraint on the one side after a previous parallel build-up to be reciprocated by the other side.

This difficulty can be overcome by the hypothesis of asymmetric response, i.e. the notion that the change in a dependent variable associated with a unit increase in an explanatory variable differs in absolute magnitude from the change effected by a unit decrease in the same variable. Applied to arms races its most extreme version would be that decreases in the adversary's capability are systematically ignored whereas increases are consistently answered by stepping up the own nation's military posture. Tailored to Richardson's model, this version of asymmetric response is formalized in (14) and (15).

$$x_t = ax_{t-1} + c(C_{t-i}y_{t-i} + |C_{t-i} - 1|u_{t-i}) + g \qquad (14)$$

$$y_t = by_{t-1} + d(D_{t-j}x_{t-j} + |D_{t-j} - 1|v_{t-j}) + h \qquad (15)$$

where

$$u_{t-i} = \max(y_m)\ (m = 1, 2, ..., t-i-1)$$

$$v_{t-j} = \max(x_n)\ (n = 1, 2, ..., t-j-1)$$

$$C_{t-i} = \begin{cases} 0 \text{ if there is a } y_m\ (m = 1, 2, ..., t-i-1) \text{ with } y_{t-i} < y_m \\ 1 \text{ otherwise} \end{cases}$$

$$D_{t-j} = \begin{cases} 0 \text{ if there is a } x_n\ (n = 1, 2, ..., t-j-1) \text{ with } x_{t-j} < x_n \\ 1 \text{ otherwise} \end{cases}$$

One disadvantage of this modification is that the additional explanatory power of the asymmetric response hypothesis can be assessed only indirectly. It can be evaluated more directly by including it into a dummy variable as in (16) and (17):

$$x_t = ax_{t-1} + cy_{t-i} + kC_{t-i}y_{t-i} + g \qquad (16)$$

$$y_t = by_{t-1} + dx_{t-j} + lD_{t-j}x_{t-j} + h \qquad (17)$$

where

$$C_{t-i} = \begin{cases} 1 \text{ if } y_{t-i} \leqslant y_{t-i-1} \\ 0 \text{ otherwise} \end{cases}$$

$$D_{t-j} = \begin{cases} 1 \text{ if } x_{t-j} \leqslant x_{t-j-1} \\ 0 \text{ otherwise} \end{cases}$$

This variant is less sophisticated than the first one as it does not precisely specify what is going to happen to the defense efforts of one side if the other side does not increase its own military preparations and vice versa. Instead, it merely asserts that the first nation's response will be *different* according to whether the second state either increases its military effort or not, i.e. holds it constant or even reduces it. The benefit of this more general hypothesis is that it is readily tested by testing the hypotheses $k = 0$ and $l = 0$.

DISTRIBUTED LAG MODELS

All arms race models proposed so far follow Richardson in assuming that each side's armaments at a given time depend upon a set of explanatory variables in just one or at most two previous points in time. The same goes for the models from the section on asymmetric response, but this assumption does not seem overly realistic. Whether it can nevertheless be justified in a particular arms race hinges upon the theoretical framework our model is derived from and upon the indicators that are assumed to characterize this competition.

Generally, however, it is reasonable to suppose that the quantitative and qualitative composition of one side's armaments in each consecutive interval – for convenience consider these intervals to be years – serves as a stimulus to the adversary's military build-up. This stimulus will usually not be responded to instantaneously and completely but will rather induce or intensify defense programs which will stretch out over several years. Thus, looking backwards, a given military posture of one nation or alliance appears as a function of a series of previous force levels of its opponent.

This suggests the application of distributed lag models to the analysis of arms races. In theory, the number of lagged values of the opponent's arms levels which may be taken into account is unlimited. In practice we will proceed more restrictively if we realize that the effect of distant armaments levels of an adversary upon a nation's military stance is negligible in comparison to more recent ones. This leads to the adoption of a geometric lag perspective which has the effects of past hostile armaments deline in geometric progression.

Geometric lag models can be deduced in two different ways (Kmenta, 1971). The first rationale is usually presented as the hypothesis of "adaptive expectations". Applied to arms races it implies that the arms level x_t of A will be a function not of the past or present *actual* capability y of its enemy B but rather of the capability y'_t which A *expects* B to acquire in t. Proceeding from Richardson's model (12) and introducing a stochastic error term $e_{1,t}$ with zero mean and finite variance for estimation purposes we obtain:

$$x_t = ax_{t-1} + cy'_t + g + e_{1,t} \tag{18}$$

Since y'_t is not readily observable we have to specify the process generating A's expectation of B's military capability. A straightforward approach is to view A as comparing the previous expectation to the actual behavior of B and then modifying current expectations accordingly:

$$y'_t = (1-p)y_{t-1} + py'_{t-1} \qquad 0 < p \leqslant 1 \tag{19}$$

Equation (19) states that the value of y expected by A in t is a weighted average of the expected and actual values of y in the preceding interval. By means of the so-called "Koyck transformation" (Koyck, 1954) (19) can be shown to be equivalent to

$$y'_t = (1-p)(y_{t-1} + py_{t-2} + p^2y_{t-3} + \ldots) \tag{20}$$

Substituting (20) into (18) we have

$$x_t = ax_{t-1} + c(1-p)(y_{t-1} + py_{t-2} + p^2y_{t-3} + \ldots) + g + e_{1,t} \tag{21}$$

Equation (21) shows that the hypothesis of adaptive expectations when introduced into Richardson's model leads to a geometric lag representation. As this equation is difficult to estimate it is simplified by lagging one period, multiplying by p, and subtracting the result from (21):

$$x_t = (a+p)x_{t-1} - pax_{t-2} + c(1-p)y_{t-1} + (1-p)g + e_{2,t} \tag{22}$$

where

$$e_{2,t} = e_{1,t} - pe_{1,t}$$

Note that (22) is exactly identified.

The second rationale for geometric lag models is often called the "partial adjustment" hypothesis. Applied to arms races it says that each side at each time has a desired or optimal arms level to which it tries to adjust its actual capability. If x''_t is A's desired potential A would compare x''_t to the actual previous armaments and then try to make good the discrepancy as much as possible by choosing x_t.

$$x_t - x_{t-1} = r(x_t'' - x_{t-1}) + e_{3,t} \quad 0 < r \leq 1 \tag{23}$$

r measures the success of A in adjusting actual to desired capabilities which again are not directly observable so that the process generating them has to be specified. A simple formulation is that the arms level currently desired by A is a linear function of B's preceding capacity, y_{t-1}:

$$x_t'' = cy_{t-1} + g + e_{4,t} \tag{24}$$

Substituting (24) into (23) yields

$$x_t = (1 - r)x_{t-1} + rcy_{t-1} + rg + e_{5,t} \tag{25}$$

where

$$e_{5,t} = re_{4,t} + e_{3,t}$$

Equation (25) is an interpretation of Richardson's classical model and again is exactly identified. To reveal its character as a geometric lag representation it has to be lagged one period, multiplied by $(1 - r)$, the result has to be subtracted from (25), and an expansion of series has to be performed. To save space the result is not shown here.

A geometric lag model of arms race behavior can also be deduced by combining partial adjustment and adaptive expectation hypotheses. If we assume that the military capability x_t'' *desired* by A at time t is not a linear function of B's *actual* arms level but rather of the capability y_t' which A *expects* B to attain in t we can write:

$$x_t'' = cy_t' + g + e_{6,t} \tag{26}$$

This model, called a compound geometric lag model, can be translated into observable variables by substituting y_t' from (20) and x_t'' from (23) into (26) which leads to

$$x_t = (1 - r)x_{t-1} + rc(1 - p)(y_{t-1} + py_{t-2} + p^2y_{t-3} + ...) + \tag{27}$$
$$+ rg + e_{7,t}$$

where

$$e_{7,t} = e_{3,t} + re_{6,t}$$

Equation (27) clearly describes the geometric decline in the influence of past values of y upon x but is not directly amenable to estimation.

Again applying the Koyck transformation yields

$$x_t = (1 - r + p)x_{t-1} - (1 - r)px_{t-2} + (1 - p)rcy_{t-1} + \qquad (28)$$
$$+ (1 - p)rg + e_{8,t}$$

where

$$e_{8,t} = e_{7,t} - pe_{7,t-1}$$

(28) is exactly identified and identical to (22) in its variables but its coefficients demonstrate that it combines partial adjustment and adaptive expectations. If $r = 1$ it is a pure adaptive expectations model, if $p = 0$ it is a pure partial adjustment model, and if both conditions are met (28) reduces to a bivariate regression equation.

As this section is concerned with the potential of econometrics for the specification of arms race models estimation problems will not be considered [9]. Suffice it to say that partial adjustment models like (25) can generally be estimated by ordinary least squares if successive errors are uncorrelated. This is not true for adaptive expectation or compound models, even if the latter condition is met. The reason is that (22) and (28) contain error terms which are correlated with the independent variable y_{t-1}. Ordinary least squares regression would thus produce inconsistent estimates of the coefficients and alternative methods have to be used.

5. Conclusion

This section is to sum up and to indicate topics of interest for future scrutiny. By borrowing econometric estimation techniques, arms race researchers have to face all the problems arising from resorting to nonlinear modeling, from the necessity of estimating models from short series of data, and from the occurrence of unstable autoregressive relationships and of autocorrelated residuals. The latter two issues appear especially discomforting as the small-sample properties of the estimates of parameters of autoregressive systems are not well known and as there are reasons to suspect bias regardless of stability and autocorrelation of residuals. Systematic studies of the small-sample properties of such estimates by Monte Carlo experiments therefore deserve high priority if the benefits of econometric estimation techniques are to be fully exploited in future arms race analyses.

Two examples have served here to illustrate the potential contribution of econometrics for developing more realistic arms race models. Allowing for distributed lags and for asymmetries in the response to an adversary's armanents eliminates unnecessarily restrictive and untenable assumptions which have been relied on by previous models. One might therefore expect that these two mechanisms – possibly in combination – will in some cases provide an explanation of empirically observed armament dynamics where other models have failed. It will take a series of empirical studies along these lines to go beyond conjecture.

Notes

1. For a proof that Richardson's classical two-nation model is actually disconfirmed by the very pre-World War I data his famous "confirmation" is performed with see Rattinger (1975).
2. For further examples see, e.g., Wonnacott and Wonnacott (1970, chapter 4).
3. An introduction to techniques of nonlinear estimation and an overview of some standard programs can be found in Draper and Smith (1966, chapter 10). Results on the asymptotic (large-sample) properties of non-linear estimation results are presented by Malinvaud (1970, chapter 9).
4. See Theil (1971, chapter 3-7).
5. This case is briefly dealt with by Rubin (1950).
6. An article by Rao (1961) contains a table summarizing available results on the asymptotic properties of estimates of stable as well as unstable autoregressive processes of first or higher order.
7. Similar results are presented by Malinvaud (1970, chapter 14-3).
8. Available techniques are discussed by Malinvaud (1970, chapter 14-6) and Hibbs (1974).
9. The problems arising in the estimation of distributed lag models in general and geometric lag models in particular are treated extensively by Malinvaud (1970, chapter 15), Theil (1971, chapters 6-5 and 8-8), and Kmenta (1971, chapter 11-4).

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