Essays in Applied Contest Theory: Round-Robin Tournaments and Innovation Competition

Inaugural Dissertation

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Chapter 1

Introduction

A contest is an interaction in which players decide on their exertion of costly and irretrievable effort to obtain a chance to outperform rivals and win some prize (Konrad, 2009; Corchón and Serena, 2018; Fu and Wu, 2019). Contest situations occur in a great many of real life interactions and either emerge naturally from existing structures or are planned and organized by a contest designer. Naturally emerging contests include litigator and defendant who pay for lawyers to win a trial, political parties who spend resources on campaigns to win elections, firms which invest in advertising to increase their market share, lobbyists who prepare reports and speeches to persuade policy makers, armies which mobilize military equipment to win territories, and employees who exert work efforts to get promoted. Planned and designed contests include students who spend time on learning to be awarded a scholarship, and firms who expend resources on the preparation of projects and calculations for a public procurement (Corchón and Serena, 2018).

Sports contests are perhaps the most obvious and often involve high stakes (Konrad, 2009). Sport itself is one of the most significant branches of the entertainment industry and the contests attract hundreds of millions of spectators (Szymanski, 2003). In sports, athletes spend time in training effort to defeat their opponents in an organized contest. Irrespective of whether an athlete or a team of athletes wins the contest or not, the training effort cannot be recovered. One famous example is *The Boat Race*, the annually staged rowing regatta between the universities of Cambridge and Oxford.

R&D-Contests are another prominent field of contests with high stakes (Konrad, 2009). Competition in research and development might foster innovational breakthroughs that are a key factor for economic growth and thus for welfare gains (Arrow, 1963). R&D-Contests mostly emerge naturally when firms race for new products and processes to earn at best some monopoly rent by irreversibly investing in research and development (Loury, 1979). Recently, the race of biotech firms for a *CoViD-19* vaccine is a much-noticed example.

From an economic perspective, such interactions are studied in the scope of contest theory. Literature on contest theory starts with the seminal contributions of Tullock (1967, 1980), Krueger (1974), Posner (1975), and Becker (1983). These articles specifically analyze rent-seeking, i.e., the unproductive strive for an additional rent provided by the state or bureaucratic institutions. Surveys on contest theory in general are provided early on by Nitzan (1994), later on by Corchón (2007) and Konrad (2009), and recently by Corchón and Serena (2018) and Fu and Wu (2019).

Formally, a contest is a game that is described by a set of players $i \in I = \{1, 2, ..., n\}$

who exert their efforts x_i at some costs to compete for a winner prize $V > 0$ and a loser prize $L \in [0, V]$ that each player values differently with $v_i > 0$ (Fu and Wu, 2019). The exact winning probability for each player is defined by a contest success function (CSF). A CSF is a function of all players' efforts and maps the players' efforts to the probability of winning (Skaperdas, 1996). Given a vector of efforts $\mathbf{x} = (x_1, ..., x_n)$, the probability that player i wins the contest is, e.g., expressed by the functional form below, which is a generalized form of the CSF by Nti (1997) based on the Tullock (1980) ratio form:

$$p_i := p_i\left(x_1, ..., x_n\right) = \frac{f_i(x_i)}{\sum_{j=1}^{n} f_j(x_j) + d},$$

where $i \in I,\ f_i(x_i)$ is a non-negative, strictly increasing function with $f_i(0) = 0$, and $d \geq 0$ is a constant.

Thus, a contest can be represented as a normal-form game where the set of efforts corresponds to the set of strategies and the expected payoffs correspond to the expected utility (Corchón and Serena, 2018). With the assumptions that marginal costs of effort equal one and that the players are risk-neutral, this yields the following objective function:

$$E_i = p_i v_i V + \left(\sum_{j \neq i}^{n} p_j\right) v_i L + \left(1 - \sum_{j=1}^{n} p_j\right) D - x_i,$$

where the probability that player i loses and some other player j wins is given by $\sum_{j \neq i}^{n} p_j$ and the probability that neither player wins is given by $1 - \sum_{j=1}^{n} p_i = \frac{d}{\sum_{j=1}^{n} f_j(x_j) + d} =: p_d$, which is here prized with $D \in (v_i L, v_i V)$.

Many contests are embedded in larger games with manifold dynamic structures where multiple decisions are made in sequential stages rather than standing alone as static onetime interaction games. In other words, within such dynamic games, a single contest is a component that follows the rules of a static contest. Eventually, the overall outcome of the large game is determined by the results in its components. It becomes obvious that in dynamic games, a player's behavior in a given contest depends on actions in previous stages and has implications on choices in future stages (Konrad, 2012).

Dynamic games that entail contests are in most occasions identical to dynamic contests where every component is a static contest itself (Konrad, 2009, Chapter 8). For instance, in elimination contests such as the English Soccer *FA Cup*, effort choices are made in sub-contests at multiple stages and stage by stage the number of players is reduced (Rosen, 1986). Another example are races in which the same players interact over several stages, and the player who wins the most sub-contests wins the whole race, as for example in the *US presidential primaries* (Harris and Vickers, 1985; Klumpp and Polborn, 2006).

Round-robin tournaments are dynamic contests that are far less investigated, despite their frequent use in practice (see, e.g., Krumer et al., 2017a,b, 2020a; Sahm, 2019). In a round-robin tournament the players are matched pair-wisely in static contests and every player meets every other player in turn. Eventually, the players are ranked and awarded according to the number of matches won. Contest designers, particularly in sports, heavily rely on round-robin tournaments in all kinds and sizes. Team sports on a large scale like the major European soccer leagues which include the English *Premier League* with up to 20 teams, or on a small scale, like the first rounds (i.e., group or pool stages) of Basketball *Olympics* (since 1948), *Rugby World Cups* (since 1987) and

FIFA Soccer World Cups (since 1950) with down to four teams. For their World Cups from 2026 on, the FIFA recently announced a transition to an initial first round with only three teams per group. Three-player round-robin tournaments are also applied in individual sports such as the *Olympic preliminaries* of Badminton (since 2012) or Wrestling (2000, 2004).

While three-player round-robin tournaments are inevitably sequential, also round-robin tournaments with more than three players often have a sequential schedule due to restrictions of the venue capacity or strategic marketing decisions by the organizers. Such a sequence of contests where different participants interact at different stages raises the question whether the structure provides equally fair conditions for all participants.

Innovation competition is often considered as a dynamic game (see, e.g., Dasgupta and Stiglitz, 1980a; Delbono and Denicolo, 1991; Marshall and Parra, 2019). It describes firms' R&D activity within the scope of market structures. In other words, there is interdependence between firms' incentives to innovate and competition. A dynamic game occurs when firms, subsequent to a successful R&D process, reap their rewards in terms of a competitive advantage on the product market. This implies that R&D-Contests naturally emerge in product market competition as firms race to be the first to successfully innovate and, thus, earn some extra rent on the product market. For example, currently, in the automotive industry, e-car producers are situated in a R&D-Contest targeting a competitive advantage in the production of batteries.

When many firms simultaneously race for new innovations, which they subsequently bring to market, it is ambiguous whether the overall investment in R&D is efficient in terms of welfare. On the one hand there might be over-investment due to a duplication of innovation effort. On the other hand there might be under-investment because single firms on their own are unable to achieve substantial advances in R&D. Eventually, the question arises whether cooperation due to a merger of firms can increase the efficiency of R&D activity in the market.

This dissertation investigates the strategic interactions of players in those two distinct dynamic fields: round-robin tournaments and innovation competition. For that purpose, I apply contest models to each and use game-theoretical equilibrium concepts to analyze them. In particular, I solve the dynamic games by backward induction for their subgame perfect equilibria. Each chapter of this dissertation consists of a self-contained article.

The initial three articles contribute to the literature on round-robin tournaments. The article "Sequential Round-Robin Tournaments with Multiple Prizes" is joint work with Christoph Laica and Marco Sahm and is forthcoming in *Games and Economic Behavior*. It extends the existing literature on round-robin tournaments in several dimensions and is the first and only work that provides a comprehensive analysis of sequential round-robin tournaments with an arbitrary number of heterogeneous players and multiple prizes. Moreover, it is the theoretical basis for the following experimental studies "Round-Robin Tournaments in the Lab: Lottery Contests vs. All-Pay Auctions" and "Intensity, Fairness and Dynamics in Round-Robin Tournaments: The Role of the Prize Structure". These studies are joint work with Marco Sahm. They are the first ever that empirically test round-robin tournaments under controlled conditions in the laboratory.

In Chapter 2, presenting the article "Sequential Round-Robin Tournaments with Multiple Prizes", we examine the fairness and intensity of sequential round-robin tournaments with multiple prizes and heterogeneous players. We define a round-robin tournament as a game in extensive form characterized by a set I of n players, a sequence of $\frac{n(n-1)}{2}$ pairwise different two-player contests (called matches) and a vector of rank-dependent prizes in descending order with the first prize normalized to one and the last prize normalized to zero such that $1 = R_1 \geq R_2 \geq ... \geq R_n = 0$. Players may be heterogeneous with respect to their motivation denoted by $v_i > 0$ which is modeled as an idiosyncratic weight on the prizes such that $v_i R_g$. We define $V = w$ as the expected continuation payoff from winning a given match and $L = l$ as the expected continuation payoff from losing a given match. For the CSF we suppose that $d = 0$ and $f_i(x_i) = x_i^r$ with the discriminatory power $r \geq 0$ that captures that degree of randomness in a contest.

Initially, we assume $n = 3$ with an exogenously fixed sequence of matches in which player 1 meets player 2 in the first match, player 1 meets player 3 in the second match and player 2 meets player 3 in the third match. We show that in equilibrium for any $r > 0$, the tournament is fair in every match if and only the second prize is valued half of the first prize. This implies that the tournament is also ex-ante fair, i.e., fair before the tournament starts. Simulations reveal that if the second prize is not valued half of the first prize, the tournament will not be fair in every match. More precisely, if players are homogeneous in their motivation and the second prize exceeds one half of the first prize, the player who competes in the last two matches is advantaged for all $r > 0$. In contrast, for a second prize below one half of the first prize it depends on the size of r which player is advantaged. Intensity in tournaments with symmetric players is maximum and dissipates the whole prize money if $r > 2$ and the second prize equals half of the first prize. Further simulations point out that intensity in tournaments with symmetric players strictly increases in the second prize if $r = 1$ and is inverted U-shaped in the second prize if $r > 2$.

Subsequently, we analyze three-player tournaments with two different endogenous sequences of matches and find similar results. Irrespective whether we implement a winner-first structure where the winner of the first match always plays in the second match or a loser-first structure where the loser of the first match always plays in the second match, the tournament is fair if and only if the second prize is valued half of the first prize.

Finally, we consider sequential tournaments with $n > 3$. We find that for $r > 0$ and an exogenously fixed sequence of matches there exists no prize scheme such that the tournament is fair. In particular, we analyze tournaments where all matches are either organized as all-pay auctions ($r = \infty$) or as lottery contests ($r = 1$). While the all-pay auction is perfectly discriminating and the player with highest effort certainly wins, the lottery contest randomly determines the winner such that a player's probability of winning is given by the ratio between her own effort and the aggregate effort of all players. Our simulations suggest that there is no prize structure for which the tournament is ex-ante fair, neither for a three-prize nor a two-prize scheme.

In the articles in Chapter 3, "Round-Robin Tournaments in the Lab: Lottery Contests vs. All-Pay Auctions", and in Chapter 4, "Intensity, Fairness, and Dynamics in Round-Robin Tournaments: The Role of the Prize Structure", we experimentally reassess our theoretical results on intensity, fairness, and dynamic behavior from Chapter 2 in the laboratory. In both articles we investigate round-robin tournaments with three symmetric players. In the former, we focus on single-prize tournaments and the effects of the discriminatory power of the CSF on the match-level. In the latter, we consider the impact of the prize structure when each match is organized as an all-pay auction.

Chapter 3 experimentally compares single-prize tournaments where each single match is organized as a lottery contest (LC-treatment) to single-prize tournaments where each single match is organized as an all-pay auction (APA-treatment). We discover that in both tournaments there is significant overdissipation. Overdissipation decreases significantly with experience only when matches are organized as all-pay auctions, and for the players competing in the first match. While theory predicts that APA-tournaments are less intense due to more pronounced discouragement effects of the late-moving player, we find no significant difference in the intensity of LC- and APA-tournaments.

Irrespective whether single matches are organized as lottery contests or as all-pay auctions, the tournament is not significantly ex-ante discriminatory. This supports the theoretical predictions for LC-treatments but contradicts the theoretical predictions for APA-treatments. The reason is that we find no evidence for the predicted discouragement effect of the late mover in the APA-treatment. Instead, a dissipation-trap occurs where players end up in an effort intense, final-like last match which significantly reduces payoffs for the late-moving player compared to the other players.

In Chapter 4, we experimentally explore how the prize structure affects intensity, fairness, and dynamic behavior in three-player round-robin tournaments where single matches are organized as all-pay auctions. We compare tournaments with a second prize equal to either 0%, 50%, or 100% of the first prize. We find that aggregate effort is highest in the 0%-tournament while theory predicts the 50%-treatment (0%-treatment) to be the most (least) intense. The main reason is the absence of the predicted discouragement effect of the late-mover in the 0%-tournament (which is equivalent to the APA-treatment).

As predicted by theory, we ascertain a fair ranking induced by the 50%-treatment. However, we must reject the hypotheses that it induces fair payoffs and fair matches. In the 0%-treatment (100%-treatment) we find discrimination that supports the predicted late mover disadvantage (advantage).

Players' dynamic behavior is characterized by momentum effects. In particular, a strategic momentum in the 0%-treatment is visible: a player increases (decreases) effort after winning (losing) the first match. Similarly, we identify a reverse momentum in the 100%-treatment: a player decreases (increases) effort after winning (losing) the first match. A reverse momentum is also detected in the 50%-treatment. However, mixed-strategy equilibrium play can only partly explain this behavior and our comprehensive analysis suggests that dynamic behavior is subject to a reverse *psychological* momentum.

Together, our experimental findings for round-robin tournaments with three players indicate the following: for single-prize tournaments, independent of the institutional character, i.e., whether, e.g., a sports contest inherently contains some randomness or not, the model with matches organized as lottery contests yields better predictions than the model with matches organized as all-pay auctions. In contrast to theoretical predictions, a prize-allocating tournament designer faces a trade-off between intensity and fairness when matches are organized as all-pay auctions.

In Chapter 5, the article "Mergers and Innovation Effort in R&D-Contests" investigates the effects of a horizontal merger between two firms on the incentives to innovate in an oligopolistic market. I assume that $v_i = 1$ and that the R&D-Contest is organized as a lottery contest that allows for a draw with $f_i(x_i) = \theta x_i$ where $\theta \geq 1$ and $d \geq 0$. The parameter θ captures the ability of a firm to conduct R&D, where $\theta > 1$ holds for the merged entity and $\theta = 1$ holds for non-merged firms. The constant d is considered as a difficulty to successfully innovate that possibly prevents an innovational breakthrough.

If $d > 0$ ($d = 0$) a successful innovation is uncertain (certain) and innovation effort is considered as (un-)productive.

I develop a new dynamic model with the innovation competition modeled as a R&D-Contest that shapes subsequent Cournot competition on the product market. It is the first study that deploys a R&D-Contest with the possibility of unsuccessful innovation (despite innovation effort) embedded in a dynamic innovation competition to analyze mergers. I show that there exists a robust domain where a merger enhances the efficiency of R&D activity in the market and, thus, total welfare.

When innovation effort is unproductive ($d = 0$), R&D-investment is considered as excessive expense as the firms' parallel investment in R&D is beyond the socially optimal level of a marginal effort by a single firm. I show that the approval of a merger always decreases the duplication of R&D expenses in triopolistic markets. However, in larger oligopolisitc markets excessive R&D expenses are only reduced by a merger if the R&D ability of the merged firm or the cost reduction in production due to the innovation is sufficiently large. Moreover, if the innovation size is large enough, I find that the profitability of a merger is a sufficient condition for a reduction in total R&D expenses in the market which in turn, also implies that the merger increases total welfare.

When innovation effort is productive ($d > 0$), I analyze triopolistic markets and mergers that double the ability to conduct R&D in relation to non-merged firms with $\theta = 2$. I prove that a merger (weakly) increases the probability of a successful innovation. Furthermore, I show that when the difficulty to successfully innovate is too high, firms stop to engage in R&D. Eventually, the approval of a merger leads to innovation effort due its synergies that increase the probability to overcome the difficulty to successfully innovate. However, such a merger not always increases total welfare as the gains of a successful innovation do not fully compensate for the loss in consumer surplus as a result of reduced competition on the product market.

In this dissertation I deploy models from contest theory to investigate the strategic interactions of players in two different fields. Chapters 2 to 4 apply contests with particular attention on the variation of discriminatory power and the prize structure on the modeling of round-robin tournaments. In chapter 5, a specific lottery contest that allows for a draw is used to model innovation competition.

The contest model applied on round-robin tournaments provides manifold insights to major sports tournaments while the contest model applied on innovation competition reveals further understanding on how mergers affect research intense industries. Our results in Chapters 2 to 4 suggest a rethinking in the exploitation of round-robin tournaments, as many of them are organized sequentially and, thus, are not neutral with respect to the order of matches. Intensity, fairness, and dynamic behavior in round-robin tournaments crucially depend on the degree of randomness in a contest, the prize structure, the number and the heterogeneity of players, and whether the sequence of matches is exogenously or endogenously given. My results in Chapter 5 propose to focus even more on the 'culture for innovation' predefined by the business environment when assessing the effects of a merger.[1]

This dissertation contributes decisive information on the targeted use of round-robin tournaments for the organization of sports contest or political debates on the one hand, and of merger policy in innovation-driven markets on the other hand. In case the *FIFA*

[1] The OECD (2016, p. 162) uses the term 'culture for innovation' to describe the difficulty to successfully innovate in a business environment where certain policies such as tax incentives promote innovation effort.

opts for a round-robin tournament with three teams in the group stage of world cups, it is well-advised to carefully examine how and how many teams qualify for the next stage. If the European Commission considers to block mergers like the one of *Siemens' and Alstom's* rail businesses to sustain competition, it should verify that a merger actually not enables greater chances for an innovational breakthrough. All in all, my dissertation demonstrates that the theory on contests helps to better understand the complex strategic interactions in round-robin tournaments and in innovation competition.

Chapter 2

Sequential Round-Robin Tournaments with Multiple Prizes

(joint with Christoph Laica and Marco Sahm)

Abstract

We examine the fairness and intensity of sequential round-robin tournaments with multiple prizes and heterogeneous players. A tournament is called fair if the winning probabilities in each match depend only on the players' characteristics but not on the sequence of matches. We show that tournaments with three players will be fair if and only if the second prize is valued half of the first prize. The optimal prize structure may, however, be subject to a trade-off between fairness and intensity (measured by the players' expected aggregate effort). By contrast, there is no prize structure for which tournaments with more than three players will be fair if the sequence of matches is exogenously fixed. Our analysis suggests that many tournaments of major sporting events are inherently unfair.

Keywords: Round-Robin Tournament; Multiple Prizes; Fairness; Intensity; Tullock Contest; All-Pay Auction

JEL classification: C72, D72, Z20

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2.1 Introduction

A round-robin tournament is an all-play-all competition format where each participant meets every other participant in turn. Round-robin tournaments are widely applied to organize sport contests on a large scale, such as the major European football leagues with up to 20 teams (like in the English Premier League), as well as on a small scale, such as the first round (group stage) of the FIFA World Cup (since 1950) or the UEFA European Championship (since 1980) with four teams per group. For their World Cups from 2026 on, the FIFA recently announced a transition to an initial group stage with only three teams per group. This modification is particularly remarkable as the schedule of matches in round-robin tournaments with three teams is inevitably sequential. But even if some of the matches could take place simultaneously, also round-robin tournaments with four or more participants often have a sequential schedule due to limited capacities of the venue or strategic marketing decisions by the organizers.

One reason for the popularity of round-robin tournaments might be the conventional wisdom that "a round-robin tournament is the fairest way to determine the champion among a known and fixed number of contestants" (Wikipedia[1], 2020). For tournaments for which the schedule of matches is sequential, however, this conventional wisdom is challenged by Krumer et al. (2017a) and Sahm (2019). The authors investigate whether *single-prize* round-robin tournaments with three and four symmetric players are fair with respect to ex-ante winning probabilities and expected payoffs. The different pairwise matches take place one after the other, players are ranked according to the number of matches won, and (only) the player with the most victories receives a prize.

Krumer et al. (2017a) assume that each single match is organized as an all-pay auction and find substantial discrimination by the order of matches in the subgame perfect equilibrium of the sequential game: depending on their position in the sequence of matches, the players have differing ex-ante winning probabilities and expected payoffs.[2] The reason is a *discouragement effect* of trailing players that has been identified in many forms of dynamic contests (Konrad, 2009, Chapter 8).

Sahm (2019) confirms the non-fairness result for round-robin tournaments in which each match is organized as a general non-perfectly discriminating Tullock contest, including the perfectly discriminating all-pay auction as a limit case. He shows, however, that the extent and direction of discrimination in the round-robin tournament depend crucially on the discriminatory power of the contest success function that shapes competition on the match level.

In many real world tournaments, such as the group stage of the FIFA World Cup where two teams earn a spot in the next round, the assumption of a single prize seems too narrow because the players have a positive valuation not only for ranking first. We therefore revisit the analysis of sequential round-robin tournaments under the assumption of *multiple prizes*.

The basic idea why the introduction of additional prizes may reduce discrimination is that they induce a second effect which counteracts the discouragement effect and which we call the *lean-back effect*: a second prize, for example, weakens the incentives for winners of early matches to provide additional effort in their later matches because this second prize reduces the difference in payoffs from ranking first and second, respectively.

[1]Accessed at https://en.wikipedia.org/wiki/Round-robin_tournament on 01/30/2020.

[2]Based on sports data from mega-events, Krumer and Lechner (2017) provide empirical evidence for such discrimination.

Our paper is the first and only to provide a comprehensive analysis of sequential round-robin tournaments with an arbitrary number of heterogeneous players and multiple prizes. In a first pass, Krumer et al. (2019) consider round-robin tournaments with three symmetric players and matches organized as all-pay auctions. They show that introducing a second prize which equals the first prize may reduce discrimination but does not lead to a completely fair competition. We extend their analysis in several dimensions.

First, we allow for additional prizes which may be smaller than the first prize. In many tournaments, the prize money for the player ranked second is positive but indeed smaller than for the player ranked first; or, as in the initial group stage of the FIFA World Cup where the winning team of each group is paired with a runner-up team of a different group in the next stage, ranking first is more valuable than ranking second.[3] We therefore consider additional prizes which equal an arbitrary proportion of the first prize and weakly decrease in the rank.

Second, we consider tournaments in which the matches are organized as a general (non-perfectly discriminating) Tullock contest. This includes the (perfectly discriminating) all-pay auction as a limit case and allows to study the impact of the discriminatory power on the fairness and intensity of the tournament.

Third, we incorporate the possibility of heterogeneous contestants.[4] For the sake of concreteness, we introduce heterogeneity in form of possibly different individual valuations (motivations), but the model also permits a reinterpretation in terms of different abilities or effort costs.

Finally, we do not restrict our analysis to tournaments with three players but allow for an arbitrary number of contestants. Indeed, our two main results reflect a fundamental difference between tournaments with three players and tournaments with more than three players.

Round-robin tournaments with three players are fair if and only if the second prize equals half of the first prize.[5] In this case, not only the player's ex-ante ranking probabilities and expected payoffs but also their winning probabilities in each single match of the tournament depend only on their individual characteristics but not on the order of matches in the schedule of the tournament. This result is very robust. It holds whether the sequence of matches is exogenously given or endogenously determined in the sense that the outcome of the first match defines the order of the following two matches. It also holds irrespective of the discriminatory power of the contest success function on the match level. If the discriminatory power is sufficiently large, the fair tournament will also exhibit maximum intensity (measured by the players' expected aggregate effort per unit of prize money). A low discriminatory power, however, may entail a trade-off

[3]Similarly, in the initial group stage of the UEFA Champions League, four teams compete in a double round-robin tournament in which the two teams ranked first and second move to the next stage, the team ranked third qualifies for the elimination phase of the UEFA Europa League, and the team ranked fourth drops out.

[4]Krumer et al. (2017b) also include heterogeneity but limit their analysis to tournaments with a single prize, perfectly discriminating matches, and three players two of which are symmetric.

[5]Dagaev and Zubanov (2017) also consider round-robin tournaments with three players and two prizes. In contrast to our model, they assume that players are symmetric and have limited resources but face no real effort costs: each player just decides how to split her resources between her two matches. Though the authors find a multiplicity of equilibria which do not allow for unambiguous predictions about the extent and direction of discrimination, the case in which the second prize equals half of the first prize plays a particular role in their model as well and, in line with our results, is capable to entail a fair tournament.

between fairness and intensity.

Unlike for three-player tournaments, there is generally no prize structure for which sequential round-robin tournaments with more than three players will be fair if the sequence of matches is exogenously given. With more than three players, the number of potential courses of the tournament increases exponentially and the number of available prizes is insufficient to simultaneously guarantee fairness in all of these courses. The analysis thus suggests that many tournaments of major sporting events are inherently discriminatory.

The remainder of this paper is organized as follows. Section 2.2 introduces a formal model of sequential round-robin tournaments. Section 2.3 provides the analysis and main results for round-robin tournaments with three players. In Section 2.4, we examine sequential round-robin tournaments with more than three players. Section 2.5 discusses some applications to the design of sports tournaments. Section 2.6 concludes.

2.2 Model

2.2.1 The tournament

Following Dagaev and Zubanov (2017), we define a round-robin tournament $T = \{I = \{1, \ldots, n\}, (k_t), p, (R_g)\}$ as a game in extensive form characterized by a set I of n players, a sequence (k_t) of the $\frac{n(n-1)}{2}$ pairwise different two-player contests (called matches) with contest success function p, and a vector of rank-dependent prizes (R_g) where the players' ranking is based on the number of matches won in descending order.

We assume that the ranking of prizes is non-trivial (not all prizes are the same) and monotonic. Without loss of generality, we normalize the first prize to one and the last prize to zero: $1 = R_1 \geq R_2 \geq \ldots \geq R_n = 0$. Players are risk-neutral and may differ with respect to their motivation. The motivation of player $i \in I$ is modeled as an idiosyncratic weight $v_i > 0$ on the prizes. Thus player i's individual valuation for attaining rank j equals $v_i R_g$. Although the assumption of rank-independent idiosyncratic weights on the prizes entails a substantial restriction compared to a model with arbitrary rank-dependent player values $(v_1^i, \ldots, v_n^i)$, it is fairly general in the following sense: it is equivalent to the assumption that players are heterogeneous with respect to individual abilities θ_i or effort costs c_i (Cornes and Hartley, 2005; Ryvkin, 2013).[6]

We focus on fully sequential round-robin tournaments. Successively, each player is matched one-to-one with each other player.[7] We abstract from draws: in each match, one player wins and the other player loses. At the end of the tournament, players are ranked according to the number of matches won. If there are $m \in \{1, \ldots, n\}$ players sharing the same number of wins and $s \in \{0, \ldots, n-m\}$ players with a larger number of wins, each of the former receives one of the prizes $R_{s+1}, \ldots, R_{s+m}$, each with probability $1/m$ (random tie breaking).[8]

[6]Notice that, in any Tullock contest with rent R, maximizing $\frac{x_i^r}{x_i^r + x_j^r} v_i R - x_i$ by the choice of x_i is equivalent to maximizing $\frac{x_i^r}{x_i^r + x_j^r} R - c_i x_i$ with $c_i := 1/v_i$ by the choice of x_i, and this, in turn, is equivalent to maximizing $\frac{\theta_i y_i^r}{\theta_i y_i^r + \theta_j y_j^r} R - y_i$ by the choice of y_i, where $y_i := x_i/v_i$ and $\theta_i := v_i^r$.

[7]Notice that, in a round-robin tournament with $n \geq 3$ players in which the $\frac{n(n-1)}{2}$ different pairwise matches are scheduled sequentially, there are $S(n) = \frac{[(1/2) \cdot n \cdot (n-1)]!}{n!}$ different exogenous sequences except for renaming players; e.g. $S(3) = 1$, $S(4) = 30$, and $S(5) = 30240$.

[8]For risk-neutral players, the tie breaking rule is equivalent to the assumption that the prize money

2.2.2 The matches

Each match of the tournament is organized as a Tullock contest between two players, A and B, with linear costs of effort, see e.g. Konrad (2009, Chapter 2.3). More specifically, player A's probability of winning match k is

$$p_A^k = \begin{cases} 1/2 & \text{if } x_A^k = x_B^k = 0, \\ \frac{(x_A^k)^r}{(x_A^k)^r + (x_B^k)^r} & \text{otherwise,} \end{cases}$$

where x_i^k denotes the effort of player $i \in \{A, B\}$ in match k, and $r \geq 0$ describes the discriminatory power of the contest, also referred to as accuracy level (Wang, 2010). In the context of rent-seeking, this type of contest success function was introduced by Tullock (1980) and given an axiomatic foundation by Skaperdas (1996). For $r = 0$ the players' winning probabilities are independent from efforts and equal $1/2$. For $r = 1$ the contest is also referred to as a *lottery contest.* For $r \to \infty$ the winning probability of the player with the higher investment approaches 1: the contest success function converges to the perfectly discriminatory all-pay auction.

Player $i \in \{A, B\}$ chooses x_i^k in order to maximize his expected payoff

$$E_i^k = p_i^k (w_i^k - x_i^k) + \left(1 - p_i^k\right)(\ell_i^k - x_i^k), \tag{2.1}$$

where w_i^k denotes player i's expected continuation payoff from winning match k and ℓ_i^k denotes his expected continuation payoff from losing match k, with $w_i^k, \ell_i^k \geq 0$.

If $w_i^k > \ell_i^k$ for all $i \in \{A, B\}$, a Nash equilibrium in match k always exists. If instead $w_i^k \leq \ell_i^k$ for some $i \in \{A, B\}$, player i's optimal effort choice is $x_i^k = 0$ for any effort level $x_j^k \geq 0$ of player $j \in \{A, B\}, j \neq i$, and player j may have no best reply. To avoid the problem that no equilibrium may exist because some player has no incentive to win match k, we introduce an additional prize $m > m^k := \max\{0, \ell_A^k - w_A^k, \ell_B^k - w_B^k\}$ on the match-level that guarantees positive winning incentives for both players.[9] We then consider the limit of equilibrium values as $m \to m^k$ from above. It is straightforward to show that, for $\min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\} \leq 0$, the (expected) equilibrium efforts of both players converge to zero and player i's winning probability converges to $p_i^k = 1/2$ if $w_i^k - \ell_i^k = w_j^k - \ell_j^k$ but to $p_i^k = 1$ if $w_i^k - \ell_i^k > w_j^k - \ell_j^k$, where $i, j \in \{A, B\}, j \neq i$. Obviously, if $\min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\} > 0$, the limit values will coincide with the equilibrium values for $m = 0$. Depending on the discriminatory power r, three types of equilibria may arise.

First, the Nash equilibrium is unique and in pure strategies if and only if the discriminatory power is sufficiently small (Nti, 1999); more precisely, if and only if $r \leq \bar{r}^k$, where for $i, j \in \{A, B\}$ with $i \neq j$ and $w_i^k - \ell_i^k = \min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\}$ the threshold

$\sum_{g=s+1}^{s+m} R_g$ is shared equally among the m players with the same number of victories.

[9] In the context of elimination tournaments and round-robin tournaments, respectively, Groh et al. (2012) and Krumer et al. (2017a,a, 2020b) propose a similar solution for the non-existence problem. Konrad and Kovenock (2009) provide a careful approach to such component contests in the context of races. As long as at least one player has positive winning incentives, the consideration of epsilon-equilibria offers an alternative solution for the non-existence problem (Sahm, 2019): If $x_i^k = 0$ and $w_j^k > \ell_j^k$, player j will have no best reply unless there is a smallest monetary unit $\varepsilon > 0$; the best reply is then $x_j^k = \varepsilon$. As $\varepsilon \to 0$, in the limit, $x_j^k \to 0$ and $p_j^k \to 1$.

$\bar{r}^k \in (1,2]$ is implicitly defined by

$$\bar{r}^k = 1 + \left(\frac{w_i^k - \ell_i^k}{w_j^k - \ell_j^k} \right)^{\bar{r}^k}. \tag{2.2}$$

The equilibrium effort levels can then be derived from the necessary conditions

$$\frac{\partial E_i^k}{\partial x_i^k} = \frac{r(x_i^k)^{r-1}(x_j^k)^r}{[(x_i^k)^r + (x_j^k)^r]^2}(w_i^k - \ell_i^k) - 1 = 0$$

yielding

$$x_i^k = r \frac{(w_i^k - \ell_i^k)^{1+r}(w_j^k - \ell_j^k)^r}{[(w_i^k - \ell_i^k)^r + (w_j^k - \ell_j^k)^r]^2} \tag{2.3}$$

for $i, j \in \{A, B\}$ with $i \neq j$. The resulting equilibrium winning probabilities equal

$$p_i^k = \frac{(w_i^k - \ell_i^k)^r}{(w_i^k - \ell_i^k)^r + (w_j^k - \ell_j^k)^r}. \tag{2.4}$$

Inserting (2.3) and (2.4) into (2.1) yields the expected equilibrium payoffs

$$E_i^k = \ell_i + \frac{(w_i^k - \ell_i^k)^{r+1}[(w_i^k - \ell_i^k)^r + (1-r)(w_j^k - \ell_j^k)^r]}{[(w_i^k - \ell_i^k)^r + (w_j^k - \ell_j^k)^r]^2}. \tag{2.5}$$

Second, for an intermediate discriminatory power $\bar{r}^k < r \leq 2$, the Nash equilibrium is unique and in semi-mixed strategies in which only one player pursues a pure strategy while the other randomizes (Wang, 2010; Ewerhart, 2017b; Feng and Lu, 2017). These studies imply that, for $i, j \in \{A, B\}$ with $i \neq j$ and $w_i^k - \ell_i^k = \min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\}$, the equilibrium (expected) efforts are

$$E(x_i^k) = f(r)\frac{(w_i^k - \ell_i^k)^2}{w_j^k - \ell_j^k} \quad \text{and} \quad x_j^k = f(r)(w_i^k - \ell_i^k), \tag{2.6}$$

the equilibrium winning probabilities are

$$p_i^k = f(r)\frac{w_i^k - \ell_i^k}{w_j^k - \ell_j^k} \quad \text{and} \quad p_j^k = 1 - p_i^k, \tag{2.7}$$

and the expected equilibrium payoffs are

$$E_i^k = \ell_i^k \quad \text{and} \quad E_j^k = w_j^k - 2f(r)(w_i^k - \ell_i^k), \tag{2.8}$$

where

$$f(r) := \frac{1}{r}(r-1)^{\frac{r-1}{r}} \quad \in [\frac{1}{2}, 1) \quad \text{for all} \quad r \in (1,2]. \tag{2.9}$$

Finally, for a high discriminatory power $r > 2$, all Nash equilibria are in mixed strategies and equivalent to the unique equilibrium of the all-pay auction[10] with respect to expected efforts, winning probabilities, expected payoffs, and expected revenues (Baye et al., 1994; Alcalde and Dahm, 2010; Ewerhart, 2017a). Round-robin tournaments with

[10]Baye et al. (1996) provide a comprehensive analysis of all-pay auctions.

matches organized as all-pay auctions have been analyzed by Krumer et al. (2017a); for $i, j \in \{A, B\}$ with $i \neq j$ and $w_i^k - \ell_i^k = \min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\}$, the equilibrium (expected) efforts are

$$E(x_i^k) = \frac{(w_i^k - \ell_i^k)^2}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad E(x_j^k) = \frac{w_i^k - \ell_i^k}{2}, \tag{2.10}$$

the equilibrium winning probabilities are

$$p_i^k = \frac{w_i^k - \ell_i^k}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad p_j^k = 1 - p_i^k, \tag{2.11}$$

and the expected equilibrium payoffs are

$$E_i^k = \ell_i^k \quad \text{and} \quad E_j^k = w_j^k - (w_i^k - \ell_i^k). \tag{2.12}$$

2.2.3 Fairness and intensity

Denote a sequential round-robin tournament with n players and matches organized as Tullock contests with discriminatory power $r \geq 0$ by $T_n[r]$. We will call the tournament $T_n[r]$ *fair* if, for each match of the tournament, the winning probabilities in this match depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament. This definition accommodates the requirement that the outcome of a certain match between two players should not be influenced by the playing schedule.

By contrast, we will call the tournament $T_n[r]$ *ex-ante fair* with respect to ranking probabilities (payoffs) if, the players' ex-ante ranking probabilities (expected payoffs) depend only on their characteristics but not on the order of matches in the sequence of the tournament.[11] Here, ex-ante means before the tournament starts. The idea is that the playing schedule should not influence the players' overall success in the tournament. Obviously, the notion of ex-ante fairness is weaker than the notion of fairness: any fair tournament is, a fortiori, ex-ante fair.[12]

The intensity of a tournament $T_n[r]$ is measured by the players' expected aggregate effort across all matches per unit of prize money. In order to compare the intensity of tournaments with differing numbers of players, we correct for the resulting differing number of matches considering the average intensity per match.

2.3 Tournaments with Three Players

We first consider round-robin tournaments with three players and an exogenous sequence in which player 1 is matched with player 2 in the first match, player 1 is matched with player 3 in the second match, and player 2 is matched with player 3 in the third

[11] These two definitions of ex-ante fairness are not equivalent: as the right panels of Figures 2.6 and 2.7 in Appendix 2.F illustrate, the threshold values r_{EP}^{end} and r_{WQP}^{end} defined in Simulation Result 2.3 (b) usually do not coincide. See also Footnote 17 in Sahm (2019).

[12] As we illustrate below, the converse does not hold generally. Notice that the analysis of Krumer et al. (2017a), Sahm (2019), and Krumer et al. (2017a) is based on these weaker notions of ex-ante fairness.

match.[13] The structure of the resulting sequential game $T_3^{ex}[r]$ with its $2^3 = 8$ potential courses is depicted in Figure 2.1. The seven nodes $k \in \{A, \ldots, F\}$ represent all combinations for which the ranking of the tournament has not yet been determined when the respective match starts. For short, $a := R_2$ denotes the second prize, such that $(R_1, R_2, R_3) = (1, a, 0)$, and $\Gamma := \frac{1+a+0}{3}$ denotes the players' expected gross payoff in case of a tie.

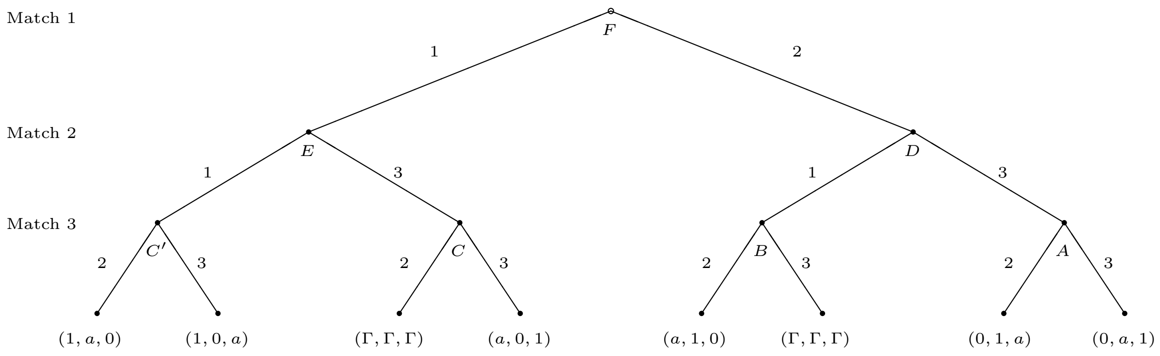


Figure 2.1: 3-player round-robin tournament with exogenous sequence

For each combination of parameters describing the characteristics of the players $(v_i)_{i=1}^3$, the prize structure of the tournament a, and the discriminatory power of its matches r, game $T_3^{ex}[r]$ can be solved by backward induction for its subgame perfect equilibrium (SPE), making repeatedly use of equations (2.3)–(2.5), (2.6)–(2.8), or (2.10)–(2.12), respectively. For symmetric players ($v_i = 1$ for all $i \in \{1, 2, 3\}$), the details of this procedure have been provided by Krumer et al. (2017a) for $(a, r) = (0, \infty)$, by Krumer et al. (2017a) for $(a, r) = (1, \infty)$, and by Sahm (2019) for $(a, r) = (0, 1)$.[14]

2.3.1 Fairness

The aforementioned contributions have shown that sequential round-robin tournaments with three players will usually not be fair if there is only a single prize ($a = 0$) or the second prize equals the first prize ($a = 1$). This raises the question whether a prize structure exists under which sequential round-robin tournaments with three players are generally fair. The question is answered by

Proposition 2.1. *For any $r > 0$, the sequential round-robin tournament with three players $T_3^{ex}[r]$ is fair if and only if the second prize is valued half of the first prize ($a = 1/2$).*

The proof can be found in Appendix 2.A. While solving game $T_3^{ex}[r]$ by backward induction for $a = 1/2$ proves the if-part, the proof of the only-if-part relies on the fact that symmetric players have equal winning probabilities in each match of a fair tournament. Notice that Proposition 2.1 (just as our entire analysis) also applies to tournaments with matches organized as all-pay auctions ($r \to \infty$).

[13] Apart form renaming players, this exogenous sequence is unique. In Section 2.3.3, we discuss the use of endogenous sequences in which the outcome of the first match determines the order of the two remaining matches.

[14] Moreover, for $a = 0$, Sahm (2019) approximates the equilibria for all $r \geq 0$ based on numerical computations. In the proof of Proposition 2.1 (see Appendix 2.A), we explicitly solve game $T_3^{ex}[r]$ by backward induction for $a = 1/2$.

The intuition behind Proposition 2.1 can be explained as follows. The first prize alone discourages trailing players (here: player 3) from providing equivalent effort. This is the so-called *discouragement effect* (e.g. Konrad, 2012; Sahm, 2019). A second prize, however, induces a second effect: it provides a positive (expected) payoff to the winner of the first match even if he loses his second match. This makes him lean back and reduce effort in his second match (here: against player 3). We thus call it the *lean-back effect*. The lean-back effect runs counter to the discouragement effect and intensifies as the second prize increases. But why do the two effects cancel out exactly at $a = 1/2$?

Suppose for a moment that the organizer of the tournament allocates no rank-dependent prizes but only prizes at the match-level: the winner of each single match receives a prize. Intuitively, fairness then requires that the prizes for the three matches be identical. In consequence, a player who wins two (zero) matches receives two (zero) match-prizes and a player who wins one match receives one match-prize. Now suppose that the organizer of the tournament wants to replicate this intuitively fair scheme using rank-dependent prizes instead of prizes at the match-level. Obviously, the only candidate is a rank-dependent scheme in which the second prize equals half of the first prize. In the cases in which one of the players wins two matches (and thus is the only player to rank first), the replication is exact. In the cases in which each player wins one match (and thus the ranking is determined randomly), the replication works at least in expectations.

Put differently, abstracting from the case of a tie, a player must win two matches to rank first and one match to rank second: he must win twice as many matches to rank first than to rank second. Only a prize structure under which the second prize is valued half of the first prize reflects this proportion and thus makes the player marginally indifferent between investing in order to win one more match and leaning back.[15]

Remember that a fair tournament is, a fortiori, ex-ante fair in the sense that the players' ex-ante ranking probabilities and expected payoffs depend only on their characteristics but not on the order of matches. Thus, we have the following

Corollary 2.1. *For any $r > 0$, the sequential round-robin tournament with three players $T_3^{ex}[r]$ will be ex-ante fair if the second prize is valued half of the first prize ($a = 1/2$).*

We have run a large number of computer based calculations suggesting that $a = 1/2$ is not only sufficient but also necessary for the tournament to be ex-ante fair. To this end, we have approximated the subgame perfect equilibrium of the tournament $T_3^{ex}[r]$ with three symmetric players for all combinations (r, a) on a grid over $[0, 2] \times [0, 1]$ with a width of 0.01.[16] Figure 2.2a depicts the extent of discrimination as measured by the standard deviation of the players' ex-ante expected normalized payoffs, i.e., payoffs per unit of prize money.[17] It illustrates that this measure is zero if and only if $a = 1/2$. By and large, the extent of discrimination is the smaller the lower the discriminatory power of the matches r and the closer the second prize a to 1/2.[18]

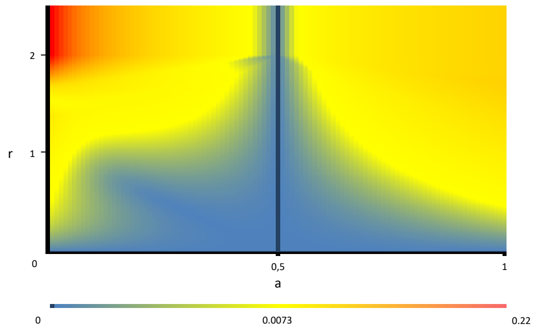
Not only the extent but also the direction of discrimination varies with both, the discriminatory power of the matches r and the value of the second prize a. Figure 2.2b

[15]More formally, the proof of Proposition 2.1 demonstrates that, for $a = 1/2$, winning any single match has a value of $v_i/2$ for each player i.

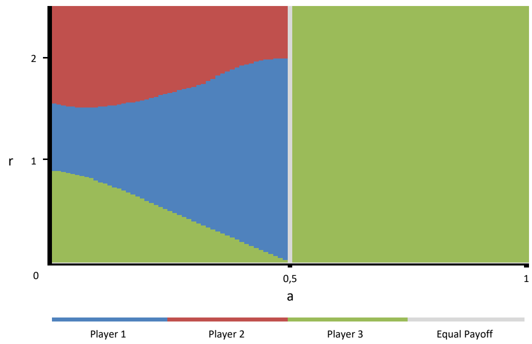
[16]Remember that for all $r \geq 2$ the equilibria are equivalent.

[17]The colors in this and the following heat maps represent the range from the lowest value (blue) to the highest value (red) and are centered around the median (yellow).

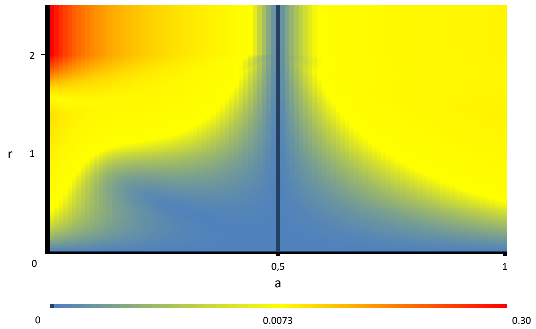
[18]Locally, we also find some non-monotonicity in both dimensions (cf. Sahm, 2019).



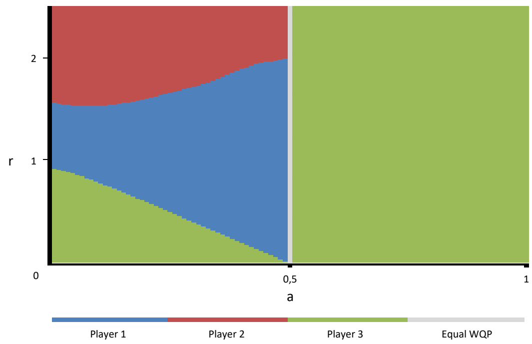
(a) Standard deviations of ex-ante expected normalized payoffs



(b) Player with highest ex-ante expected payoff



(c) Standard deviation of ex-ante WQP



(d) Player with highest ex-ante WQP

Figure 2.2: Extent and direction of discrimination in $T_3^{ex}(r)$

depicts which of three symmetric players has the highest ex-ante expected payoff. In particular, it illustrates that the player who competes in the last two matches (player 3) will always be advantaged if the second prize exceeds half of the first prize because then the lean-back effect dominates.

A very similar picture emerges in Figures 2.2c and 2.2d where we use the standard deviation of ex-ante weighted qualification probabilities (WQP) as an alternative measure of discrimination. We define the WQP as the normalized aggregate probability to rank first ($P^{1^{st}}$) or second ($P^{2^{nd}}$), where the probability to rank second is weighted by the second prize as a proportion of the first prize:[19] WQP $= \frac{P^{1^{st}} + a \cdot P^{2^{nd}}}{1+a}$. We summarize our observations in

Simulation Result 2.1. *If the second prize is not valued half of the first prize ($a \neq 1/2$), the tournament $T_3^{ex}[r]$ will not be ex-ante fair. In particular, if players are symmetric and the second prize exceeds half of the first prize ($a > 1/2$), the player who competes in the last two matches (player 3) will be advantaged for all $r > 0$.*

2.3.2 Intensity

The previous results imply that a contest designer should choose a second prize a that equals half of the first prize if he wants to maximize the fairness of the tournament.

[19]In light of the example of the initial group stage of the FIFA World Cup, the WQP measures the overall probability to qualify for the next round adjusted for the fact that ranking second reduces the winning probability in the next round compared to ranking first.

Besides fairness, organizers of tournaments are often interested in the intensity of competition because the contest is more attractive for spectators and sponsors not only the closer it is but also the higher the participants' effort. Another important question is thus how variations of the second prize a affect expected aggregate effort per unit of prize money ρ.[20] In this subsection, we illustrate that the optimal prize structure may be subject to a trade-off between fairness and intensity. To this end, we focus on symmetric players ($v_1 = v_2 = v_3$).

We first show that such a trade-off will not arise if the discriminatory power in tournament $T_3^{ex}[r]$ is sufficiently large. For $r \geq 2$, a second prize that equals half of the first prize ($a = 1/2$) also implies maximum intensity in the sense that ex-ante expected aggregate effort equals aggregate prize money or, in the language of the rent seeking literature, the rent is fully dissipated ($\rho = 1$). We formally prove this statement in Appendix 2.B solving game $T_3^{ex}[r]$ by backward induction for its SPE.

Proposition 2.2. *For three symmetric players and $r \geq 2$, the intensity of tournament $T_3^{ex}[r]$ will be maximum ($\rho = 1$) if the second prize is valued half of the first prize ($a = 1/2$).*

To illustrate that Proposition 2.2 will generally not hold if the discriminatory power is low, we have computed the subgame perfect equilibrium of tournament $T_3^{ex}[r]$ with three symmetric players for $r = 1$ and $r = \infty$ increasing a from 0 to 1 in steps of 0.01. Interpolating the respective results, the graphs in Figure 2.3 depict the intensity of these tournaments, measured by the ex-ante expected aggregate effort per unit of prize money, as functions of the second prize a. As the example of the tournament $T_3^{ex}[1]$ with matches organized as lottery contests (LC) illustrates, a trade-off between fairness and intensity may arise: for second prizes below $a = 1/2$, the contest designer will have to accept a decrease in expected aggregate effort per unit of prize money if he wants to increase the fairness of the tournament by increasing the second prize a. By contrast, such a trade-off does not exist in tournaments with sufficiently large discriminatory power such as $T_3^{ex}[\infty]$ with matches organized as all-pay auctions (APA): a second prize $a = 1/2$ maximizes both, fairness and intensity.

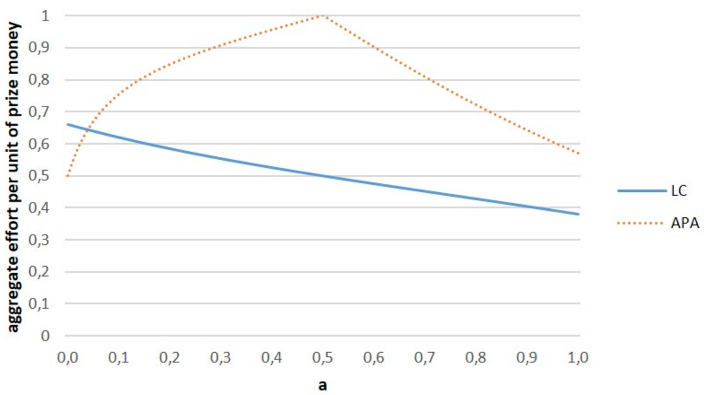


Figure 2.3: Ex-ante expected aggregate effort per unit of prize money

[20]In the literature, ρ is also referred to as *rent dissipation rate* as it measures how much of the economic rent is dissipated during the contest in form of the players' efforts.

Simulation Result 2.2. *With three symmetric players ($v_1 = v_2 = v_3$), the intensity of tournament $T_3^{ex}[r]$ (as measured by the expected aggregate effort per unit of prize money ρ)*

(a) strictly decreases in the second prize a if $r = 1$,

(b) is inverted U-shaped in the second prize a and peaks at $a = 1/2$ if $r \geq 2$.

2.3.3 Endogenous Sequences

Besides tournaments $T_3^{ex}[r]$ with an exogenously fixed sequence of matches, we may consider two different endogenous sequences in which the order of the last two matches depends on the outcome of the first match: we denote the tournament in which player 3 is always matched first with the winner (loser) of the first match by $T_3^{WF}[r]$ ($T_3^{LF}[r]$). For a graphical representation, see Figure 2.4 (2.5) in Appendix 2.C. Notice that an endogenous sequence makes the players of the first match strategically symmetric: for $i \in \{1, 2\}$, player i's strategic investment incentives do not depend on the label i.

Sahm (2019) shows that for tournaments with a single prize ($a = 0$) and matches organized as either lottery contests or all-pay auctions ($r \in \{1, \infty\}$), the WF-structure $T_3^{WF}[r]$ is, ex-ante, fairer and more intense than the exogenous schedule $T_3^{ex}[r]$. For tournaments with either a single prize ($a = 0$) or a second prize that equals the first prize ($a = 1$) and matches organized as all-pay auctions ($r = \infty$), Krumer et al. (2017a) compare the ex-ante fairness and intensity of the three schedules, $T_3^{ex}[\infty]$, $T_3^{WF}[\infty]$, and $T_3^{LF}[\infty]$.

We now provide a complete picture demonstrating that our previous results largely hold also for tournaments with an endogenous sequence of matches. We start with the exact analog of Proposition 2.1. The proof can be found in Appendix 2.C.

Proposition 2.3. *For any $r > 0$, the sequential round-robin tournament $T_3^{end}[r]$ with an endogenous sequence of matches ($end \in \{WF, LF\}$) and three players is fair if and only if the second prize is valued half of the first prize ($a = 1/2$).*

Second, we revisit the question of ex-ante fairness replicating the numerical calculations from above for $T_3^{WF}[r]$ and $T_3^{LF}[r]$. Figures 2.6 and 2.7 in Appendix 2.F illustrate the results. Again, if players are symmetric and the second prize exceeds half of the first prize, player 3 will always be advantaged. Unlike the exogenous structure, however, the endogenous structures feature instances for which the second price is smaller than half of the first price but symmetric players still have the same ex-ante expected payoff (WQP).[21] More formally, we record the following observations.

Simulation Result 2.3. *Consider a tournament $T_3^{end}[r]$ between three symmetric players ($v_1 = v_2 = v_3$) with an endogenous sequence of matches ($end \in \{WF, LF\}$).*

(a) If the second prize is larger than half of the first prize ($a > 1/2$), the player who competes in the last two matches (player 3) will be advantaged for all $r > 0$.

[21] In the right panels of Figures 2.6 and 2.7 in Appendix 2.F, for $a < 1/2$ such instances occur at the margin between the green area (where player 3 is advantaged) and the blue area (where players 1 and 2 are equally advantaged).

(b) *For each second prize smaller than half of the first prize ($a < 1/2$), there is exactly one discriminatory power $0 < r^{end}_{EP} < 2$ ($0 < r^{end}_{WQP} < 2$) for which all three players have the same ex-ante expected payoff (WQP). Below (above) this threshold, player 3 (players 1 and 2) will be advantaged. The threshold r^{end}_{EP} (r^{end}_{WQP}) is a decreasing function of $a \in [0, 1/2)$.*

(c) *Under the WF-structure, all three players will have the same ex-ante expected payoff of zero if the second prize is smaller than half of the first prize ($a < 1/2$) and matches are sufficiently discriminatory ($r \geq 2$).*

Next, we proceed with the exact analog of Proposition 2.2. The proof can be found in Appendix 2.D.

Proposition 2.4. *For three symmetric players and $r \geq 2$, the intensity of tournament $T^{end}_3[r]$ with an endogenous sequence of matches ($end \in \{WF, LF\}$) will be maximum ($\rho = 1$) if the second prize is valued half of the first prize ($a = 1/2$).*

Just as Proposition 2.2, Proposition 2.4 will not hold if the discriminatory power is sufficiently low. Then, again, a trade-off between fairness and intensity may arise (see Figure 2.8 in Appendix 2.F).

2.4 Tournaments with More than Three Players

The previous section has demonstrated that there is a particular prize scheme which guarantees fairness in sequential round-robin tournaments with three players. In this section, we show that such a prize scheme usually does not exist in larger tournaments: sequential round-robin tournaments with more than three players will never be fair if the sequence of matches is exogenously fixed. Before we formally state and prove this general result, we illustrate it by means of a concrete example. Moreover, our simulations suggest that ex-ante fairness can never be reached.

2.4.1 An example with four players

Figure 2.9 in Appendix 2.F depicts the example of a round-robin tournament with four players and the following exogenous sequence of matches:[22] first player 1 is matched with player 2, second player 3 with player 4, third player 1 with player 3, fourth player 2 with player 4, fifth player 1 with player 4, and sixth player 2 with player 3. The prizes are $R_1 = 1$ for the first rank, $R_2 = a$ for the second rank, $R_3 = b$ for the third rank, and $R_4 = 0$ for the last rank with $0 \leq b \leq a \leq 1$; for short we write $\Delta := \frac{1+a}{2}$, $\Theta := \frac{a+b}{3}$, and $\Omega := \frac{1+a+b}{3}$.

We now show that the tournament cannot be fair. The proof is by contradiction. To this end, consider symmetric players ($v_1 = v_2 = v_3 = v_4 = 1$). Fairness then implies that they have equal winning probabilities (of one half) in each match and every potential course of the tournament. In particular, players 2 and 3 have equal winning probabilities in the last match of the tournament whatever its previous course, for example in node A as well as in node B (see Figure 2.9 in Appendix 2.F).

[22]With four players, there are 30 different exogenous sequences of matches (Sahm, 2019). The exogenous sequence of our example has been referred to as *case A* by Krumer et al. (2017a) and Sahm (2019).

For $0 < r \leq \min\{\bar{r}^A, \bar{r}^B\}$ or $2 < r$, equal odds in node A imply

$$p_2^A = p_3^A = \frac{1}{2} \quad \Leftrightarrow \quad w_2^A - \ell_2^A = w_3^A - \ell_3^A \quad \Leftrightarrow \quad \Theta = a - \Theta \quad \Leftrightarrow \quad 2b = a$$

by equations (2.4) or (2.11), respectively. Similarly, equal odds in node B imply

$$p_2^B = p_3^B = \frac{1}{2} \quad \Leftrightarrow \quad w_2^B - \ell_2^B = w_3^B - \ell_3^B \quad \Leftrightarrow \quad \frac{1}{2}b = \Omega - \frac{1}{2}b \quad \Leftrightarrow \quad 2b = 1 + a,$$

a contradiction. For $\min\{\bar{r}^A, \bar{r}^B\} < r \leq 2$, a similar contradiction can be constructed.[23]

2.4.2 The general case

We can generalize the argument of the above example to demonstrate that round-robin tournaments with more than three players and an exogenously fixed sequence of matches are never fair. The complete proof can be found in Appendix 2.E.[24]

Proposition 2.5. *Consider a sequential round-robin tournament $T_n^{ex}[r]$ with $n > 3$ players, discriminatory power $r > 0$, and an exogenously fixed sequence of matches. There is no prize scheme $(R_g)_{g=1}^n$ such that $T_n^{ex}[r]$ is fair.*

Intuitively, the number of available instruments for contest design (i.e., the number of prizes) is too small compared to the number of potential courses of the tournament. While three prizes are sufficient to guarantee fairness in all $2^3 = 8$ potential courses of the tournament with three players, four (n) prizes are not enough to guarantee fairness in all $2^6 = 64$ ($2^{\frac{1}{2}n(n-1)}$) potential courses of the tournament with four (n) players. The prize structure required to guarantee fairness in one particular course of the tournament may not be suitable to guarantee fairness in a different one.

2.4.3 Ex-ante fairness

For matches organized as all-pay auctions ($r = \infty$) and lottery contests ($r = 1$), respectively, Krumer et al. (2017a) and Sahm (2019) show that sequential round-robin tournaments with a single prize, four players, and an exogenous schedule are not ex-ante fair. This raises the question whether introducing additional prizes can restore (or at least improve) the ex-ante fairness of tournaments with four or more players (even though complete fairness is out of reach).

To shed some light on this issue, we reconsider the four player tournament depicted in Figure 2.9 (see Appendix 2.F) and compute its subgame prefect equilibrium for a large number of a-b combinations, both for $r = \infty$ and $r = 1$. Our simulations suggest that there is no prize-structure for which the tournament is ex-ante fair, regardless of whether matches are organized as all-pay auctions or lottery contests.

Figure 2.10 depicts the players' ex-ante probabilities of ranking first or second as well as their ex-ante expected payoff per unit of prize money for the case with only two

[23]The details are provided in the proof of Proposition 2.5 (see Appendix 2.E).

[24]Notice that the proof of Proposition 2.5 does not require the tournament to be *fully* sequential: it only makes use of the fact that the players of the last match know the results of all previous matches. Proposition 2.5 thus holds for all round-robin tournaments with an exogenously fixed sequence of matches provided that there is a single last match (including tournaments in which some of the previous matches take place simultaneously).

prizes ($b = 0$).[25] It shows that no ex-ante fair tournament exists for any value of the second prize a, neither if matches are organized as lottery contests (LC) nor as all-pay auctions (APA). However, consistent with the findings by Sahm (2019) for the case of a single prize, the lottery contest is less discriminatory than the all-pay auction for any given value of the second prize a.

Moreover, the right panels of Figure 2.10 illustrate that the extent and direction of discrimination in tournaments with matches organized as all-pay auctions are very sensitive to variations of the second prize a. In a recent working paper, Krumer et al. (2020b) consider the same environment and observe that, with two identical prizes ($a = 1$), even adverse incentives may arise: depending on the outcome of the first matches, in the course of the tournament a player may have a higher expected continuation value from losing a subsequent match than from winning it.[26] Our computations reveal that such adverse incentives will arise whenever the second prize is sufficiently large.[27]

2.5 Applications

Our model immediately applies to many sports tournaments in which the matches never end in a draw and are scheduled sequentially.[28]

One example is the announced reform of the FIFA World Cup. Effective from 2026, it affects the organization of the first round of the tournament and consists in switching from eight groups with four teams each to 16 groups with three teams each.[29] At the same time, the FIFA proposes the avoidance of draws by penalty shoot-out already in this group stage. As before, two teams per group will advance to the next round (elimination phase). In this second round, the winner of each group is paired with the runner-up of another group. Expecting winners of a group to be stronger opponents, teams usually prefer ranking first over ranking second. According to our analysis (Proposition 2.1), the tournaments in the group stage will be fair if the teams value ranking first twice as much as ranking second.[30]

A second example are the Olympic Basketball Tournaments for Men (since 1976, except 1980) and Women (since 1996). In the first round (group stage), the twelve participating teams are divided into two groups of six and compete in a fully sequential round-robin tournament within their group. In both groups the top four teams qualify for a subsequent elimination phase.[31] According to our analysis (Proposition 2.5), the

[25] This may seem reasonable for many sports tournaments like the current group stage of the FIFA World Cup where only two teams advance to the next round.

[26] This is, e.g., the case for Player 1 in Vertex 13 of Figure 1 in their paper, which corresponds to node M in our Figure 2.9.

[27] More precisely, our numerical calculations underlying the right panels of Figure 2.10 suggest that Player 1 prefers to lose in node M of Figure 2.9 if and only if the second prize exceeds about 62% of the first prize.

[28] Organizers are often not able or willing to schedule some matches simultaneously, for example because they have only one playing field available or want to offer spectators the opportunity to watch all matches.

[29] Notice that this reform may also induce (additional) incentives for collusive behavior (Guyon, 2020) from which we abstract away in the present analysis.

[30] Historically, in FIFA World Cups since 1998 with four teams per group, a group winner has won the second round (elimination) match with an average frequency of 77.5%, which would correspond to ranking first being valued $77.5/22.5 \approx 3.4$ times as much as ranking second.

[31] The same format also applies to the Olympic Volleyball Tournaments for Men and Women since 1996. Notice that Basketball and Volleyball competitions feature no draws.

tournaments in the group stage are inherently discriminatory.

To obtain a fair tournament including a group-stage, we have to implement a three player round-robin tournament and a prize structure that reflects a second prize equal to one half of the first prize. We achieve this for twelve participating teams with a first stage composed of four groups with three teams each. The four first ranked teams immediately advance to the second stage whereas the four second ranked teams compete each in a wild card elimination game such that only two out of the four advance to the second stage. This structure reflects a second prize equal to one half of the first prize as a second ranked team has a 50 percent chance of staying in the tournament.

Eventually, in the second stage the six remaining teams are again divided into two groups of three and play round-robin tournaments. Now the first ranked teams immediately advance to the gold medal match and the second ranked teams advance to the bronze medal match. Under the assumption that the primary goal of many athletes and sport associations is *getting an Olympic medal no matter what colour it is*[32], this structure also reflects a second prize equal to roughly one half: while the first ranked teams win a medal for sure, the second ranked teams have a 50 percent chance of winning a medal.

2.6 Conclusion

We have examined the fairness and intensity of sequential round-robin tournaments with multiple prizes and heterogeneous players. We call a tournament fair if, in each of its matches, the winning probabilities depend only on the players' characteristics but not on the order of matches.

We have shown that round-robin tournaments with three players will be fair if and only if the second prize is valued half of the first prize. This result is robust to both, variations in the discriminatory power of the matches and the choice between an exogenous or endogenous match schedule.

We have illustrated, however, that the decision on the prize structure may be subject to a trade-off between fairness and intensity (measured by the players' expected aggregate effort per unit of prize money), e.g., if the matches are organized as lottery contests. Instead, if the discriminatory power of the matches is sufficiently large, a fair tournament will entail also maximum intensity. Also this result is robust to the choice between an exogenous or endogenous match schedule.

Moreover, we have shown that there is no prize structure for which round-robin tournaments with more than three players will be fair if the schedule of matches is sequential and exogenously fixed. Whether this problem can be solved by the use of endogenous match schedules or schemes which schedule certain matches simultaneously is beyond the scope of this paper and left for future research.

[32]Mo Farah on facebook on April 25, 2019 (https://eo-eo.facebook.com/LaureusSportforGood/posts/if-i-was-capable-of-getting-an-olympic-medal-no-matter-what-colour-it-is-would-y/10156195276235777/, accessed on 09/03/20); similar statements can be found by many other athletes, e.g., Kristy Coventry (https://www.newzimbabwe.com/kirsty-one-more-medal-it-doesnt-matter-what-colour/, accessed on 09/03/20) or Claudia Pechstein (https://www.main-post.de/sport/ueberregional/wintersport/deutschesteam/news/Olympia-Partys-der-Pechstein-10-Medaille-waere-Platin;art9577,7960317?wt_ref=https%3A%2F%2Fwww.google.com%2F&wt_t=-1599137898757, accessed on 09/03/20), and associations, e.g., the German Olympic Sports Confederation (https://rp-online.de/sport/olympia-sommer/farbe-der-medaille-in-zukunft-egal_aid-16150479, accessed on 09/03/20).

Our results imply that many present tournament formats of major sports competitions are not neutral with respect to the sequence of matches. Furthermore, they challenge the conventional wisdom according to which "a round-robin tournament is the fairest way to determine the champion among a known and fixed number of contestants" (Wikipedia[33], 2020): under certain conditions, we may find fairer tournament structures.[34]

Our analysis also contributes to a better understanding of complex tournament games with an arbitrary number of players and (possibly) multiple stages as considered by Vong (2017): In each stage, the remaining players are divided into groups. In each group, a round-robin tournament takes place and leads to a ranking within the group. The author shows that the overall tournament is non-manipulable (in the sense that no player ever has an incentive to shirk) if and only if, in each stage, only the players ranking first in their group qualify for the next stage (or for the prize money in the final stage). Together with this finding, our results imply that the only non-manipulable and fair structure for sequential tournaments is the elimination tournament (with groups of two at each stage).

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Appendix

2.A Proof of Proposition 2.1

The proof is structured in two parts. In the first part we show that $a = 1/2$ is sufficient for fairness. In the second part we show that it is also necessary. In each part, we distinguish between the cases with $0 < r \leq 2$ and $r > 2$, respectively. Notice that $a = 1/2$ implies $\Gamma = 1/2$.

2.A.1 Sufficiency

To show that $T_3^{ex}[r]$ is fair if $a = 1/2$, we solve the game by backward induction. With respect to the players' valuations, in principle, six different scenarios may arise:

[33] Accessed at https://en.wikipedia.org/wiki/Round-robin_tournament on 01/30/2020

[34] For example, there is no prize-scheme for which a sequential round-robin tournament with four symmetric players is fair (Proposition 2.5). By contrast, consider a tournament with four symmetric players and two rounds each of which consists of two (sequential) matches. In the first round, players are randomly matched in pairs. In the second round, winners (losers) of the first round are matched. The player who wins in both (none of the) rounds ranks first (last), the player who wins only in the first (second) round ranks second (third). It is straightforward to show that then, for any sequence of matches within rounds and any prize structure $0 = R_4 < R_3 < R_2 < R_1 = 1$, each player has a winning probability of $1/2$ in each match because incentives are always symmetric.

(a) $v_1 \leq v_2 \leq v_3$,

(b) $v_1 \leq v_3 < v_2$,

(c) $v_2 < v_1 \leq v_3$,

(d) $v_2 < v_3 < v_1$,

(e) $v_3 < v_1 \leq v_2$,

(f) $v_3 < v_2 < v_1$.

In Scenario (a), for example, the weakest players meet in the first match and the strongest players meet in the last match, whereas the opposite holds in Scenario (f). Below, we will exemplarily compute the equilibrium for Scenario (a), the other scenarios can be treated in an analogous manner.

Case $r > 2$

For $r > 2$ the outcome of each match is equivalent to that of an all-pay auction and characterized by equations (2.10)–(2.12). In Scenario (a), backward induction thus proceeds as follows.

Stage 3: Player 2 vs. Player 3

In node A, we have $E_2^A = \frac{v_2}{2}$ and $E_3^A = v_3 - \frac{v_2}{2}$ by (2.12), and $p_2^A = \frac{v_2/2}{2\cdot(v_3/2)} = \frac{v_2}{2v_3}$ and $p_3^A = 1 - p_2^A = \frac{2v_3-v_2}{2v_3}$ by (2.11). Moreover, we compute $E_1^A = p_2^A \cdot 0 + p_3^A \cdot 0 = 0$.

Similarly, in node B, we have $E_2^B = \frac{v_2}{2}$ and $E_3^B = \frac{v_3}{2} - \frac{v_2}{2}$, and $p_2^B = \frac{v_2}{2v_3}$ and $p_3^B = \frac{2v_3-v_2}{2v_3}$. Moreover, we compute $E_1^B = p_2^B \cdot \frac{v_1}{2} + p_3^B \cdot \frac{v_1}{2} = \frac{v_1}{2}$.

In node C, we have $E_2^C = 0$ and $E_3^C = v_3 - \frac{v_2}{2}$, and $p_2^C = \frac{v_2}{2v_3}$ and $p_3^C = \frac{2v_3-v_2}{2v_3}$. Moreover, $E_1^C = \frac{v_1}{2}$.

Finally, in node C', we have $E_2^{C'} = 0$ and $E_3^{C'} = \frac{v_3}{2} - \frac{v_2}{2}$, and $p_2^{C'} = \frac{v_2}{2v_3}$ and $p_3^{C'} = \frac{2v_3-v_2}{2v_3}$. Moreover, $E_1^{C'} = v_1$.

Stage 2: Player 1 vs. Player 3

In node D, we have $E_1^D = E_1^A = 0$ and $E_3^D = E_3^A - (E_1^B - E_1^A) = v_3 - \frac{v_2}{2} - \frac{v_1}{2}$ by (2.12), and $p_1^D = \frac{v_1/2}{2\cdot(v_3/2)} = \frac{v_1}{2v_3}$ and $p_3^D = 1 - p_1^D = \frac{2v_3-v_1}{2v_3}$ by (2.11). Moreover, we compute $E_2^D = p_1^D \cdot \frac{v_2}{2} + p_3^D \cdot \frac{v_2}{2} = \frac{v_2}{2}$.

Similarly, in node E, we have $E_1^E = \frac{v_1}{2}$ and $E_3^E = v_3 - \frac{v_2}{2} - \frac{v_1}{2}$, and $p_1^E = \frac{v_1}{2v_3}$ and $p_3^E = \frac{2v_3-v_1}{2v_3}$. Moreover, $E_2^E = 0$.

Stage 1: Player 1 vs. Player 2

In node F, we have $E_1^F = E_1^D = 0$ and $E_2^F = E_2^D - (E_1^E - E_1^D) = \frac{v_2}{2} - \frac{v_1}{2}$ by (2.12), and $p_1^F = \frac{v_1/2}{2\cdot(v_2/2)} = \frac{v_1}{2v_2}$ and $p_2^F = 1 - p_1^F = \frac{2v_2-v_1}{2v_2}$ by (2.11). Moreover, we compute $E_3^F = p_1^F \cdot v_3 - \frac{v_2}{2} - \frac{v_1}{2} + p_2^F \cdot v_3 - \frac{v_2}{2} - \frac{v_1}{2} = v_3 - \frac{v_2}{2} - \frac{v_1}{2}$.

Result

As these computations show, in each match of the tournament between two players $i \neq j \in \{1,2,3\}$ with $v_i \leq v_j$, the winning probability of the (weakly) weaker player equals $\frac{v_i}{2v_j}$ and the winning probability of the (weakly) stronger player equals $\frac{2v_j - v_i}{2v_j}$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, in each match, the winning probabilities depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

Case $0 < r \leq 2$

For $0 < r \leq 2$, we have to check in each match whether $r \leq \bar{r}^k$ as defined by equation (2.2). If the condition is met, then its outcome will be characterized by equations (2.3)–(2.5), otherwise by equations (2.6)–(2.8). In Scenario (a), backward induction thus proceeds as follows.

Stage 3: Player 2 vs. Player 3

Notice that $w_2^k - \ell_2^k = v_2/2$ and $w_3^k - \ell_3^k = v_3/2$ and thus, by equation (2.2), $\bar{r}^k = 1 + (v_2/v_3)^r$ in all nodes $k \in \{A, B, C, C'\}$ of match 3. We first consider the case in which $r \leq \bar{r}^k$ and equations (2.3)–(2.5) apply.

In node A, we have $p_2^A = \frac{(v_2/2)^r}{(v_2/2)^r + (v_3/2)^r} = \frac{v_2^r}{v_2^r + v_3^r}$ and $p_3^A = \frac{v_3^r}{v_2^r + v_3^r}$ by equation (2.4), and $E_2^A = \frac{v_2}{2} + \frac{(v_2/2)^{r+1}[(v_2/2)^r + (1-r)(v_3/2)^r]}{((v_2/2)^r + (v_3/2)^r)^2} = \frac{v_2}{2} + \frac{v_2^{r+1}[v_2^r + (1-r)v_3^r]}{2(v_2^r + v_3^r)^2}$ and $E_3^A = \frac{v_3}{2} + \frac{v_3^{r+1}[v_3^r + (1-r)v_2^r]}{2(v_2^r + v_3^r)^2}$ by (2.5). Moreover, we compute $E_1^A = p_2^A \cdot 0 + p_3^A \cdot 0 = 0$.

Similarly, in node B, we have $p_2^B = \frac{v_2^r}{v_2^r + v_3^r}$ and $p_3^B = \frac{v_3^r}{v_2^r + v_3^r}$, and $E_2^B = \frac{v_2}{2} + \frac{v_2^{r+1}[v_2^r + (1-r)v_3^r]}{2(v_2^r + v_3^r)^2}$ and $E_3^B = \frac{v_3^{r+1}[v_3^r + (1-r)v_2^r]}{2(v_2^r + v_3^r)^2}$. Moreover, $E_1^B = \frac{v_1}{2}$.

In node C, we have $p_2^C = \frac{v_2^r}{v_2^r + v_3^r}$ and $p_3^C = \frac{v_3^r}{v_2^r + v_3^r}$, and $E_2^C = \frac{v_2^{r+1}[v_2^r + (1-r)v_3^r]}{2(v_2^r + v_3^r)^2}$ and $E_3^C = \frac{v_3}{2} + \frac{v_3^{r+1}[v_3^r + (1-r)v_2^r]}{2(v_2^r + v_3^r)^2}$. Moreover, $E_1^C = \frac{v_1}{2}$.

Finally, in node C', we have $p_2^{C'} = \frac{v_2^r}{v_2^r + v_3^r}$ and $p_3^{C'} = \frac{v_3^r}{v_2^r + v_3^r}$, and $E_2^{C'} = \frac{v_2^{r+1}[v_2^r + (1-r)v_3^r]}{2(v_2^r + v_3^r)^2}$ and $E_3^{C'} = \frac{v_3^{r+1}[v_3^r + (1-r)v_2^r]}{2(v_2^r + v_3^r)^2}$. Moreover, $E_1^{C'} = v_1$.

We now consider the case in which $r > \bar{r}^k$ and equations (2.6)–(2.8) apply.

In node A, we have $p_2^A = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{(v_2/2)}{(v_3/2)} = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_2}{v_3}$ and $p_3^A = 1 - p_2^A$ by equation (2.7), and $E_2^A = \frac{v_2}{2}$ and $E_3^A = v_3 - \frac{2}{r}(r-1)^{\frac{r-1}{r}} \frac{v_2}{2} = v_3 - \frac{v_2}{r}(r-1)^{\frac{r-1}{r}}$ by (2.8). Moreover, we compute $E_1^A = p_2^A \cdot 0 + p_3^A \cdot 0 = 0$.

Similarly, in node B, we have $p_2^B = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_2}{v_3}$ and $p_3^B = 1 - p_2^B$, and $E_2^B = \frac{v_2}{2}$ and $E_3^B = \frac{v_3}{2} - \frac{v_2}{r}(r-1)^{\frac{r-1}{r}}$. Moreover, $E_1^B = \frac{v_1}{2}$.

In node C, we have $p_2^C = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_2}{v_3}$ and $p_3^C = 1 - p_2^B$, and $E_2^C = 0$ and $E_3^C = v_3 - \frac{v_2}{r}(r-1)^{\frac{r-1}{r}}$. Moreover, $E_1^C = \frac{v_1}{2}$.

Finally, in node C', we have $p_2^{C'} = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_2}{v_3}$ and $p_3^{C'} = 1 - p_2^B$, and $E_2^{C'} = 0$ and $E_3^{C'} = \frac{v_3}{2} - \frac{v_2}{r}(r-1)^{\frac{r-1}{r}}$. Moreover, $E_1^{C'} = v_1$.

Stage 2: Player 1 vs. Player 3

Notice that $w_1^D - \ell_1^D = E_1^B - E_1^A = v_1/2$ and $w_1^E - \ell_1^E = E_1^{C'} - E_1^C = v_1/2$ as well as $w_3^D - \ell_3^D = E_3^A - E_3^B = v_3/2$ and $w_3^E - \ell_3^E = E_3^C - E_3^{C'} = v_3/2$, independent from the type of equilibrium in match 3. Thus, by equation (2.2), $\bar{r}^k = 1 + (v_1/v_3)^r$ in all nodes $k \in \{D, E\}$ of match 2. We first consider the case in which $r \leq \bar{r}^k$ and equations (2.3)–(2.5) apply.

In node D, we have $p_1^D = \frac{(v_1/2)^r}{(v_1/2)^r+(v_3/2)^r} = \frac{v_1^r}{v_1^r+v_3^r}$ and $p_3^D = \frac{v_3^r}{v_1^r+v_3^r}$ by equation (2.4), and $E_1^D = 0 + \frac{(v_1/2)^{r+1}[(v_1/2)^r+(1-r)(v_3/2)^r]}{((v_1/2)^r+(v_3/2)^r)^2} = \frac{v_1^{r+1}[v_1^r+(1-r)v_3^r]}{2(v_1^r+v_3^r)^2}$ and $E_3^D = E_3^B + \frac{v_3^{r+1}[v_3^r+(1-r)v_1^r]}{2(v_1^r+v_3^r)^2}$ by (2.5). Moreover, $E_2^D = p_1^D \cdot E_2^B + p_3^D \cdot E_2^A = E_2^A$ because $E_2^B = E_2^A$ for either type of equilibrium in match 3.

Similarly, in node E, we have $p_1^E = \frac{v_1^r}{v_1^r+v_3^r}$ and $p_3^E = \frac{v_3^r}{v_1^r+v_3^r}$, and $E_1^E = \frac{v_1}{2} + \frac{v_1^{r+1}[v_1^r+(1-r)v_3^r]}{2(v_1^r+v_3^r)^2}$ and $E_3^D = E_3^{C'} + \frac{v_3^{r+1}[v_3^r+(1-r)v_1^r]}{2(v_1^r+v_3^r)^2}$. Moreover, $E_2^E = p_1^E \cdot E_2^{C'} + p_3^E \cdot E_2^C = E_2^C$ because $E_2^C = E_2^{C'}$ for either type of equilibrium in match 3.

Notice that $E_3^B = E_3^{C'}$ for either type of equilibrium in match 3 and thus $E_3^D = E_3^E$.

We now consider the case in which $r > \bar{r}^k$ and equations (2.6)–(2.8) apply.

In node D, we have $p_1^D = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{(v_1/2)}{(v_3/2)} = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_1}{v_3}$ and $p_3^D = 1 - p_1^D$ by equation (2.7), and $E_1^D = 0$ and $E_3^D = E_3^A - \frac{2}{r}(r-1)^{\frac{r-1}{r}} \frac{v_1}{2} = E_3^A - \frac{v_1}{r}(r-1)^{\frac{r-1}{r}}$ by (2.8). Moreover, $E_2^D = p_1^D \cdot E_2^B + p_3^D \cdot E_2^A = E_2^A$ because $E_2^B = E_2^A$ for either type of equilibrium in match 3.

Similarly, in node E, we have $p_1^E = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_1}{v_3}$ and $p_3^E = 1 - p_1^E$, and $E_1^E = \frac{v_1}{2}$ and $E_3^E = E_3^C - \frac{v_1}{r}(r-1)^{\frac{r-1}{r}}$. Moreover, $E_2^E = p_1^E \cdot E_2^{C'} + p_3^E \cdot E_2^C = E_2^C$ because $E_2^C = E_2^{C'}$ for either type of equilibrium in match 3.

Notice that $E_3^A = E_3^C$ for either type of equilibrium in match 3 and thus, again, $E_3^D = E_3^E$.

Stage 1: Player 1 vs. Player 2

Notice that $w_1^F - \ell_1^F = E_1^E - E_1^D = v_1/2$ for either type of equilibrium in match 2 and $w_2^F - \ell_2^F = E_2^D - E_2^E = E_2^A - E_2^C = v_2/2$ for either type of equilibrium in match 3. Thus, by equation (2.2), $\bar{r}^F = 1 + (v_1/v_2)^r$ in match 1 (in node F).

We first consider the case in which $r \leq \bar{r}^F$ and equations (2.3)–(2.5) apply. We then have $p_1^F = \frac{(v_1/2)^r}{(v_1/2)^r+(v_2/2)^r} = \frac{v_1^r}{v_1^r+v_2^r}$ and $p_2^F = \frac{v_2^r}{v_1^r+v_2^r}$ by equation (2.4), and $E_1^F = E_1^D + \frac{(v_1/2)^{r+1}[(v_1/2)^r+(1-r)(v_2/2)^r]}{((v_1/2)^r+(v_2/2)^r)^2} = E_1^D + \frac{v_1^{r+1}[v_1^r+(1-r)v_2^r]}{2(v_1^r+v_2^r)^2}$ and $E_2^F = E_2^E + \frac{v_2^{r+1}[v_2^r+(1-r)v_1^r]}{2(v_1^r+v_2^r)^2}$ by (2.5). Moreover, we compute $E_3^F = p_1^F \cdot E_3^E + p_2^F \cdot E_3^D = E_3^D$ (remember that $E_3^D = E_3^E$ for either type of equilibrium in match 2).

We finally consider the case in which $r > \bar{r}^F$ and equations (2.6)–(2.8) apply. We then have $p_1^F = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{(v_1/2)}{(v_2/2)} = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_1}{v_2}$ and $p_2^F = 1 - p_1^F$ by equation (2.7), and $E_1^F = E_1^D$ and $E_2^F = E_2^D - \frac{2}{r}(r-1)^{\frac{r-1}{r}} \frac{v_1}{2} = E_2^D - \frac{v_1}{r}(r-1)^{\frac{r-1}{r}}$ by (2.8). Moreover, as above, $E_3^F = E_3^D$.

Result

As these computations show, in each match of the tournament between two players $i \neq j \in \{1, 2, 3\}$ with $v_i \leq v_j$, the winning probability of the (weakly) weaker player equals $p_i = \frac{v_i^r}{v_i^r+v_j^r}$ if $r \leq 1 + (v_i/v_j)^r$ and $p_i = \frac{1}{r}(r-1)^{\frac{r-1}{r}} \frac{v_i}{v_j}$ if $r > 1 + (v_i/v_j)^r$, and the winning probability of the (weakly) stronger player equals $p_j = 1 - p_i$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, for any given r, the winning probabilities in each match depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

2.A.2 Necessity

To show that $T_3^{ex}[r]$ is fair only if $a = 1/2$, we use the fact that, for symmetric players ($v_1 = v_2 = v_3$), fairness implies that each player's winning probability equals 1/2 in each single match of the tournament, particularly in match 3.

Case $r > 2$

For $r > 2$ the outcome of each match is equivalent to that of an all-pay auction and characterized by equations (2.10)–(2.12). Particularly, in node B of match 3, equation (2.11) implies for symmetric players that either, for $a \leq 1/2$,

$$p_3^B = \frac{\Gamma - 0}{2 \cdot (1 - \Gamma)} = \frac{1 + a}{4 - 2a} = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}$$

or, for $a \geq 1/2$,

$$p_2^B = \frac{1 - \Gamma}{2 \cdot (\Gamma - 0)} = \frac{2 - a}{2 + 2a} = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

Case $0 < r \leq 2$

Notice that $\Gamma = \frac{1+a+0}{3} \leq \frac{1}{2}$ if and only if $a \leq \frac{1}{2}$.

First suppose $\Gamma \leq \frac{1}{2}$. For symmetric players we then have $w_2^B - \ell_2^B = 1 - \Gamma \geq \Gamma - 0 = w_3^B - \ell_3^B$. According to equation (2.2), if $r \leq 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r$, then equation (2.4) applies and the fairness requirement implies for symmetric players that

$$p_2^B = p_3^B \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} = \frac{(\Gamma - 0)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow a = \frac{1}{2}.$$

If

$$r > 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r \quad \Leftrightarrow \quad \Gamma < \frac{(r-1)^{1/r}}{1 + (r-1)^{1/r}}, \tag{2.13}$$

then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{\Gamma}{1-\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2f(r) + 1}. \tag{2.14}$$

Using $\Gamma = (1 + a)/3$, fairness in nodes A and B implies

$$\begin{aligned} w_1^D - \ell_1^D &= (\frac{1}{2}a + \frac{1}{2}\Gamma) - 0 = 2\Gamma - \frac{1}{2} \\ w_3^D - \ell_3^D &= (\frac{1}{2} + \frac{1}{2}a) - (\frac{1}{2}\Gamma + 0) = \Gamma \end{aligned}$$

and thus $w_1^D - \ell_1^D \leq w_3^D - \ell_3^D$ since $\Gamma \leq \frac{1}{2}$. If $r \leq 1 + \left(\frac{2\Gamma - \frac{1}{2}}{\Gamma}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_1^D = p_3^D \quad \Leftrightarrow \quad \frac{(2\Gamma - \frac{1}{2})^r}{\Gamma^r + (2\Gamma - \frac{1}{2})^r} = \frac{\Gamma^r}{\Gamma^r + (2\Gamma - \frac{1}{2})^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{2\Gamma - \frac{1}{2}}{\Gamma}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{2\Gamma - \frac{1}{2}}{\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{4f(r) - 1}. \tag{2.15}$$

However, (2.15) contradicts (2.14) unless $f(r) = 1 \Leftrightarrow r = 1$. But for $r = 1$ we have $\frac{(r-1)^{1/r}}{1+(r-1)^{1/r}} = 0$ which contradicts (2.13).

Now suppose $\Gamma \geq \frac{1}{2}$. For symmetric players we then have $w_2^B - \ell_2^B = 1 - \Gamma \leq \Gamma - 0 = w_3^B - \ell_3^B$. According to equation (2.2), if $r \leq 1 + \left(\frac{1-\Gamma}{\Gamma}\right)^r$, then equation (2.4) applies and the fairness requirement implies for symmetric players that

$$p_2^B = p_3^B \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} = \frac{(\Gamma - 0)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{1-\Gamma}{\Gamma}\right)^r$, then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{1-\Gamma}{\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{2f(r)}{2f(r) + 1}. \tag{2.16}$$

Using $\Gamma = (1+a)/3$, fairness in nodes A and B again implies

$$\begin{aligned} w_1^D - \ell_1^D &= (\frac{1}{2}a + \frac{1}{2}\Gamma) - 0 = 2\Gamma - \frac{1}{2} \\ w_3^D - \ell_3^D &= (\frac{1}{2} + \frac{1}{2}a) - (\frac{1}{2}\Gamma + 0) = \Gamma \end{aligned}$$

and thus $w_1^D - \ell_1^D \geq w_3^D - \ell_3^D$ since $\Gamma \geq \frac{1}{2}$. If $r \leq 1 + \left(\frac{\Gamma}{2\Gamma - \frac{1}{2}}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_1^D = p_3^D \quad \Leftrightarrow \quad \frac{(2\Gamma - \frac{1}{2})^r}{\Gamma^r + (2\Gamma - \frac{1}{2})^r} = \frac{\Gamma^r}{\Gamma^r + (2\Gamma - \frac{1}{2})^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{\Gamma}{2\Gamma - \frac{1}{2}}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{\Gamma}{2\Gamma - \frac{1}{2}} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{4 - 4f(r)}. \tag{2.17}$$

However, (2.17) contradicts (2.16) unless $f(r) = 1/2 \Leftrightarrow r = 2$. But for $f(r) = 1/2$, (2.16) and (2.17) again imply $\Gamma = 1/2$ and, thus, $a = 1/2$.

2.B Proof of Proposition 2.2

Suppose $a = 1/2$ and $r \geq 2$. With three symmetric players ($v_1 = v_2 = v_3$), the calculations in the first part of Appendix 2.A then imply that, in node F, we have $E_i^F = 0$ for $i \in \{1, 2, 3\}$: each player's ex-ante expected payoff is zero.[35] Consequently, the ex-ante expected aggregate payoff equals zero as well and thus, in expectations, the rent is fully dissipated: the ex-ante expected aggregate effort per unit of prize money equals $\rho = 1$.

2.C Proof of Proposition 2.3

We consecutively consider $T_3^{WF}[r]$ and $T_3^{LF}[r]$. In either case, we proceed in exactly the same way as in the proof of Proposition 2.1. Considering sufficiency, again, we exemplarily compute the equilibria for Scenario (a) in which $v_1 \leq v_2 \leq v_3$, the other scenarios can be treated in an analogous manner.

2.C.1 Winner first (WF)

The WF-structure is depicted in Figure 2.4.

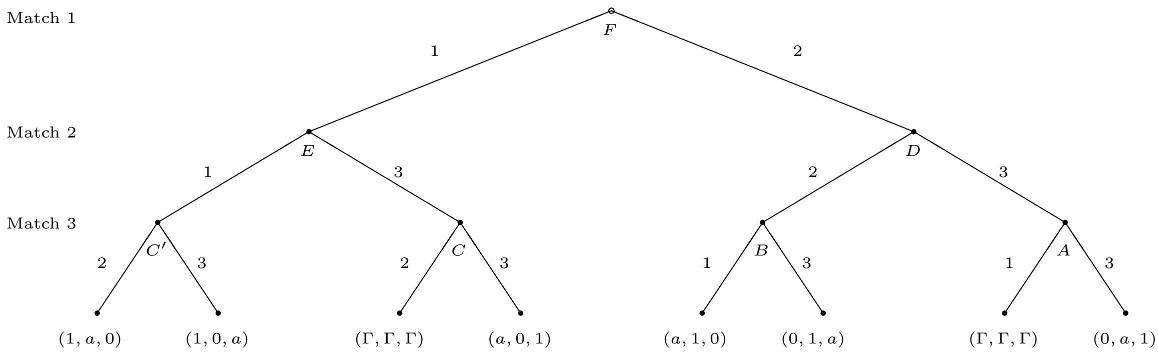


Figure 2.4: 3-player round-robin tournament with WF-sequence

Case $r > 2$

For $r > 2$ the outcome of each match is equivalent to that of an all-pay auction and characterized by equations (2.10)–(2.12).

To show that $a = 1/2$ is sufficient for fairness, we solve $T_3^{WF}[r]$ by backward induction. For Scenario (a), the results are given in Table 2.1. As the entries show, in each match of the tournament between two players $i \neq j \in \{1, 2, 3\}$ with $v_i \leq v_j$, the winning probability of the (weakly) weaker player equals $\frac{v_i}{2v_j}$ and the winning probability of the (weakly) stronger player equals $\frac{2v_j - v_i}{2v_j}$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, in each match, the winning probabilities depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

[35]Notice that for $r = 2$ (and thus $f(r) = 1/2$) the winning probabilities and expected payoffs as given by equations (2.7) and (2.8) coincide with those for $r > 2$ as given by equations (2.11) and (2.12).

Node	p_1	p_2	p_3	E_1	E_2	E_3
A	$\frac{v_1}{2v_3}$		$\frac{2v_3-v_1}{2v_3}$	0	$\frac{v_2}{2}$	$v_3-\frac{v_1}{2}$
B	$\frac{v_1}{2v_3}$		$\frac{2v_3-v_1}{2v_3}$	0	v_2	$\frac{v_3}{2}-\frac{v_1}{2}$
C		$\frac{v_2}{2v_3}$	$\frac{2v_3-v_2}{2v_3}$	$\frac{v_1}{2}$	0	$v_3-\frac{v_2}{2}$
C'		$\frac{v_2}{2v_3}$	$\frac{2v_3-v_2}{2v_3}$	v_1	0	$\frac{v_3}{2}-\frac{v_2}{2}$
D		$\frac{v_2}{2v_3}$	$\frac{2v_3-v_2}{2v_3}$	0	$\frac{v_2}{2}$	$v_3-\frac{v_1}{2}-\frac{v_2}{2}$
E	$\frac{v_1}{2v_3}$		$\frac{2v_3-v_1}{2v_3}$	$\frac{v_1}{2}$	0	$v_3-\frac{v_1}{2}-\frac{v_2}{2}$
F	$\frac{v_1}{2v_2}$	$\frac{2v_2-v_1}{2v_2}$		0	$\frac{v_2}{2}-\frac{v_1}{2}$	$v_3-\frac{v_1}{2}-\frac{v_2}{2}$

Table 2.1: Backward induction of $T_3^{WF}[r]$ for $r > 2$, $a = 1/2$, and $v_1 \leq v_2 \leq v_3$

To show that $a = 1/2$ is necessary for fairness, we again use the fact that, for symmetric players ($v_1 = v_2 = v_3$), fairness implies that each player's winning probability equals 1/2 in each single match of the tournament. Particularly, in node A of match 3, equation (2.11) implies for symmetric players that either, for $a \leq 1/2$,

$$p_1^A = \frac{\Gamma - 0}{2\cdot(1-\Gamma)} = \frac{1+a}{4-2a} = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}$$

or, for $a \geq 1/2$,

$$p_3^A = \frac{1-\Gamma}{2\cdot(\Gamma - 0)} = \frac{2-a}{2+2a} = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

Case $0 < r \leq 2$

For $0 < r \leq 2$, we have to check in each match whether $r \leq \bar{r}^k$ as defined by equation (2.2). If the condition is met, then its outcome will be characterized by equations (2.3)–(2.5), otherwise by equations (2.6)–(2.8).

To show that $a = 1/2$ is sufficient for fairness, we solve $T_3^{WF}[r]$ by backward induction. For Scenario (a), the results are given in Table 2.2.[36] As the entries show, in each match of the tournament between two players $i \neq j \in \{1,2,3\}$ with $v_i \leq v_j$, the winning probability of the (weakly) weaker player equals $p_i = \frac{v_i^r}{v_i^r+v_j^r}$ if $r \leq 1 + (v_i/v_j)^r$ and $p_i = f(r)\frac{v_i}{v_j}$ if $r > 1+(v_i/v_j)^r$, and the winning probability of the (weakly) stronger player equals $p_j = 1-p_i$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, for any given r, the winning probabilities in each match depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

To show that $a = 1/2$ is necessary for fairness, we again use the fact that, for symmetric players ($v_1 = v_2 = v_3$), fairness implies that the players have equal winning probabilities in each single match of the tournament. Notice that $\Gamma = \frac{1+a+0}{3} \leq \frac{1}{2}$ if and only if $a \leq \frac{1}{2}$.

First suppose $\Gamma \leq \frac{1}{2}$. For symmetric players we then have $w_3^A - \ell_3^A = 1-\Gamma \geq \Gamma - 0 = w_1^A - \ell_1^A$. According to equation (2.2), if $r \leq 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r$, then equation (2.4) applies

[36]It is straightforward to show that, in each node k, we have $w_i^k - \ell_i^k = v_i/2$ for both players i deciding in k, independent from the type of equilibria in potentially previous matches, i.e. for all $0 < r \leq 2$.

Node	$r \le \bar{r}^k$	p_1	p_2	p_3	E_1	E_2	E_3
A	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$V_{13}(r)$	$\frac{v_2}{2}$	$\frac{v_3}{2}+V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1-f(r)\frac{v_1}{v_3}$	0	$\frac{v_2}{2}$	$v_3 - f(r)v_1$
B	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$V_{13}(r)$	v_2	$V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1-f(r)\frac{v_1}{v_3}$	0	v_2	$\frac{v_3}{2} - f(r)v_1$
C	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	$\frac{v_1}{2}$	$V_{23}(r)$	$\frac{v_3}{2}+V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1-f(r)\frac{v_2}{v_3}$	$\frac{v_1}{2}$	0	$v_3 - f(r)v_2$
C'	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	v_1	$V_{23}(r)$	$V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1-f(r)\frac{v_2}{v_3}$	v_1	0	$\frac{v_3}{2} - f(r)v_2$
D	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	$E_1^A = E_1^B$	$\frac{v_2}{2}+V_{23}(r)$	$E_3^B + V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1-f(r)\frac{v_2}{v_3}$	$E_1^A = E_1^B$	$\frac{v_2}{2}$	$E_3^A - f(r)v_2$
E	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$\frac{v_1}{2}+V_{13}(r)$	$E_2^C = E_2^{C'}$	$E_3^{C'} + V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1-f(r)\frac{v_1}{v_3}$	$\frac{v_1}{2}$	$E_2^C = E_2^{C'}$	$E_3^C - f(r)v_1$
F	yes	$\frac{v_1^r}{v_1^r+v_2^r}$	$\frac{v_2^r}{v_1^r+v_2^r}$				
	no	$f(r)\frac{v_1}{v_2}$	$1-f(r)\frac{v_1}{v_2}$				

Notes: $V_{ij}(r) := \frac{v_i^{r+1}[v_i^r+(1-r)v_j^r]}{2(v_i^r+v_j^r)^2}$

Table 2.2: Backward induction of $T_3^{WF}[r]$ for $0 < r \le 2$, $a = 1/2$, and $v_1 \le v_2 \le v_3$

and the fairness requirement implies for symmetric players that

$$p_3^A = p_1^A \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma-0)^r+(1-\Gamma)^r} = \frac{(\Gamma-0)^r}{(\Gamma-0)^r+(1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r$,then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{\Gamma}{1-\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2f(r)+1}. \tag{2.18}$$

Using $\Gamma = (1+a)/3$, fairness in nodes A and B implies

$$\begin{aligned} w_2^D - \ell_2^D &= 1 - (\frac{1}{2}a + \frac{1}{2}\Gamma) = \frac{3}{2} - 2\Gamma \\ w_3^D - \ell_3^D &= (\frac{1}{2} + \frac{1}{2}\Gamma) - (\frac{1}{2}a + 0) = 1 - \Gamma \end{aligned}$$

and thus $w_2^D - \ell_2^D \ge w_3^D - \ell_3^D$ since $\Gamma \le \frac{1}{2}$. If $r \le 1 + \left(\frac{1-\Gamma}{\frac{3}{2}-2\Gamma}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_2^D = p_3^D \quad \Leftrightarrow \quad \frac{(\frac{3}{2}-2\Gamma)^r}{(\frac{3}{2}-2\Gamma)^r+(1-\Gamma)^r} = \frac{(1-\Gamma)^r}{(\frac{3}{2}-2\Gamma)^r+(1-\Gamma)^r} \quad \Leftrightarrow \Gamma = \frac{1}{2} \Leftrightarrow a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{1-\Gamma}{\frac{3}{2}-2\Gamma}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{1-\Gamma}{\frac{3}{2}-2\Gamma}=\frac{1}{2} \quad \Leftrightarrow \quad \Gamma=\frac{\frac{3}{4}-f(r)}{1-f(r)}. \tag{2.19}$$

However, (2.19) contradicts (2.18) unless unless $f(r)=1/2 \Leftrightarrow r=2$. But for $f(r)=1/2$, (2.18) and (2.19) again imply $\Gamma=1/2$ and, thus, $a=1/2$.

Now suppose $\Gamma \geq \frac{1}{2}$. For symmetric players we then have $w_3^A-\ell_3^A=1-\Gamma \leq \Gamma-0=w_1^A-\ell_1^A$. According to equation (2.2), if $r \leq 1+\left(\frac{1-\Gamma}{\Gamma}\right)^r$, then equation (2.4) applies and the fairness requirement implies for symmetric players that

$$p_3^A=p_1^A \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma-0)^r+(1-\Gamma)^r}=\frac{(\Gamma-0)^r}{(\Gamma-0)^r+(1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma=\frac{1}{2} \quad \Leftrightarrow a=\frac{1}{2}.$$

If $r>1+\left(\frac{1-\Gamma}{\Gamma}\right)^r$,then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{1-\Gamma}{\Gamma}=\frac{1}{2} \quad \Leftrightarrow \quad \Gamma=\frac{2f(r)}{2f(r)+1}. \tag{2.20}$$

Using $\Gamma=(1+a)/3$, fairness in nodes A and B implies

$$\begin{aligned} w_2^D-\ell_2^D &= 1-(\frac{1}{2}a+\frac{1}{2}\Gamma)=\frac{3}{2}-2\Gamma \\ w_3^D-\ell_3^D &= (\frac{1}{2}+\frac{1}{2}\Gamma)-(\frac{1}{2}a+0)=1-\Gamma \end{aligned}$$

and thus $w_2^D-\ell_2^D \leq w_3^D-\ell_3^D$ since $\Gamma \geq \frac{1}{2}$. If $r \leq 1+\left(\frac{\frac{3}{2}-2\Gamma}{1-\Gamma}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_2^D=p_3^D \quad \Leftrightarrow \quad \frac{(\frac{3}{2}-2\Gamma)^r}{(\frac{3}{2}-2\Gamma)^r+(1-\Gamma)^r}=\frac{(1-\Gamma)^r}{(\frac{3}{2}-2\Gamma)^r+(1-\Gamma)^r} \quad \Leftrightarrow \Gamma=\frac{1}{2} \Leftrightarrow a=\frac{1}{2}.$$

If $r>1+\left(\frac{\frac{3}{2}-2\Gamma}{1-\Gamma}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{\frac{3}{2}-2\Gamma}{1-\Gamma}=\frac{1}{2} \quad \Leftrightarrow \quad \Gamma=\frac{1-3f(r)}{1-4f(r)}. \tag{2.21}$$

However, (2.21) contradicts (2.20) unless $f(r)=1/2 \Leftrightarrow r=2$ or $f(r)=1 \Leftrightarrow r=1$. But for $f(r)=1/2$, (2.20) and (2.21) again imply $\Gamma=1/2$ and, thus, $a=1/2$, whereas $r=1$ contradicts $r>1+\left(\frac{\frac{3}{2}-2\Gamma}{1-\Gamma}\right)^r$ for all feasible Γ.

2.C.2 Loser first (LF)

The LF-structure is depicted in Figure 2.5.

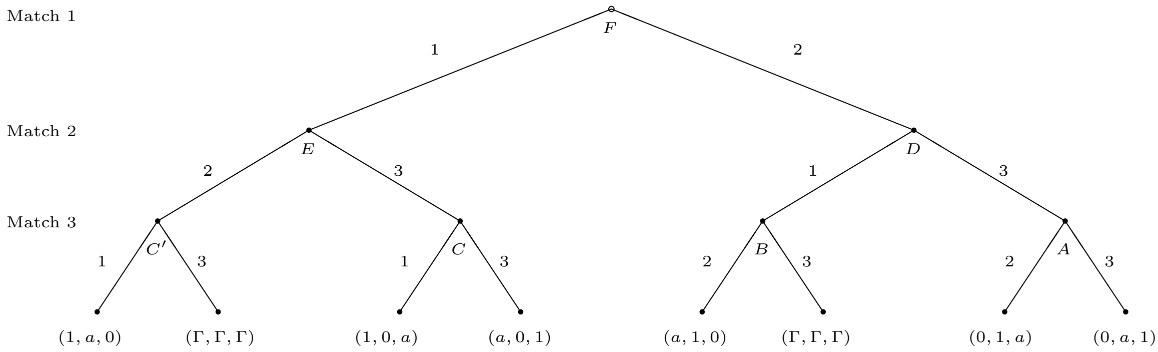


Figure 2.5: 3-player round-robin tournament with LF-sequence

Case $r > 2$

For $r > 2$ the outcome of each match is equivalent to that of an all-pay auction and characterized by equations (2.10)–(2.12).

To show that $a = 1/2$ is sufficient for fairness, we solve $T_3^{LF}[r]$ by backward induction. For Scenario (a), the results are given in Table 2.3. As the entries show, in each match of the tournament between two players $i \neq j \in \{1, 2, 3\}$ with $v_i \leq v_j$, the winning probability of the (weakly) weaker player equals $\frac{v_i}{2v_j}$ and the winning probability of the (weakly) stronger player equals $\frac{2v_j - v_i}{2v_j}$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, in each match, the winning probabilities depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

Node	p_1	p_2	p_3	E_1	E_2	E_3
A		$\frac{v_2}{2v_3}$	$\frac{2v_3 - v_2}{2v_3}$	0	$\frac{v_2}{2}$	$v_3 - \frac{v_2}{2}$
B		$\frac{v_2}{2v_3}$	$\frac{2v_3 - v_2}{2v_3}$	$\frac{v_1}{2}$	$\frac{v_2}{2}$	$\frac{v_3}{2} - \frac{v_2}{2}$
C	$\frac{v_1}{2v_3}$		$\frac{2v_3 - v_1}{2v_3}$	$\frac{v_1}{2}$	0	$v_3 - \frac{v_1}{2}$
C'	$\frac{v_1}{2v_3}$		$\frac{2v_3 - v_1}{2v_3}$	$\frac{v_1}{2}$	$\frac{v_2}{2}$	$\frac{v_3}{2} - \frac{v_1}{2}$
D	$\frac{v_1}{2v_3}$		$\frac{2v_3 - v_1}{2v_3}$	0	$\frac{v_2}{2}$	$v_3 - \frac{v_2}{2} - \frac{v_1}{2}$
E		$\frac{v_2}{2v_3}$	$\frac{2v_3 - v_2}{2v_3}$	$\frac{v_1}{2}$	0	$v_3 - \frac{v_1}{2} - \frac{v_2}{2}$
F	$\frac{v_1}{2v_2}$	$\frac{2v_2 - v_1}{2v_2}$		0	$\frac{v_2}{2} - \frac{v_1}{2}$	$v_3 - \frac{v_1}{2} - \frac{v_2}{2}$

Table 2.3: Backward induction of $T_3^{LF}[r]$ for $r > 2$, $a = 1/2$, and $v_1 \leq v_2 \leq v_3$

To show that $a = 1/2$ is necessary for fairness, we again use the fact that, for symmetric players ($v_1 = v_2 = v_3$), fairness implies that each player's winning probability equals 1/2 in each single match of the tournament. Particularly, in node B of match 3, equation (2.11) implies for symmetric players that either, for $a \leq 1/2$,

$$p_2^B = \frac{\Gamma}{2(1-\Gamma)} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}$$

or, for $a \geq 1/2$,

$$p_3^A = \frac{1-\Gamma}{2\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

Case $0 < r \le 2$

For $0 < r \le 2$, again we have to check in each match whether $r \le \bar{r}^k$ as defined by equation (2.2). If the condition is met, then its outcome will be characterized by equations (2.3)–(2.5), otherwise by equations (2.6)–(2.8).

To show that $a = 1/2$ is sufficient for fairness, we solve $T_3^{LF}[r]$ by backward induction. For Scenario (a), the results are given in Table 2.4.[37] As the entries show, in each match of the tournament between two players $i \ne j \in \{1,2,3\}$ with $v_i \le v_j$, the winning probability of the (weakly) weaker player equals $p_i = \frac{v_i^r}{v_i^r+v_j^r}$ if $r \le 1 + (v_i/v_j)^r$ and $p_i = f(r)\frac{v_i}{v_j}$ if $r > 1+(v_i/v_j)^r$, and the winning probability of the (weakly) stronger player equals $p_j = 1-p_i$. In the same manner, one easily checks that this holds true also in Scenarios (b) – (f). Thus, for any given r, the winning probabilities in each match depend only on the characteristics of the matched players but not on the position of the match in the sequence of the tournament: the tournament is fair.

Node	$r \le \bar{r}^k$	p_1	p_2	p_3	E_1	E_2	E_3
A	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	0	$\frac{v_2}{2} + V_{23}(r)$	$\frac{v_3}{2} + V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1 - f(r)\frac{v_2}{v_3}$	0	$\frac{v_2}{2}$	$v_3 - f(r)v_2$
B	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	$\frac{v_1}{2}$	$\frac{v_2}{2} + V_{23}(r)$	$V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1 - f(r)\frac{v_2}{v_3}$	$\frac{v_1}{2}$	$\frac{v_2}{2}$	$\frac{v_3}{2} - f(r)v_2$
C	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$\frac{v_1}{2} + V_{13}(r)$	0	$\frac{v_3}{2} + V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1 - f(r)\frac{v_1}{v_3}$	$\frac{v_1}{2}$	0	$v_3 - f(r)v_1$
C'	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$\frac{v_1}{2} + V_{13}(r)$	$\frac{v_2}{2}$	$V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1 - f(r)\frac{v_1}{v_3}$	$\frac{v_1}{2}$	$\frac{v_2}{2}$	$\frac{v_3}{2} - f(r)v_1$
D	yes	$\frac{v_1^r}{v_1^r+v_3^r}$		$\frac{v_3^r}{v_1^r+v_3^r}$	$V_{13}(r)$	$E_2^A = E_2^B$	$E_3^B + V_{31}(r)$
	no	$f(r)\frac{v_1}{v_3}$		$1 - f(r)\frac{v_1}{v_3}$	0	$E_2^A = E_2^B$	$E_3^A - f(r)v_1$
E	yes		$\frac{v_2^r}{v_2^r+v_3^r}$	$\frac{v_3^r}{v_2^r+v_3^r}$	$E_1^C = E_1^{C'}$	$V_{23}(r)$	$E_3^{C'} + V_{32}(r)$
	no		$f(r)\frac{v_2}{v_3}$	$1 - f(r)\frac{v_2}{v_3}$	$E_1^C = E_1^{C'}$	0	$E_3^C - f(r)v_2$
F	yes	$\frac{v_1^r}{v_1^r+v_2^r}$	$\frac{v_2^r}{v_1^r+v_2^r}$				
	no	$f(r)\frac{v_1}{v_2}$	$1 - f(r)\frac{v_1}{v_2}$				

Notes: $V_{ij}(r) := \frac{v_i^{r+1}[v_i^r+(1-r)v_j^r]}{2(v_i^r+v_j^r)^2}$

Table 2.4: Backward induction of $T_3^{LF}[r]$ for $0 < r \le 2$, $a = 1/2$, and $v_1 \le v_2 \le v_3$

To show that $a = 1/2$ is necessary for fairness, we again use the fact that, for symmetric players ($v_1 = v_2 = v_3$), fairness implies that the players have equal winning probabilities in each single match of the tournament. Notice that $\Gamma = \frac{1+a+0}{3} \le \frac{1}{2}$ if and only if $a \le \frac{1}{2}$.

First suppose $\Gamma \le \frac{1}{2}$. For symmetric players we then have $w_2^B - \ell_2^B = 1 - \Gamma \ge \Gamma - 0 = w_3^B - \ell_3^B$. According to equation (2.2), if $r \le 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r$, then equation (2.4) applies and the fairness requirement implies for symmetric players that

$$p_2^B = p_3^B \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma-0)^r + (1-\Gamma)^r} = \frac{(\Gamma-0)^r}{(\Gamma-0)^r + (1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow a = \frac{1}{2}.$$

[37]Again, it is straightforward to show that, in each node k, we have $w_i^k - \ell_i^k = v_i/2$ for both players i deciding in k, independent from the type of equilibria in potentially previous matches, i.e. for all $0 < r \le 2$.

If $r > 1 + \left(\frac{\Gamma}{1-\Gamma}\right)^r$,then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{\Gamma}{1-\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2f(r)+1}. \tag{2.22}$$

Using $\Gamma = (1+a)/3$, fairness in nodes A and B implies

$$\begin{aligned} w_1^D - \ell_1^D &= \frac{1}{2}a + \frac{1}{2}\Gamma = 2\Gamma - \frac{1}{2} \\ w_3^D - \ell_3^D &= (\frac{1}{2} + \frac{1}{2}a) - (\frac{1}{2}\Gamma + 0) = \Gamma \end{aligned}$$

and thus $w_1^D - \ell_1^D \leq w_3^D - \ell_3^D$ since $\Gamma \leq \frac{1}{2}$. If $r \leq 1 + \left(\frac{2\Gamma - \frac{1}{2}}{\Gamma}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_1^D = p_3^D \quad \Leftrightarrow \quad 2\Gamma - \frac{1}{2} = \Gamma \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{2\Gamma - \frac{1}{2}}{\Gamma}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{2\Gamma - \frac{1}{2}}{\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{f(r)}{4f(r)-1}. \tag{2.23}$$

However, (2.23) contradicts (2.22) unless $f(r) = 1/2 \Leftrightarrow r = 2$ or $f(r) = 1 \Leftrightarrow r = 1$. But for $f(r) = 1/2$, (2.22) and (2.23) again imply $\Gamma = 1/2$ and thus $a = 1/2$, whereas $r = 1$ contradicts $r > 1 + \left(\frac{2\Gamma - \frac{1}{2}}{\Gamma}\right)^r$ for all feasible Γ.

Now suppose $\Gamma \geq \frac{1}{2}$. For symmetric players we then have $w_2^B - \ell_2^B = 1 - \Gamma \leq \Gamma - 0 = w_3^B - \ell_3^B$. According to equation (2.2), if $r \leq 1 + \left(\frac{1-\Gamma}{\Gamma}\right)^r$, then equation (2.4) applies and the fairness requirement implies for symmetric players that

$$p_2^B = p_3^B \quad \Leftrightarrow \quad \frac{(1-\Gamma)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} = \frac{(\Gamma - 0)^r}{(\Gamma - 0)^r + (1-\Gamma)^r} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \Leftrightarrow a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{1-\Gamma}{\Gamma}\right)^r$,then equation (2.7) applies and the fairness requirement implies for symmetric players that

$$f(r)\frac{1-\Gamma}{\Gamma} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{2f(r)}{2f(r)+1}. \tag{2.24}$$

Using $\Gamma = (1+a)/3$, fairness in nodes A and B implies

$$\begin{aligned} w_1^D - \ell_1^D &= \frac{1}{2}a + \frac{1}{2}\Gamma = 2\Gamma - \frac{1}{2} \\ w_3^D - \ell_3^D &= (\frac{1}{2} + \frac{1}{2}a) - (\frac{1}{2}\Gamma + 0) = \Gamma \end{aligned}$$

and thus $w_1^D - \ell_1^D \geq w_3^D - \ell_3^D$ since $\Gamma \geq \frac{1}{2}$. If $r \leq 1 + \left(\frac{\Gamma}{2\Gamma - \frac{1}{2}}\right)^r$, then equation (2.4) applies and fairness in node D implies for symmetric players that

$$p_1^D = p_3^D \quad \Leftrightarrow \quad \Gamma = 2\Gamma - \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{2} \quad \Leftrightarrow \quad a = \frac{1}{2}.$$

If $r > 1 + \left(\frac{\Gamma}{2\Gamma - \frac{1}{2}}\right)^r$, then equation (2.7) applies and fairness in node D implies for symmetric players that

$$f(r)\frac{\Gamma}{2\Gamma - \frac{1}{2}} = \frac{1}{2} \quad \Leftrightarrow \quad \Gamma = \frac{1}{4 - 4f(r)}. \tag{2.25}$$

However, (2.25) contradicts (2.24) unless $f(r) = 1/2 \Leftrightarrow r = 2$. But for $f(r) = 1/2$, (2.24) and (2.25) again imply $\Gamma = 1/2$ and, thus, $a = 1/2$.

2.D Proof of Proposition 2.4

Suppose $a = 1/2$ and $r \geq 2$. With three symmetric players ($v_1 = v_2 = v_3$), the results in the last row of Tables 2.1 and 2.3 then imply that, in node F, we have $E_i^F = 0$ for $i \in \{1, 2, 3\}$: each player's ex-ante expected payoff is zero.[38] Consequently, the ex-ante expected aggregate payoff equals zero as well and thus, in expectations, the rent is fully dissipated: the ex-ante expected aggregate effort per unit of prize money equals $\rho = 1$.

2.E Proof of Proposition 2.5

Suppose that all $n > 3$ players in tournament $T_n^{ex}[r]$ are symmetric ($v_i = v = 1$ for all $i \in \{1, \ldots, n\}$). Denote the players who meet in the last match of the tournament, match $\frac{n}{2}(n-1)$, by i and j, respectively. Let $\tau := \max\{s \in \{1, \ldots, n-1\} | R_s = 1\}$ be the last rank for which the prize money equals 1 (which must exist since $R_1 = 1$ and $R_n = 0$) and define $t := \min\{\tau, n-3\}$. Independent of the exact exogenous sequence, there is always a partition of the set of players

$$\{1, \ldots, n\} = \{\omega_1, \ldots, \omega_{t-1}\} \cup \{i, j, k, l\} \cup \{\lambda_1, \ldots, \lambda_{n-4-(t-1)}\}$$

and a certain course of the tournament such that the last match starts in a node, called node A, at which

(a) player ω_α has won all matches but those against players ω_β with $\beta < \alpha$; thus player ω_α has won $n - \alpha \geq n + 1 - t$ matches,

(b) player λ_γ has lost all matches but those against players λ_δ with $\delta > \gamma$; thus player λ_γ has has won $n - 4 - (t-1) - \gamma \leq n - 4 - t$ matches,

(c) player i has lost all prior matches but those against players λ_δ for all $\delta \in \{1, \ldots, n-4-(t-1)\}$; thus player i has won exactly $n - 3 - t$ matches before,

[38] Notice again that for $r = 2$ (and thus $f(r) = 1/2$) the winning probabilities and expected payoffs as given by equations (2.7) and (2.8) coincide with those for $r > 2$ as given by equations (2.11) and (2.12).

(d) player j has lost all prior matches but those against players λ_δ for all $\delta \in \{1, \dots, n-4-(t-1)\}$ and player k; thus player j has won exactly $n-2-t$ matches before,

(e) player k has lost all matches but those against players λ_δ for all $\delta \in \{1, \dots, n-4-(t-1)\}$ and player i; thus player k has won exactly $n-2-t$ matches,

(f) player l has won all matches but those against players ω_β for all $\beta \in \{1, \dots, t-1\}$; thus player l has won exactly $n-t$ matches.

Similarly, there are also courses such that the last match starts in a node, called node B, at which properties (a)–(d) hold and, additionally,

(e') player k has won all matches but those against players ω_β for all $\beta \in \{1, \dots, t-1\}$ and player j; thus player k has won exactly $n-1-t$ matches,

(f') player l has won all matches but those against players ω_β for all $\beta \in \{1, \dots, t-1\}$ and player k; thus player l has won exactly $n-1-t$ matches,

or in a node, called node D, at which properties (a)–(b) hold and, additionally,

(c") player i has lost all prior matches but those against players λ_δ for all $\delta \in \{1, \dots, n-4-(t-1)\}$ and player k; thus player i has won exactly $n-2-t$ matches before,

(d") player j has won all prior matches but those against players ω_β for all $\beta \in \{1, \dots, t-1\}$; thus player j has won exactly $n-1-t$ matches before,

(e") player k has lost all matches but those against players λ_δ for all $\delta \in \{1, \dots, n-4-(t-1)\}$ and player l; thus player k has won exactly $n-2-t$ matches,

(f") player l has lost all matches but those against players λ_δ for all $\delta \in \{1, \dots, n-4-(t-1)\}$ and player i; thus player k has won exactly $n-2-t$ matches.

Notice that, in all these cases, for all $\beta \in \{1, \dots, t-1\}$ ($\delta \in \{1, \dots, n-4-(t-1)\}$), player ω_β (λ_δ) will have more (less) wins than any player from $\{i, j, k, l\}$. In all three nodes, the last match will thus determine only the allocation of prizes $(R_t, R_{t+1}, R_{t+2}, R_{t+3})$ among players i, j, k, and l.

A fair prize structure would imply that players i and j have equal winning probabilities in the last match, whether it starts in node A or B or D. To show that such a prize structure does not exist, we distinguish between two cases.

2.E.1 Case 1:

If $t = n-3$, define $a := R_{n-2}$ and $b := R_{n-1}$. In node A, we have $w_i^A = \frac{a+b}{3} = \Theta$, $\ell_i^A = 0$, $w_j^A = a$, and $\ell_j^A = \frac{a+b}{3} = \Theta$. In node B, we have $w_i^B = \frac{b}{2}$, $\ell_i^B = 0$, $w_j^B = \frac{1+a+b}{3} = \Omega$, and $\ell_j^B = \frac{b}{2}$. As $2b \leq 1 + a$, we have $\frac{1}{2}b \leq \Omega - \frac{1}{2}b$, and thus

$$\bar{r}^B = 1 + \left(\frac{3b}{2+2a-b}\right)^{\bar{r}^B}. \tag{2.26}$$

In node D, we have $w_i^D = \frac{1+a}{2} = \Delta$, $\ell_i^D = \frac{a+b}{3} = \Theta$, $w_j^D = 1$, and $\ell_j^D = \frac{1+a}{2} = \Delta$. As $2a \geq b$, we have $\Delta - \Theta \geq 1 - \Delta$, and thus

$$\bar{r}^D = 1 + \left(\frac{3-3a}{3+a-2b}\right)^{\bar{r}^D}. \tag{2.27}$$

Case 1.a

For $0 < r \leq 1$ or $2 < r$, argue exactly as in Section 2.4.1, substituting player-index 2 by i and 3 by j.

Case 1.b

For $1 < r \leq 2$, the contradiction can be constructed in three steps.

Step 1: Suppose $r \leq \bar{r}^B$. Equal odds in node B then imply $2b = 1 + a$ by equation (2.4). As $0 \leq b \leq a \leq 1$, it follows that $a = b = 1$. In node D, this implies $\bar{r}^D = 1 < r$ by equation (2.27). Equal odds in node D then imply

$$\frac{1}{2} = f(r)\frac{3-3a}{3+a-2b} = 0$$

by equation (2.7), a contradiction.

Step 2: Suppose $r \leq \bar{r}^D$. Equal odds in node D then imply $2a = b$ by equation (2.4). As $0 \leq b \leq a \leq 1$, it follows that $a = b = 0$. In node B, this implies $\bar{r}^B = 1 < r$ by equation (2.26). Equal odds in node B then imply

$$\frac{1}{2} = f(r)\frac{3b}{2+2a-b} = 0$$

by equation (2.7), a contradiction.

Step 3: Now suppose $\max\{\bar{r}^B, \bar{r}^D\} < r$. Equal odds in node B then imply

$$p_i^B = p_j^B = \frac{1}{2} \quad \Leftrightarrow \quad 2f(r)(w_i^B - \ell_i^B) = w_j^B - \ell_j^B \quad \Leftrightarrow \quad b = \frac{2}{6f(r)+1}(1+a) \tag{2.28}$$

and equal odds in node D imply

$$p_i^D = p_j^D = \frac{1}{2} \Leftrightarrow 2f(r)(w_j^D - \ell_j^D) = w_i^D - \ell_i^D \Leftrightarrow b = \frac{3 - 6f(r) + [6f(r)+1]a}{2} \tag{2.29}$$

by equation (2.7). Now distinguish between the subcases $r \leq \bar{r}^A$ and $\bar{r}^A < r$.

Subcase $r \leq \bar{r}^A$: Equal odds in node A imply $b = a/2$ by equation (2.4). Together with equations (2.28) and (2.29) this implies

$$a = \frac{4}{6f(r)-3} \quad \text{and} \quad a = \frac{2f(r)-1}{2f(r)},$$

respectively. This, however, implies a contradiction, because the equation $\frac{4}{6x-3} = \frac{2x-1}{2x}$ has no real solution.

Subcase $\bar{r}^A < r$: If $a \geq 2b$ then $a - \theta \geq \theta$ and equal odds in node A imply

$$p_i^A = p_j^A = \frac{1}{2} \quad \Leftrightarrow \quad 2f(r)(w_i^A - \ell_i^A) = w_j^A - \ell_j^A \quad \Leftrightarrow \quad b = \frac{2 - 2f(r)}{1 + 2f(r)} a$$

by equation (2.7). Together with equation (2.28), this implies $a = \frac{1+2f(r)}{3f(r)(1-2f(r))} < 0$ for all $r \in (1, 2]$, a contradiction.[39]

Instead, if $a < 2b$ then $a - \theta < \theta$ and equal odds in node A imply

$$p_i^A = p_j^A = \frac{1}{2} \quad \Leftrightarrow \quad 2f(r)(w_j^A - \ell_j^A) = w_i^A - \ell_i^A \quad \Leftrightarrow \quad b = \frac{4f(r) - 1}{2f(r) + 1} a.$$

Together with equations (2.28) and (2.29) this implies

$$a = \frac{2[2f(r) + 1]}{3[8f(r)^2 - 2f(r) - 1]} \quad \text{and} \quad a = \frac{4f(r)^2 - 1}{4f(r)^2 + 1},$$

respectively. It is straightforward to show that there is a unique $r_0 \in (1, 2] \Leftrightarrow f(r_0) \in [\frac{1}{2}, 1)$ such that both equations hold simultaneously. However,

$$r_0 < \bar{r}^A = 1 + \left(\frac{2a - b}{a + b}\right)^{\bar{r}^A},$$

a contradiction.[40]

2.E.2 Case 2:

If $t = \tau$, define $a := R_{t+1}$, $b := R_{t+2}$, and $c := R_{t+3}$. In node A, we have $w_i^A = \frac{a+b+c}{3}$, $\ell_i^A = c$, $w_j^A = a$, and $\ell_j^A = \frac{a+b+c}{3}$. In node B, we have $w_i^B = \frac{b+c}{2}$, $\ell_i^B = c$, $w_j^B = \frac{1+a+b}{3}$, and $\ell_j^B = \frac{b+c}{2}$. As $2b \leq 1 + a$ we have $\frac{b+c}{2} - c \leq \frac{1+a+b}{3} - \frac{b+c}{2}$, and thus

$$\bar{r}^B = 1 + \left(\frac{3(b - c)}{2 + 2a - b - 3c}\right)^{\bar{r}^B}. \tag{2.30}$$

In node D, we have $w_i^D = \frac{1+a}{2}$, $\ell_i^D = \frac{a+b+c}{3}$, $w_j^D = 1$, and $\ell_j^D = \frac{1+a}{2}$. As $2a \geq b + c$ we have $\frac{1+a}{2} - \frac{a+b+c}{3} \geq 1 - \frac{1+a}{2}$, and thus

$$\bar{r}^D = 1 + \left(\frac{3 - 3a}{3 + a - 2b - 2c}\right)^{\bar{r}^D}. \tag{2.31}$$

Case 2.a

For $0 < r \leq 1$ or $2 < r$, equal odds in node A imply

$$p_i^A = p_j^A = \frac{1}{2} \quad \Leftrightarrow \quad w_i^A - \ell_i^A = w_j^A - \ell_j^A \quad \Leftrightarrow \quad a = 2b - c$$

[39] Remember that $r \in (1, 2] \Leftrightarrow f(r) \in [\frac{1}{2}, 1)$.

[40] Approximately, $f(r_0) \approx 0.89$, $a \approx 0.52$, $b \approx 0.48$, and thus $r_0 \approx 1.03 < 1.44 \approx \bar{r}^A$.

by equations (2.4) or (2.11), respectively. Similarly, equal odds in node B imply

$$p_i^B = p_j^B = \frac{1}{2} \quad \Leftrightarrow \quad w_i^B - \ell_i^B = w_j^B - \ell_j^B \quad \Leftrightarrow \quad a = 2b - 1.$$

The two equations imply $c = 1$, a contradiction since $c = R_{t+3} \leq b = R_{t+2} \leq a = R_{t+1} < R_t = 1$.

Case 2.b

For $1 < r \leq 2$, the contradiction can again be constructed in three steps.

Step 1: Suppose $r \leq \bar{r}^B$. Equal odds in node B then imply $2b = 1 + a$ by equation (2.4). As $0 \leq b \leq a \leq 1$, it follows that $a = b = 1$, a contradiction since $b = R_{t+2} \leq a = R_{t+1} < R_t = 1$.

Step 2: Suppose $r \leq \bar{r}^D$. Equal odds in node D then imply $2a = b + c$ by equation (2.4). As $0 \leq c \leq b \leq a < 1$, it follows that $a = b = c$. In node B, this implies $\bar{r}^B = 1 < r$ by equation (2.30). Equal odds in node B then imply

$$\frac{1}{2} = f(r)\frac{3(b-c)}{2 + 2a - b - 3c} = 0$$

by equation (2.7), a contradiction.

Step 3: Now suppose $\max\{\bar{r}^B, \bar{r}^D\} < r$. Equal odds in node B then imply

$$p_2^B = p_3^B = \frac{1}{2} \quad \Leftrightarrow \quad 2f(r)(w_2^B - \ell_2^B) = w_3^B - \ell_3^B \quad \Leftrightarrow \quad c = \frac{2 + 2a - [1 + 6f(r)]b}{3 - 6f(r)}, \tag{2.32}$$

and equal odds in node D imply

$$p_2^B = p_3^B = \frac{1}{2} \Leftrightarrow 2f(r)(w_3^D - \ell_3^D) = w_2^D - \ell_2^D \Leftrightarrow c = \frac{3 - 6f(r) + [6f(r) + 1]a - 2b}{2}. \tag{2.33}$$

by equation (2.7). The equality of (2.32) and (2.33) implies

$$b = \frac{4 - [3 - 6f(r)]^2 + [6f(r) - 1]^2 a}{4[6f(r) - 1]}. \tag{2.34}$$

Now distinguish between the subcases $r \leq \bar{r}^A$ and $\bar{r}^A < r$.

Subcase $r \leq \bar{r}^A$: Equal odds in node A imply $c = 2b - a$ by equation (2.4). Together with equation (2.33) this implies

$$b = \frac{1 - 2f(r) + (2f(r) + 1)a}{2}.$$

Together with equation (2.34), the last equality implies $a = 1$ for all $r \in (1, 2]$, a contradiction since $a = R_{t+1} < R_t = 1$.

Subcase $\bar{r}^A < r$: If $2b - c \leq a$ then $\frac{a+b+c}{3} - c \leq a - \frac{a+b+c}{3}$ and equal odds in node A imply

$$p_2^A = p_3^A = \frac{1}{2} \Leftrightarrow 2f(r)(w_2^A - \ell_2^A) = w_3^A - \ell_3^A \Leftrightarrow c = \frac{2[f(r) - 1]a + [2f(r) + 1]b}{4f(r) - 1}. \tag{2.35}$$

The equality of (2.33) and (2.35) implies

$$b = \frac{6f(r) - 8f(r)^2 - 1 + [1 - 2f(r) + 8f(r)^2]a}{4f(r)}, \tag{2.36}$$

and the equality of (2.34) and (2.36) yields $a = 1$ for all $r \in (1, 2]$, a contradiction since $a = R_{t+1} < 1$.

Instead, if $2b - c > a$ then $\frac{a+b+c}{3} - c > a - \frac{a+b+c}{3}$ and equal odds in node A imply

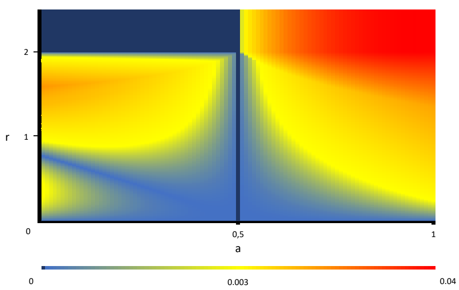
$$p_2^A = p_3^A = \frac{1}{2} \Leftrightarrow 2f(r)(w_3^A - \ell_3^A) = w_2^A - \ell_2^A \Leftrightarrow c = \frac{[4f(r) - 1]a - [2f(r) + 1]b}{2[f(r) - 1]}. \tag{2.37}$$

The equality of (2.33) and (2.37) implies

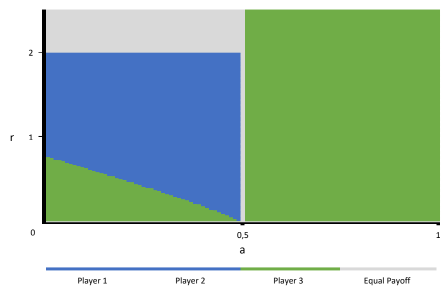
$$b = 1 - f(r)[3 - 2f(r)] + f(r)[3 - 2f(r)]a, \tag{2.38}$$

and the equality of (2.34) and (2.38) yields $a = 1$ for all $r \in (1, 2]$, a contradiction since $a = R_{t+1} < 1$.

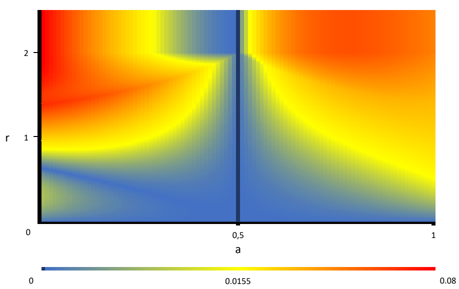
2.F Additional Figures



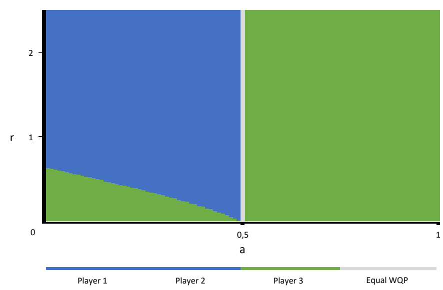
(a) Standard deviations of ex-ante expected normalized payoffs



(b) Player with highest ex-ante expected payoff

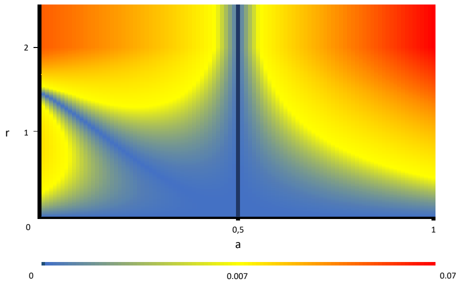


(c) Standard deviation of ex-ante WQP

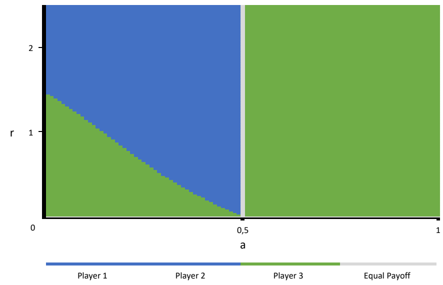


(d) Player with highest ex-ante WQP

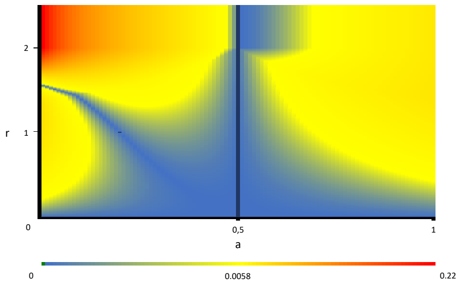
Figure 2.6: Extent and direction of discrimination in $T_3^{WF}(r)$



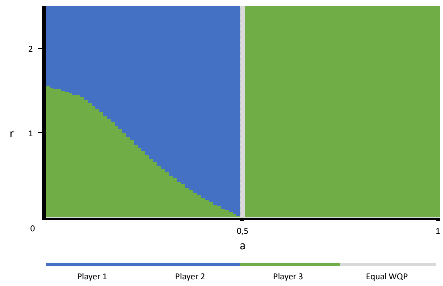
(a) Standard deviations of ex-ante expected normalized payoffs



(b) Player with highest ex-ante expected payoff

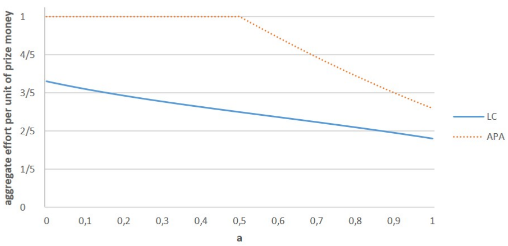


(c) Standard deviation of ex-ante WQP

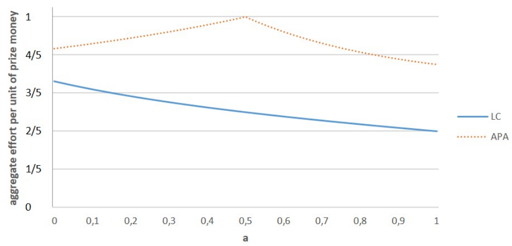


(d) Player with highest ex-ante WQP

Figure 2.7: Extent and direction of discrimination in $T_3^{LF}(r)$



(a) WF-structure



(b) LF-structure

Figure 2.8: Ex-ante expected aggregate effort per unit of prize money

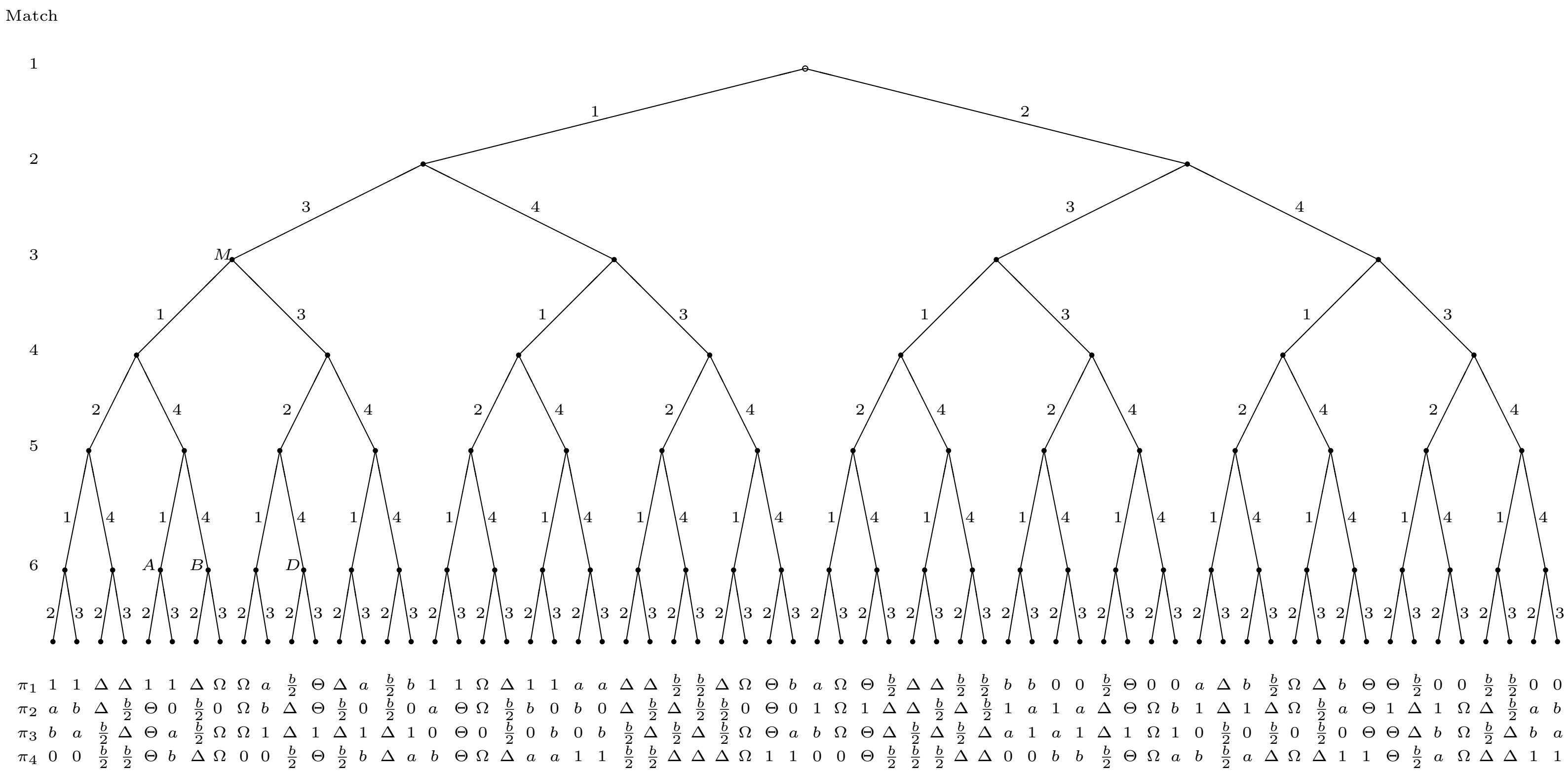
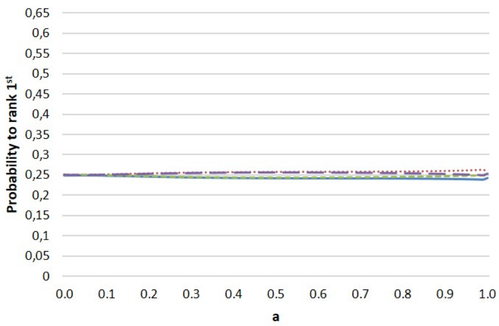
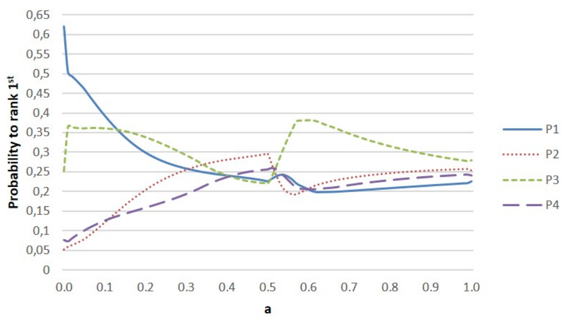


Figure 2.9: Example of a four-player round-robin tournament with an exogenous sequence of matches

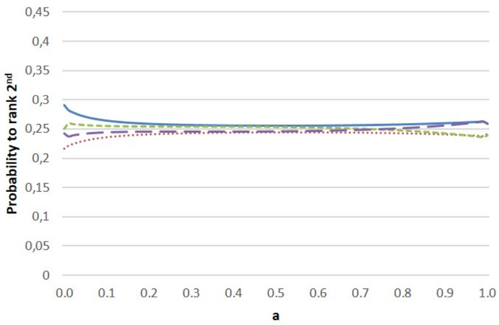
LC **APA**

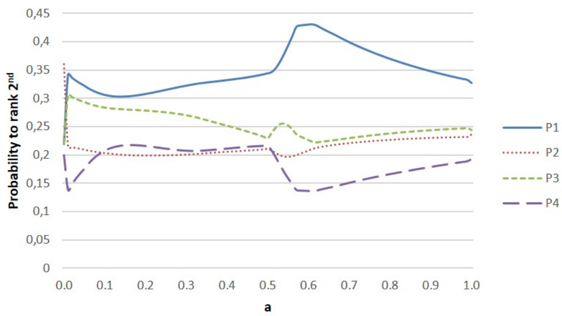
Probability to rank 1^{st}



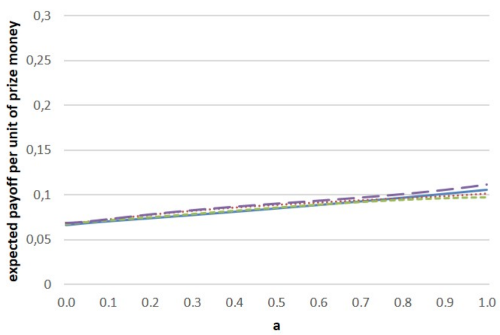


Probability to rank 2^{nd}





Expected payoffs per unit of prize money



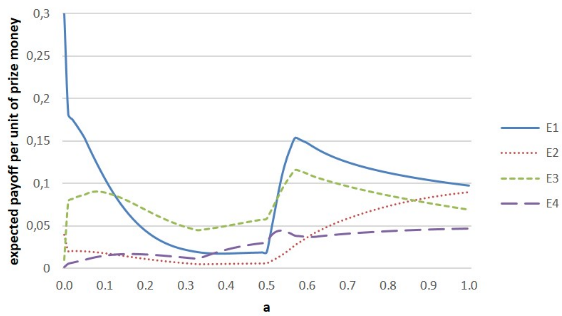


Figure 2.10: Four-player round-robin tournament with $b = 0$

Chapter 3

Round-Robin Tournaments in the Lab: Lottery Contests vs. All-Pay Auctions

(joint with Marco Sahm)

Abstract

We conduct a laboratory experiment to compare the fairness and intensity of round-robin tournaments with three symmetric players, a single prize, and two alternative match formats: either lottery contests or all-pay auctions. Irrespective of the match format, we do not find any significant discrimination with respect to overall winning probabilities. This is in line with the equilibrium predictions for lottery contests, but at odds with those for all-pay auctions. Instead of the predicted discouragement effect in tournaments with all-pay auctions, we observe a dissipation-trap: players end up in an effort-intense, final-like last match which significantly reduces the payoffs of the late-mover. We thus neither find significant differences in the intensity: irrespective of the match format, we observe similar over-dissipation of aggregate efforts.

Keywords: Sequential Round-Robin Tournament; Lottery Contest; All-Pay Auction; Laboratory Experiment; Discouragement Effect; Dissipation Trap

JEL classification: C72, C91, D72, Z20

3.1 Introduction

Randomness is an inherent element of most if not all contests. The role of luck, though, varies from case to case. The random component is, explicitly, more influential in a poker tournament than a chess tournament. But, implicitly, luck may also play a more important role in outdoor ski racing (with possibly changing weather conditions) than indoor ice skating. In the theory of contests, randomness is formally captured by the contest success function (CSF). It relates the contestant's efforts to their winning probabilities. As the above examples illustrate, this relationship is often determined by the natural or legal environment. For instance, as sports differ in their institutional character and, therewith, the scope of randomness, the CSF is predefined by the rules of the game.[1] To some extent, however, a contest designer may be able to manipulate these rules and tailor the CSF in order to incorporate more or less randomness.[2]

Two of the most prominent CSFs in the contest literature are particular versions of the famous Tullock contest (Tullock, 1980), namely the lottery contest and the all-pay auction.[3] While the all-pay auction is perfectly discriminating and always awards the contest prize to the contestant with the highest effort, the lottery contest awards the prize randomly such that a contestant's probability of winning is given by the ratio between her own effort and the aggregate effort of all contestants.

Early experimental studies on contests by Millner and Pratt (1989), Davis and Reilly (1998), and Potters et al. (1998) focus on the question how a lottery contest compared to an (equilibrium-equivalent to the) all-pay auction empirically affects effort in static or simple two-stage contests. In line with equilibrium predictions, these studies reveal that average effort is lower for the lottery contest than for the all-pay auction. Many contests, however, feature a more complex dynamic structure where the grand contest is composed of a sequence of many component contests. Prominent examples are races or elimination tournaments; see Konrad (2009, Chapter 8) for a survey of theoretical contributions and Dechenaux et al. (2015, p.629 ff) for a survey of experimental contributions.

Round-robin tournaments are another form of dynamic contests. Despite their frequent use in practice, they have received much less attention in the literature so far. A round-robin tournament arranges competition in an all-play-all-style such that each participant meets every other participant in turn. Contest designers, particularly in sports, heavily rely on round-robin tournaments of all kinds and sizes, e.g., for organizing team events on a large scale like the major European soccer leagues (including the English Premier League) with up to 20 teams, or on a small scale, like the first rounds (group or pool stages) of Basketball Olympics (since 1948), Rugby World Cups (since 1987) and FIFA Soccer World Cups (since 1950) with down to only four teams. For their World Cups from 2026 on, the FIFA recently announced a transition to an initial first round with only three teams per group. Three-player round-robin tournaments are also applied in individual sports such as the Olympic preliminaries of Badminton (since 2012) or Wrestling (2000, 2004). The three-player round-robin tournament is particularly remarkable because its structure is inevitably sequential.

[1] Szymanski (2003) provides a survey that discusses the role of the contest success function in sports.

[2] Examples include table tennis (increase in the ball seize), football (implementation of the three-point rule and the video assistant referee (VAR)), and many others.

[3] For comparisons, see e.g Ellingsen (1991), Che and Gale (1997), Fang (2002), Alcalde and Dahm (2010), Epstein et al. (2011, 2013), Franke et al. (2014), or Konrad (2009, Chapter 2) for a survey.

The goals of a contest designer are diverse. Especially in sports, a contest designer wants to maximize productive effort on an aggregate level, whereas in other fields she might want to maximize intensity on an individual level (like in R&D races or promotional competition), or to minimize wasteful effort (like in rent-seeking). Fairness and competitive balance are other important objectives: while in some contests all participants should have equal opportunities, in other contests between unequal contestants it is sometimes required to level the playing field to increase the interest of a third party (like that of spectators and sponsors of sports). The choice of the CSF may thus be an important feature of contest design.

Indeed, recent theoretical contributions on the strategic behavior in sequential round-robin tournaments show that, in general, the intensity and fairness critically depend on the discriminatory power of the CSF on the match level (Laica et al., 2021). In particular, Krumer et al. (2017a) and Sahm (2019) consider sequential round-robin tournaments with three symmetric players, which are ranked according to the number of matches won, and a single prize for the player ranked first. Krumer et al. (2017a) assume that each single match is organized as an all-pay auction (APA) and show that such APA-tournaments are not fair: depending on their position in the sequence of matches, the players have differing ex-ante winning probabilities and expected payoffs (with a considerable advantage for the player who competes in the first and the last match).[4] The reason are pronounced asymmetries in intermediate continuation payoffs causing a strong *discouragement effect* for player 3 who competes in the last two matches.[5] Sahm (2019) assumes that each match is organized as a general Tullock contest including the all-pay auction and the lottery contest (LC) as particular cases. He shows that the LC-tournament is close to be ex-ante fair (with a slight disadvantage for the player who competes in the first and the last match) because the discouragement effect is much less pronounced than in the APA-tournament. This also explains why the aggregate expected effort in the subgame perfect equilibrium of APA-tournaments is smaller than in LC-tournaments – in contrast to static contests.

Our experimental study is the first that empirically tests the intensity and fairness of three-player round-robin tournaments under controlled conditions in the laboratory. The experiment offers the opportunity to control for the players' abilities. Furthermore, we are able to observe actual effort levels and can thus examine whether the predicted institutional influence of the organization of matches is identifiable. In particular, we test whether LC-tournaments are more intense than APA-tournaments in terms of expected aggregate effort, and whether APA-tournaments (LC-tournaments) are ex-ante unfair (fair) in terms of expected payoffs and winning probabilities. In addition, our experimental setting allows us to explicitly search for evidence of a discouragement effect. For that, we implement a between subject experiment with two treatments, one incorporating LC-tournaments (LC-treatment) and the other incorporating APA-tournaments (APA-treatment).

We find that subjects significantly overbid compared to the theoretical predictions in both treatments. In the LC-treatment, this level is very stable even when players gain experience. By contrast, in the APA-treatment, we observe learning effects that reduce overbidding with experience for the two players who compete in the first match. We cannot confirm that the tournaments differ in intensity: a subject's overall effort

[4] Based on sports data from Olympic Wrestling competitions, Krumer and Lechner (2017) provide empirical evidence for such non-fairness results.

[5] Similar effects arise in most dynamic contest, see, e.g., Konrad (2012).

choice per tournament in the LC-treatment is not significantly different from the one in the APA-treatment.

We find that LC-tournaments are close to be ex-ante fair. The experimental evidence accurately replicates the theoretical predictions. Ex-ante winning probabilities show only small, non-significant deviations from the equilibrium predictions. In line with the theory, the player who competes in the first and the last match has significantly lower payoffs than the other players.

For APA-tournaments, in contrast to theoretical predictions, we do not observe any significant discrimination in ex-ante winning probabilities either, even if we control for subjects' heterogeneous characteristics. The reason is that we find no evidence for the predicted discouragement effect. Quite to the contrary, we discover that the player who competes in the last two matches tries to exploit, in his first match, a negative momentum of the first match loser and therefore chooses a significantly higher effort than predicted. As a result, he often wins and ends up in a final-like, effort-intense last match (instead of always ending up in a low-effort match as theory predicts). We call this a *dissipation-trap*. It explains, on the one hand, why the late-moving player has significantly lower payoffs than the other players, and, on the other hand, why the APA-tournament is not less intense than the LC-tournament.

The remainder of this paper is organized as follows: after a short summary of the related literature in Section 3.2, Section 3.3 provides our hypotheses that are derived from the basic models of three-player round-robin tournaments with matches organized as lottery contests and all-pay auctions, respectively. In Section 3.4, we summarize our experimental design and procedures. The experimental results are presented in section 3.5. Section 3.6 concludes.

3.2 Related Literature

Our experiment examines the impact of different contest success functions in a dynamic framework. It thus combines and contributes to two strands of the literature: general comparisons of lottery contests and all-pay auctions on the one hand, and experimental studies of dynamic contests on the other hand. Moreover, it is, to the best of our knowledge, the first experimental study that addresses round-robin tournaments in a controlled environment. Our work thus also complements empirical field studies of round-robin tournaments.

In the theory of contests, the comparison of the lottery contest and the all-pay auction has received considerable attention. If players are symmetric, the equilibrium intensity (in terms of expected aggregate effort) is larger in all-pay auctions than in lottery contests. By contrast, if two players are asymmetric (in their valuations), theory predicts that lottery contests may be more intense than all-pay auctions because more randomness helps leveling the playing field (Nti, 1999). Similarly, Fang (2002) shows that, in the context of lobbying, when prize valuations among more than two contestants are heterogeneous enough, the lottery contest is more intense than the all-pay auction. These findings for static contests already suggest that similar results may prevail in dynamic contests if the sequential structure implies (ex-interim) asymmetric continuation values.

Experimentally, Millner and Pratt (1989) compare static symmetric two-player lottery contests and a highly discriminatory Tullock contest that is equilibrium-equivalent to the all-pay auction and find that aggregate effort is larger in the highly discriminatory contest. Likewise, Potters et al. (1998) compare symmetric two-player all-pay auctions and lottery contests in a static setting. They find that subjects significantly deviate from equilibrium behavior in both contest types. On average effort choices and rent-dissipation are significantly lower for lottery contests. Difference in the variance of effort choices is only marginally significant with less dispersion in lottery contests.

Davis and Reilly (1998) use a laboratory experiment to compare static single-prize lottery contests and all-pay auctions with four symmetric players. In their baseline treatments they find that intensity in all-pay auctions is higher than in lottery contests. Moreover, they compare treatments where they add a fifth player with a higher prize valuation to their four-player single-prize contest. In this asymmetric five-player game, aggregate effort with all-pay auctions is smaller than with lottery contests which is consistent with equilibrium predictions.

In experimental studies with static contests organized as all-pay auctions, a discouragement effect finds clear support in Davis and Reilly (1998) and Llorente-Saguer et al. (2019) and mixed support in Fehr and Schmid (2018).

A dissent for the prevalence of a strong discouragement effect is discovered by Mago and Sheremeta (2017) in dynamic contests. They study all-pay auctions in the setting of a best-of-3 race with budget constraints. In the first match average aggregate effort choices are significantly higher in each period than in the equilibrium solution. Moreover, they detect that the probability of a first-match-winner to win the whole race was significantly lower than theory predicts.

In races where matches are organized as lottery contests, Zizzo (2002) finds a discouragement effect. He experimentally tests two-player long distance races with budget constraints. The results report that a leader does not choose significantly higher effort than the trailing player unless the advantage of the leader is considerable. This suggests the existence of a critical point where a follower's discouragement prevents him from competing. However, this effect is smaller than theory predicts.

Similarly Mago et al. (2013) analyze a lottery contest within a best-of-3 race and observe significant overdissipation which decreases with experience but never fully vanishes. They find evidence for a "strategic moment" rather than a "psychological moment" in which the winner of the first match submits significantly more in the following match of the tournament. Mago and Sheremeta (2019) find no empirical evidence for a discouragement effect of a match-one-loser in match two in an experiment with best-of-3 races where matches are organized as lottery contests.

Based on sports data, empirical evidence for a discouragement effect in best-of races is, for example, detected by Malueg and Yates (2010) and Iqbal and Krumer (2019) in professional Tennis and by Sonnabend (2020) in professional Beach Volleyball. The authors find that a player who lags by one victory reduces effort and has significantly lower chances to win the whole race.

Krumer and Lechner (2017) provide empirical evidence for discrimination in three-player round-robin tournaments exploring data of the Olympic Wrestling competitions. The wrestler who competes in the first and in the last match has a significantly higher probability to win the tournament, but to a much smaller extent than theory predicts when matches are organized as all-pay auction. Moreover, they do not find that the discouragement effect induced by the player who competes in the last two matches is responsible for that.

For round-robin tournaments there exists empirical evidence that a player's current effort choice is influenced by forward looking behavior. Investigating data from the National Basketball Association NBA (Taylor and Trogdon, 2002) and the National Hockey League NHL (Fornwagner, 2019), the authors find that teams which are already eliminated from playoff consideration strategically reduce effort to increase the probability of higher future payoffs due to an early pick in the annual entry draft. Deutscher et al. (2019) use data from the German Soccer Bundesliga and observe that teams save resources for the upcoming match if the asymmetry between the two teams in a given match is sufficiently large.

3.3 Theoretical Model and Predictions

In this section, we sketch the theory on round-robin tournaments with a single prize and three symmetric, risk-neutral players. For a general theoretical analysis of round-robin tournaments see Laica et al. (2021). We summarize the main results in form of our hypotheses.

Successively, each player is matched one-to-one with each other player in a sequence of three pairwise matches. The final ranking is determined according to the number of victories:[6] if there is a player with two victories, this player wins the prize; if there is a tie because each player has won one match, the prize is assigned randomly with equal probabilities of 1/3 for each player. For risk-neutral players, the tie-breaking rule is equivalent to sharing the prize equally. The value of winning the prize is identical for all players and normalized to 1.

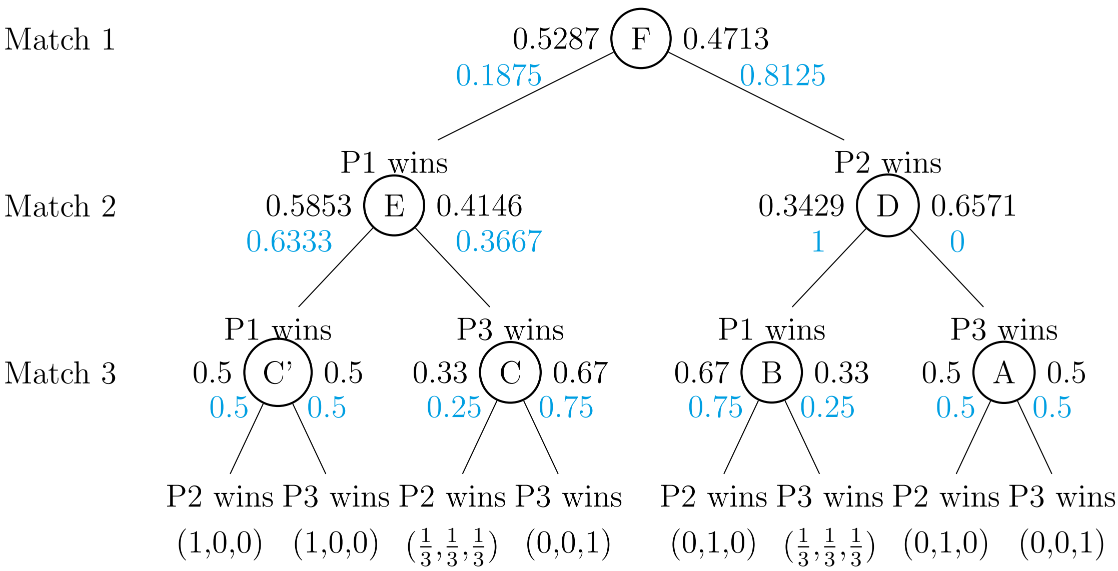
Without loss of generality, we consider an exogenous sequence in which player 1 meets player 2 in the first match, player 1 meets player 3 in the second match, and player 2 meets player 3 in the third match. Apart form renaming players, this exogenous sequence is unique. In the following analysis, we distinguish between two types of tournaments that differ in the organization of their matches. We recollect the theoretical predictions by Sahm (2019) for LC-tournaments, where each match is organized as a lottery contest, and by Krumer et al. (2017a) for APA-tournaments, where each match is organized as an all-pay auction between two players, A and B, with linear costs of effort. The structure of the resulting sequential game with its $2^3 = 8$ potential courses is depicted in Figure 3.1 (henceforth game tree). The seven nodes $k \in \{\mathrm{A}, \ldots, \mathrm{F}\}$ of the game tree represent all combinations for which the ranking of the tournament has not yet been determined when the respective match starts.

In match k, player $i \in \{A, B\}$ chooses effort x_i^k in order to maximize his expected payoff

$$E_i^k = p_i^k (w_i^k - x_i^k) + \left(1 - p_i^k\right) (\ell_i^k - x_i^k), \tag{3.1}$$

where w_i^k denotes player i's expected continuation payoff from winning the match and ℓ_i^k denotes his expected continuation payoff from losing it, with $w_i^k \geq \ell_i^k \geq 0$ for $i \in \{A, B\}$. The probability p_i^k that player $i \in \{A, B\}$ wins match k is a function of the effort choices of both players. This so-called contest success function depends on how the matches are organized: in LC-tournaments, player A's probability of winning

[6] As customary in the theoretical literature on contests, we abstract from draws. Many sports waive draws, and even for the FIFA World Cups from 2026 on, one possible scenario is to abolish draws completely.



Note: black $\triangleq$ LC; light blue $\triangleq$ APA.

Figure 3.1: Game Tree: Winning Probabilities

match k is defined by (Tullock, 1980; Skaperdas, 1996)

$$p_A^k = \begin{cases} 1/2 & \text{if } x_A^k = x_B^k = 0, \\ \frac{x_A^k}{x_A^k + x_B^k} & \text{else,} \end{cases}$$

and in APA-tournaments, it is defined by (Baye et al., 1996)

$$p_A^k = \begin{cases} 1 & \text{if} \quad x_A^k > x_B^k, \\ 1/2 & \text{if} \quad x_A^k = x_B^k, \\ 0 & \text{if} \quad x_A^k < x_B^k. \end{cases}$$

For $w_A^k = \ell_A^k$, the optimal choice is $x_A^k = 0$ for any $x_B^k \geq 0$. If $x_A^k = 0$ and $w_B^k > \ell_B^k$, player B will have no best reply unless there is a smallest monetary unit $\epsilon > 0$. As $\epsilon \to 0$, in the limit, $x_B^k \to 0$ and $p_B^k \to 1$. Otherwise, in match k for $i, j \in \{A, B\}$ with $i \neq j$ and $w_i^k - \ell_i^k = \min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\}$, a unique Nash-equilibrium always exists.

In LC-tournaments, the Nash equilibrium is in pure strategies and has the following properties (Sahm, 2019): the equilibrium efforts are

$$x_i^k = \frac{(w_i^k - \ell_i^k)^2 (w_j^k - \ell_j^k)}{(w_i^k - \ell_i^k + w_j^k - \ell_j^k)^2}, \tag{3.2}$$

the equilibrium winning probabilities are

$$p_i^k = \frac{(w_i^k - \ell_i^k)}{(w_i^k - \ell_i^k) + (w_j^k - \ell_j^k)}, \tag{3.3}$$

and the expected equilibrium payoffs are

$$E_i^k = \ell_i + \frac{(w_i^k - \ell_i^k)^3}{(w_i^k - \ell_i^k + w_j^k - \ell_j^k)^2}. \tag{3.4}$$

In APA-tournaments, the Nash Equilibrium is in mixed strategies has the following properties (Krumer et al., 2017a): for $(w_i^k - \ell_i^k) \leq (w_j^k - \ell_j^k)$ the expected equilibrium efforts are

$$E(x_i^k) = \frac{(w_i^k - \ell_i^k)^2}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad E(x_j^k) = \frac{w_i^k - \ell_i^k}{2}, \tag{3.5}$$

the equilibrium winning probabilities are

$$p_i^k = \frac{w_i^k - \ell_i^k}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad p_j^k = 1 - p_i^k, \tag{3.6}$$

and the expected equilibrium payoffs are

$$E_i^k = \ell_i^k \quad \text{and} \quad E_j^k = w_j^k - (w_i^k - \ell_i^k). \tag{3.7}$$

The tournament represents a sequential game that can be solved by backward induction for its subgame perfect equilibrium (SPE), making repeatedly use of equations (3.2)-(3.4) for LC-tournaments and equations (3.5)-(3.7) for APA-tournaments. The details of this procedure have been provided by Sahm (2019) for LC-tournaments and by Krumer et al. (2017a) for APA-tournaments.

Krumer et al. (2017a) show that the APA-tournament is highly discriminatory (to the favor of player 2), whereas Sahm (2019) shows that the LC-tournament is almost fair, as illustrated in Table 3.1. The columns contain the equilibrium values of the players' ex-ante winning probabilities, expected payoffs, and expected efforts for the APA-tournament (APA) and the LC-tournament (LC), respectively. Additionally, we report the corresponding relative standard deviations (RSD), defined as the ratio of standard deviation to the mean, and aggregate values.

Table 3.1: APA-tournament vs. LC-tournament

	Win. Prob.		Exp. Pay.		Exp. Eff.	
	APA	LC	APA	LC	APA	LC
P1	0.1927	0.3518	0.0833	0.1239	0.1094	0.2262
P2	0.6828	0.3049	0.4167	0.0978	0.2661	0.2070
P3	0.1245	0.3433	0.0000	0.1181	0.1245	0.2253
RSD	0.7443	0.0612	1.0801	0.1027	0.4236	0.0387
Σ	1	1	0.5	0.3415	0.5	0.6585

In the APA-tournament, player 2, who competes in the first and last match, has, by far, the highest winning probability (more than 68 percent) and the highest expected payoff (more than 41 percent of the prize value), whereas player 3, who competes in the last two matches, has the lowest winning probability (about 12 percent) and the lowest expected payoff (0). Discrimination in winning probabilities is high with a RSD of about 0.74. By contrast, LC-tournaments will be close to ex-ante fair: the least (most) favored where player 2 (player 1) has a winning probability of about 30 percent (35 percent) and the RSD is only about 0.06.

The reason is a discouragement effect of the late-moving player 3 in node D of the APA-tournament as shown by Krumer et al. (2017a). The sequential structure of the tournament implies asymmetric continuation payoffs in intermediate matches such that players reduce their effort compared to the symmetric benchmark (Nti, 1999).

The larger the discriminatory power of the CSF, the stronger is this effect (Sahm, 2019). Hence, APA-tournaments provoke stronger discouragement effects than LC-tournaments. In APA-tournaments, player 3 has an expected continuation payoff of zero in node D and, therefore, exerts no effort at all. In contrast, in LC-tournaments player 3 has an expected continuation payoff larger than zero in node D and, therefore, exerts positive effort. Figures 3.9 and 3.8 in Appendix 3.A.1 visualize the expected efforts for each node and player in APA-tournaments and LC-tournaments, respectively.

This more pronounced discouragement effect is also the reason why Sahm (2019) finds that intensity, i.e. expected aggregate effort measured by the sum of all players' expected effort, is higher for LC-tournaments (66 percent of the first prize) than for APA-tournaments (50 percent of the first prize).

These results give rise to the following testable hypotheses.

Hypothesis 3.1. *APA-tournaments are less intense than LC-tournaments.*

Hypothesis 3.2. *LC-tournaments are almost ex-ante fair.*

Hypothesis 3.3. *APA-tournaments are ex-ante discriminatory to the favor of player 2.*

Hypothesis 3.4. *APA-tournaments are characterized by a discouragement effect of the late-moving player 3 after player 2 wins the first match.*

3.4 Experimental Design and Procedure

We test the hypotheses outlined in Section 3.3 with the help of a laboratory experiment. This enables us to investigate the influence of the sequential structure combined with the institutional character under controlled conditions. Here we describe the design and the procedure of the experiment.

3.4.1 Design

We design a between subject experiment which consists of two separate treatments, a lottery-contest treatment (LC-treatment) and an all-pay auction treatment (APA-treatment). Irrespective of the treatment, the experiment is segmented in three parts. For the first part we adopt a test for risk-preferences by Holt and Laury (2002). Part 2 is the main part of the experiment and differs between the treatments. In this part subjects interact in round-robin tournaments where matches are either organized as all-pay auctions or as lottery contests as described in Section 3.3. Finally, in the last part, we implement a cognitive reflection test (CRT) similar to Frederick (2005).

At the beginning of part 2 in each treatment each subject is randomly and anonymously assigned a player number. That means a subject can either take the role of player 1, player 2, or player 3. This role is fixed throughout part 2 to provide player-specific learning opportunities. After the assignment of roles each subject participates in 20 independent repetitions of round-robin tournaments (periods henceforth).

In each period subjects are randomly matched in three-player round-robin tournaments. Within a round-robin tournament players compete in the following sequence: in match 1 player 1 meets player 2, in match 2 player 1 meets player 3, and in match

3 player 2 meets player 1. Each time a new tournament starts subjects are again randomly and anonymously reassembled in round-robin tournaments to avoid collusion. Then once again players compete according to the above sequence.

For each round-robin tournament each player receives an initial endowment of $I = 600$ points which he can use to invest in his two matches to gain a round-robin tournament prize of $R = 600$ points.[7] Hence, in his first match a player can invest any number of integer points $Q_1 \in [0, 600]$ and in his second match any number of the remaining integer points $Q_2 \in [0, 600 - Q_1]$.

The winner of a single match is determined by an all-pay auction or a lottery contest depending on the treatment. While at the end of a match the winner is announced to all players of the particular round-robin tournament, only the players who actually participate in a match are informed about the points chosen in that particular match.[8] During the tournament subjects are briefed on their current account of points, results of matches and all points chosen in each match they participated, and the current standings. Player number, match plan and prize value are continuously displayed.

The overall winner of a round-robin tournament is the player with most wins, i.e. two wins in matches. In case all players win one match, the round-robin tournament winner is randomly determined by the computer such that every player has the same chance to win the tournament.

The final payoffs of a player are either $\pi_{\mathrm{W}} = I - Q_1 - Q_2 + R$ in case he wins the tournament or $\pi_{\mathrm{NW}} = I - Q_1 - Q_2$ otherwise. At the end of a tournament each subject learns his final payoffs and whether he is a univocal or randomly determined round-robin tournament winner.

3.4.2 Procedure

Four sessions were conducted for each treatment. The sessions took place at the experimental laboratory of the department of social sciences at the University of Bamberg ("BLER") from November 2016 to May 2017. Participants were invited via the ORSEE recruitment system (Greiner, 2015). Either 15 or 18 subjects participated in a session, in total 69 subjects per treatment. On average a session lasted 90 minutes. The experimental sessions were computerized by using zTree (Fischbacher, 2007).

Upon arrival, subjects were randomly assigned to cubicles with a single computer that did not allow for any visual communication between them. At each cubicle, beside a pen, paper-based basic instructions which contained information on the general laboratory rules of behavior, show-up fee and point to cash conversion rate were placed right from the beginning (translated into English in Appendix 3.B). Once all participants were seated, they were also verbally enlightened that during the experiment verbal communication between participants was permitted. Eventually, subjects were informed about the procedure of the experiment.

Each experimental session was partitioned into three parts. At the beginning of each of the first two parts, paper-based instructions in German language (reproduced

[7]An initial endowment per period which is equal for all subjects is supposed to not account for significant individual distortions in each subject's effort choices (see, e.g., Sheremeta, 2011; Price and Sheremeta, 2011, 2015).

[8]That way we prevent players from exploiting budget constraints of other players. Otherwise it should not influence effort choices in the equilibrium anyhow. In practice, although there are measures indicating the intensity of a match, some intensity is never observable and perceived only as a participant.

in English in Appendix 3.B) were distributed to the subjects. These were read aloud by one of the experimenters after each subject privately studied them at their own pace. Questions were permitted all the time during instruction phases. After the second part, instructions for part three were only read out aloud by one of the experimenters.

In part 1, we elicited subjects' risk preferences by their choices in 10 simple lotteries, similar to Holt and Laury (2002). On the computer screen, each subject was presented with a table of ten ordered decisions between a safe amount of 180 points and a risky lottery of 400 points or 0 points and was asked to submit his choices via the computer. Only one out of the ten decisions was paid. The payoff-relevant row as well as the payoff of the risky lottery were each randomly determined by a ten-sided dice at the end of the experiment.

After the instructions for part 2, paper-based control questions were handed out to verify the participants' full understanding of the instructions. Subjects had time to fill them out at their own pace. Afterwards the experimenters revised each subject's answers individually and explained the correct answer if necessary. Then each subject competed in matches of 20 consecutive repetitions of round-robin tournaments where the treatment defines how the winner of a match is determined as described above. Hence, each subject made a total of 40 effort choice decisions which they submitted via the computer. Only one out of 20 periods in the second part was paid. The payoff-relevant period was randomly selected by a twenty-sided dice at the end of the experiment.

Finally, in part 3, subjects were trialed in a simple three-question cognitive reflection test by Frederick (2005) to examine impulsive behavior.[9] The questions reproduced in English are in Appendix 3.C. For each of the three questions, subjects had 40 seconds to enter their answer into the computer. Afterwards in a fourth question on the computer, subjects were asked how many of these questions had already been known to them before the experiment.

At the end of the experiment, the payoff-relevant decision in part 1 and the payoff-relevant period in part 2 were listed on the screen. Points were converted into cash at the rate 1 Point = EUR 0.01. This amount was added to the cash accumulated in part 3 where subjects could earn EUR 0.50 per correct answer and to a show-up fee of EUR 4.00. On average an amount of EUR 13.18 was earned per subject. Before subjects were collecting their earnings, we had been asking them to fill out a short questionnaire at the computer consisting of some demographic questions and some questions related to the experiment, also translated into English in Appendix 3.C.

3.5 Results

3.5.1 Choice Dynamics and Rent-dissipation

In many experimental studies on contests learning and overbidding is observed. Hence, we first examine whether this is also true for round-robin tournaments. Figure 3.2 illustrates the average overdissipation per player type in all periods for the LC-Treatment (APA-treatment) on the left (right) side.[10]

[9] Sheremeta (2016) finds a negative correlation between average effort choices and correct answers in a cognitive reflection test.

[10] Average overdissipation per player type per period is defined as the mean of each player's difference between her aggregate effort choices in a tournament and the path-dependent theoretical equilibrium

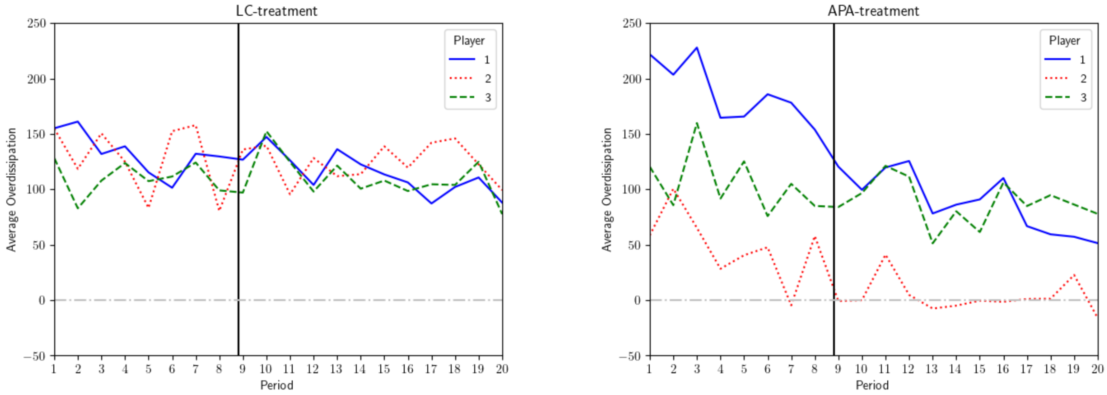


Figure 3.2: Overdissipation

In both treatments we find overbidding as the average overdissipation is positive. In the LC-treatment we observe overdissipation to be significant (Wilcoxon signed-rank test, p-value < 0.01) with player 1 revealing a slight downward trend. In contrast in the APA-treatment we indeed find overdissipation, but, it highlights clear downward trends for player 1 and 2. Across the first eight periods average overdissipation of player 1 (player 2) is 187.8 (49.3) points. In the last twelve periods it is reduced to 88.9 (3.3) for player 1 (player 2). A Kolmogorov-Smirnov test supports that distributions of average overdissipations across the first eight and the last twelve periods are not equal for each of the two players.

To obtain statistical evidence for the unveiled effects of downward trends, we estimate panel regression models with robust standard errors clustered at the session level and a subject-specific random effects error structure to account for multiple decisions made by a subject (PE).[11] Dependent variable is a subject's overdissipation in a tournament. As explanatory variables the models include dummies for player 2 and player 3, the inverse of the period and interactions. The results are presented in Table 3.2. We find clear evidence for a significant decline of overdissipation for player 1 in both treatments and for player 2 in the APA-treatment.

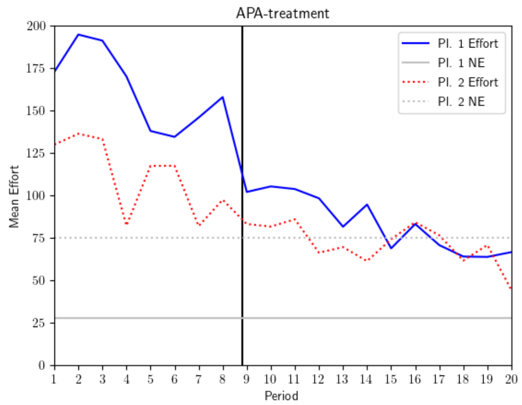


Figure 3.3: Effort in Node F

Our findings lead us to the conclusion that particularly in the APA-treatment subjects of player type 1 and 2 need time to adapt to the setting. Effort choices in the path-breaking first match of the APA-treatment consolidate this finding. Figure 3.3 shows the juxtaposition of the player types' average effort choices and expected effort

effort predictions in that tournament.

[11]From now on we use the abbreviation PE to refer to a panel regression model with robust standard errors clustered at the session level and that includes a subject-specific random effects error structure to account for the multiple decisions made by a subject.

Table 3.2: Panel Estimation for periodical changes of overdissipation

Dep. Variable	Overdissipation			
Treatment	LC		APA	
	Coef.	SE	Coef.	SE
Constant	111.532***	(25.703)	96.123**	(38.629)
Player				
2	9.154	(15.635)	-90.684**	(37.177)
3	-3.422	(38.794)	-8.123	(33.485)
Period trend [=1/Period]				
× *Player 1*	57.535***	(19.710)	179.586***	(34.494)
× *Player 2*	28.829	(38.350)	90.331***	(33.262)
× *Player 3*	9.925	(44.052)	40.649	(68.383)
Observations	1380		1380	
Subjects	69		69	
R^2	0.004		0.071	

Note: Robust errors in parentheses, clustered at the session level.
Significance level: *** (1%), ** (5%), * (10%)

predictions in the first match (Node F). Both players' distributions of effort choices in the first eight periods are significantly different from that in the last 12 periods indicating higher effort at the beginning (Kolmogorov -Smirnov test, p-values < 0.01).

The significant decrease of effort and overdissipation, respectively, at least in the APA-treatment suggests that with experience not neglectable learning effects occur. As subjects of one player type who learn across the first periods are enough to distort a tournament outcome, our main analysis focuses on late decisions, i.e. decisions made in the last twelve periods 9 to 20 (L12). However, for the LC-treatment we also consider decisions across all periods (A20). Exceptions will be mentioned.

To sum up, regarding only L12, we still find overdissipation in the LC-Treatment for all players and in the APA-treatment for player 1 and 3 (Wilcoxon signed-rank test, p-values < 0.01).

Additional Result 3.1. *Overdissipation occurs in both, APA- and LC-Tournaments. However, for player 1 in both tournaments the level decreases with experience. For player 2 in APA-tournaments the level also decreases and even vanishes with experience.*

3.5.2 Intensity

Rent-dissipation plays a crucial role in the analysis of contests: how much of the prize money is dissipated in form of aggregate effort in the course of the tournament? From the viewpoint of the designer of round-robin tournaments, aggregate effort is a valuable input that intensifies competition and attracts spectators. In this subsection we investigate aggregate effort of round-robin tournaments as a measure of intensity.

Table 3.3 compares the expected effort in the subgame perfect equilibrium and the corresponding averages of the empirical results from the experiment in A20 and L12, respectively, for both treatments. For comparability the empirical averages of effort points are stated as percentage of the tournament prize R.

Columns T reveal that in LC-tournaments the expected aggregate effort equals almost two thirds of the prize value, but only one half of the prize value in APA-tournaments. Thus, in theory LC-tournaments are more intense than APA-tournaments mainly due to the less pronounced discouragement effect which is illustrated in node D of Figures 3.8 and 3.9 in Appendix 3.A.1. In this subsection we compare average aggregate effort choices per subject per tournament for the LC- and APA-treatment.

Table 3.3: Effort per prize money: theory vs. empiricism

	LC					APA				
	T	TA20	A20	TL12	L12	T	TA20	A20	TL12	L12
P1	0.226	0.229	0.432	0.226	0.417	0.109	0.248	0.462	0.238	0.387
P2	0.207	0.203	0.413	0.206	0.414	0.266	0.298	0.334	0.317	0.322
P3	0.225	0.222	0.405	0.224	0.407	0.125	0.327	0.485	0.339	0.484
RSD	0.039	0.050	0.027	0.041	0.011	0.424	0.113	0.155	0.146	0.168
$\sum$	0.656	0.654	1.250	0.657	1.237	0.5	0.873	1.281	0.893	1.193

Note: T $\triangleq$ equilibrium predictions; TA20 $\triangleq$ theoretical predictions for all 20 periods; A20 $\triangleq$ all 20 periods; TL12 $\triangleq$ theoretical predictions for last 12 periods; L12 $\triangleq$ last 12 periods; RSD $\triangleq$ relative deviation

In the LC-treatment on average, effort choices almost double the theoretical predictions. In total effort choices are about 1.25 (1.24) of the prize money in A20 (L12). However, as theory predicts, differences in effort choices are small as the RSD of chosen effort is below (around) 0.03 (0.01) for A20 (L12). In the APA-treatment aggregate effort choices are about 1.28 (1.19) of the prize money in A20 (L12) and, thus, much higher than in equilibrium.

In contrast to the predictions in equilibrium, we cannot reject that average aggregate effort choices per subject per tournament are equal in both treatments (Mann-Whitney-U Test $p > 0.1$). In support we estimate a PE for A20 and L12 with aggregate effort choice per subject per tournament as dependent variable. As explanatory variables we include LC-treatment as a dummy variable and the inverse of the period. The results are presented in Table 3.10 in Appendix 3.A.2.

Hypothesis 3.1 is rejected. We find that the treatment has no significant impact on the aggregate effort choice per subject per tournament. This result is stable even when we control for other factors such as demographics, skills and self-evaluated preferences.

Result 3.1. *APA- and LC-tournaments are equally intense.*

To detect the determinants of intensity we include several variables. Firstly, we account for subjects' decisions in part 1 and part 3 of the experiment. For part 1 that means heterogeneity in risk aversion measured by the number of safe choices subjects selected in part 1. In this part subjects participated in in a lottery choice experiment containing 10 choices: subjcts where asked whether they preferred a safe option S or a risky option L; option S ensures a payment of EUR 1.80 while Option L results in either EUR 4.00 or EUR 0 where the chances of winning EUR 4 increase by steps of 10 percentage points from 10 percent in the first to 100 percent in the last. For Part 3 non-impulsive behavior is measured. We consider a subject to have non-impulsive behavior if it gives a correct answer to at least one of the three questions in a cognitive reflection test (CRT) similar to Frederick (2005) (see Appendix 3.C) without knowing any of these questions.

Secondly, we include data of the subjects' self-evaluated preferences on a scale from 1 to 7 which we collected by a Questionnaire. How risky they consider themselves

(Q1), how often they gamble (Q2), how often they play parlor games (Q3), how much ambition they have (Q4), how generous they see themselves (Q5), how important the payment (Q6) and winning (Q7) is to them.[12] The precise questions are stated in Appendix 3.C. Lastly, we introduce selected demographic information collected in the questionnaire, namely, gender and economics/business administration study or business experience.

We find that irrespective of the treatment, the player type has a significant impact on the effort level. In both treatments in L12 player 1 chooses significantly less effort than player 2. In contrast effort choices of player 3 compared to player 2 significantly depend on the treatment. In L12 player 3 in the APA-treatment (LC-treatment) chooses significantly more (less) effort than player 2 by about 68.8 (9.6 plus the insignificant coefficient -4.9 of LC-treatment dummy variable).

On the other hand highly significant is the "Importance of Winning" which raises a subject's effort by about 41.3 for one additional point in self-evaluation in L12. A negative significant impact is attributed to the "Importance of Payment" and "parlor games" which reduce effort by about 33.4 and 21.4 for one additional point in self-evaluation, respectively, in L12.

Additional Result 3.2. *Intensity in both, APA- and LC-tournaments, increases (decreases) in the self-evaluated "Importance of Winning" ("Importance of Payment" and the participation in parlor games).*

3.5.3 LC-tournaments

Table 3.4 compares the theoretical predictions of player types' ex-ante winning probabilities and expected payoffs in the subgame perfect equilibrium and the corresponding averages of the empirical results from the experiment for A20 and L12, respectively. For comparability the empirical averages of payoffs are stated as percentage of the tournament prize R (summary table henceforth). It presents the values for the LC-tournament.

To summarize the results for the LC-tournament: the theoretical ex-ante winning probabilities in the equilibrium are virtually correctly approved by the empirical results. The tournament is almost fair in terms of winning probabilities with a relative deviation of 0.0757 (0.1024) across A20 (L12). Indeed player 2 has the lowest winning probability and payoffs which is manifested with experience in L12.

We formally test our Hypothesis 3.2 by performing a PEs to provide statistical evidence for the reported descriptive statistics. In one model the dependent variable is "Tournament Winner" which is binary and states whether a subject is a winner of a tournament or not or. In the other model the dependent variable is "Payoffs". At first, in the basic model, we only include as explanatory variables the dummies player 1 and player 3.

By the results on the left side of Table 3.5 for A20 (L12), we actually observe that player 2, given by the constant, has the lowest winning probability with 0.298 (0.286) compared to player 1 and player 3 whose winning probabilities are 0.057 (0.062) and 0.05 (0.08) percentage points greater, respectively. However, the winning probability

[12]This is a procedure, e.g. similar to Malhotra (2010) who inquires subjects to state how important their "desire to win" on scale from 1 = "Not at all Important" to 7 = "Extremely Important" was, after they submitted their bids in a charity auction. In auctions, desire to win is defined as the urge to invest effort beyond the valuation to defeat an opponent even though it provides no strategic benefit.

Table 3.4: LC-tournament: theory vs. empiricism

	Winning Probability			Payoffs		
	T	A20	L12	T	A20	L12
P1	0.352	0.348	0.348	0.126	-0.084	-0.069
P2	0.305	0.298	0.286	0.098	-0.115	-0.128
P3	0.343	0.354	0.366	0.118	-0.051	-0.041
RSD	0.061	0.076	0.102	0.103	0.313	0.459
$\sum$	1	1	1	0.342	-0.250	-0.237

Note: T $\triangleq$ equilibrium predictions; A20 $\triangleq$ all 20 periods; L12 $\triangleq$ last 12 periods; RSD $\triangleq$ relative deviation

of player 2 is not significantly different to those of player 1 and 3.[13] The outcome is robust when we control for other variables revealed by the results presented in Table 3.11 in Appendix 3.A.

The right side of Table 3.5 reports as the dependent variable the payoffs. We find that in A20 (L12) player 2 has (significantly) lower payoffs compared to player 1 and 3.

Table 3.5: Panel Estimations for LC-Treatment

Dep. Variable	Tournament Winner				Payoffs			
Treatment	LC A20		LC L12		LC A20		LC L12	
	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE
Constant	0.298***	(0.037)	0.286***	(0.024)	531.00***	(28.64)	523.42***	(31.39)
Player								
1	0.057	(0.058)	0.062	(0.042)	18.68	(27.06)	35.25**	(17.98)
3	0.050	(0.065)	0.080	(0.071)	38.37	(16.71)	52.27**	(25.75)
Observations	1380		828		1380		828	
Subjects	69		69		69		69	
R^2	0.003		0.005		0.004		0.007	

Note: Robust errors in parentheses, clustered at the session level.
Significance level: *** (1%), ** (5%), * (10%)

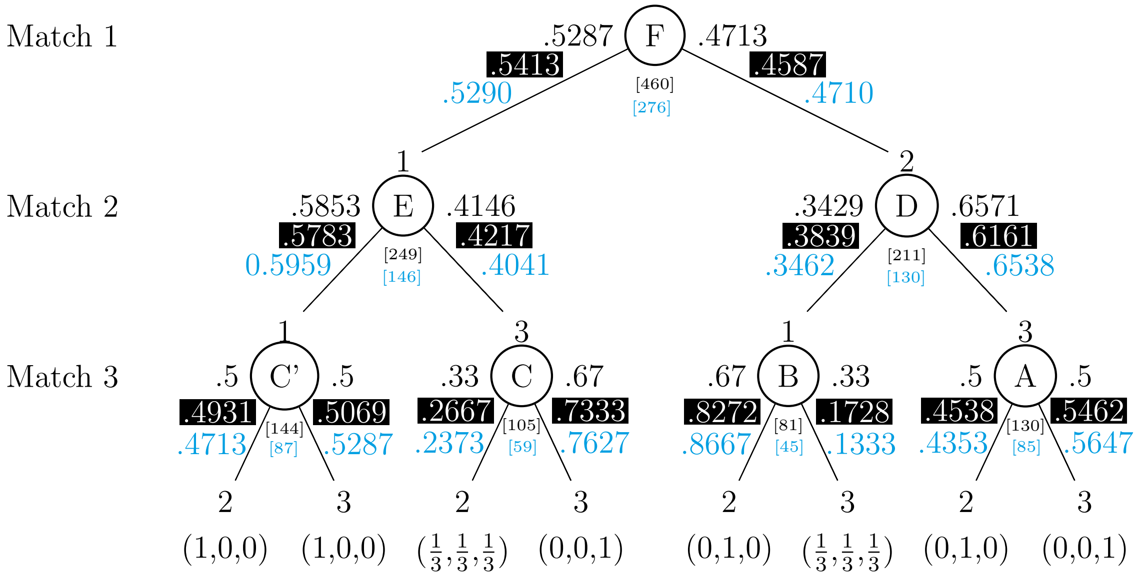
This approves the data of descriptive statistics by verifying that player 2 is disadvantaged compared to the other players at least in terms of payoffs. Together, as there is no evidence for one player having a higher or lower probability to win the tournament and as there is evidence for player 2 having significantly lower payoffs than the other players, we cannot fully reject our Hypothesis 3.2.

A look at each node of the game tree supports these findings very well. Figure 3.4 confronts the theoretical equilibrium winning probabilities in black with average empirical winning probabilities across A20 in black boxes and L12 in light blue. Values at each branch belong to the (player) number above the following node. Additionally, below each node the absolute frequencies, with which this particular node is reached in the experiment across A20 and L12, respectively, are quoted in squared brackets.

Apart from nodes B and C where winning probabilities differ by about 10 percentage points from the theoretical predictions, it reveals that irrespective of experience, the empirical winning probabilities almost perfectly replicate the theoretical predictions.

Result 3.2. *LC-tournaments are ex-ante fair in terms of winning probabilities, but significantly discriminate player 2 in terms of payoffs.*

[13] A logit PE validates this finding.



Note: black $\triangleq$ Theory; black box $\triangleq$ A20; light blue $\triangleq$ L12.

Figure 3.4: LC-tournament: Winning Probabilities

3.5.4 APA-tournaments

Table 3.6 shows a summary table for the APA-tournament. Regarding the winning probabilities it is obvious that player 2's distinct theoretical advantage is not confirmed. Throughout A20 player 2 even has the lowest winning probability on average with about 27 percent while player 1 has the highest with 41 percent. Qualitatively, this persists when only considering L12 where player 2 is the least favored together with player 3. In general, it is apparent that discrimination measured by the RSD of winning probabilities is substantially lower with about 0.17 across A20 than in theoretical equilibrium with more than 0.74. Moreover, supporting the learning effects, discrimination is reduced with a RSD of less than 0.11 in L12.

Table 3.6: APA-tournament: theory vs. empiricism

	Winning Probability			Payoffs		
	T	A20	L12	T	A20	L12
P1	0.193	0.411	0.384	0.083	-0.051	-0.002
P2	0.683	0.274	0.308	0.417	-0.060	-0.014
P3	0.125	0.315	0.308	0.000	-0.169	-0.176
RSD	0.744	0.172	0.108	1.080	0.572	1.233
$\sum$	1	1	1	0.500	-0.281	-0.193

Note: T $\triangleq$ equilibrium predictions; A20 $\triangleq$ all 20 periods; L12 $\triangleq$ last 12 periods; RSD $\triangleq$ relative deviation

We perform again PEs analogue to the analysis of the LC-Treatment to formally test our Hypothesis 3.3. By the results in Table 3.7 for A20 and L12 we indeed observe that player 2, given by the constant, has not the highest winning probability. However, the winning probability is not significantly different to those of player 1 and 3.[14] Addi-

[14] A logit PE validates this finding.

tionally, we find that player 3 has significantly lower payoffs than player 2. In L12 this difference is about 97.2.

Table 3.7: Panel Estimations for APA-treatment

Dependent Variable	Tournament Winner				Payoffs			
Treatment	APA A20		APA L12		APA A20		APA L12	
	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE
Constant	0.274***	(0.047)	0.308***	(0.071)	563.86***	(23.12)	591.45***	(32.42)
Player								
1	0.137	(0.098)	0.076	(0.116)	5.28	(24.43)	7.08	(28.14)
3	0.041	(0.073)	0.000	(0.103)	-65.45***	(14.03)	-97.21**	(43.43)
Observations	1380		828		1380		828	
Subjects	69		69		69		69	
R^2	0.015		0.006		0.016		0.036	

Note: Robust errors in parentheses, clustered at the session level.
Significance level: *** (1%), ** (5%), * (10%)

As players differ in their extent of overdissipation, we also control for heterogeneous preferences and skills. On the right side of Table 3.11 in Appendix 3.A are the results when we control for these and demographics. We find that actually player 2 has the highest winning probability. However, it is not significantly different to those of player 1 and 3.

Result 3.3. *APA-tournaments are not significantly ex-ante discriminatory in terms of winning probabilities, but significantly discriminate player 3 in terms of payoffs.*

Remarkable in Table 3.6 and Table 3.3 are the RSDs of payoffs and efforts since they indicate that mainly player 3 is responsible for the theory conflicting empirical results. In A20 they are less compared to only L12, and in L12 they are larger than theory predicts. Particularly, if we regard player 3 we find no increase in payoffs and no significant decrease in effort with experience by the comparison of A20 and L12. This supports the findings of Section 3.5.1 that player 3 reveals no learning effects.

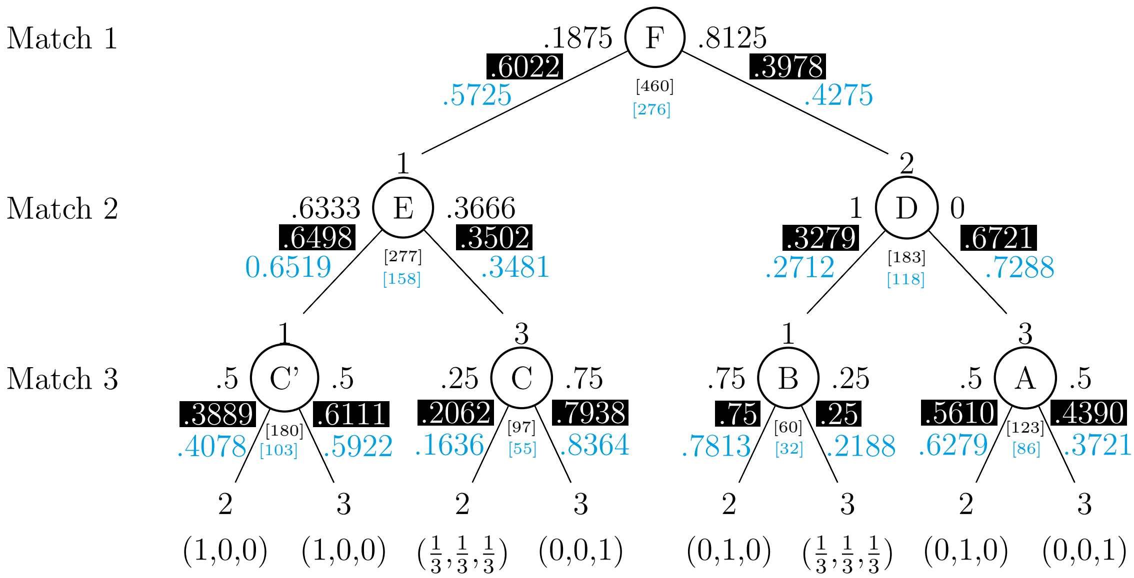
To understand why even with experience player 3 behaves different and has significantly lower payoffs than players 1 and 2, we consider the game tree in Figure 3.5 that reports theoretical and empirical winning probabilities at each node for the APA-Tournamant. In contrast to the LC-tournament, empirical results deviate drastically from the theoretical predictions. If we solve this game backwards, it is apparent that in the last stage at nodes B and C empirical observations and theoretical predictions are almost identical. In nodes A and C' observations and predictions are at least close. In the second stage in node E theory and empiricism also coincide almost perfectly.

However, in node D theoretical predictions and empirical observations differ fundamentally. Instead of a predicted sure loss of player 3, he wins more than two thirds of the matches. Moreover, considering only L12, this trend is even slightly amplified with an average winning probability of almost 73 percent.

We statistically test whether these differences are significant by a PE where "Winner in node D" is the binary dependent variable and the independent variable is a dummy with either player 1 or 3. Table 3.8 illustrates the results.[15] We find clear evidence for significant higher chances for player 3 to win the match in node D.[16] While across

[15] Only 45 subjects are observed as one player 1 always wins his first match in node F, thus never reaches node D.

[16] A logit PE validates this finding.



Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 3.5: APA-tournament: Winning Probabilities

A20 the winning probability is 66.3 percent for player 3, in L12 the difference is more pronounced with 72.5 percent. This result fundamentally contradicts the theoretical predictions in the equilibrium where player 1 is expected to win every match in node D.

Table 3.8: Panel Estimation for winning probability in Node D

Dependent Variable	Winner in Node D			
Treatment	APA A20		APA L12	
	Coef.	SE	Coef.	SE
Constant	0.346***	(0.030)	0.296***	(0.019)
Player				
3	0.327***	(0.053)	0.431***	(0.033)
Observations	366		236	
Subjects (of 46)	45		45	
R^2	0.119		0.209	

Note: Robust errors in parentheses, clustered at the session level.
Significance level: *** (1%), ** (5%), * (10%)

The deviation in winning probabilities from theoretical predictions can be explained on the basis of effort choices in Match 2 of the game tree. Figure 3.6 and 3.7 illustrate the effort choices in node E and D by player 1 and 3. They plot the cumulative distribution function (CDF) of effort choices considering both, A20 and L12, against the CDF of dedicated theoretical Nash equilibrium (NE) predictions.[17]

In node E, both players are very close to the mixed equilibrium solution irrespective of the underlying range of periods. Both players seem to randomize their effort choices according to the theoretical equilibrium interval $[0, 400]$. Whereas player 3's effort choices exhibit a tendency to be bimodal with many choices around 0 and 300 points,

[17]See Appendix 3.A.1 Figure 3.9 for the tree compares theoretical and empirical effort for each node.

player 1 almost perfectly replicates theory.

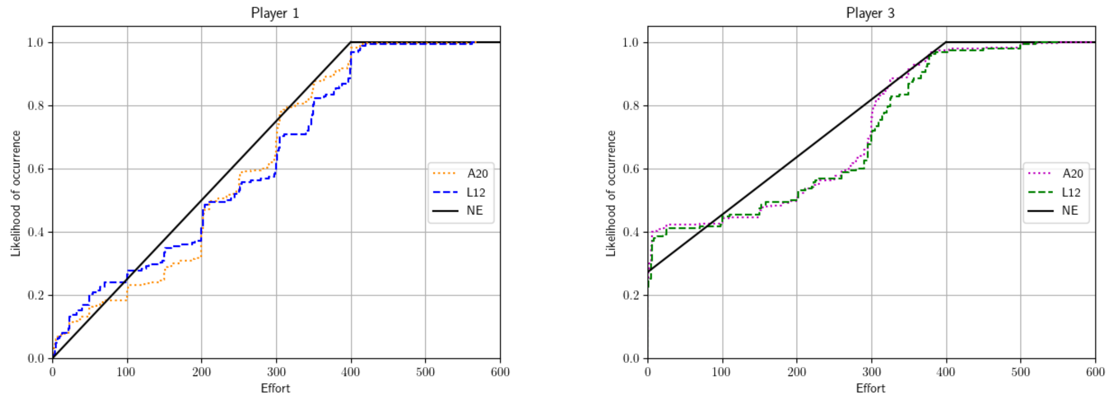


Figure 3.6: CDFs of effort choices in Node E

In node D it becomes apparent that this is the crucial point for the theory-conflicting results in terms of discrimination. Player 3 fundamentally departs from the equilibrium effort choice with actually only having less than 40 percent of his choices around the equilibrium choice of 0 points.

Player 1 compared to his optimal choice of 1 point also departs, but this can be justified by some kind of 'bounded rational' behavior. Player 1 responds almost optimal to player 3's observed deviation from the optimal behavior. More precisely, player 1's expected continuation payoffs in node D are 50 points as $w_1^E = p_3^B \cdot 1/3 = 1/4 \cdot 1/3 = 1/12$. That means if player 1 expects player 3 to choose effort greater than zero in node D, randomizing on the interval $[0, 50]$ seems reasonable. Indeed about 90 percent of player 1's effort choices with experience in L12 are in that range.

This finding clearly indicates that no *discouragement effect* for player 3 occurs in node D other than theory predicts. Instead player 3 seems to identify a negative psychological momentum by player 1 that he tries to exploit.[18] Thus, he chooses effort larger than zero.

An intuition follows from a comparison of effort choices of player 3 in nodes D and E. In a certain way player 3 behaves not completely nonstrategic as he actually seems to consider the information of the winner of the first match as the CDF in node E reveals a higher percentage of high effort choices compared to D.

This is confirmed by the results of a PE with effort choices in the second stage as dependent and an independent dummy variable that states either node E or node D. Table 3.9 shows that in node D player 3's effort choice is about 121.4 whereas in node E it is more than 170. In L12 this margin is even widened with about 112.2 compared to 179.9. That suggests that player 3 at least incorporates in his choice that player 1 is expected to choose substantially less effort in node D than as a winner of the first match in node E.

Player 3 thereby seems to neglect to look forward to what he will expect in case of a win. Hence, player 3 frequently does not anticipate that with a costly win in node D he ends up in a *dissipation-trap*: an effort-intense, final-like last match in node A against

[18] As Cohen-Zada et al. (2017) simplify it, a "psychological momentum is the tendency of an outcome to be followed by a similar outcome not caused by any strategic incentive of the player". In our case player 3 might consider player 1 as a discouraged loser who experiences negative psychological momentum after the loss in Match 1.

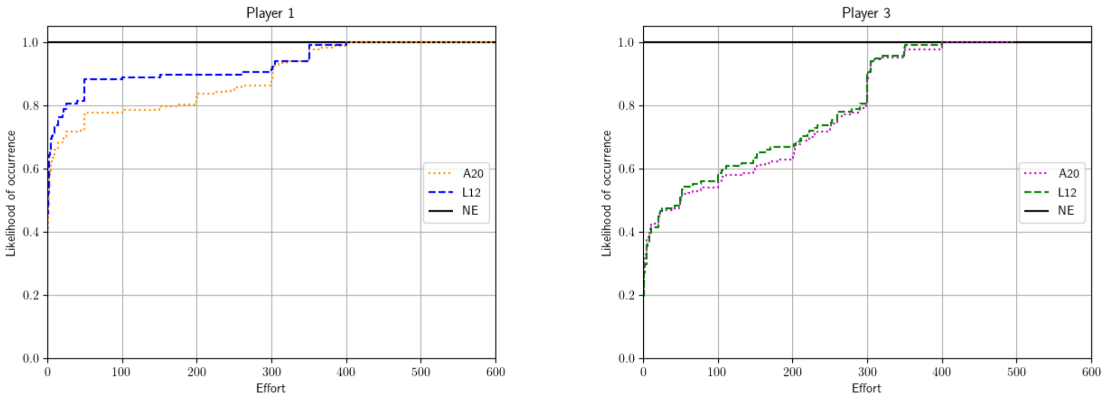


Figure 3.7: CDFs of effort choices in Node D

player 2 who is the winner of Match 1. Actually, an experienced player 3 in L12 has significantly lower payoffs with about 67.9 less in case of a win compared to a loss in node D (PE with p-value < 0.1).

Table 3.9: Panel Estimation for Effort Choices of Player 3 in Match 2

Dependent Variable	Effort in Match 2			
Treatment	APA A20		APA L12	
	Coef.	SE	Coef.	SE
Constant	121.381***	(20.647)	112.193***	(18.275)
Node E	48.848**	(19.667)	67.669***	(21.940)
Observations	460		276	
Subjects	23		23	
R^2	0.020		0.034	

Note: Robust errors in parentheses, clustered at the session level.
Significance level: *** (1%), ** (5%), * (10%)

To sum up, we find no discrimination in ex-ante winning probabilities in the APA-treatment which stems from a strategic backward looking player 3 in node D who erroneously exploits a negative momentum of player 1. Thus, he shows no effects of discouragement and ignores that a less costly win against player 1 will direct him to a dissipation trap in node A. Therefore, his payoffs are significantly lower than those of the other players.

Result 3.4. *APA-tournaments are not characterized by a discouragement effect of the late-moving player 3 after player 2 wins the first match. Instead player 3 ends up in a dissipation-trap.*

3.5.5 Determinants of Winning Probabilities

To find possible determinants for a round-robin tournament winner we account for heterogeneous preferences and skills of subjects. Therefore we extent our analyses from the previous subsections 3.5.3 and 3.5.4 by including the additional control variables introduced in Section 3.5.2.

Table 3.11 in Appendix 3.A shows the results for the LC-treatment (APA-treatment) on the left (right) side. Regarding the LC-treatment it becomes obvious that

"Importance of Winning" has a significant positive effect on the winning probability whereas "Games" has a negative effect on the winning probability. In case we consider only L12 also more risk-aversion measured by the number "Safe Choices" has a significant positive effect.[19] Non-impulsiveness measured by the dummy "CRT" has no significant effects.

Additional Result 3.3. *The winning probability in LC-tournaments decreases (increases) in the self-evaluated participation in parlor games ("Importance of Winning"). With experience the winning probability is increased by more risk-aversion.*

In the APA-treatment we also find that the "Importance of Winning" is very pronounced and highly significant for a tournament winner.[20] [21] Additionally, "Importance of Payment" has a significant negative effect on the winning probability. Non-impulsiveness significantly decreases the winning probability by around 10 percentage points. Another interesting finding is that, in contrast to the LC-treatment, risk-aversion reduces the probability of winning significantly by more than 3 percentage points. Remarkable is also that generosity has a negative and economics/business administration background has positive impact on the probability to win.

Additional Result 3.4. *The winning probability in APA-tournaments decreases (increases) in the self-evaluated generosity and "Importance of payment" ("Importance of winning"). Additionally, the winning probability is increased by (decreased) non-impulsive behavior (more risk-aversion).*

3.6 Conclusion

We have examined the intensity and fairness of sequential round-robin tournaments with three symmetric players in a controlled laboratory environment. Two treatments that differ in the discrimination power of their contest success function are considered, an all-pay auction treatment and a lottery contest treatment.

For LC-tournaments, the existence and persistence of strong overdissipation of all players is identified. In APA-tournaments overdissipation indeed also markedly exists but significantly decreases with experience for the players competing in the first match.

In contrast to experimental findings on static contests with symmetric players which feature that lottery contests are less intense than all-pay auctions, we do not observe that the intensity in LC-tournaments differs significantly from that in APA-tournaments.

[19] This is in contrast to March and Sahm (2018) who find that in two-player lottery contests an increase in risk-aversion reduces the winning probability.

[20] This is not a surprise for the fully discriminatory all-pay auction as we already figured out that the desire to win a significant effect on effort choices and winning probabilities are naturally correlated with the winning probability.

[21] To provide some evidence that subjects did not adjust their evaluation of winning importance retrospectively to balance cognitive dissonance we use a procedure similar to Cason et al. (2020). They measure the subjects' winning importance by their effort choices in an all-pay auction with a prize valued 0. In node C' we actually have such a scenario where at least player 2 and player 3 compete for a prize of value 0. We estimate a panel regression for the APA-treatment with the effort choices in node C' as dependent variable and find that subjects who stated the highest and second highest value of "Importance of Winning" choose significantly more effort than subjects who stated winning is not important at all.

The reason is that we find no evidence for a discouragement effect in APA-tournaments. This is not inconsistent with the empirical results by Krumer and Lechner (2017) who alike discover no discouragement effect in three-player round-robin tournaments. Moreover, it is in line with the experimental results in races by Mago and Sheremeta (2017, 2019).

Instead we detect a dissipation-trap for the late-moving player. In this player's first match, this player seems to exploit a negative psychological momentum of the first match loser since the chosen effort is significantly beyond the expected equilibrium level. As a result the late-mover often ends up in a final-like, effort intense last match rather than always ending up in a low-effort match as theory predicts.

Due to this dissipation-trap in all-pay auction round-robin tournaments, we find a significant disadvantage for the late-moving player in terms of payoffs. However, we cannot confirm the theoretical predictions by Krumer et al. (2017a) who show a major advantage for the player who competes in the first and last match. Our findings reveal no significant discrimination in terms of ex-ante winning probabilities.

For LC-tournaments our results fully confirm the theoretical predictions by Sahm (2019). That means tournaments are very close to be ex-ante fair with slightly lower payoffs for the player who competes in the first and the last match. In fact, at each match in every node, the empirical observation accurately replicate the theoretical predictions in the subgame perfect equilibrium.

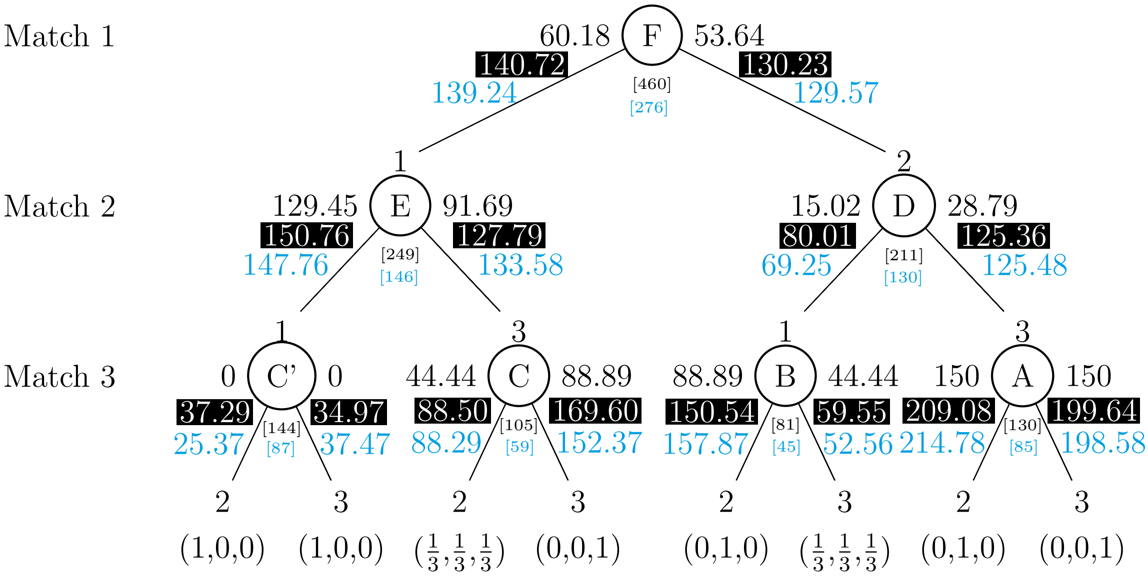
Our experimental results indicate that independent of the actual institutional character, i.e. whether for example a sports contest inherently contains some source of randomness or not, the model with matches organized as lottery contests yields better predictions for the empirical outcome of a round-robin tournament.

This paper acts as a benchmark for further experimental investigations on round-robin tournaments and on the influence of the contest success function's discriminatory power in dynamic contests in general. Obvious is the extension of the experiment to a round-robin tournament setting with multiple prizes.

Appendix

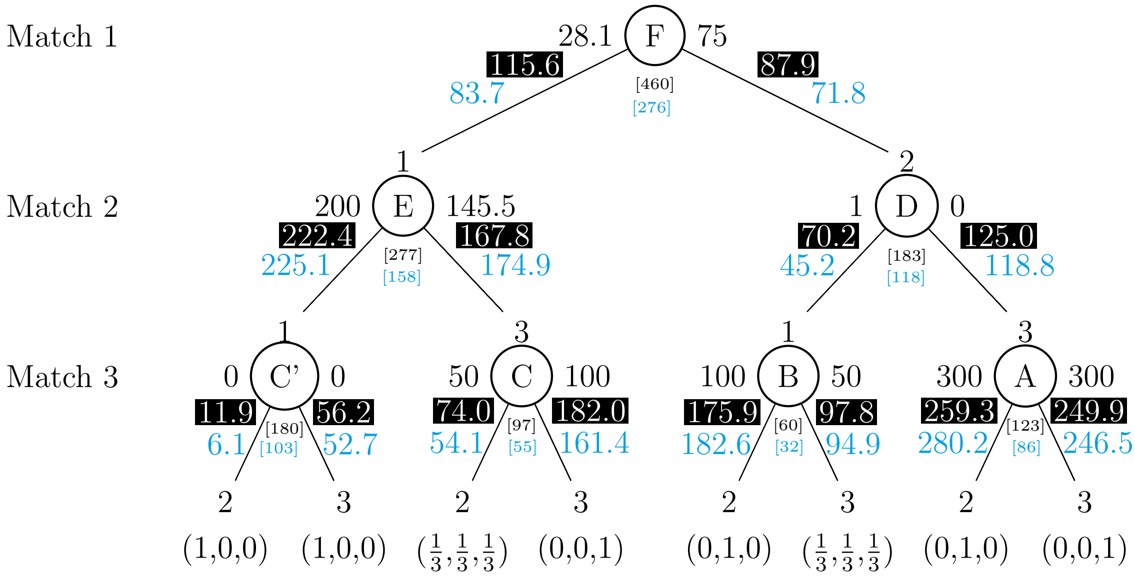
3.A Experimental Results: Figures and Tables

3.A.1 Effort Trees



Note: black ≙ Theory; black box ≙ A20; light blue ≙ L12.

Figure 3.8: LC-tournament: Effort



Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 3.9: APA-tournament: Effort

3.A.2 Determinants of Intensity and Fairness

Table 3.10: Panel Estimation for determinants of total effort per subject

Dependent Variable	Total Effort per Subject per Tournament							
Specification	(1)				(2)			
Periods	A20		L12		A20		L12	
	OR	SE	OR	SE	OR	SE	OR	SE
Constant	246.205***	(27.419)	206.687***	(29.246)	377.714***	(57.058)	311.053***	(51.836)
Period Trend	55.566**	(23.692)	435.158**	(214.774)	55.476**	(23.793)	432.231**	(217.199)
LC-treatment	-6.213	(33.993)	8.812	(36.197)	-16.851	(31.495)	-4.916	(31.698)
Player								
1					-7.695	(23.350)	-40.096**	(19.615)
3					63.191**	(24.878)	68.812**	(31.339)
LC-treatment								
× Player 1					-5.850	(43.047)	19.305	(40.689)
× Player 3					-71.225***	(23.804)	-78.439***	(29.494)
CRT					11.421	(17.572)	9.146	(18.720)
# SafeChoices					-12.671	(10.601)	-5.536	(10.073)
Female					13.356	(20.959)	-3.374	(21.940)
Economic/BA Student					-6.343	(17.207)	-6.689	(17.398)
Participation Lotteries					11.203	(15.845)	13.207	(15.053)
Participation Games					-20.081***	(6.193)	-21.414***	(6.618)
Ambition					3.336	(6.808)	6.546	(7.621)
Generosity					-2.372	(8.222)	-4.582	(7.468)
Importance Payment					-34.103***	(6.568)	-33.375***	(6.951)
Importance Winning					41.886***	(4.761)	41.316***	(6.179)
Observations	2760		1656		2760		1656	
Subjects	138		138		138		138	
R^2	0.004		0.002		0.219		0.218	

Note: Robust errors clustered at the session level in parentheses. Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 3.11: Panel Estimation for determinants of Tournament Winning Probability

Dependent Variable	Tournament Winner							
Treatment	LC A20		LC L12		APA A20		APA L12	
	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE
Constant	0.190**	(0.094)	0.132	(0.104)	0.573***	(0.124)	0.698***	(0.231)
Player 1	0.010	(0.036)	0.033	(0.026)	-0.038	(0.064)	-0.087	(0.093)
Player 3	0.042	(0.038)	0.068	(0.047)	-0.003	(0.048)	-0.041	(0.103)
CRT	0.001	(0.033)	0.020	(0.050)	-0.090**	(0.038)	-0.110**	(0.043)
# SafeChoices	0.014	(0.016)	0.033*	(0.018)	-0.034***	(0.011)	-0.033***	(0.012)
Female	0.034	(0.028)	0.002	(0.041)	0.002	(0.015)	-0.020	(0.043)
Economic/BA Student	-0.028	(0.026)	-0.034	(0.034)	0.075***	(0.023)	0.081***	(0.031)
Participation Lotteries	0.024	(0.024)	0.025	(0.032)	-0.016	(0.025)	-0.016	(0.020)
Participation Games	-0.035***	(0.006)	-0.019***	(0.003)	-0.007	(0.011)	-0.005	(0.016)
Ambition	0.015	(0.016)	0.003	(0.018)	0.003	(0.018)	-0.001	(0.024)
Generosity	0.003	(0.018)	0.004	(0.027)	-0.030**	(0.014)	-0.033***	(0.008)
Importance Payment	-0.000	(0.018)	-0.011	(0.018)	-0.027***	(0.003)	-0.035***	(0.012)
Importance Winning	0.024**	(0.011)	0.020*	(0.011)	0.059***	(0.009)	0.051***	(0.009)
Observations	1360		828		1360		828	
Subjects	69		69		69		69	
R^2	0.035		0.035		0.109		0.092	

Note: Robust errors clustered at the session level in parentheses. Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

3.B English Translation of the Experimental Instructions

3.B.1 General Instructions

This is an experiment in strategic decision-making. Thank you for your participation. To compensate you for showing up on time you will receive

$\boxed{\text{4 Euro}}$

If you follow these instructions, you can earn additional money depending on your own decisions, the decisions of the other participants, and chance. At the end of the experiment the total amount of money that you have earned will be paid out to you privately in cash.

From now on, we ask you to remain seated quietly at your computer desk. You may use the computer only for the experiment. Please do not communicate with other participants during the experiment. If you have any questions during the experiment, please raise your hand and wait for an experimenter to come to you. Participants who intentionally violate these rules will be asked to leave the experiment without being financially compensated.

During the experiment your decisions determine a score expressed in points. At the end of the experiment, the points you have earned in some of your decisions will determine your earnings according to the following rule:

$\boxed{\text{1 Point = 1 Cent}}$ $\boxed{\text{100 Points = 1 Euro}}$

The experiment consists of 3 parts and a questionnaire at the end. On the next pages you initially receive detailed information on the first part of the experiment. Once part 1 is finished additional information on the second part follows. After part 2 is finished, you receive instructions for the third part.

3.B.2 Instructions for Part 1

In the first part of the experiment, your earnings only depend on your own decisions and chance. You have to submit **10 decisions** in this part. These are listed in the following table:

Choice	Option S Points	Option L Points	Option L Dice Score	Your Choice	
1	**180**	**400**, if **0**, if	1 2, 3, 4, 5, 6, 7, 8, 9, 10	S	L
2	**180**	**400**, if **0**, if	1, 2 3, 4, 5, 6, 7, 8, 9, 10	S	L
3	**180**	**400**, if **0**, if	1, 2, 3 4, 5, 6, 7, 8, 9, 10	S	L
4	**180**	**400**, if **0**, if	1, 2, 3, 4 5, 6, 7, 8, 9, 10	S	L
5	**180**	**400**, if **0**, if	1, 2, 3, 4, 5 6, 7, 8, 9, 10	S	L
6	**180**	**400**, if **0**, if	1, 2, 3, 4, 5, 6 7, 8, 9, 10	S	L
7	**180**	**400**, if **0**, if	1, 2, 3, 4, 5, 6, 7 8, 9, 10	S	L
8	**180**	**400**, if **0**, if	1, 2, 3, 4, 5, 6, 7, 8 9, 10	S	L
9	**180**	**400**, if **0**, if	1, 2, 3, 4, 5, 6, 7, 8, 9 10	S	L
10	**180**	**400**, if **0**, if	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 - - -	S	L

In each decision, you have a choice between two options, Option S and Option L:

- Option S yields a secure final score of 180 points.
- The final score of option L depends on the throw of a 10-sided dice. For example, in the first decision option L yields 400 points, if the dice result is 1, and it yields 0 points if the result is 2, 3, 4, 5, 6, 7, 8, 9, or 10. For the other decisions, the final score of option L is determined analogously, with the probability of receiving 400 points increasing as you move down the table. Indeed, in the last decision option L yields a secure final score of 400 points.

Only one of the 10 decisions will count towards your final earnings. To determine your earnings for the first part of the experiment, one of the participants will throw a 10-sided dice twice at the end of the experiment. The result of the first throw determines the number of the decision which counts towards your earnings. Your earnings for the first part are then determined as follows:

- If you have chosen option S in the selected decision, you earn the money equivalent of 180 points.
- If you have chosen option L in the selected decision, your earnings depend on the result of the second throw of the dice. You earn the money equivalent of the points related to the result.

Please remain quiet until all participants have finished reading the instructions. An experimenter will then read them aloud. Afterwards, the first part of the experiment will begin.

3.B.3 Instructions for Part 2

In the second part of the experiment you participate as a player in 20 independent tournaments. For each single tournament you are randomly allocated into groups of three. On that point you are randomly assigned a player from 1 to 3, i.e. you are either player 1, player 2, or player 3. Your player number remains the same across all 20 tournaments. Within a tournament you sequentially interact with every other player (opponent)in your group. The sequence is the following:

MATCH 1:	Player 1 vs. Player 2
MATCH 2:	Player 1 vs. Player 3
MATCH 3:	Player 2 vs. Player 3

In those matches the participating players make decisions, for each decision you have 35 seconds. In the meantime the non-participating player pauses but has to confirm with a click on 'OK'. Once all matches in a group are completed and consequently the tournament is finished, a new tournament, independent from the previous tournament, starts. For that, you are again randomly assigned with your player number into new composed groups of three. The players interact all over again in the illustrated sequence.

Your decisions in each tournament

You and your opponent compete in matches for a *tournament prize* which equals

$$\boxed{R = 600 \text{ Punkte.}}$$

At the beginning of each tournament, that means after each group allocation, each of you receives, independently of the outcomes of previous tournaments, an *initial endowment* of

$$\boxed{I = 600 \text{ Punkten.}}$$

You can use this endowment to submit it in matches with your opponent. For this purpose you can submit any number of positive integer points Q_1 between 0 and your initial endowment of 600. In your second match you can submit any number of positive integer points Q_2 between 0 and your remaining endowment of $600 - Q_1$.

The winner...

LC-treatment

...of a match: Once you and your opponent made your decisions the winner will be determined as follows. If neither you nor your opponent submitted any points, a computerized fair coin toss determines the winner. Otherwise, the computer randomly draws an integer number bewteen 1 and the total number of points submitted by yourself and your opponent. Each of those numbers is equally likely to be drawn. You receive the prize,

- if you possess the *lower* player number and the drawn number is at most as large as the number of points submitted by yourself.
- if you possess the *higher* player number and the drawn number is larger than the number of points submitted by your opponent.

APA-treatment

...of a match: After you and your opponent have made your decisions the winner will determined in the as follows. You win the match, if you submitted a larger number of points than your opponent. In case you and your opponent have submitted an identical number of points, a computerized fair coin toss determines the winner.

...of a tournament: Tournament winner is the player who has the most wins in all matches, i.e. overall 2 wins. In case that all players in one group have won the same number of matches (1 win per player), the computer randomly draws a tournament winner. That means, each player's probability to win the tournament is identical.

Your final score at the end of a tournament

The points you have submitted in match are deducted from your endowment irrespective of the outcome of the match and of the tournament. You keep the remaining endowment. Your final score at the end of a tournament therefore equals

$$\boxed{finalscore = \begin{cases} I - Q_1 - Q_2 + R, & \text{if you win the tournament.} \\ I - Q_1 - Q_2, & \text{if you do not win the tournament.} \end{cases}}$$

At the end of each match, as a *participating* player you are informed about (i) the number of points you and your opponent submitted, (ii) the winner of the match, (iii) your total number of wins in matches,(iv) your current endowment. As a *non-particiapting* player you are only informed which of the participating player has won the match. Generally, the display is updated such that you are always informed about (i) your current endowment, (ii) the outcomes in previous matches, (iii) the standings. Exemplary, Figure 3.10 shows a input-screen and Figure 3.11 (Figure 3.12) shows an outcome-screen. At the end of each tournament you are informed if you have won the tournament and about your final score.

Example

The following tables illustrate the decision situation in the second part of the experiment with the help of a fictious example.

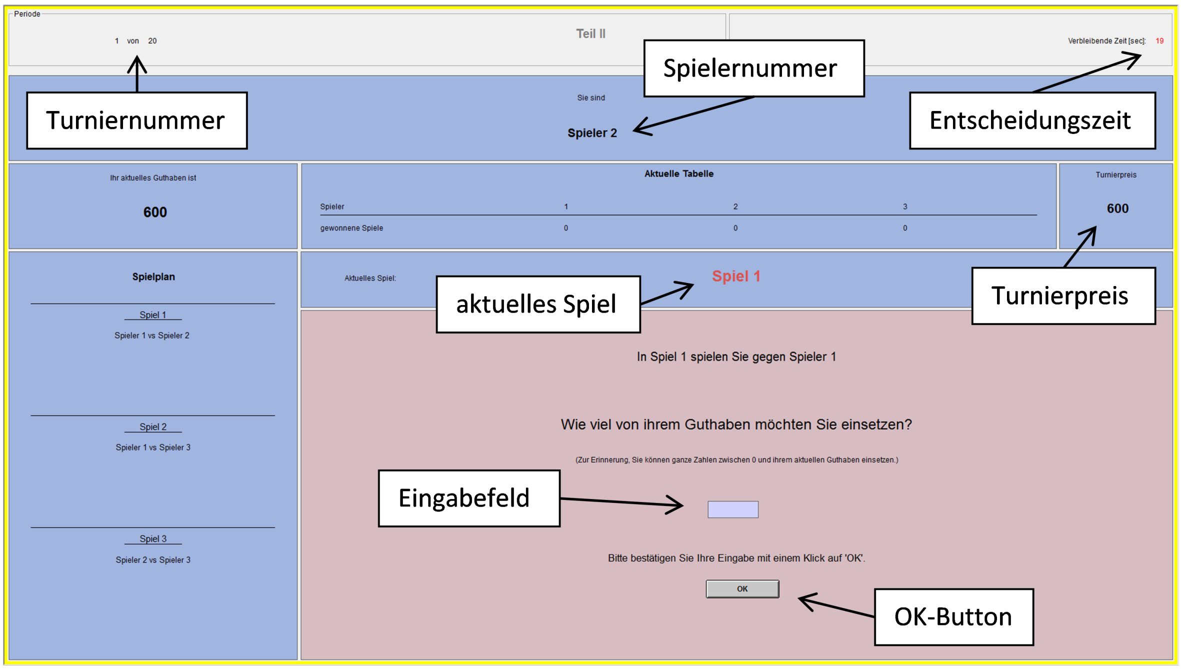


Figure 3.10: Input-screen

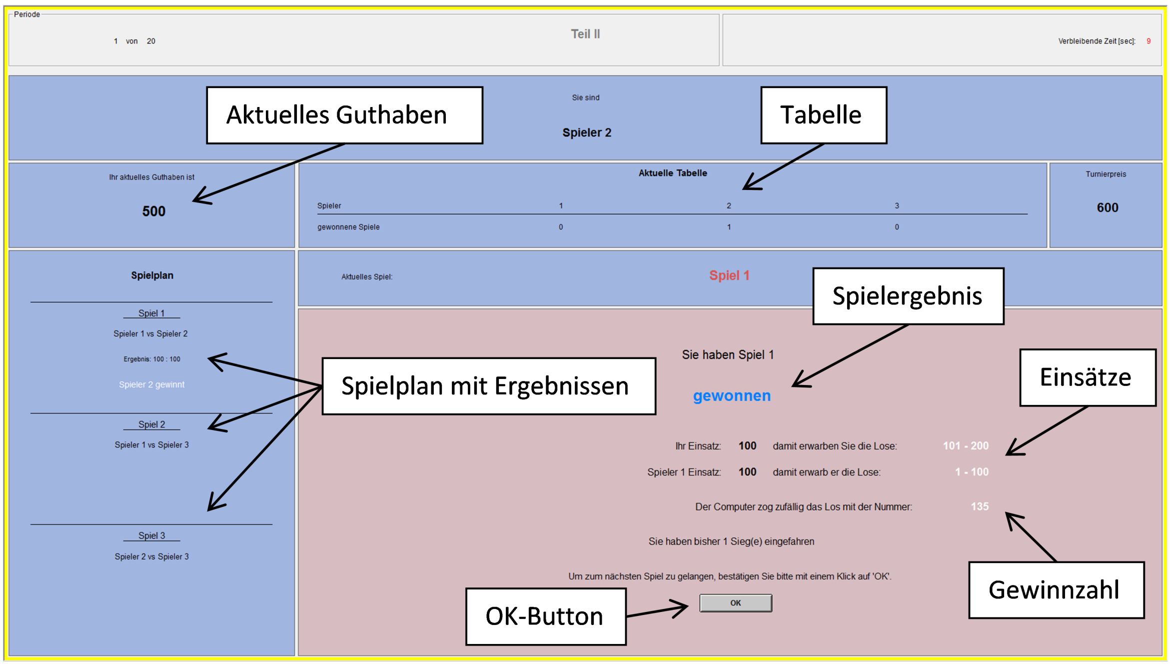


Figure 3.11: LC-treatment Outcome-screen

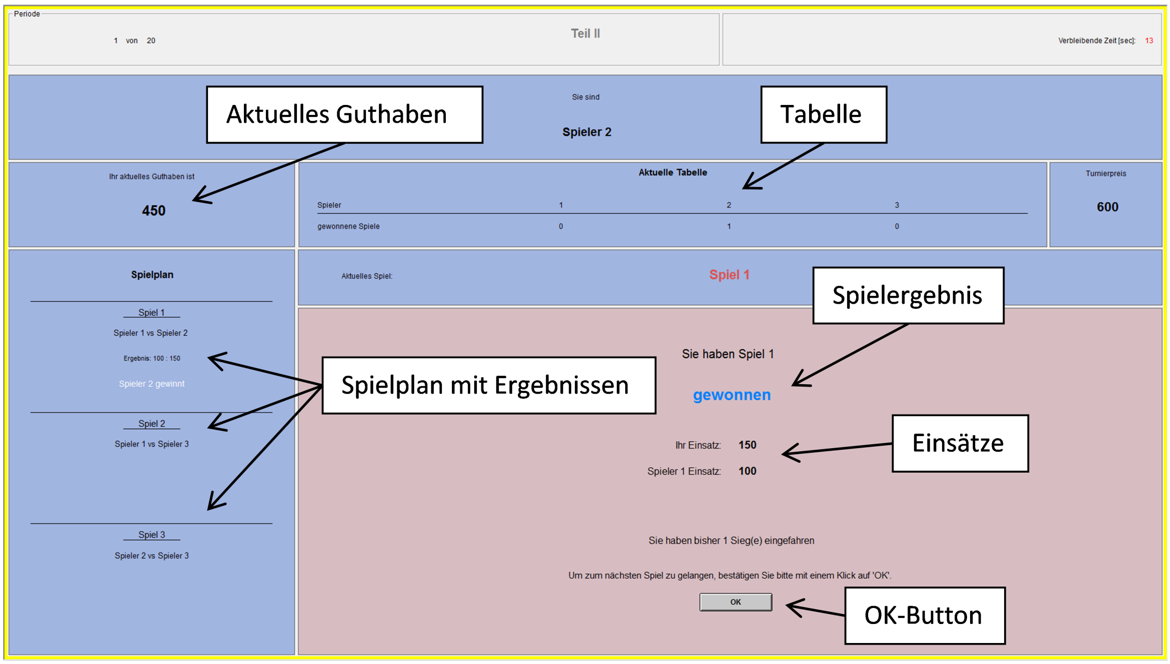


Figure 3.12: APA-treatment Outcome-screen

LC-treatment

Match	Player	Submitted Points	Winning numbers	Drawn number	Winner	Current Endowment
1	Player 1 Player 2	100 100	1,2,...,100 101,102,...,200	135	Player 2	600 − 100 = 500 600 − 100 = 500
2	Player 1 Player 3	200 100	1,2,...,200 201,102,...,300	269	Player 3	500 − 200 = 300 600 − 100 = 500
3	Player 2 Player 3	200 100	1,2,...,200 201,102,...,300	47	Player 2	500 − 200 = 300 500 − 100 = 400

Player	Wins	Finalscore
1	0	600 − 100 − 200 = 300
2	$\boxed{\mathbf{2}}$	600 − 100 − 200 + 600 = 900
3	1	600 − 100 − 100 = 400

APA-treatment

Match	Player	Submitted Points	Winner	Current Endowment
1	Player 1	100	Player 2	$600 - 100 = 500$
	Player 2	150		$600 - 150 = 450$
2	Player 1	200	Player 1	$500 - 200 = 300$
	Player 3	100		$600 - 100 = 500$
3	Player 2	200	Player 2	$450 - 200 = 250$
	Player 3	100		$500 - 100 = 400$

Consequently the standings, with each player's final score and player 2 as a winner due to two wins in matches, yield as follows.

Player	Wins	Finalscore
1	1	$600 - 100 - 200 = 300$
2	$\boxed{\mathbf{2}}$	$600 - 150 - 200 + 600 = 850$
3	0	$600 - 100 - 100 = 400$

Your earnings in the second part

At the end of the 20 tournaments one tournament will be selected. Only the finalscore of this tournament determines your earnings in the second part of the experiment. To deterimene this tournament, one randomly selected participant will throw a 20-sided dice once. The score of this throw determines the tournament relevant for the earnings.

3.B.4 Control Questions

The following questions are intended to ensure that you have understood the instructions. Please answer to the best of your knowledge and raise your hand once you are finished. An experimenter will then come to you and peruse the answers with you.

LC-treatment

1. Which of the following statements is true?
   □ For every tournament you are the same player and you play in the same group.
   □ For every tournament you are the same player and you are newly drawn to a random group.
   □ For every tournament you are a newly drawn a random player number and you play in the same group.
   □ For every tournament you are a newly drawn a random player number and you are newly drawn to a random group.

2. What is your likelihood of winning a match, if you submit exactly half as many points as your opponent?
   □ 0 □ 1/2 □ 1/3

3. Who wins the match, if you obtain the lower player number, you and your opponent have each submitted 93 points, and the computer randomly draws the number 97?
   ☐ You. ☐ Your opponent.

4. Who wins the match, if you submit 0 points and your opponent submits 1 point?
   ☐ You for sure.
   ☐ Your opponent for sure.
   ☐ Depending on the random draw of the computer, either of us may win.

5. Is it possible to win a tournament with one win in matches?
   ☐ Yes. ☐ No.

6. What is your final score, if you submit your entire endowment in a tournament and you do not win in the tournament?
   ☐ 0 points. ☐ 600 points.

7. If you won a tournament and you have submitted 300 points. How large is your endowment in the next tournament?
   ☐ 1200 points. ☐ 900 points. ☐ 600 points.

8. And if you lost it with a submission of 300 points?
   ☐ 600 points. ☐ 300 points. ☐ 0 points.

APA-treatment

1. Which of the following statements is true?
   ☐ For every tournament you are the same player and you play in the same group.
   ☐ For every tournament you are the same player and you are newly drawn to a random group.
   ☐ For every tournament you are a newly drawn a random player number and you play in the same group.
   ☐ For every tournament you are a newly drawn a random player number and you are newly drawn to a random group.

2. What is your likelihood of winning a match, if you submit exactly half as many points as your opponent?
   ☐ 0 ☐ 1/2 ☐ 1/3

3. Is it possible to win a tournament with one win in matches?
   ☐ Yes. ☐ No.

4. What is your final score, if you submit your entire endowment in a tournament and you do not win in the tournament?
   ☐ 0 points. ☐ 600 points.

5. If you won a tournament and you have submitted 300 points. How large is your endowment in the next tournament?
   ☐ 1200 points. ☐ 900 points. ☐ 600 points.

6. And if you lost it with a submission of 300 points?
   ☐ 600 points. ☐ 300 points. ☐ 0 points.

3.C Experiment

3.C.1 Part 3

Please answer the following questions. For each question you have 40 seconds to respond. For the first three questions you recieve EUR 0.50 per correct answer additionally to your final score. The last question in not relevant for the earnings but ask you to answer to the best of your knowledge.

(1) A pencil and an eraser cost EUR 1.10 in total. The pencil costs EUR 1.00 more than the eraser. How much does the eraser cost? ________________ cents

(2) It takes 5 machines 5 minutes to make 5 widgets. How long does it take 100 machines to make 100 widgets? ________________ minutes

(3) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. It takes 48 days for the patch to cover the entire lake. How long does it take for the patch to cover half of the lake? ________________ days

Across how many of these questions you had come already before this experiment?
☐ 0 ☐ 1 ☐ 2 ☐ 3

3.C.2 Questionnaire [extract of Experiment related questions]

Q1. How do you describe yourself as a person in terms of willing to take risks from 1 (not willing to take risks at all) to 7 (very willing to take risks)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q2. How often do you participate in gambling (e.g. lotteries, casinos, online betting) from 1 (never) to 7 (very often)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q3. How much do you like playing parlor games (e.g. Chess, Monopoly, cards) from 1 (not at all) to 7 (very much)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q4. How ambitious are you from 1 (not ambitious at all) to 7 (highly ambitious)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q5. How do you describe your money management from 1 (very spare) to 7 (very generous)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q6. How important was it to you that you achieve earnings as high as possible from 1 (not important at all) to 7 (very important)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Q7. How important was it to you that you win as many tournaments as possible from 1 (not important at all) to 7 (very important)?

☐	☐	☐	☐	☐	☐	☐
1	2	3	4	5	6	7

Chapter 4

Intensity, Fairness, and Dynamics in Round-Robin Tournaments: The Role of the Prize Structure

(joint with Marco Sahm)

Abstract

We experimentally investigate how the prize structure affects intensity, fairness, and dynamic behavior in round-robin tournaments with three players. We compare tournaments with a second prize equal to either 0%, 50%, or 100% of the first prize. While theory predicts the 50%-treatment to be most intense, we find that aggregate effort is highest in the 0%-treatment. Our evidence supports, though, the predicted late mover disadvantage (advantage) in the 0%-treatment (100%-treatment) and fair ranking in the 50%-treatment. Also in line with the theory, we identify a strategic (reverse) momentum: after winning the first match, a player increases (decreases) effort in the second match of the 0%-treatment (100%-treatment). Additional findings suggest, however, that dynamic behavior is also subject to a reverse psychological momentum.

Keywords:Sequential Round-Robin Tournament; All-pay Auction; Fairness; Intensity; Strategic Momentum; Psychological Momentum

JEL classification: C72, C91, D72, Z20

4.1 Introduction

The goals of contest design are diverse. The organizer of a contest often targets at intensity: she may want to maximize productive effort, either on an aggregate level (like in sports or sales contests) or on an individual level (like in R&D races), or minimize wasteful effort (like in rent-seeking). Other frequent objectives are fairness and competitive balance: while contestants with similar characteristics should have similar prospects of success, contests between unequal participants are sometimes required to level the playing field in order to overcome selection biases (like in promotion contests) or increase the interest of third parties (like that of spectators and sponsors of sports, music, and arts competitions). For dynamic battles, which are composed of a sequence of component contests, this includes also the requirement that, e.g., a sports tournament should not be decided before its final match to maintain suspense.

One of the most important features in contest design is the allocation of prizes. While other elements like the contest success function or the dynamic structure are often fixed by the natural or legal environment, the organizer of a contest can frequently allocate prizes from a given pool at her own discretion. The prize structure shapes an incentive scheme that determines the contestants' decisions on entry and effort. It is thus decisive for individual and aggregate investment levels. Moreover, in dynamic contests, like elimination tournaments or races, the allocation of prizes affects also the intertemporal effort decisions and, thereby, the fairness and competitive balance.

One particular form of a dynamic contest is the round-robin tournament, in which participants compete against each other in a sequence of pairwise matches and are ranked according to the number of matches won. Round-robin tournaments are widely used to organize competitions in sports: entire championships, such as the major national football leagues in Europe with up to 20 teams (like in the English Premier League), but also small components of larger tournaments, such as the early rounds (group stages) of the Olympic wrestling tournaments (2000 and 2004) or the second stage of the FIFA World Cup in Spain (1982) with only three contestants per group. While in both of these examples only the first-ranked contestants advanced to the next stage, from 2026 on, the initial group stage of the FIFA World Cups will be organized in the form of round-robin tournaments with three teams per group two of which qualify for the next stage. As a winner of one group will always be matched with a runner-up of a different group at the next stage, ranking first may, however, be more valuable than ranking second. More generally, organizers of round-robin tournaments often employ several rank-dependent prizes.

Despite their frequent use in practice, a formal analysis of round-robin tournaments that accounts for the strategic effects of their dynamics has been neglected until recently. The main theoretical results from the related literature can be summarized as follows. Sequential round-robin tournaments with a *single prize* for the player ranked first are not fair: depending on their position in the sequence of matches, the players have differing ex-ante winning probabilities and expected payoffs (Krumer et al., 2017a; Sahm, 2019). The reason is a discouragement effect of trailing players that implies a strategic momentum (see, e.g., Mago et al., 2013) and has been identified in many forms of dynamic contests (Konrad, 2009, Chapter 8). Allowing for *multiple prizes*, Laica et al. (2021) show that no rank-dependent prize structure exists for which sequential round-robin tournaments with more than three players are fair, and round-robin tournaments with three players are fair if and only if the second prize equals half of the first prize.

Only in this case, the participant's intertemporal effort decisions are not distorted either: in each single match, they only depend on the players' characteristics but not on the position of the match in the schedule of the tournament. Moreover, in a round-robin tournament with three symmetric players, a second prize equal to half of the first prize will also maximize expected aggregate effort if the discriminatory power of the contest success function that shapes competition on the match-level is sufficiently high.

In this article, we present a laboratory experiment that enables us to test how the prize structure influences the intensity, fairness, and dynamics of round-robin tournaments in a controlled environment. Three players compete against each other in an exogenous sequence of pairwise matches by choosing effort from a given budget. Each single match is organized as an all-pay auction. The players are ranked according to the number of matches won and receive rank-dependent prizes. The third prize is normalized to zero. We compare three different treatments: tournaments with a second prize equal to either 0%, 50%, or 100% of the first prize.

While theory predicts the 50%-tournament (0%-tournament) to be most (least) intense, we find that aggregate effort is highest in the 0%-tournament. The main reason is that the observed discouragement effect for the late mover (player 3) in the 0%-tournament is much weaker than predicted (see also Mago and Sheremeta, 2017, 2019). This result is surprising because previous experimental studies of contest environments, in which multiple prizes are predicted to elicit more aggregate effort than a single prize, mostly support these predictions (see, e.g., Lim et al., 2009; Müller and Schotter, 2010; Freeman and Gelber, 2010). In line with the theory, aggregate effort is lower in the 100%-tournament than in the 50%-tournament, which can be explained by lean-back effects (Laica et al., 2021): if the second prize equals the first prize, a player who has won her first match will lose much of her incentives to provide additional effort in her second match.

We consider three notions of fairness: a tournament induces i) fair payoffs if the players have the same ex ante expected payoffs, ii) a fair ranking if they have the same weighted qualification probabilities (WQP, defined as the sum of the ex ante probability to rank first and the ex ante probability to rank second times the weight of the second prize), and iii) fair matches if they have the same winning probabilities of 1/2 in each single match. By and large, we confirm the predicted late mover disadvantage (advantage) in the 0%-tournament (100%-tournament). While we also confirm the theoretical prediction that the 50%-tournament induces a fair ranking, we must (partly) reject the hypothesis that it induces fair payoffs (matches). Instead, we find that player 1, who competes in the first two matches of the tournament, has a significantly higher ex ante expected payoff. Moreover, not all the matches of the 50%-tournament are found to be fair.

Considering the players' dynamic behavior, we identify a strategic momentum in the 0%-tournament: after winning (losing) her first match, a player increases (decreases) effort in her second match. Similarly, we observe a reverse strategic momentum in the 100%-tournament: after winning (losing) her first match, a player decreases (increases) effort in her second match. While these observations can be explained by the dynamic incentives implied by the respective prize structure, such strategic effects are absent in the 50%-tournament. We still observe a reverse momentum in the 50%-tournament that cannot be fully explained by (mixed-strategy) equilibrium play. This finding suggests that the players' dynamic behavior is guided also by psychological effects. Our complementary analysis supports the presence of a reverse psychological momentum.

The remainder of this article is structured as follows. Section 4.2 surveys the related literature. Section 4.3 provides the theoretical foundations. We explain our experimental design and procedures in Section 4.4 and present our results in Section 4.5. Section 4.6 concludes.

4.2 Related Literature

Many authors have studied the role of the prize structure in static and dynamic contests. Sisak (2009) and Chowdhury et al. (2020, Chapter 4.4) provide surveys of the related literature. Here, we focus on the most closely related articles with comparable assumptions.

Barut and Kovenock (1998) show that for static all-pay auctions with an arbitrary number of symmetric risk-neutral players, linear costs of effort, and the sum of prize money fixed, all prize structures with a last prize (for the worst performer) of zero yield the same expected aggregate effort. Although this theoretical neutrality result implies, in particular, that a single prize is sufficient to maximize expected aggregate effort, Harbring and Irlenbusch (2003) provide experimental evidence that average effort increases (and the number of zero-bidders decreases) in the number of winner prizes. Moldovanu and Sela (2001) show that a winner-take-all prize structure is also optimal in static all-pay auctions with linear effort costs if players have private information about their (symmetrically distributed) valuations and Müller and Schotter (2010) provide experimental evidence supporting this result.[1] By contrast, if players are ex-ante asymmetric (in their valuations), multiple prizes may be optimal even if effort costs are linear (Glazer and Hassin, 1988; Clark and Riis, 1998; Cohen and Sela, 2008; Dahm, 2018). These findings for static contests already suggest that multiple prizes may be optimal in dynamic contests if the sequential structure implies (ex-interim) asymmetric continuation values, as it is the case in sequential round-robin tournaments.

One persistent phenomenon in many forms of dynamic contests is the so-called discouragement effect (Konrad, 2009, Chapter 8): low continuation values undermine the players' incentives to provide effort in early stages. The effect may stem from (ex-interim) asymmetries like, e.g., in races when one player is ahead of the other (Harris and Vickers, 1985). But it may also arise in symmetric structures due to the anticipation that an initial win will only lead to further battles in which much of the rent will be dissipated like, e.g., in elimination contests (Rosen, 1986). It is a stable finding in the theory of dynamic contests that multiple prizes may be suitable to mitigate such discouragement effects. This holds for both, additional prizes or penalties on the level of single component contests in races (see, e.g., Konrad and Kovenock, 2009; Gelder, 2014; Sela and Tsahi, 2020) or multi-stage battles (see, e.g., Sela, 2012; Feng and Lu, 2018; Clark and Nilssen, 2018a,b, 2020, 2021) and rank-dependent prizes in elimination tournaments (see, e.g., Rosen, 1986) and round-robin tournaments (the respective literature is discussed below in more detail). There is also some experimental evidence for the predicted impact of different prize structures in races (Mago et al., 2013; Mago and Sheremeta, 2017; Gelder and Kovenock, 2017) and elimination contests (Stracke et al., 2014; Delfgaauw et al., 2015). To the best of our knowledge, however, the role of the prize structure in round-robin tournaments has not yet been investigated

[1] Instead, multiple prizes may be optimal if effort costs are convex or players are risk averse; see, e.g., Fang et al. (2020) and Kalra and Shi (2001).

in a controlled experiment.

The reason may be that, despite their frequent use in practice, a formal analysis of round-robin tournaments that accounts for the strategic effects of their dynamics has been neglected until recently. Krumer et al. (2017a) and Sahm (2019) consider sequential round-robin tournaments with three or four symmetric players, which are ranked according to the number of matches won, and a single-prize for the player ranked first. Krumer et al. (2017a) assume that each single match is organized as an all-pay auction and show that such tournaments are not fair: depending on their position in the sequence of matches, the players have differing ex-ante winning probabilities and expected payoffs.[2] Sahm (2019) assumes that each match is organized as a general Tullock contest (including the perfectly discriminating all-pay auction as a limit case) and shows that the extent and direction of discrimination in the round-robin tournament depend crucially on the discriminatory power of the contest success function that shapes competition on the match level. For round-robin tournaments with three symmetric players and matches organized as all-pay auctions, Krumer et al. (2017a) compare two discrete prize structures: if the sequence of matches is exogenous (endogenous), two identical prizes generate more (less) expected aggregate effort than a single prize. Krumer et al. (2020b) illustrate that in round-robin tournaments with four players, matches organized as all-pay auctions, and two identical prizes a player may even have adverse ex-interim incentives in the sense that he may prefer losing over winning some match depending on the course of the tournament. Laica et al. (2021) extend the analysis to sequential round-robin tournaments with an arbitrary number of heterogeneous players, matches organized as general Tullock contests, and multiple arbitrary rank-dependent prizes. They show that a tournament with three players is fair if and only if the second prize equals the first prize;[3] this prize structure also maximizes expected aggregate effort if matches are organized as all-pay auctions. By contrast, with more than three players, no prize structure exists for which a tournament with a fully sequential exogenous match schedule is fair.[4] Our experimental design allows us to test the main theoretical predictions by Laica et al. (2021) for round-robin tournaments with three players.

Most experimental and empirical studies of behavior in dynamic contests try to identify a so-called momentum. They distinguish between a strategic momentum and a psychological momentum. A strategic momentum arises due to different effort incentives as a result of asymmetric continuation values in component contests. The discouragement effect is an example for a strategic momentum. On the contrary, a psychological momentum considers past performance to be causal for the players subsequent behavior. In other words, how a stage was actually reached changes a player's perception as it has a direct effect on confidence, motivation, competitiveness and, thus, effort provision (see, e.g., Meier et al., 2020).

Evidence for a strategic momentum is mixed. In single-prize laboratory experiments with best-of-n races and single matches organized as lottery contests, Mago and Razzolini (2019) and Mago and Sheremeta (2019) find evidence for a strategic momentum.

[2]Based on sports data from mega-events, Krumer and Lechner (2017) provide empirical evidence for such non-fairness results.

[3]Dagaev and Zubanov (2017) show that such a prize structure is capable to entail a fair tournament also if players have no real effort costs but just decide how to split their given resources between their two matches.

[4]As Caglayan et al. (2020) show, though, round-robin tournaments with four players and three rounds, in each of which two matches (organized as lottery contests) take place simultaneously, are ex ante fair.

However, in best-of-three races where matches are organized as all-pay auctions, Mago and Sheremeta (2017) find no significant discouragement of a first match loser in the second match. Empirical studies on tennis confirm the existence of a strategic momentum (Malueg and Yates, 2010; Gauriot and Page, 2019), whereas Ferrall and Smith Jr. (1999) only find negligible strategic effects in best-of-n races in basketball, baseball, and hockey. When intermediate prizes are introduced, Mago et al. (2013) confirm the existence of a strategic momentum for a first match winner in a best-of-three laboratory experiment where matches are organized as lottery contests. Iqbal and Krumer (2019) empirically investigate best-of-five tennis competitions of nations that include several combinations of pairwise matches and find that intermediate prizes mitigate discouragement effects of trailing nations. We contribute to this literature identifying a (reverse) strategic momentum also in sequential round-robin tournaments.

A psychological momentum is usually considered as bi-directional affecting both, winners and losers, and as equal-directional meaning that the tendency of an outcome is more likely to be confirmed subsequently: "success breeds success" (see, e.g. Mago et al., 2013; Cohen-Zada et al., 2017; Gauriot and Page, 2018, 2019; Mago and Razzolini, 2019; Meier et al., 2020). In their laboratory experiments, Mago et al. (2013) and Mago and Razzolini (2019) find no evidence for a psychological momentum in best-of-n races where matches are organized as lottery contests. In their empirical studies on basketball and tennis, Gilovich et al. (1985), Morgulev et al. (2019), and Gauriot and Page (2018) find no support for a psychological momentum either. By contrast, Cohen-Zada et al. (2017) and Meier et al. (2020) provide empirical evidence for a psychological momentum in judo and tennis.

Contrary to the "success breeds success" pattern, the opposite effect that falling behind incentivizes laggards is also observed, both in experiments (Eriksson et al., 2009; Gelder and Kovenock, 2017) and field studies (Berger and Pope, 2011). Based on their laboratory experiments, Tong and Leung (2002, p.404) even suggest a "hare-tortoise" decision heuristic for dynamic contests which is line with reference-dependent objectives in prospect theory (Kahneman and Tversky, 1979). It can be understood as a reverse psychological momentum in the sense that the trailing player will exert more effort to catch up whereas the leading player slacks off. Similar effects arise in other (field) experiments (Casas-Arce and Martínez-Jerez, 2009; Kuhnen and Tymula, 2012). Fu et al. (2015) find support for this heuristic in a real-effort laboratory experiment on single-prize best-of-three races between two parties.[5] Our experimental findings suggest that the dynamic behavior in sequential round-robin tournaments is also subject to such a reverse psychological momentum.

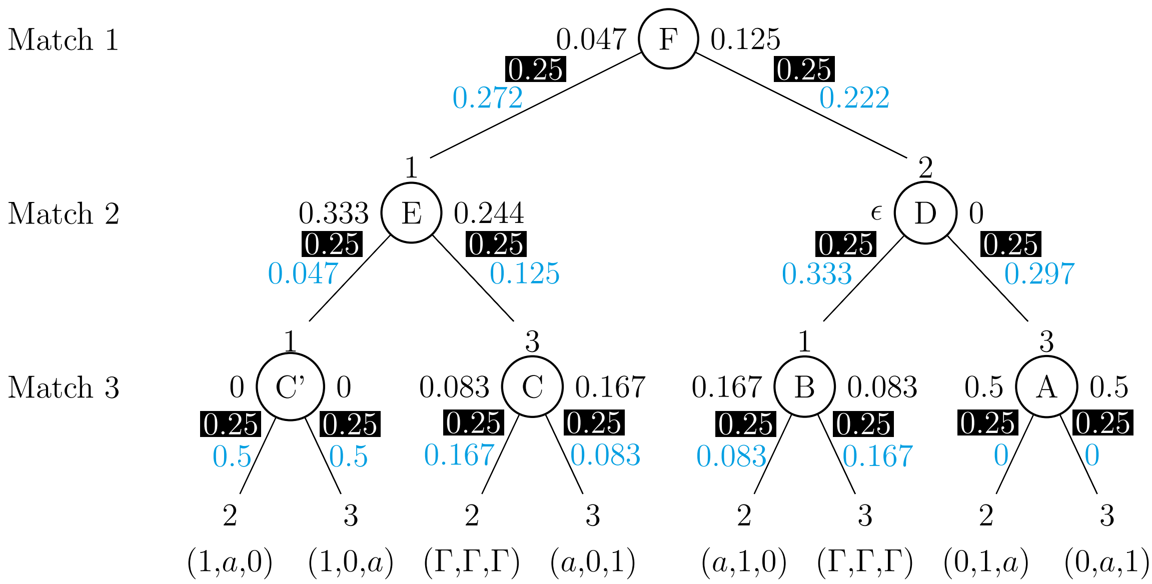
4.3 Theoretical Model

We consider round-robin tournaments with three symmetric, risk-neutral players and an exogenous sequence in which player 1 is matched with player 2 in the first match, player 1 is matched with player 3 in the second match, and player 2 is matched with player 3 in the third match.[6] We abstract from draws: in each match, one player wins and

[5]In contrast to the theoretical predictions, they observe that the race is not more likely to end after two than three matches because losers (winners) of the first match increase (decrease) their effort in the second match.

[6]Apart form renaming players, this exogenous sequence is unique. Laica et al. (2021) show that the use of endogenous sequences in which the outcome of the first match determines the order of the two

the other player loses. At the end of the tournament, players are ranked in descending order according to the number of matches won and receive rank-dependent prizes (R_j). Without loss of generality, we normalize the first prize to one and the last prize to zero, and denote the second prize by a: $1 = R_1 \geq R_2 = a \geq R_3 = 0$. If all three players have won one match, each player receives one of the prizes $R_1, \dots, R_3$ with probability 1/3 (random tie breaking)[7], which yields an expected payoff of $\Gamma := (1 + a)/3$. The structure of the tournament with its $2^3 = 8$ potential courses is depicted in Figure 4.1. The seven nodes $k \in \{A, \dots, F\}$ represent all combinations for which the ranking of the tournament has not yet been determined when the respective match starts.



Note: black $\triangleq$ $(a = 0)$; black box $\triangleq$ $(a = 1/2)$; light blue $\triangleq$ $(a = 1)$; $\Gamma := \frac{1+a}{3}$.

Figure 4.1: Tournament Structure and Expected Efforts

Each match of the tournament is organized as an all-pay auction between two players, A and B, with linear costs of effort, see e.g. Konrad (2009, Chapter 2.1). More specifically, player A's probability of winning match k is

$$p_A^k = \begin{cases} 0 & \text{if } x_A^k < x_B^k, \\ 1/2 & \text{if } x_A^k = x_B^k, \\ 1 & \text{if } x_A^k > x_B^k, \end{cases}$$

where x_i^k denotes the effort of player $i \in \{A, B\}$ in match k. Player $i \in \{A, B\}$ chooses x_i^k in order to maximize his expected payoff

$$E_i^k = p_i^k (w_i^k - x_i^k) + \left(1 - p_i^k\right) \left(\ell_i^k - x_i^k\right), \tag{4.1}$$

where w_i^k denotes player i's expected continuation payoff from winning match k and ℓ_i^k denotes his expected continuation payoff from losing match k, with $w_i^k \geq \ell_i^k \geq 0$. If $w_i^k = \ell_i^k$ for some $i \in \{A, B\}$, player i's optimal effort choice is $x_i^k = 0$ for any effort

remaining matches leads to similar results.

[7]For risk-neutral players, the tie breaking rule is equivalent to the assumption that the aggregate prize money $1 + a$ is shared equally among the three players.

level $x_j^k \geq 0$ of player $j \in \{A, B\}, j \neq i$. Thus, for a positive continuation payoff, player j will have no best reply unless there is a smallest monetary unit $\varepsilon > 0$; the best reply is then $x_j^k = \varepsilon$. As $\varepsilon \to 0$, in the limit, $x_j^k \to 0$ and $p_j^k \to 1$.[8] If instead $w_i^k > \ell_i^k$ for all $i \in \{A, B\}$, a Nash equilibrium (in mixed strategies) in match k always exists and has the following properties (Krumer et al., 2017a, 2020a; Laica et al., 2021):[9] For $i, j \in \{A, B\}$ with $i \neq j$ and $w_i^k - \ell_i^k = \min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\}$, the expected equilibrium efforts are

$$E(x_i^k) = \frac{(w_i^k - \ell_i^k)^2}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad E(x_j^k) = \frac{w_i^k - \ell_i^k}{2}, \tag{4.2}$$

the equilibrium winning probabilities are

$$p_i^k = \frac{w_i^k - \ell_i^k}{2(w_j^k - \ell_j^k)} \quad \text{and} \quad p_j^k = 1 - p_i^k, \tag{4.3}$$

and the expected equilibrium payoffs are

$$E_i^k = \ell_i^k \quad \text{and} \quad E_j^k = w_j^k - (w_i^k - \ell_i^k). \tag{4.4}$$

The tournament represents a sequential game that can be solved by backward induction for its subgame perfect equilibrium (SPE), making repeatedly use of equations (4.2)–(4.4). The details of this procedure have been provided by Krumer et al. (2017a) for $a = 0$, by Krumer et al. (2020a) for $a = 1$, and by Laica et al. (2021) for $a = 1/2$. The solutions provide theoretical predictions about both, the ex ante expected outcome of the tournament and the players' dynamic behavior.

Table 4.1: Ex Ante Expected SPE-Values

	$a = 1$	$a = \frac{1}{2}$	$a = 0$
EX_1	0.436	0.5	0.109
EX_2	0.377	0.5	0.266
EX_3	0.328	0.5	0.125
$\sum EX_i/(1+a)$	0.557	1	0.5
WQP_1	0.642	0.5	0.193
WQP_2	0.647	0.5	0.683
WQP_3	0.711	0.5	0.125
RSD	0.047	0	0.746
$E\pi_1$	0.206	0	0.083
$E\pi_2$	0.271	0	0.417
$E\pi_3$	0.378	0	0
$\sum E\pi_i/(1+a)$	0.430	0	0.5
RSD	0.255	0	1.080

Note: RSD$\triangleq$ relative standard deviation

Table 4.1 displays the players' ex ante expected overall efforts (EX), payoffs ($E\pi$), and weighted qualification probabilities (WQP) for the three different prize structures.

[8]As an alternative tie breaking rule, introducing an additional (marginal) prize on the match level has the same implications (Laica et al., 2021).

[9]Baye et al. (1996) provide a comprehensive analysis of all-pay auctions.

The WQP is defined as the sum of the probability to rank first and the probability to rank second times the weight of the second prize: WQP $= \text{Prob}\{1st\} + a\text{Prob}\{2nd\}$. Only the prize structure for which the second prize equals half of the first prize ($a = 1/2$) leads to symmetric continuation payoffs in each and every match (Laica et al., 2021). It thus induces not only a fair tournament in the sense that the players have identical ex ante expected payoffs, WQPs, and winning probabilities of 1/2 in each single match. It also maximizes the intensity of the tournament, measured by ex ante expected aggregate effort per unit of prize money. By contrast, any other prize structure entails asymmetric continuation payoffs in some of the matches reducing investment incentives and provoking discrimination. Intuitively, if only the player ranking first receives a prize ($a = 0$), a *discouragement effect* occurs which reduces the investment incentives of trailing players and thus disadvantages the late mover (player 3). If instead the second prize equals the first prize ($a = 1$), a player who has won her first match will lose much of her incentives to provide additional effort in her second match. Such *lean-back effects* favor the late mover (player 3). We summarize these theoretical predictions in the following hypotheses.

Hypothesis 4.1. *If the second prize equals half of the first prize (zero), the tournament is most (least) intense.*

Hypothesis 4.2. *A prize structure for which the second prize equals half of the first prize induces*

(a) a fair ranking,

(b) fair payoffs,

(c) fair matches.

Hypothesis 4.3. *The late moving player 3 will be*

(a) advantaged if the second prize equals the first prize,

(b) disadvantaged if the second prize equals zero.

The solutions of the subgame perfect equilibrium also allow predictions about the players' dynamic behavior depending on the course of the tournament. Figure 4.1 displays the players' expected equilibrium efforts in each single match. In particular, we observe a *strategic momentum* for the early movers (players 1 and 2) if the second prize equals zero (discouragement effect) and a *reverse strategic momentum* if it equals the first prize (lean-back effect); more precisely:

Hypothesis 4.4. *After winning the first match,*

(a) each player will decrease effort in her second match if the second prize equals the first prize,

(b) player 1 and 2 will increase effort in their second match if the second prize equals zero.

4.4 Experimental Design and Procedure

We test the hypotheses outlined in Section 4.3 with the help of a laboratory experiment. This enables us to investigate the impact of the prize structure on intensity, fairness and dynamics in round-robin tournaments with three players under controlled conditions. We extend the design of the experiment in Chapter 3 by a multiple prize structure while the procedure remains the same. In this section, we describe the extended design and the unaltered procedure.

4.4.1 Design

We extent the between subject experiment of Chapter 3 by another two treatments. In addition to their APA-treatment (which we call here 0%-treatment), we design a 50%-treatment and a 100%-treatment. While the 0%-treatment incorporates a single prize structure, each, the 50%- and the 100%-treatment, incorporate a two-prize structure.

Irrespective of the treatment, the experiment is trichotomous. For the first part we adopt a test for risk-preferences by Holt and Laury (2002). Part 2 is the main part of the experiment and differs between the treatments. In this part subjects interact in round-robin tournaments where matches are organized as all-pay auctions under the three different prize structures as described in Section 4.3, but with a constant prize budget. Finally, in the last part, we implement a cognitive reflection test (CRT) similar to Frederick (2005).

At the beginning of part 2 in each treatment each subject is randomly and anonymously assigned a player number. That means a subject can either take the role of player 1, player 2, or player 3. This role is fixed throughout part 2 to provide player-specific learning opportunities. After the assignment of roles each subject participates in 20 independent repetitions of round-robin tournaments (periods henceforth).

In each period subjects are randomly matched in three-player round-robin tournaments. Within a round-robin tournament players compete in a sequential sequence: in match 1 player 1 meets player 2, in match 2 player 1 meets player 3, and in match 3 player 2 meets player 1. Each time a new tournament starts subjects are again randomly and anonymously assigned into consequently newly assembled round-robin tournaments to avoid collusion. Then once again players compete according to the sequential sequence.

For each round-robin tournament each subject receives an initial endowment of $I = 600$ points which he can use to invest in his two matches to gain a prize.[10] Hence, in his first match a subject can invest any number of integer points $Q_1 \in [0, 600]$ and in his second match any number of the remaining integer points $Q_2 \in [0, 600 - Q_1]$. A prize is awarded according to the final ranking of the round-robin tournament. In each treatment a prize money of $R = R(1) + R(2) = 600$ points consisting of prizes for the first rank $R(1)$ and the second rank $R(2)$ is distributed.

The prize structure depends on the treatment. In all three treatments the third ranked receives no prize. In the single prize 0%-treatment only the first ranked receives a prize of $R(1)^{0\%} = 600$ points whereas the second ranked receives $R(1)^{0\%} = 0$. In the 50%-treatment a first ranked receives a prize of $R(1)^{50\%} = 400$ and the second ranked

[10] An initial endowment is supposed to, although collectively decreasing or increasing the average level of subjects' effort choices, not account for significant individual distortions in each subject's effort choices and thus, is independent for the determination of a contest winner (Sheremeta, 2011).

receives a prize of $R(2)^{50\%} = 200$ points. In the 100%-treatment both, first and second ranked, each receives a prize of $R(1)^{100\%} = R(2)^{100\%} = 300$ points.

The winner of a single match is determined by an all-pay auction. That means the player who chooses more points wins. In case both choose the same amount of points, the computer randomly selects the winner by coin flip. While at the end of a match the winner is announced to all players of the particular round-robin tournament, only the players who actually participate in a match are informed about the points chosen in that particular match.[11] During the tournament subjects are briefed on their current account of points, results of matches and all points chosen in each match they participated, and the current standings. Player number, match plan and prize values are continuously displayed.

The winner of the first rank in a round-robin tournament is the subject with two wins in matches. The winner of the second rank is the only subject with one win in matches. In case all players win one match, the round-robin tournament ranks are randomly determined by the computer such that every player has the same chances to rank first, second or third. The final payoffs of a player in case of a first rank are $\pi(1) = I - Q_1 - Q_2 + R(1)$ and in case of a second rank are $\pi(2) = I - Q_1 - Q_2 + R(2)$. Otherwise the payoffs are $\pi(3) = I - Q_1 - Q_2$. At the end of a tournament each subject learns his final payoffs and whether he is a univocal or randomly determined first, second, or third ranked.

4.4.2 Procedure

Four sessions were conducted for the 0%-treatment and three sessions were conducted for each, the 50%- and the 100%-treatment. The former proceeded from November to December 2016 whereas the latter between May and July 2019. All the sessions took place at the experimental laboratory of the department of social sciences at the University of Bamberg ("BLER"). Participants were invited via the ORSEE recruitment system (Greiner, 2015). Either 15 or 18 subject participated in a session which lasted on average 90 minutes. In total 174 subjects participated and earned on average an amount of EUR 14.42 per subject. The experimental sessions were computerized by using the software zTree (Fischbacher, 2007).

The procedure of the actual experiment is identical to Chapter 3.4.2. Therefore the general instructions (Chapter 3.B.1), instructions for part one (Chapter 3.B.2), questions of part three (Chapter 3.C.1), and relevant extracts of the questionnaire (Chapter 3.C.2) can be found in Chapter 3. The new instructions and control questions for part two of each, the 50%-treatment and 100%-treatment, are provided translated into English in Appendix 4.B.3.

4.5 Results

We present our results in four steps: First, we analyze the evolution of effort choice decisions across the course of the experiment. Second, we show the summary statistics and discuss intensity and fairness. Third, we explicitly examine the treatments for

[11]That way we prevent players from exploiting budget constraints of other players. Otherwise it should not influence effort choices in the equilibrium anyhow. In practice, although there are measures indicating the intensity of a match, some intensity is never observable and perceived only as a participant.

dynamic effects within a tournament. Finally, we provide a complementary analysis on dynamics across tournaments where single matches are organized as all-pay auctions and within tournaments where single matches are organized as lottery contests.

4.5.1 Choice Evolutions

To reliably test our theory which is based on the subgame perfect equilibrium concept, we have to ensure that behavior has stabilized. Figure 4.2 shows the evolutions of total average effort levels in a tournament per player type across the periods of the experiment.[12] The upper (middle, lower) panels contain the results for the 100%-treatment (50%-treatment, 0%-treatment) where subjects compete for a first prize of size $R(1)^{100\%} = 300$ ($R(1)^{0\%} = 400$, $R(1)^{0\%} = 600$) and a second prize of size $R(2)^{100\%} = 300$ ($R(2)^{50\%} = 200$, $R(1)^{0\%} = 0$).

Additionally, we differentiate the sample for our main analysis. The left (right) panels record the unadjusted (adjusted) sample which comprises (not) all subjects. In the adjusted sample we exclude subjects that apparently have not been understanding the task even not with experience. More precisely, we exclude subjects that fully invest their initial endowment in at least the first 16 periods and invest at least 90 percent of their endowment across all 20 periods.[13]

We find a clear parallel downward shift of average total effort per tournament of player 2 in the adjusted compared to the unadjusted sample in both two-prize treatments. Average total effort of player 2 per tournament across all periods in the unadjusted (adjusted) sample of the 100%-treatment is 177.7 (124.9) and of the 50%-treatment is 177.0 (152.2). Moreover, in the adjusted sample, the effort level of player 2 is not in stark contrast to the effort levels of both other player types which are unaffected compared to the unadjusted sample.

Irrespective of the sample, we find a clear downward trend in effort for subjects of each player type in both two-prize treatments. In the 100%-treatment we find mixed results as the effort of player 1 is only slightly decreased whereas player 2 and 3 substantially reduce effort. Table 4.2 and Table 4.3 report the average total effort per player type per tournament across the first eight periods (F8) and the last twelve periods (L12) and the theoretical predictions (T) of the unadjusted and the adjusted sample, respectively.

In the 100%-treatment player 1 (player 3) reduces his average total effort per tournament from F8 to L12 by 14.1 (19.7) irrespective of the sample. Player 2 reduces effort by 62.0 (64.4) in the unadjusted (adjusted) sample. In the 50%-treatment player 1 (player 3) reduces effort from F8 to L12 by 10.7 (44.2) irrespective of the sample. Player 2 reduces effort by 22.3 (23.6) in the unadjusted (adjusted) sample. In the 0%-treatment player 1 decreases his effort by 113.7 (120.0) and player 2 decreases by 17.9 (18.8). Player 3's effort remains stable at about 291 (261) in the unadjusted (adjusted) sample. .

We estimate panel regression models with the chosen effort levels per subject per tournament as dependent variable to provide statistical evidence for the reported effects.

[12] An analogous analysis that separates each player's choice evolutions for his first and second match is provided in Appendix 4.A.1.

[13] Subjects that are excluded are ID=9, ID=15, ID=17, and ID=21 in 0%-treatment; ID=149 in 50%-treatment; ID=199 and ID=233 in 100%-treatment, and ID=247, ID=248, ID=265, and ID=269 in LC-50%-treatment.

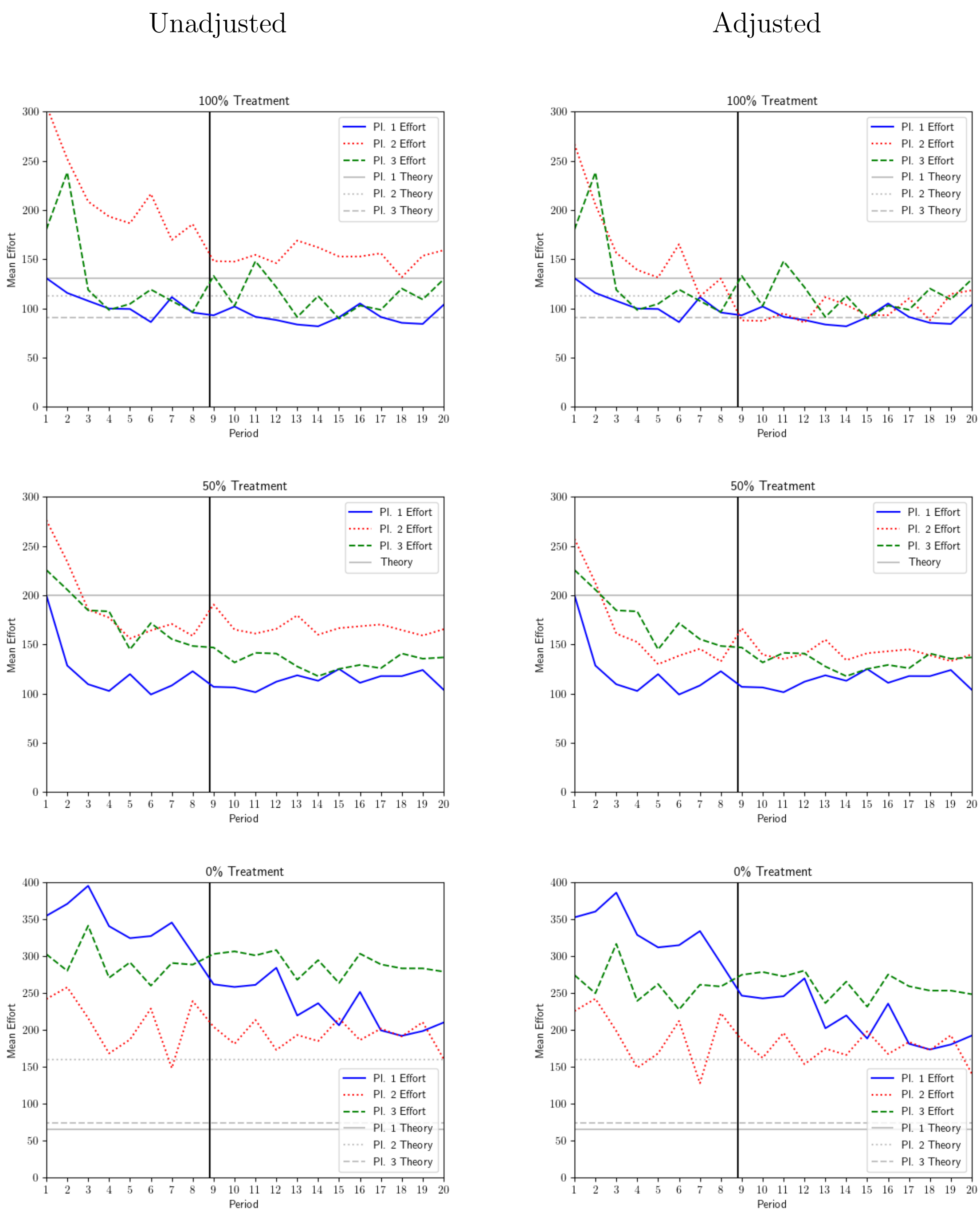


Figure 4.2: Average effort per Tournament

Table 4.2: Average total effort per player type

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	106.0	91.9	130.7	123.9	113.2	200	345.6	231.9	65.6
P2	214.9	152.9	113.0	190.4	168.1	200	211.2	193.3	159.7
P3	133.0	113.3	90.5	177.6	133.4	200	291.0	290.5	74.7
$\sum$	151.3	119.3	111.4	164.0	138.3	200	282.6	238.6	100

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum} \triangleq$ mean of the sum

Table 4.3: Average total effort per player type (adjusted)

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	106.0	91.9	130.7	123.9	113.2	200	335.2	215.2	65.6
P2	163.5	99.1	113.0	166.3	142.7	200	193.6	174.8	159.7
P3	133.0	113.3	90.5	177.6	133.4	200	261.6	261.1	74.7
$\sum$	133.0	101.5	111.4	155.8	129.5	200	263.5	216.4	100

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\sum\triangleq$ mean of the sum

To account for the multiple decisions made by a subject, all models include a subject-specific random effects error structure (PER).[14] Each model contains as base the effort choice of player 1 and as explanatory variables the inverse of the period, dummies for player 2 and 3, and interactions. For each treatment we estimate a separate model. The results are presented in Table 4.4 (4.5) for the unadjusted (adjusted) sample.

Table 4.4: Panel Estimation for changes of efforts across periods

Dep. Variable	Total Effort					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	89.50***	(13.40)	103.51***	(11.69)	244.06***	(51.21)
Player						
2	56.65**	(25.16)	51.83***	(16.20)	-55.26	(62.82)
3	13.37	(21.97)	26.87***	(7.80)	-43.69	(47.07)
Period trend [=1/Period]						
×*Player 1*	44.39	(35.64)	78.00***	(28.90)	185.23***	(51.69)
×*Player 2*	175.16***	(27.07)	120.63***	(32.68)	64.96	(48.87)
×*Player 3*	102.01***	(34.09)	114.93**	(47.42)	16.51	(74.26)
Observations	1020		1080		1380	
Subjects	51		54		69	
R^2	0.097		0.056		0.041	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

In all models we find clear evidence for a significant decrease of efforts across periods for at least one player type irrespective of the sample. Distributed over all models each player type is at least twice affected. In the 100%-treatment (50%-treatment, 0%-treatment) player 2 and 3 (all players, player 1) significantly reduce their effort choices in the course of the experiment. A notable difference between unadjusted and adjusted sample is that player 2 does not significantly choose more effort than player 1 in the adjusted sample.

Our results demonstrate two aspects we have to consider for our main analysis: For one thing, in each treatment at least subjects belonging to one player type need time

[14]From now on we use the abbreviation PER (PEF) to refer to a panel regression model with robust standard errors clustered at the session level and a subject-specific random (fixed) effects error structure to account for the multiple decisions made by a subject.

Table 4.5: Panel Estimation for changes of efforts across periods (adjusted)

Dep. Variable	Total Effort					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	89.50***	(13.40)	103.51***	(11.69)	226.93***	(38.47)
Player						
2	1.35	(11.10)	51.83***	(16.20)	-56.81	(35.88)
3	13.37	(21.97)	26.87***	(7.80)	31.09	(39.25)
Period trend [=1/Period]						
×*Player 1*	44.39	(35.64)	78.00***	(28.90)	201.51***	(37.54)
×*Player 2*	189.13***	(27.07)	127.73***	(38.86)	67.91	(51.58)
×*Player 3*	102.01***	(34.09)	114.93**	(47.42)	18.09	(81.81)
Observations	980		1060		1300	
Subjects	49		53		65	
R^2	0.080		0.050		0.044	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

to adapt to the setting. We therefore focus on late decisions, i.e. decisions made in the last twelve periods from 9 to 20 (L12). For another thing, in both two-prize treatments, but especially in the 100%-treatment, player 2 subjects who choose effort close to their entire endowment can distort the results. That sometimes justifies the exclusion of subjects who invest their initial endowment almost completely. Hence, although we analyze fairness and intensity on the basis of the unadjusted sample, we occasionally refer to the adjusted sample. Throughout the analysis of dynamic behavior we consider the adjusted sample. Most of the times when we use a certain sample, we refer as robustness check to the results of the other sample provided in the Appendix.

4.5.2 Intensity and Fairness

For each treatment, Tables 4.6, 4.7, and 4.8 summarize the WQP, payoffs per unit of prize money R, and effort per unit pf prize money R for each player type . The former two indexes we consider as measure for fairness whereas the latter as a measure for intensity. Each table compares the theoretical predictions T and the empirical observations across all twenty periods (A20) as well as the empirical observations across the last twelve periods (L12). Additionally, the relative standard deviation (RSD) and the sum of all players' ($\sum$) averages are reported.

Intensity measured by the aggregate average effort per R in L12 deviates from theoretical predictions in different degrees dependent on the treatment. In the 100%-treatment (50%-treatment, 0%-treatment), intensity with 0.597 (0.691, 1.193) is close to (not close to, far away from) the equilibrium solution with 0.557 (1, 0.5). Compared with one another, the 0%-treatment is the most intense followed by the 50%-treatment and the 100%-treatment. This contradicts theoretical predictions where the 0%-treatment is the least intense and the 50%-treatment the most intense.

To formally test our Hypothesis 4.1, we estimate a PER, again with the chosen effort levels per subject per tournament as dependent variable. However, this time the the basic model contains as base the effort in 50%-treatment and as dummies the 0%- and

Table 4.6: Summary: 100%-treatment

	WQP			Payoffs per R			Effort per R		
	T	A20	L12	T	A20	L12	T	A20	L12
P1	0.642	0.568	0.554	0.103	0.121	0.124	0.218	0.162	0.153
P2	0.647	0.741	0.696	0.135	0.075	0.093	0.188	0.296	0.255
P3	0.711	0.691	0.750	0.189	0.144	0.186	0.151	0.202	0.189
RSD	0.047	0.109	0.124	0.255	0.254	0.287	0.148	0.255	0.212
$\sum$	2	2	2	0.430	0.339	0.403	0.557	0.661	0.597

Note: T $\triangleq$ equilibrium predictions; A20 $\triangleq$ all 20 periods; L12 $\triangleq$ last 12 periods; RSD $\triangleq$ relative standard deviation

Table 4.7: Summary: 50%-treatment

	WQP			Payoffs per R			Effort per R		
	T	A20	L12	T	A20	L12	T	A20	L12
P1	0.5	0.478	0.493	0	0.123	0.140	0.33	0.196	0.189
P2	0.5	0.513	0.523	0	0.047	0.069	0.33	0.295	0.280
P3	0.5	0.510	0.484	0	0.088	0.100	0.33	0.252	0.222
RSD	0	0.032	0.034	0	0.362	0.284	0	0.164	0.164
$\sum$	1.5	1.5	1.5	0	0.257	0.284	1	0.743	0.691

Note: T $\triangleq$ equilibrium predictions; A20 $\triangleq$ all 20 periods; L12 $\triangleq$ last 12 periods; RSD $\triangleq$ relative standard deviation

Table 4.8: Summary: 0%-treatment

	WQP			Payoffs per R			Effort per R		
	T	A20	L12	T	A20	L12	T	A20	L12
P1	0.193	0.411	0.384	0.083	-0.051	-0.002	0.109	0.462	0.387
P2	0.683	0.274	0.308	0.417	-0.060	-0.014	0.266	0.334	0.322
P3	0.125	0.315	0.308	0.000	-0.169	-0.176	0.125	0.485	0.484
RSD	0.746	0.172	0.108	1.080	0.572	1.233	0.423	0.155	0.168
$\sum$	1	1	1	0.5	-0.281	-0.193	0.5	1.281	1.193

the 100%-treatment. Table 4.28 (Table 4.29) shows the results of the basic model for the unadjusted (adjusted) sample. In an extended model we control for some variables that consider individual preferences and characteristics such as risk-aversiveness and impulsivity (see Tables 4.28 and 4.29 in Appendix 4.A.2).

Table 4.9: Panel Estimations for Intensity: L12

Dep. Variable	Total Effort			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	138.26***	(12.73)	129.55***	(9.95)
Treatment				
0%	100.34***	(29.05)	86.807***	(32.86)
100%	-18.92	(18.07)	-28.03**	(11.19)
Observations	2088		2004	
Subjects	174		167	
R^2	0.091		0.097	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

We find that effort in the 0%-treatment is about 100.3 (86.8) higher whereas in the 100%-treatment 18.3 (28.0) lower than in the 50%-treatment when we consider the unadjusted (adjusted) sample. In the unadjusted (adjusted) sample the former (both) effect(s) is (are) significant. Hence, our findings confirm the impact of the prize structure on intensity. However, the dimension and the direction of this impact is not in line with theoretical predictions.

Result 4.1. *The tournament is most (least) intense if the second prize equals zero (the first prize).*

These effects are stable when we control for individual preferences and characteristics. Additionally, we find that subjects who assign themselves more generous or a higher importance to win a tournament invest more. In contrast, subjects who assign a higher importance to the final payment invest less.

Fairness and Discrimination, respectively, are measured by either the WQP or the payoffs per R. In terms of WQP in L12, both two-prize treatments are quantitatively and qualitatively close to the theoretical predictions. In the 100%-treatment player 3 (player 1) has the highest (lowest) WQP with 0.750 (0.554). In the 50%-treatment the RSD is very close to zero with 0.034. However, with regard to payoffs, only in the 100%-treatment dispersion with 0.287 is close to theoretical predictions with 0.255. The 50%-treatment is much more dispersed with 0.284 rather than zero what theory predicts.

In contrast, in the 0%-treatment both, WQPs and payoffs, quantitatively and qualitatively deviate from theoretical predictions. WQPs are much less dispersed than theory predicts and player 2 has the shared lowest with 0.308 rather than the highest with 0.683 as theory predicts. Likewise, payoffs in the experiment are negative rather than positive. Indeed, player 3 has the lowest payoffs with -0.176 but player 2 has not the highest payoffs with -0.014 (for more details see Chapter 3).

We formally test our Hypotheses 4.2 and 4.3 by performing a PER where the dependent variable is either the WQP or the payoffs. In the 100%-treatment and in the 0%-treatment the WQP is a binary variable: the first rank is valued with one and the third rank is valued with zero. Dependent on the treatment, the second rank is either valued with one in the 100%-treatment or zero in the 0%-treatment. In the 50%-treatment the WQP is an ordered ternary variable: the first rank is valued with one, the second rank is valued with one half, and the third rank is valued with zero.[15]

Payoffs are defined by a subject's payoffs per tournament. All models' base is player 3 and as explanatory variables the dummies player 1 and player 2 are included. Tables 4.10 (4.30) and 4.12 (4.31) report the results for the unadjusted (adjusted) sample (in Appendix 4.A.3).

Table 4.10: Panel Estimations for Discrimination by WQP: L12

Dep. Variable	WQP					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	0.750***	(0.008)	0.484***	(0.007)	0.308***	(0.038)
Player						
1	-0.196***	(0.007)	0.009	(0.012)	0.076	(0.052)
2	-0.054*	(0.030)	0.039	(0.029)	0.000	(0.103)
Observations	612		648		828	
Subjects	51		54		69	
R^2	0.031		0.002		0.006	

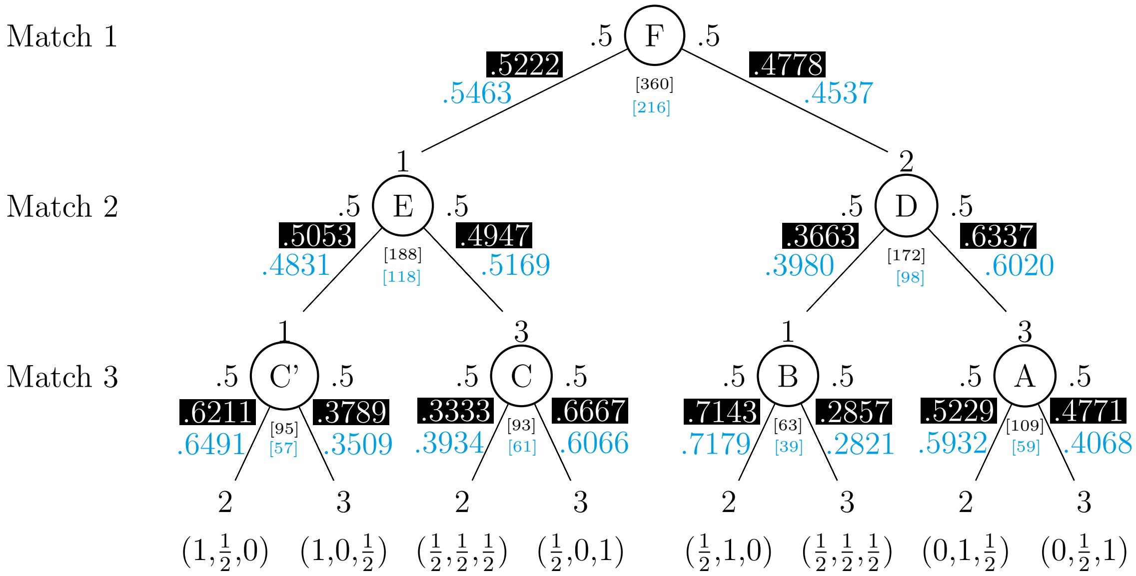
Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Hypothesis 4.2(a) cannot be rejected. The WQP in the 50%-treatment for player 3 is close to the fair 0.5 and not significantly different from those of player 1 or player 2. However, Hypothesis 4.2(b) is rejected as the payoffs of player 3 are significantly lower than those of player 1. Qualitatively, this persists when we consider the adjusted sample.

Figure 4.3 compares the theoretical winning probabilities of 0.5 in the subgame perfect equilibrium (black numbers) and the empirical observation in A20 (black boxes) and in L12 (blue numbers), respectively, for players in matches of all possible tournament outcomes. Regarding L12, it becomes apparent that in nodes D, C', C, and B, the difference between the two players' winning probabilities exceeds 20 percentage points. In nodes F, E, and A, the difference is smaller than 20 percentage points.

To statistically test Hypothesis 4.2(c), we perform a binomial test of proportions (BTP). The findings are mixed and presented in Table 4.11. We cannot fully reject the hypothesis of a fair match in nodes F, E, C, and A as the p values are larger than 0.1 and, thus, that the probability that a player wins the match is not significantly different from 0.5. For nodes D, C', and B the hypothesis of a fair tournament is rejected as p values are smaller than 0.1 and, thus, that the probability that a player wins the match is significantly different from 0.5.

[15]By and large, for all models with binary or ternary dependent variables in this article, the findings are validated by logit panel regressions if not stated otherwise. The results are provided upon request.



Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 4.3: 50%-treatment: Winning Probabilities

Table 4.11: p values for binomial test of proportions (BTP) at each node

Test/Node	F	E	D	C'	C	B	A
BTP	> 0.1	> 0.1	$= 0.05$	$= 0.03$	> 0.1	< 0.01	> 0.1

Result 4.2. *A prize structure for which the second prize equals half of the first prize induces*

(a) a fair ranking,

(b) significantly higher payoffs for player 1,

(c) fair matches only in nodes F, E, C, and A.

Our observations provide support for Hypothesis 4.3. The WQP for Player 3 in the 100%-treatment is significantly higher than those for player 1 or player 2. While the difference between player 3 and player 1 is only significant on the 10%-level in the unadjusted sample, it becomes highly significant on the 1%-level in the adjusted sample. Differences in payoffs reveal lower levels of significance, but still occur, even in the adjusted sample.

In the 0%-treatment, at least compared to player 1, player 3's WQP is lower, but not significantly.[16] However, player 3 has significantly lower payoffs by 104.3 compared to player 1 and by 97.2 compared to player 2. These significant differences persist with regard to the adjusted sample.

Result 4.3. *The late moving player 3 will be*

[16] A logit panel regression with random effects reveals a significant difference on the 10%-level between the WQP of player 3 and player 1 in the adjusted and unadjusted sample of the 0%-treatment.

(a) *advantaged (in terms of payoffs and WQP) if the second prize equals the first prize,*

(b) *disadvantaged (in terms of payoffs, but not in terms of WQP) if the second prize equals zero.*

Table 4.12: Panel Estimations for Discrimination by Payoffs: L12

Dep. Variable	Payoffs					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	711.66***	(7.64)	660.14***	(13.17)	494.24***	(36.00)
Player						
1	-37.31**	(16.17)	23.80***	(8.58)	104.30***	(23.82)
2	-55.68*	(32.33)	-19.01	(20.42)	97.21**	(43.43)
Observations	612		648		828	
Subjects	51		54		69	
R^2	0.021		0.016		0.036	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

For completion of our fairness analysis, we additionally estimate all the basis models and control for individual preferences and characteristics in the unadjusted and the adjusted sample, respectively. The results are provided in Tables 4.32, 4.33, 4.34, and 4.35 in Appendix 4.A.4. By and large, the estimations confirm the stability of the results.

Additionally, we observe in the adjusted sample of all treatments that subjects who assign a higher importance to winning a tournament have a higher WQP. However, those subjects do not necessarily have higher payoffs. Surprisingly, females' payoffs are significantly lower than those of males in all treatments except for the 50%-treatment. In the 100%-treatment less impulsive subjects, i.e. those who have answered at least one question correctly in the CR-test, invest less, similar to the findings by Sheremeta (2016).

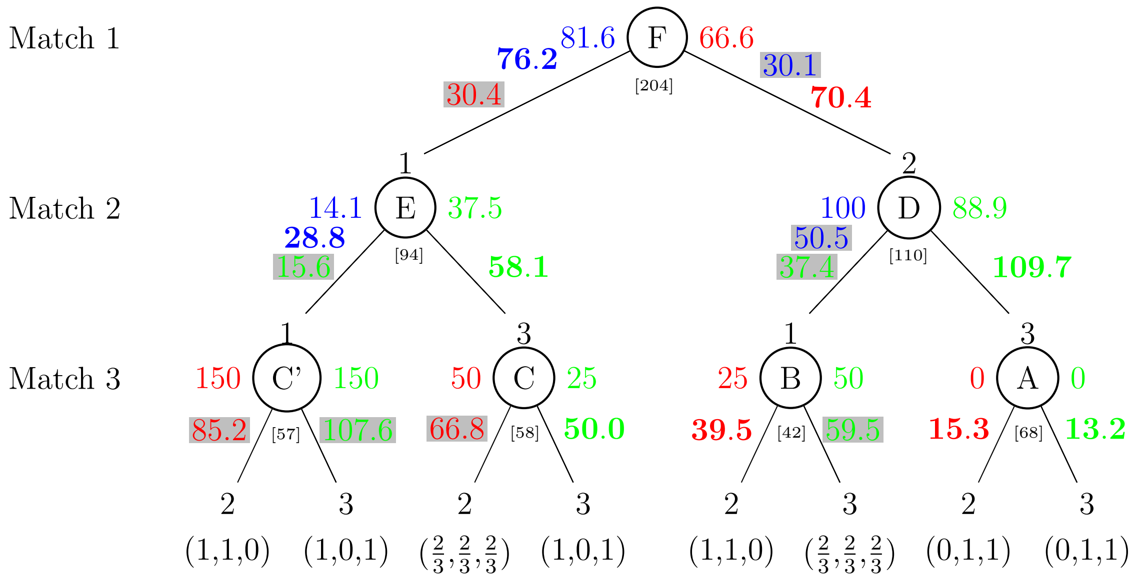
4.5.3 Dynamics

To investigate causes for our findings on intensity and fairness, we have a closer look on the dynamics within tournaments. We define dynamic behavior as the adjustment of effort choice strategies in the course of the tournament dependent on previous match outcomes. For this purpose we consider for each treatment the structure of the sequential round-robin tournament that captures all potential tournament outcomes. In each game structure, equilibrium predictions and empirical observations are opposed.

100%-treatment Figure 4.4 (Figure 4.12) illustrates in normal letters directly to the left and to the right of each node the expected effort in equilibrium for the adjusted (unadjusted) sample (in Appendix 4.A.5). Red (violet, blue) numbers represent player 1 (player 2, player 3). Numbers in bold (gray boxes) express the average effort choices of subjects who win (lose) their first match. Bracket numbers below each node state how many times this node was reached across all tournaments. Henceforth we call this a win/lose structure.[17]

For example, effort choices of player 3 subjects that win (lose) their first match are on average 58.1 (15.1) in node E and 109.7 (37.4) in node D. Player 3 winners of node E (node D) advance to node C (node A) and choose on average effort of 50.0 (13.2). Losers of node E (node D) advance to node C' (node B) and choose on average effort of 107.6 (59.5). For player 1 and player 2 this procedure works analogously.

We clearly detect lean-back effects of all players, not just for player 3, that are in line with theoretical predictions. Subjects who win their first match reduce effort in their second match. Moreover, subjects who lose their first match increase effort in their second match. Player 1 subjects as first match winners (losers) increase (reduce) effort from 76.2 (30.1) to 28.8 (50.5). Player 2 subjects as first match winners (losers) increase effort from 70.4 (30.4) to 39.5 in node B (85.2 in node C') and 15.3 in node A (66.8 in node C), respectively.



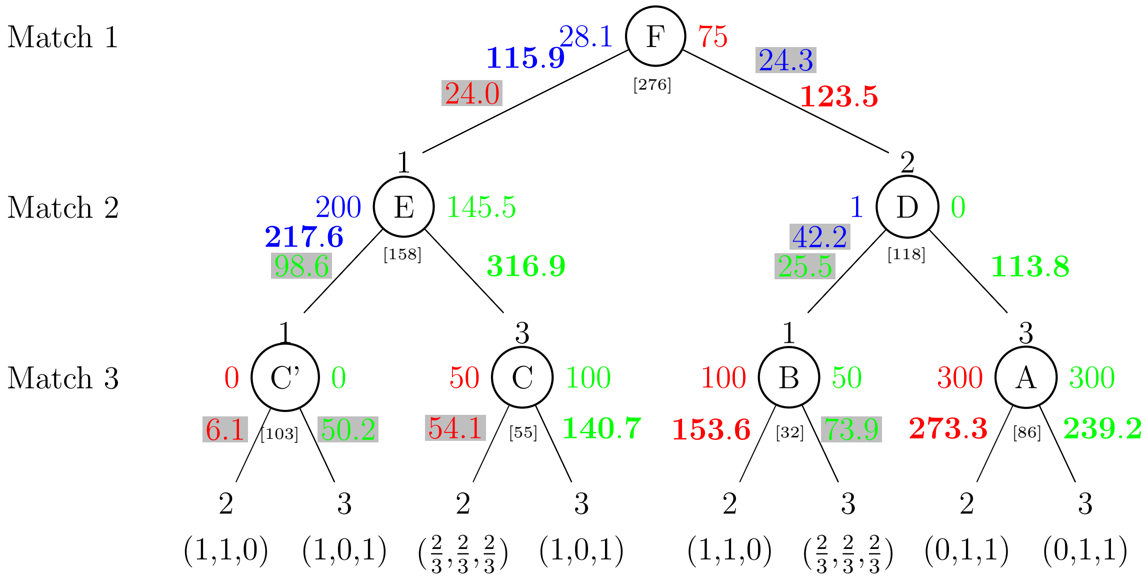
Note: blue $\triangleq$ player 1 mean effort; red $\triangleq$ player 2 mean effort; green $\triangleq$ player 3 mean effort; **bold** $\triangleq$ mean effort of subjects who **win** their first match; gray box $\triangleq$ mean effort of subjects who lose their first match; normal $\triangleq$ equilibrium predictions; bracketed $\triangleq$ absolute frequency.

Figure 4.4: 100%-treatment: win/lose mean effort in L12 (adjusted)

0%-treatment Figure 4.5 (Figure 4.16) exposes the win/lose structure for the 0%-treatment and the adjusted (unadjusted) sample (in Appendix 4.A.5). We identify a clear strategic momentum for each, player 1 and player 2. For player 3 we observe mixed results that are in line with theory. The non-occurrence of a discouragement effect by player 3 in node D discovered in Chapter 3 also exists for those player 3 subjects who lose in node D as their effort choices are larger than zero.

[17]Additionally, we provide other game structures for each treatment in Appendix 4.A.5. In Figures 4.13, 4.14, 4.17, 4.18, 4.21, and 4.22 where we oppose expected effort and empirical efforts for each, the unadjusted and the adjusted sample. In Figures 4.15 and 4.19 we oppose theoretical winning probabilities and emprirical winning probabilities for the unadjusted sample.

Player 1 subjects who win (lose) their first match increase (decrease) effort choices from 115.9 (24.3) to 217.6 (42.2) in their second match. Player 2 subjects who win (lose) their first match increase (decrease) effort choices from 123.5 (24.0) to 153.6 in node B and 273.3 in node A, respectively. Player 3 subsequent to node D (node E) always increases (decreases) effort choices, irrespective of a win or a loss. However, paths where player 3 adjustments reveal reverse effects after a win in node E and loss in node D only account for less than one third of all tournaments played.

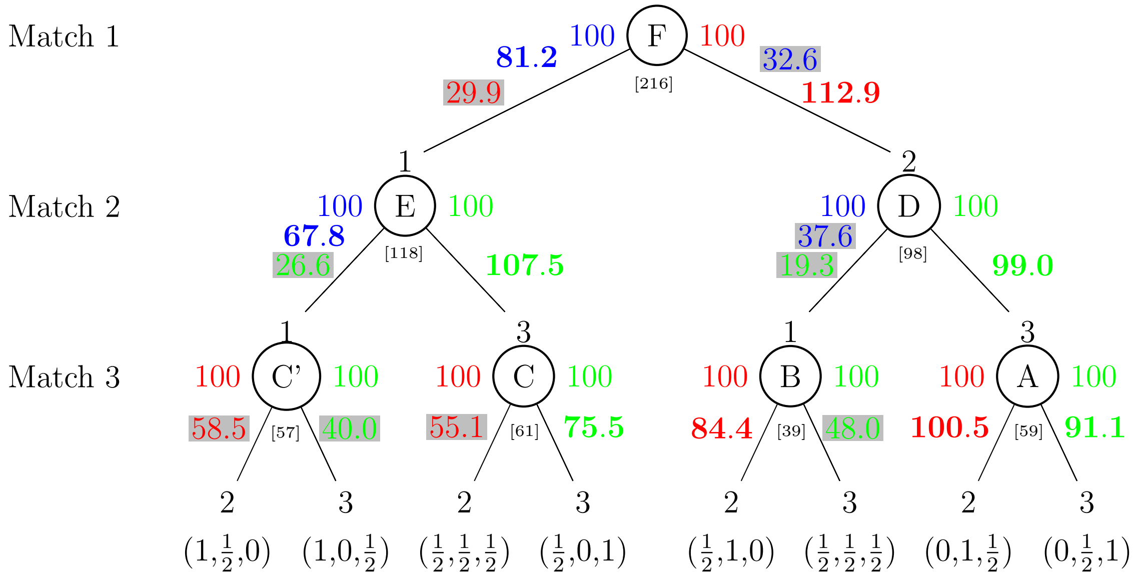


Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

Figure 4.5: 0%-treatment: win/lose mean effort in L12 (adjusted)

50%-treatment In Figure 4.6 (Figure 4.20) the win/lose structure for the 50%-treatment and the adjusted (unadjusted) sample is revealed (in Appendix 4.A.5). It becomes apparent that subjects of all player types exhibit a reverse momentum. Player 1 subjects after a win (loss) in Match 1 decrease (increase) their effort choice from 81.2 (32.6) to 67.8 (37.6) in Match 2. Player 2 subjects after a win in Match 1 decrease (increase) their effort choice from 112.9 (29.9) to 84.4 in node B (58.5 in node C') and 100.5 in node A (55.1 in node C), respectively, in Match 3. Player 3 subjects in node E after a win (loss) decrease (increase) their effort choice from 107.5 (26.6) to 75.5 in node C (40.0 in node C') and in node D after a win (loss) decrease (increase) their effort choice from 99.0 (19.3) to 91.1 in node A (48.0 in node B).

Statistical evidence To test Hypothesis 4.4, we estimate a PEF for each treatment. The dependent variable is defined as the difference between a subject's effort in his second and in his first match, *effort_2 - effort_1* . Hence, in case this difference is positive (negative) a subject increases (decreases) effort in her second match compared to her first match. As independent variable a dummy is utilized which states whether a subject is a winner in his first match or not, *Winner_1*. Table 4.13 reports the estimation results for all adjusted treatments separately.



Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

Figure 4.6: 50%-treatment: win/lose mean effort in L12 (adjusted)

Table 4.13: Panel Estimations for Effort Dynamics: L12 (adjusted)

Dep. Variable	effort_2 - effort_1					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	35.83**	(8.28)	19.95***	(0.92)	-21.61*	(7.52)
Winner_1	-85.03**	(15.55)	-39.58***	(1.76)	116.15***	(15.69)
Observations	588		648		780	
Subjects	49		53		65	
R^2	0.275		0.075		0.064	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

The results indicate that Hypothesis 4.4(a) cannot be rejected. As theory predicts, we find significant evidence for a lean-back effect: Winners decrease effort in their second match by about 49.2 points. Moreover, we find evidence for a reverse strategic momentum: Losers significantly increase their effort by about 35.8 points. Additionally, an asymmetry between the winners' negative adjustment and the losers' positive adjustment in absolute terms becomes obvious: Winners reduce effort by more than losers increase it.

Our observations also support Hypothesis 4.4(b). As theory predicts, player 1 and player 2 increase effort after a win in Match 1 consistent with a strategic momentum. The results of an analogue panel estimation that excludes player 3 is provided in Table 4.37 in Appendix 4.A.5. Yet, we additionally observe that all three players not only significantly increase their effort after a win (by about 94.5 points) but also decrease their effort after a loss (by about 21.6 points).

Result 4.4. *(a) In the 100%-tournament, a reverse strategic momentum occurs: all players significantly decrease (increase) their effort after a win (loss) in their first match.*

(b) In the 0%-tournament, a strategic momentum occurs: all players significantly increase (decrease) their effort after a win (loss) in their first match.

Additionally, we find that in the 50%-treatment, after a win (loss) subjects significantly decrease (increase) effort in their subsequent match by about 19.6 (20.0) points.

Although no strategic effects exist in equilibrium, qualitatively, this behavior can be explained by mixed strategy equilibrium play. Players uniformly randomize between their pure strategies on the same interval in every node. Conditional on the outcome of the match, a match winner's (loser's) expected effort is thus greater (smaller) than the unconditional expected effort of all players, winners and losers combined. Hence, as players still uniformly randomize on the same interval in their second match, the probability that a winner (loser) chooses an effort below (above) the level of his first match is increased.

However, when we consider a subject's average effort in the first match, mixed strategy equilibrium play can only partly account for those dynamics. Both, winners' and losers' adjustments exceed the predicted adjustment in absolute terms relative to the base which is defined as the effort in a subject's first match. Hence, our findings suggest that the players' dynamic behavior is guided also by psychological effects.

Additional Result 4.1. *In the 50%-tournament, a reverse momentum occurs: all players significantly decrease (increase) their effort after a win (loss), but to a higher extent than mixed-strategy equilibrium play predicts.*

Such a reverse momentum is documented by Tong and Leung (2002) who suggest to call it a "hare-tortoise" heuristic and is supported by findings in Casas-Arce and Martínez-Jerez (2009), Kuhnen and Tymula (2012), and Fu et al. (2015).

To provide further evidence that dynamic behavior in three-player round-robin tournaments depends on the prize structure, we estimate a pooled PER of the difference in chosen effort levels between a subject's second and first match. The model is pooled because it integrates as explanatory variables dummies for the 0%-treatment and the 100%-treatment such that the base is the 50%-treatment. Additionally, it includes a dummy whether a subject wins her first match or not and interactions. Table 4.14 shows the estimation results for each, the unadjusted and the adjusted sample.

Table 4.14: Pooled Panel Estimations for Effort Dynamics: L12

Dep. Variable	effort_2 - effort_1			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	30.75***	(16.88)	19.31***	(4.71)
Treatment				
0%	-50.30***	(18.27)	-36.60**	(15.19)
100%	19.88	(18.72)	16.86**	(8.00)
Winner_1	-41.86***	(3.25)	-38.37***	(1.51)
Treatment×Winner_1				
0%× Winner_1	148.76***	(14.62)	145.50***	(13.54)
100%× Winner_1	-40.73***	(11.91)	-47.29***	(12.63)
Observations	2088		2004	
Subjects	174		167	
R^2	0.081		0.121	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure.
Significance level: *** (1%), ** (5%), * (10%)

The findings fully confirm the previous results especially in the adjusted sample. Compared to a first match loser in the 50%-treatment, the difference between effort in his second and his first match of a first match loser in the 0%-treatment (100%-treatment) is significantly decreased (increased). More precisely, a subject who loses his first match in the 0%-treatment decreases (increases) effort by about 17.3 (36.1) and, thus, reverses (intensifies) the increase of 19.3 in the 50%-treatment.

Likewise, the difference between effort in his second and his first match compared of a first match winner in the 0%-treatment (100%-treatment) is significantly increased (decreased) in comparison to a first match winner in the 50%-treatment. In other words, a subject who wins his first match in the 0%-treatment increases (decreases) effort by about 89.8 (49.5) and, thus, reverses (intensifies) the reduction of 19.0 in the 50%-treatment.

Additional Result 4.2. *Dynamic behavior in 0%-tournaments (100%-tournaments) is reversed (intensified) compared to 50%-tournaments.*

4.5.4 Complementary Analysis on Dynamics

Since our findings in the 50%-treatment suggest that dynamic behavior is guided also by psychological effects, we provide a complementary analysis on dynamics. Firstly, we investigate whether dynamic effects occur not only within but also between round-robin tournaments in the 50%-treatment. Finally, we conduct another treatment, which we call LC-50%-treatment, where we maintain the prize structure of the 50%-treatment but change the contest success function from an all-pay auction to a standard lottery contest.

Inter-tournament

In our experiment we explicitly mention that subjects interact in twenty consecutive round-robin tournaments which are independent. That means in every round-robin tournaments subjects start to compete against new opponents with their full endowment irrespective of previous match and tournament outcomes. Hence, a subject's final rank in a tournament should not have any strategic effect on the subsequent tournament.

Complementary Hypothesis 4.1. *The outcome of a tournament has no effect on the players' effort in the following tournament.*

Table 4.15: Panel Estimation for Effort Dynamics between Tournaments: L12

Dep. Variable	T_effort_(t) - T_effort_($t-1$)			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	-26.923***	(2.347)	-27.670***	(2.084)
Rank($t-1$)				
2	24.739**	(3.510)	25.529**	(2.853)
3	54.072**	(5.820)	54.950**	(5.733)
Observations	648		636	
Subjects	54		53	
R^2	0.055		0.056	

Note: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific fixed effects error structure. Significance level: *** (1%), ** (5%), * (10%)

To statistically test our Complementary Hypothesis 4.1, we estimate a PEF with lagged variables and a subject-specific fixed effects error structure. Dependent variable is the difference between a subjects total tournament effort in period t and the subject's total tournament effort in period $t-1$, *T_effort_*(t) - *T_effort_*($t-1$). As independent variables the model includes a dummy for the final tournament rank two in period $t-1$ and for the final tournament rank three in period $t-1$. The estimated results are presented in Table 4.15 for each, the adjusted and unadjusted sample.

Our findings lead us to reject our Complementary Hypothesis 1. We find clear evidence that subjects change their total tournament effort in a round-robin tournament dependent on their rank in the previous tournament. In the adjusted sample, after a first rank (third) rank, subjects decrease (increase) total tournament effort by about 27.7 (27.3). After a second rank, subjects total tournament effort almost remains unchanged with a slight reduction by 2.1.

In the absence of strategic effects such dynamic behavior is characterized as a psychological effect. Hence, we clearly identify a reversed psychological momentum. Since the reversed psychological momentum between tournaments has the same direction as the dynamic adjustments within tournaments, this argues for the existence of a psychological momentum also within tournaments.

Complementary Result 4.1. *A reversed psychological momentum occurs between tournaments: a first (third) ranked player decreases (increases) total tournament effort in the subsequent tournament.*

Lottery Contest Treatment

To test whether unambiguous psychological effects occur in the absence of strategic effects, we implement a treatment with a prize structure identical to the 50%-treatment but without a theoretical prediction in mixed-strategies. For that we conduct a LC-50%-treatment with a contest success function analog to the LC-Treatment in Chapter 3. It deploys a standard lottery contest which implies equilibrium behavior in pure strategies.

Design and procedure of the experiment remains the same. The only difference is that the winner of a single match in part two of the experiment is now determined as described in the instructions for the LC-Treatment in Chapter 3.B.3. Therefore the control questions are also adjusted accordingly. Two sessions with 15 subjects each were conducted in November 2019. Further sessions were suspended due to regulations related to the CoVid-19 pandemic. In the following we present the preliminary results.

Unadjusted | Adjusted

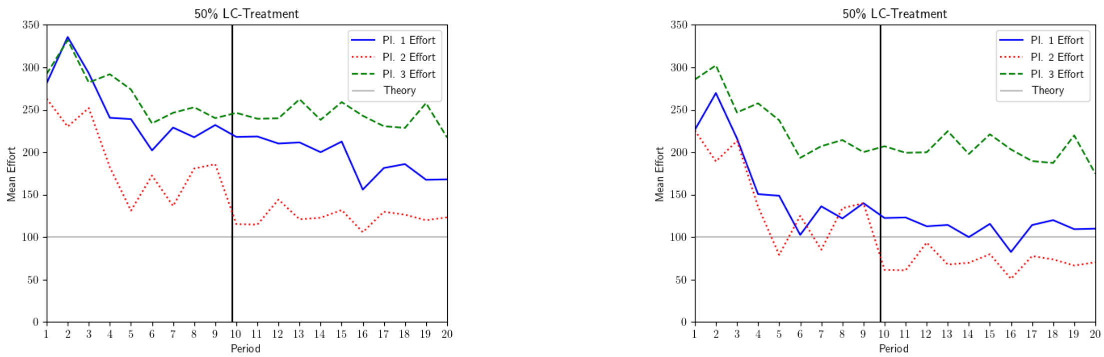


Figure 4.7: Mean effort LC-50%-treatment

Figure 4.7 illustrates the choice evolutions for each, the unadjusted and the adjusted sample. A clear downward trend is observed for all players irrespective of the sample. In the adjusted sample a parallel downward shift for all player types is visible compared to the unadjusted sample. In particular for player 2 we find that effort is stabilized only after nine periods. Table 4.16 shows great differences between the average effort for each player type in the first nine period (F9) and in the last eleven periods (L11).

Table 4.16: Average total effort per player type in LC-50%

	LC 50%-treatment					
	Unadjusted			Adjusted		
	F9	L11	T	F9	L11	T
P1	252.2	193.6	100	168.0	111.3	100
P2	192.7	123.2	100	147.5	70.2	100
P3	271.7	242.0	100	238.3	202.2	100
$\sum$	238.9	186.3	100	185.2	128.6	100

Note: F9 $\triangleq$ first 9 periods; L11 $\triangleq$ last 11 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum} \triangleq$ mean of the sum

Table 4.17: Panel Estimation for changes of efforts across periods in LC-50%

Dep. Variable	Total Effort			
Treatment	LC-50%			
Dataset	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	194.522***	(50.312)	106.157**	(48.062)
Player				
2	-71.163	(125.450)	-35.768	(23.651)
3	45.643	(156.987)	91.097***	(22.632)
Period trend				
[=1/Period]				
×*Player 1*	141.496*	(80.253)	170.590***	(63.836)
×*Player 2*	173.059***	(60.034)	192.288***	(45.279)
×*Player 3*	84.527*	(47.161)	117.925***	(14.034)
Observations	600		520	
Subjects	30		26	
R^2	0.062		0.148	

Note to Table 4.17: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

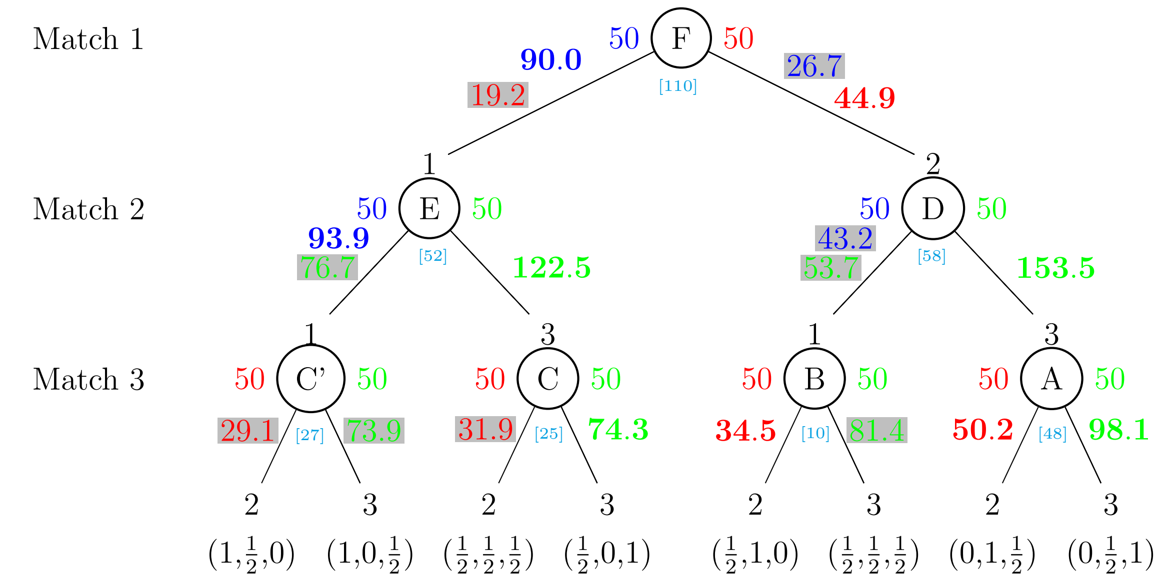
To provide statistical evidence, we estimate a panel PER analogue to the ones in Table 4.4. Table 4.17 reports the results. We find significant downwards trends across periods for all player types especially distinct in the adjusted sample. Therefore we focus our analysis here on L11 in the adjusted sample.

For a detailed insight in within tournament dynamics, Figure 4.8 (Figure 4.23) shows a win/lose structure for the adjusted sample (unadjusted sample in Appendix 4.A.6). It reveals mixed results for all player types. After a loss in her match each player type increases effort except for player 3 in node C'. In contrast, after a win player 1 and player 2 in node A slightly increase effort whereas player 3 always decreases effort.

To statistically test these findings, we estimate a PEF similar to one in Table 4.13. Table 4.18 reports the results for each, the unadjusted and adjusted sample. Actually, a winner decreases effort and a loser increases effort in the following match. However, these effects are insignificant due to the small sample size.

Additionally, we investigate dynamics between tournaments with a PEF similar to the one in Table 4.15. Table 4.19 shows the results. We detect that a tournament outcome affects effort choice in the subsequent tournament. A tournament winner (loser) decreases (increases) effort. Likewise, however, the coefficients are not significant.

By and large, our preliminary results of the LC-50%-treatment strengthen the perception of a reverse psychological momentum while strategic effects are absent. In the LC-50%-tournaments equilibrium play is characterized by pure strategies that are identical for every player type in every match irrespective of previous match outcomes. In contrast, in the 50%-treatment the equilibrium consists of mixed strategies and randomization is required. However, as we discover similar effects in both treatments, this strongly suggests that dynamic behavior in three-player round-robin tournaments in general is guided by psychological effects.



Note: blue $\triangleq$ player 1 mean effort; red $\triangleq$ player 2 mean effort; green $\triangleq$ player 3 mean effort; **bold** $\triangleq$ mean effort of subjects who **win** their first match; gray box $\triangleq$ mean effort of subjects who lose their first match; normal $\triangleq$ equilibrium predictions; bracketed $\triangleq$ absolute frequency.

Figure 4.8: LC-50%-treatment: win/lose effort (adjusted)

Table 4.18: Panel Estimations for Effort Dynamics: LC-50% L11

Dep. Var.	effort_2 - effort_1			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	9.795	(6.278)	5.532	(2.778)
Winner_1	-19.833	(11.321)	-21.161	(5.517)
Obs.	330		286	
Subjects	30		26	
R^2	0.013		0.060	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific fixed effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.19: Panel Estimations for Effort Dynamics between Tournaments: LC-50% L11

Dep. Variable	T_effort_(t) - T_effort_($t-1$)			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	-26.989*	(3.567)	-29.778	(7.82)
Rank($t-1$)				
2	24.234	(6.528)	26.186	(12.528)
3	43.150*	(4.172)	44.961	(9.344)
Observations	330		286	
Subjects	30		26	
R^2	0.016		0.017	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific fixed effects error structure. Significance level: *** (1%), ** (5%), * (10%)

4.6 Conclusion

In this article we have introduced a laboratory experiment that investigates how the prize structure influences the intensity, fairness, and dynamic behavior in round-robin tournaments with three players as a frequently used form of dynamic contests. Each single match is organized as an all-pay auction and the third prize is normalized to zero. We compare three treatments: tournaments with a second prize equal to either 0%, 50%, or 100% of the first prize.

While theory predicts the 50%-treatment to be most intense, we find that aggregate effort is highest in the 0%-treatment. Our evidence supports, though, the predicted late mover disadvantage (advantage) in the 0%-treatment (100%-treatment) and fair ranking in the 50%-treatment. Together, these experimental findings suggest that a prize-allocating contest designer of three-player round-robin tournaments faces a trade-off between maximizing intensity and fairness (although theory does not predict such a trade-off if matches are organized as all-pay auctions, see Laica et al., 2021).

Moreover, in line with the theory, we identify a *strategic* (reverse) momentum in the players' dynamic behavior: after winning the first match, a player increases (decreases) effort in the second match of the 0%-treatment (100%-treatment). Several findings suggest, however, that dynamic behavior is also subject to a reverse *psychological* momentum. Additional experiments that include a more comprehensive analysis of strategically neutral 50%-treatments with pure strategy equilibria (like in the case of matches organized as lottery contests) could help to determine whether this "hare-tortoise" heuristic proposed by Tong and Leung (2002) generally applies in round-robin tournaments.

In view of the announced transition from a four-player group stage to a three-player group stage in the first round of the FIFA World Cup 2026, the experimental study of round-robin tournaments with four players and multiple prizes could provide further insights. Besides the investigation of momentum effects, an open question is whether the theoretical prediction that sequential round-tournaments with more than three players are never ex-ante fair (Laica et al., 2021) also holds empirically.

Appendix

4.A Additional Figures and Tables

4.A.1 Player's Match 1 and 2 Choice Evolutions

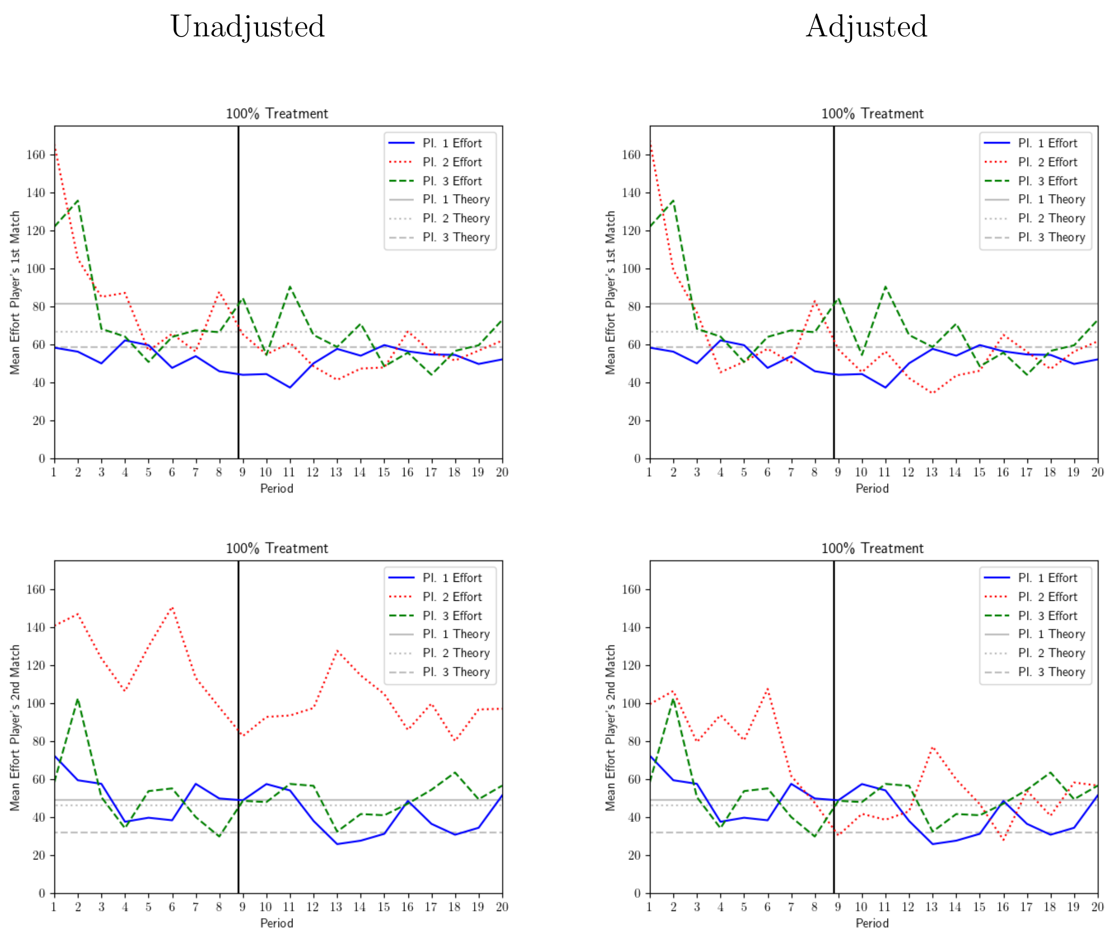


Figure 4.9: 100%-treatment: Mean effort Player's 1st and 2nd Match

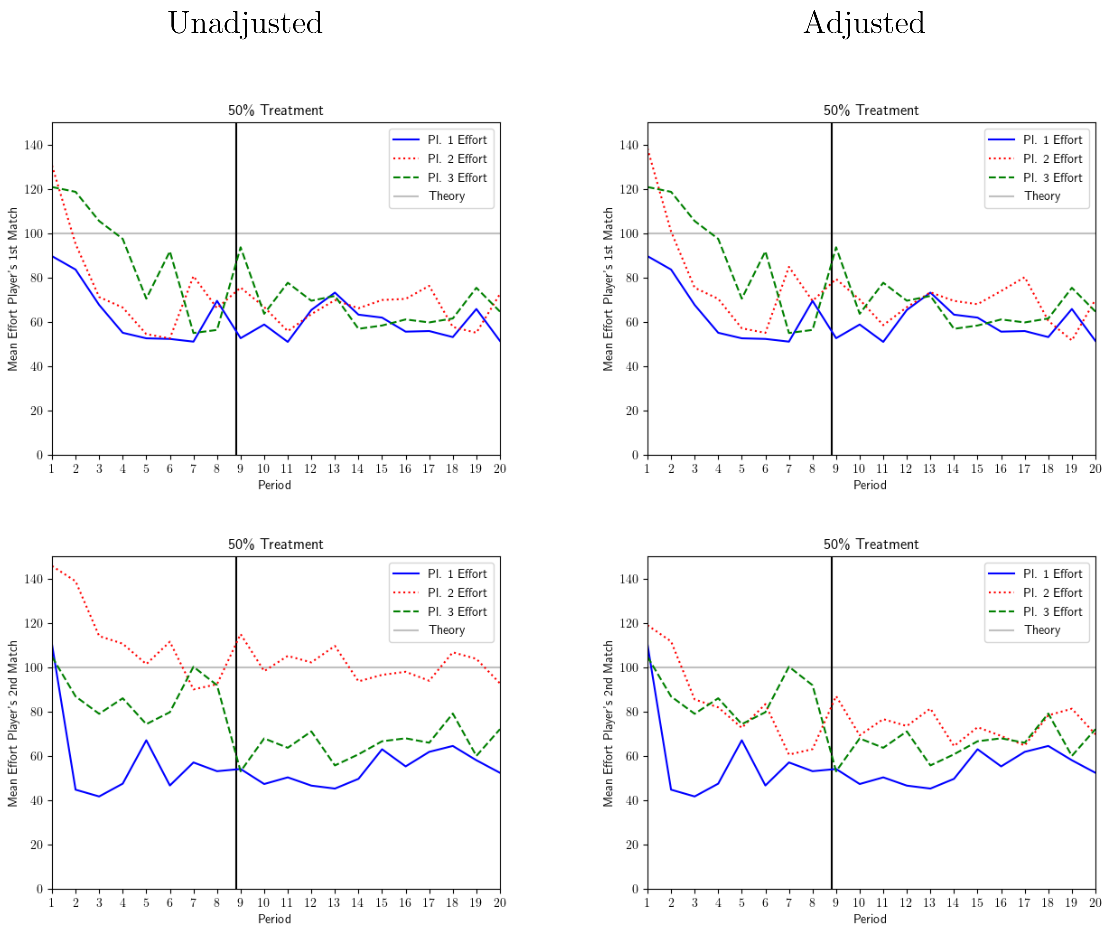


Figure 4.10: 50%-treatment: Mean effort Player's 1st and 2nd Match

Table 4.20: Individual mean player's 1st match effort choices per player type

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	54.3	51.3	81.6	65.3	59.1	100	163.3	83.7	28.1
P2	88.7	55.0	66.6	77.3	66.7	100	112.1	71.8	75.0
P3	79.9	63.5	58.5	89.6	67.9	100	150.6	150.9	37.5
$\sum$	74.3	56.6	68.9	77.4	64.6	100	142.0	102.2	46.9

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum} \triangleq$ mean of the sum

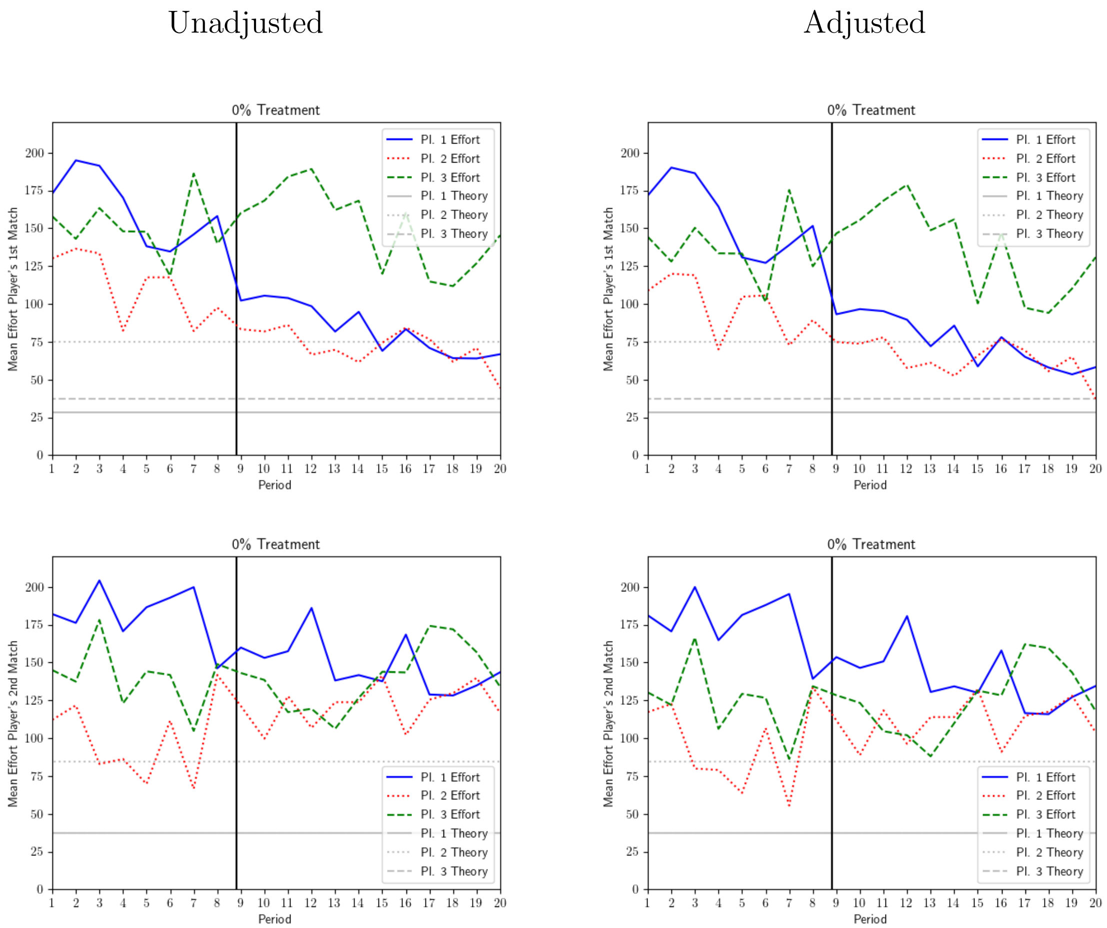


Figure 4.11: 0%-treatment: Mean effort Player's 1st and 2nd Match

Table 4.21: Individual mean player's 1st match effort choices per player type (adjusted)

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	54.3	51.3	81.6	65.3	59.1	100	157.6	75.3	28.1
P2	78.9	51.1	66.6	81.5	68.6	100	98.8	64.0	75.0
P3	79.9	63.5	58.5	89.6	67.9	100	136.4	136.2	37.5
$\sum$	70.7	55.5	68.9	78.8	65.2	100	130.8	91.14	46.9

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum}\triangleq$ mean of the sum

Table 4.22: Individual mean player's 2nd match effort choices per player type

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	51.7	40.5	49.1	58.6	54.1	100	182.3	148.2	37.5
P2	126.2	97.8	46.4	113.2	101.4	100	99.1	121.5	84.7
P3	53.1	49.8	32.0	88.0	65.4	100	140.4	139.6	37.2
$\sum$	77.0	62.7	42.4	86.6	73.7	100	140.6	136.4	53.1

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum} \triangleq$ mean of the sum

Table 4.23: Individual mean player's 2nd match effort choices per player type (adjusted)

	100%-treatment			50%-treatment			0%-treatment		
	F8	L12	T	F8	L12	T	F8	L12	T
P1	51.7	40.5	49.1	58.6	54.1	100	177.6	139.9	37.5
P2	84.6	48.0	46.4	84.8	74.1	100	94.8	110.9	84.7
P3	53.1	49.8	32.0	88.0	65.4	100	125.2	124.9	37.2
$\sum$	62.3	46.0	42.4	77.0	64.4	100	132.6	125.2	53.1

Note: F8 $\triangleq$ first 8 periods; L12 $\triangleq$ last 12 periods; T $\triangleq$ equilibrium predictions; $\bar{\sum} \triangleq$ mean of the sum

Table 4.24: Panel Estimation for changes of efforts across periods: Player's 1st Match

Dep. Variable	Effort Player's 1st Match					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	51.10***	(7.91)	55.23***	(4.38)	91.19***	(22.14)
Player						
2	-3.77	(6.69)	4.00	(6.12)	-18.19	(21.28)
3	5.19	(18.24)	8.17***	(2.87)	58.02*	(31.77)
Period trend						
[=1/Period]						
×*Player 1*	7.90	(29.72)	35.46***	(9.90)	135.41***	(21.52)
×*Player 2*	117.67***	(35.53)	65.22***	(19.62)	82.98**	(35.86)
×*Player 3*	76.60***	(21.66)	73.55**	(29.64)	8.79	(50.96)
Observations	1020		1080		1380	
Subjects	51		54		69	
R^2	0.072		0.035		0.062	

Note: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.25: Panel Estimation for changes of efforts across periods: Player's 1st Match (adjusted)

Dep. Variable	Effort Player's 1st Match					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	51.10***	(7.91)	55.23***	(4.38)	82.45***	(19.01)
Player						
2	-10.16	(8.01)	5.55	(4.70)	-16.59	(10.36)
3	5.19	(18.24)	8.17***	(2.87)	51.97	(36.96)
Period trend						
[=1/Period]						
×*Player 1*	7.90	(29.72)	35.46***	(9.90)	143.34***	(17.63)
×*Player 2*	118.26***	(43.06)	72.22***	(22.11)	66.84**	(17.43)
×*Player 3*	76.60***	(21.66)	73.55**	(29.64)	10.21	(55.98)
Observations	980		1060		1300	
Subjects	49		53		65	
R^2	0.077		0.038		0.062	

Note: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.26: Panel Estimation for changes of efforts across periods: Player's 2nd Match

Dep. Variable	Effort Player's 2nd Match					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	38.40***	(6.03)	48.29***	(7.32)	152.87***	(32.80)
Player						
2	60.41*	(30.95)	47.84**	(22.05)	-37.07	(43.72)
3	8.18	(7.79)	18.70***	(5.06)	-14.33	(27.91)
Period trend						
[=1/Period]						
×*Player 1*	36.49	(25.23)	42.53**	(19.20)	49.82	(38.63)
×*Player 2*	57.49**	(28.07)	55.41***	(18.21)	-18.03	(34.46)
×*Player 3*	25.41	(33.90)	41.38**	(17.82)	7.72	(30.64)
Observations	1020		1080		1380	
Subjects	51		54		69	
R^2	0.073		0.054		0.021	

Note: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific random effects error structure, with the individual subject as the random effect. Significance level: *** (1%), ** (5%), * (10%)

Table 4.27: Panel Estimation for changes of efforts across periods: Player's 2nd Match (adjusted)

Dep. Variable	Effort Player's 2nd Match					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	38.40***	(6.03)	48.29***	(7.32)	152.87***	(32.80)
Player						
2	11.52	(7.44)	20.13**	(9.65)	-37.07	(43.72)
3	8.18	(7.79)	18.70***	(5.06)	-14.33	(27.91)
Period trend						
[=1/Period]						
×*Player 1*	36.49	(25.23)	42.53**	(19.20)	49.82	(38.63)
×*Player 2*	70.87***	(24.78)	55.51***	(19.37)	-18.03	(34.46)
×*Player 3*	25.41	(33.90)	41.38**	(17.82)	7.72	(30.64)
Observations	980		1060		1380	
Subjects	49		53		69	
R^2	0.028		0.038		0.021	

Note: Robust standard errors clustered at the session level in parentheses. Both Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

4.A.2 Intensity Determinants

Table 4.28: Panel Estimations for Intensity: L12

	Total Effort			
Constant	138.259***	(12.734)	160.548*	(86.958)
0%-treatment	100.335***	(29.047)	96.240***	(28.804)
100%-treatment	-18.923	(18.073)	-10.600	(21.989)
Correct CRT Answer			-21.112*	(11.104)
# SafeChoices			3.647	(7.079)
Female			6.546	(17.055)
Economic/BA Student			0.336	(18.864)
# Siblings			-13.915*	(7.705)
Participation Loteries			-0.893	(7.828)
Participation Games			-3.254	(9.798)
Ambition			2.786	(9.285)
Generosity			8.782**	(4.223)
Importance Payment			-25.553**	(10.273)
Importance Winning			25.886***	(6.871)
Observations	2088		2088	
Subjects	174		174	
R^2	0.091		0.235	

Note: Robust standard errors clustered at the session level in parentheses. Model include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.29: Panel Estimations for Intensity: L12 (adjusted)

	Total Effort			
Constant	129.547***	(9.952)	140.578**	(70.533)
0%-treatment	86.807***	(32.862)	80.971**	(35.911)
100%-treatment	-28.027**	(11.191)	-28.482*	(15.281)
Correct CRT Answer			2.396	(13.897)
# SafeChoices			-6.418	(4.542)
Female			16.815	(10.950)
Economic/BA Student			5.023	(13.215)
# Siblings			-14.279*	(7.576)
Participation Loteries			2.625	(4.420)
Participation Games			-1.033	(8.266)
Ambition			13.640**	(6.905)
Generosity			8.940***	(3.381)
Importance Payment			-27.302***	(7.630)
Importance Winning			20.761***	(6.249)
Observations	2004		2004	
Subjects	167		167	
R^2	0.097		0.241	

Note: Robust standard errors clustered at the session level in parentheses. Model include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

4.A.3 Fairness

Table 4.30: Panel Estimations for discrimination by WQP: L12 (adjusted)

Dep. Variable	WQP					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	0.750***	(0.008)	0.484***	(0.007)	0.290***	(0.053)
Player						
1	-0.196***	(0.007)	0.009	(0.012)	0.089*	(0.052)
2	-0.094***	(0.024)	0.028	(0.037)	-0.006	(0.105)
Observations	588		636		780	
Subjects	49		53		65	
R^2	0.029		0.001		0.009	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure.
Significance level: *** (1%), ** (5%), * (10%)

Table 4.31: Panel Estimations for discrimination by Payoffs: L12 (adjusted)

Dep. Variable	Payoffs					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	711.66***	(7.64)	660.14***	(13.17)	512.73***	(10.01)
Player						
1	-37.31**	(16.17)	23.80***	(8.58)	99.37***	(20.01)
2	-14.11	(16.20)	2.04	(2.33)	82.87**	(34.01)
Observations	588		636		780	
Subjects	49		53		65	
R^2	0.012		0.007		0.032	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure.
Significance level: *** (1%), ** (5%), * (10%)

4.A.4 Fairness Determinants

Table 4.32: Winner determinants: L12

	WQP					
	100%		50%		0%	
Constant	0.819*	(0.430)	-0.072	(0.138)	0.618***	(0.159)
Player 1	-0.192***	(0.040)	0.019	(0.041)	-0.021	(0.022)
Player 2	-0.100***	(0.013)	0.031	(0.038)	0.046	(0.111)
Correct CRT Answer	0.053	(0.089)	-0.055	(0.061)	-0.019	(0.031)
# SafeChoices	0.011	(0.013)	0.005	(0.004)	-0.029*	(0.015)
Female	-0.000	(0.085)	-0.035	(0.053)	-0.032	(0.030)
Economic/BA Student	0.022	(0.033)	-0.006	(0.065)	0.067*	(0.039)
# Siblings	-0.014	(0.016)	-0.008	(0.009)	-0.019	(0.025)
Participation Lotteries	0.029	(0.034)	0.005	(0.020)	-0.011	(0.026)
Participation Games	-0.031	(0.029)	0.043***	(0.004)	-0.006	(0.015)
Ambition	0.021	(0.036)	0.015	(0.015)	-0.005	(0.024)
Generosity	-0.022	(0.028)	0.032**	(0.014)	-0.027**	(0.012)
Importance Payment	-0.033**	(0.013)	-0.037**	(0.018)	-0.034**	(0.014)
Importance Winning	0.039***	(0.002)	0.097***	(0.007)	0.057***	(0.009)
Observations	612		648		828	
Subjects	51		54		69	
R^2	0.079		0.250		0.084	

Note: Robust standard errors clustered at the session level in parentheses. Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.33: Winner determinants: L12 (adjusted)

	WQP					
	100%		50%		0%	
Constant	0.764**	(0.364)	-0.055	(0.112)	0.457**	(0.187)
Player 1	-0.209***	(0.060)	0.020	(0.040)	-0.011	(0.040)
Player 2	-0.165***	(0.024)	0.031	(0.036)	0.023	(0.103)
Correct CRT Answer	0.054	(0.102)	-0.050	(0.066)	-0.009	(0.031)
# SafeChoices	-0.017	(0.026)	0.002	(0.007)	-0.028*	(0.015)
Female	0.010	(0.095)	-0.029	(0.060)	-0.046	(0.029)
Economic/BA Student	0.023	(0.034)	-0.005	(0.065)	0.072*	(0.043)
# Siblings	0.004	(0.019)	-0.007	(0.007)	-0.032	(0.025)
Participation Lotteries	0.024	(0.026)	0.005	(0.022)	-0.010	(0.026)
Participation Games	-0.022	(0.024)	0.042***	(0.002)	-0.002	(0.018)
Ambition	0.026	(0.028)	0.018	(0.017)	0.012	(0.030)
Generosity	-0.021	(0.025)	0.031**	(0.014)	-0.014	(0.009)
Importance Payment	-0.017	(0.015)	-0.040**	(0.018)	-0.027**	(0.011)
Importance Winning	0.042***	(0.007)	0.096***	(0.007)	0.051***	(0.012)
Observations	588		636		780	
Subjects	49		53		65	
R^2	0.081		0.249		0.083	

Note:Robust standard errors clustered at the session level in parentheses. Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.34: Payoff determinants: L12

	Payoffs					
	100%		50%		0%	
Constant	654.64***	(62.16)	692.13***	(17.51)	521.86***	(65.81)
Player 1	-42.63***	(9.95)	30.25***	(11.68)	95.69***	(18.43)
Player 2	-75.05***	(23.36)	10.57	(15.60)	99.70**	(42.50)
Correct CRT Answer	9.59	(35.93)	41.69***	(11.82)	-28.20	(17.96)
# SafeChoices	-16.01	(10.64)	-15.90	(9.77)	-4.72	(5.89)
Female	-39.29***	(12.64)	29.07***	(0.86)	-44.87**	(21.94)
Economic/BA Student	7.01	(42.37)	4.69	(35.99)	36.50	(22.44)
# Siblings	20.85***	(7.54)	-1.68	(9.05)	15.31	(12.00)
Participation Loteries	0.66	(2.91)	3.80	(7.13)	-2.95	(13.44)
Participation Games	3.35	(5.54)	-2.53	(9.49)	13.20***	(2.65)
Ambition	5.16	(8.96)	9.44	(14.22)	-15.15***	(4.70)
Generosity	-8.73***	(2.03)	2.20	(3.08)	-24.31	(17.23)
Importance Payment	14.58*	(8.67)	-11.65	(10.32)	21.15***	(7.39)
Importance Winning	8.23	(7.72)	3.66	(7.35)	-3.55	(4.18)
Observations	612		648		828	
Subjects	51		54		69	
R^2	0.089		0.081		0.094	

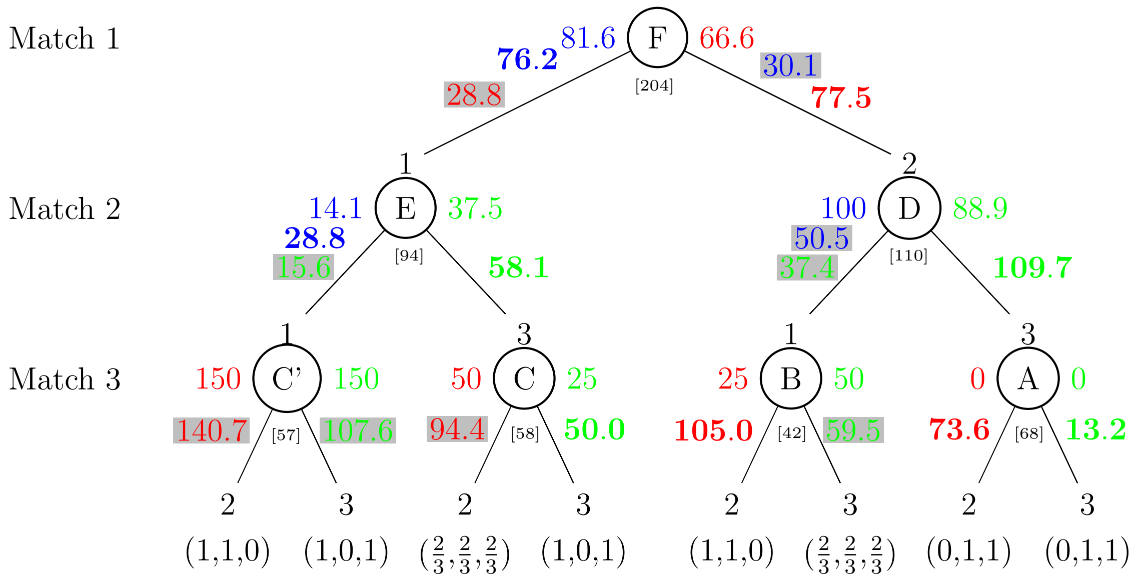
Note: Robust standard errors clustered at the session level in parentheses. Model include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

Table 4.35: Payoff determinants: L12 (adjusted)

	Payoffs					
	100%		50%		0%	
Constant	656.86***	(35.06)	605.06***	(32.21)	558.14***	(102.55)
Player 1	-32.01***	(10.72)	26.17**	(11.10)	79.14***	(16.87)
Player 2	-26.76***	(10.10)	12.11***	(2.41)	85.43***	(33.06)
Correct CRT Answer	-2.70	(25.03)	20.69*	(11.62)	-48.02**	(23.51)
# SafeChoices	0.59	(4.33)	-2.70	(7.22)	-2.89	(8.26)
Female	-39.49***	(5.87)	-0.21	(16.51)	-33.50*	(17.69)
Economic/BA Student	10.27	(23.36)	0.18	(23.39)	34.85	(25.09)
# Siblings	8.43	(7.71)	-5.60	(3.87)	19.41***	(6.57)
Participation Loteries	5.27	(3.81)	0.36	(2.09)	-9.97	(9.47)
Participation Games	-1.38	(5.81)	3.47	(4.88)	6.82*	(4.14)
Ambition	4.59	(5.95)	-5.41*	(2.91)	-16.11***	(6.22)
Generosity	-8.15	(5.79)	6.11***	(0.70)	-22.96	(14.65)
Importance Payment	5.98**	(2.98)	3.99	(2.59)	22.46***	(6.92)
Importance Winning	8.26**	(3.62)	6.15	(5.32)	0.92	(5.28)
Observations	588		636		780	
Subjects	49		53		65	
R^2	0.055		0.046		0.084	

Note: Robust standard errors clustered at the session level in parentheses. Model include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)

4.A.5 Dynamics



Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

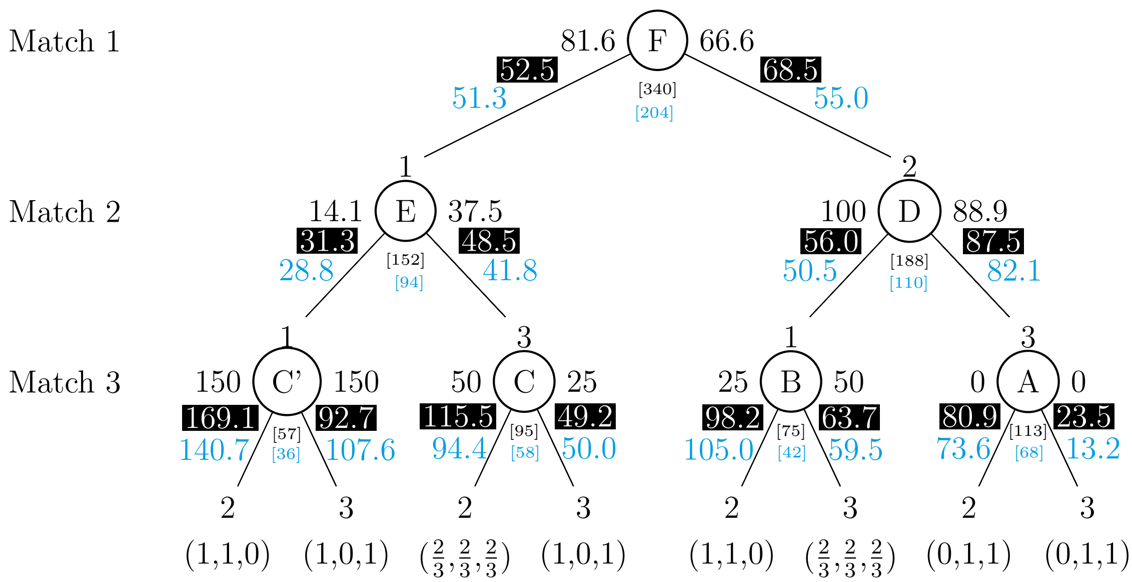
Figure 4.12: 100%-treatment: win/lose mean effort in L12

Table 4.36: Panel Estimations for Effort Dynamics: L12

Dep. Variable	effort_2 - effort_1					
Treatment	100%		50%		0%	
	Coef.	SE	Coef.	SE	Coef.	SE
Constant	50.61**	(6.95)	30.40***	(1.25)	-21.88*	(8.15)
Winner_1	-82.55**	(12.90)	-41.19***	(2.41)	111.53***	(16.18)
Observations	612		648		828	
Subjects	51		54		69	
R^2	0.119		0.058		0.060	

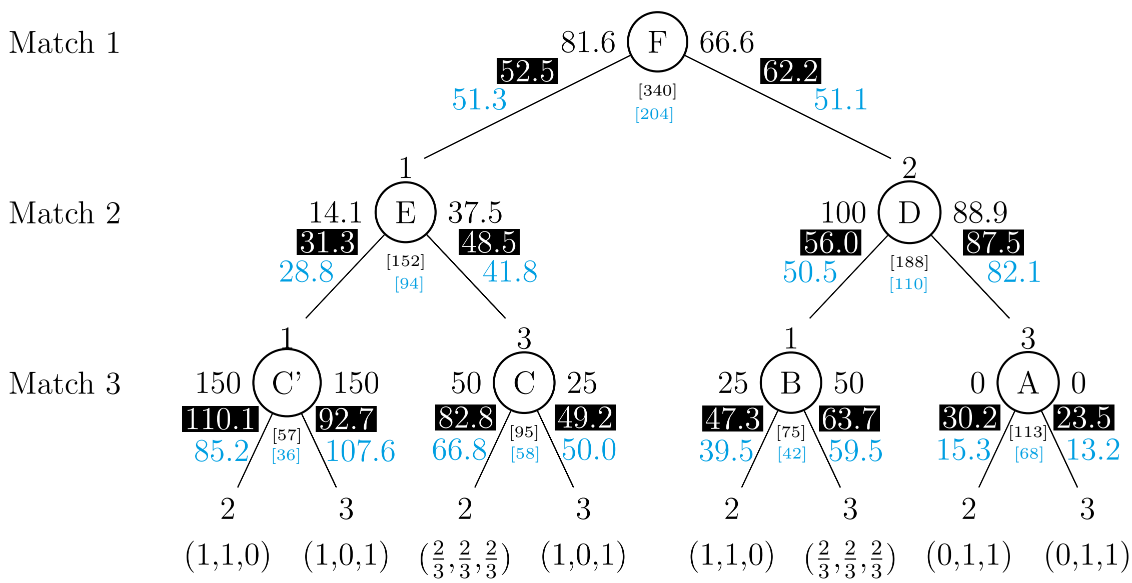
Note:Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific fixed effects error structure. Significance level: *** (1%), ** (5%), * (10%)

4.A.6 Complementary Analysis on Dynamics



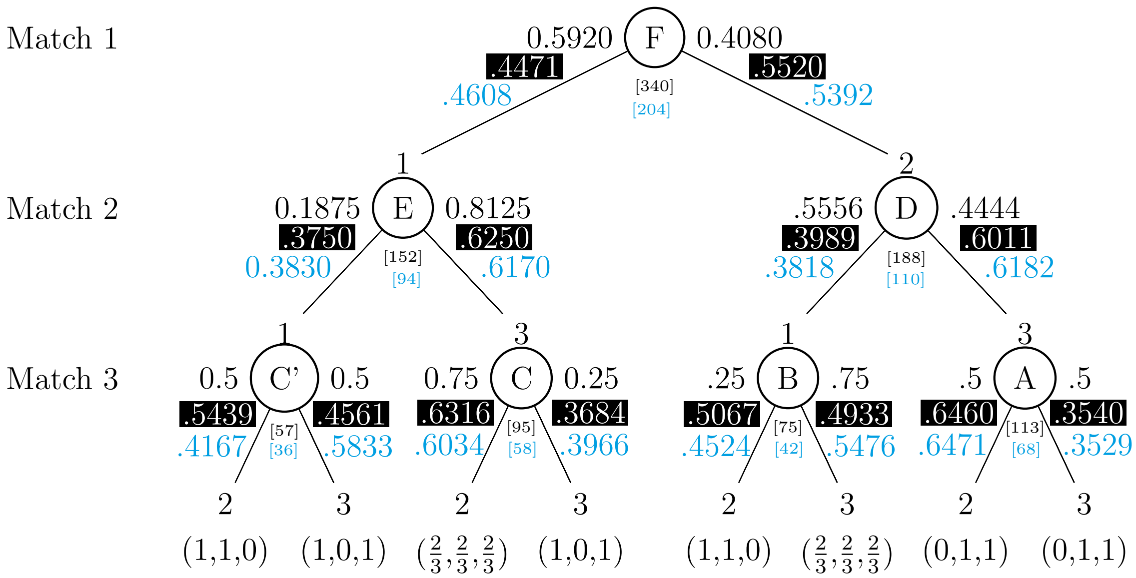
Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 4.13: 100%-treatment: mean effort



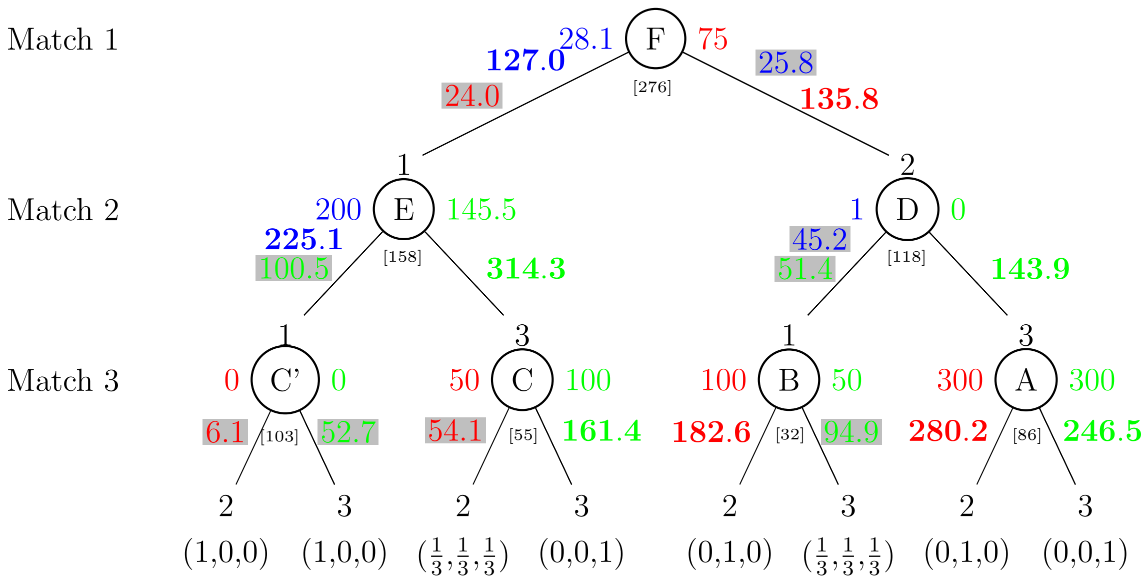
Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 4.14: 100%-treatment: mean effort (adjusted)



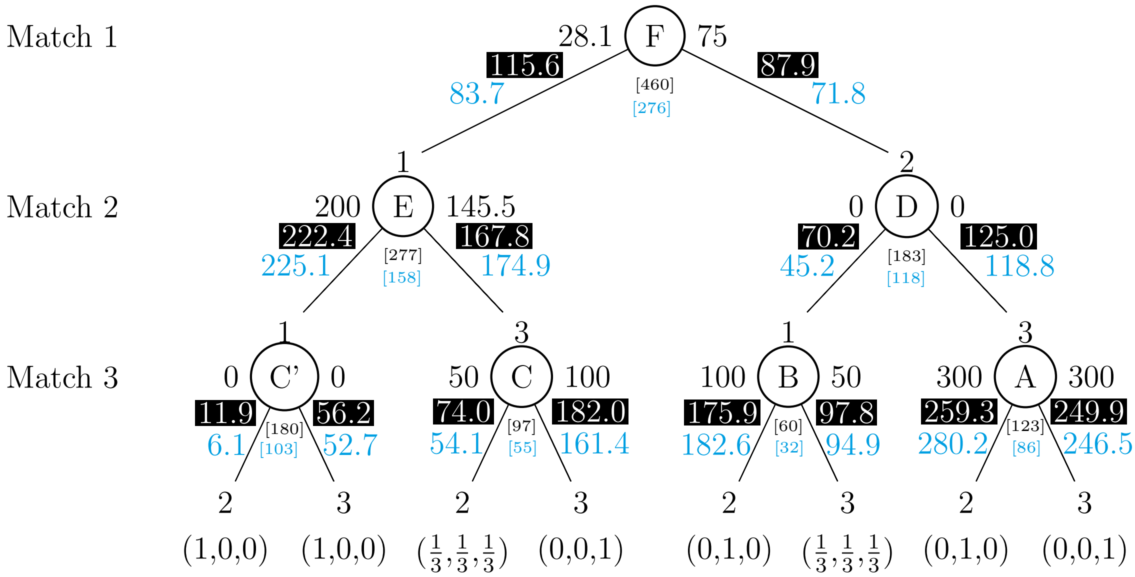
Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 4.15: 100%-treatment: Winning Probabilities



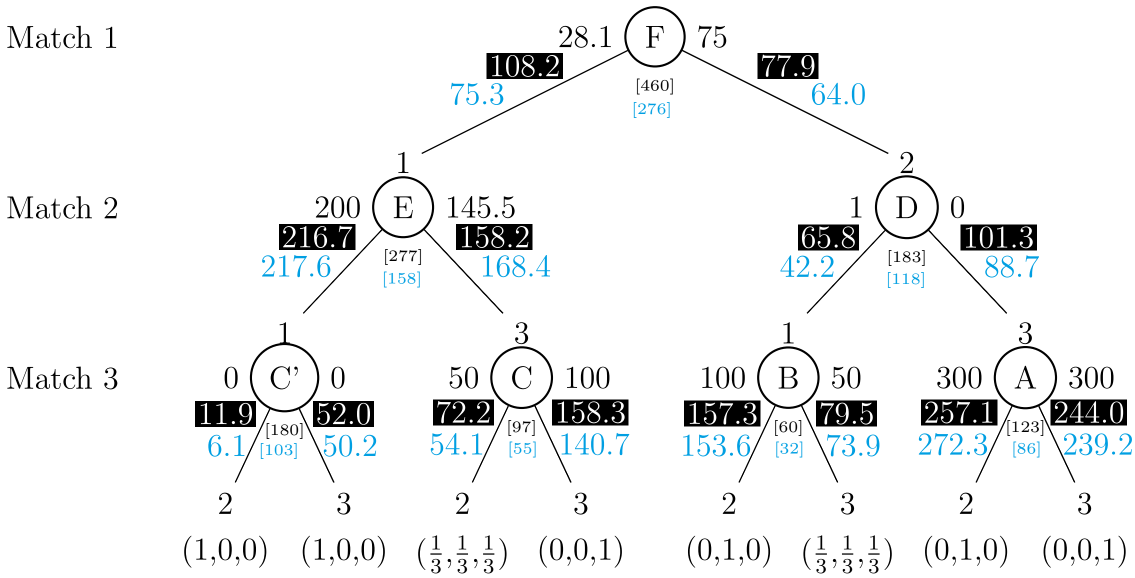
Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

Figure 4.16: 0%-treatment: win/lose mean effort in L12



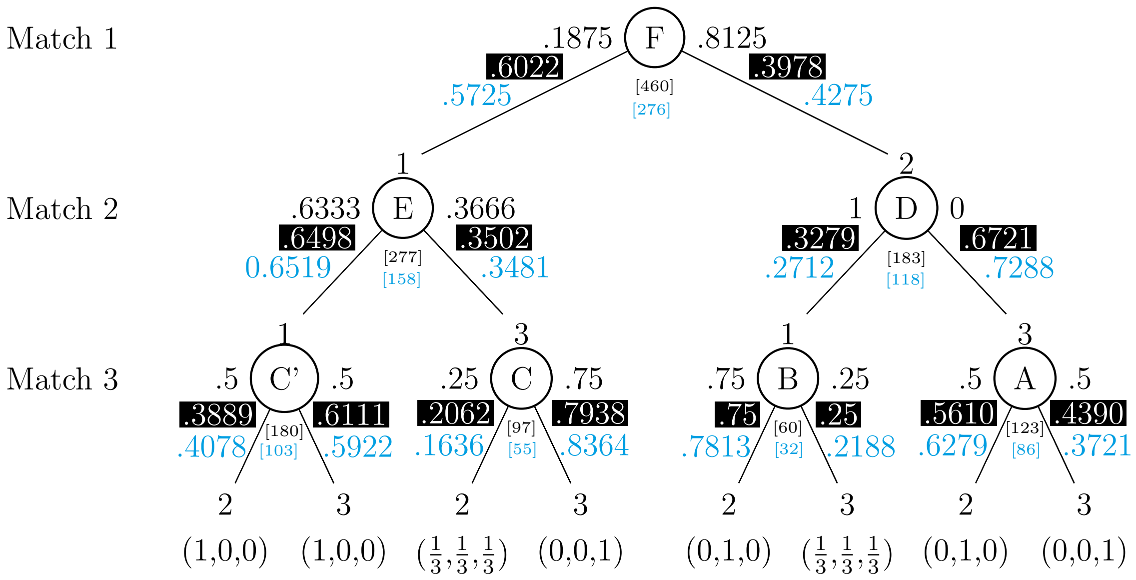
Note: black $\triangleq$ equilibrium predictions; black box $\triangleq$ A20; light blue $\triangleq$ L12.

Figure 4.17: 0%-treatment: mean effort



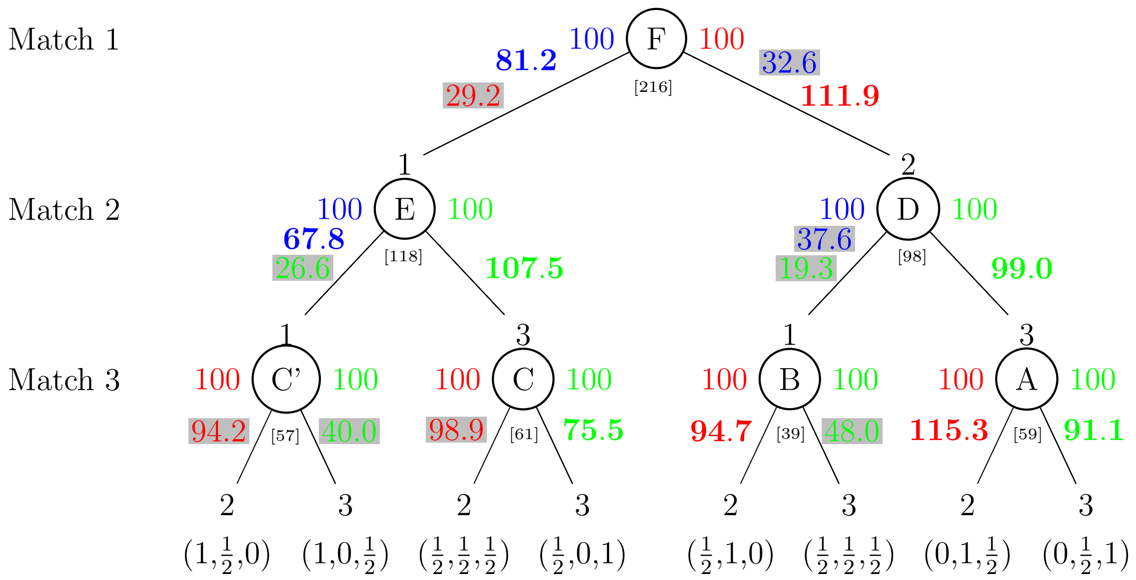
Note: black $\triangleq$ equilibrium predictions; black box $\triangleq$ A20; light blue $\triangleq$ L12.

Figure 4.18: 0%-treatment: mean effort (adjusted)



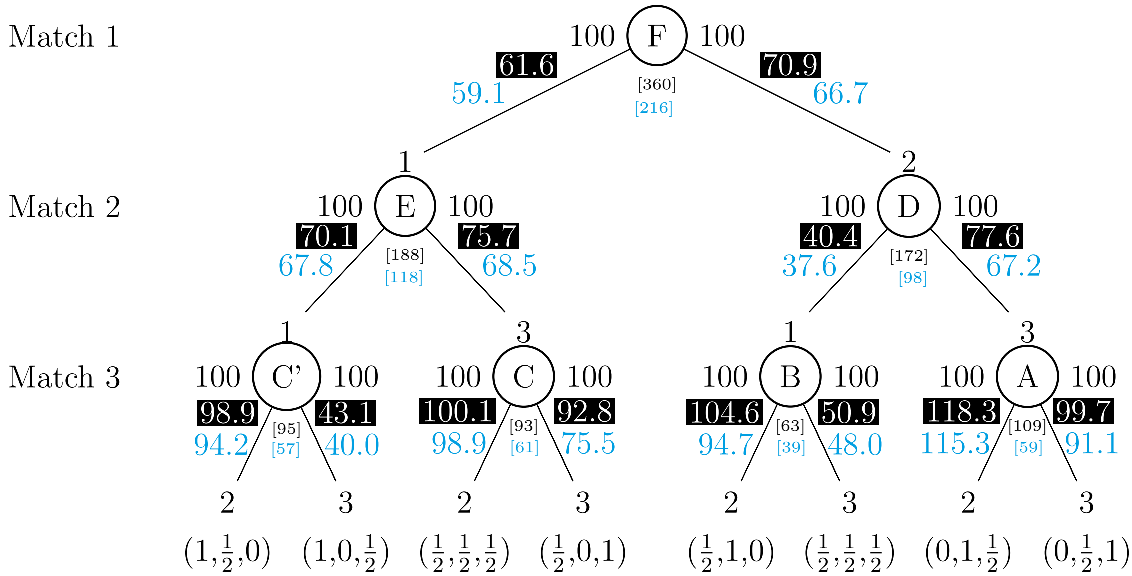
Note: black ≙ equilibrium predictions; black box ≙ A20; light blue ≙ L12.

Figure 4.19: 0%-treatment: Winning Probabilities



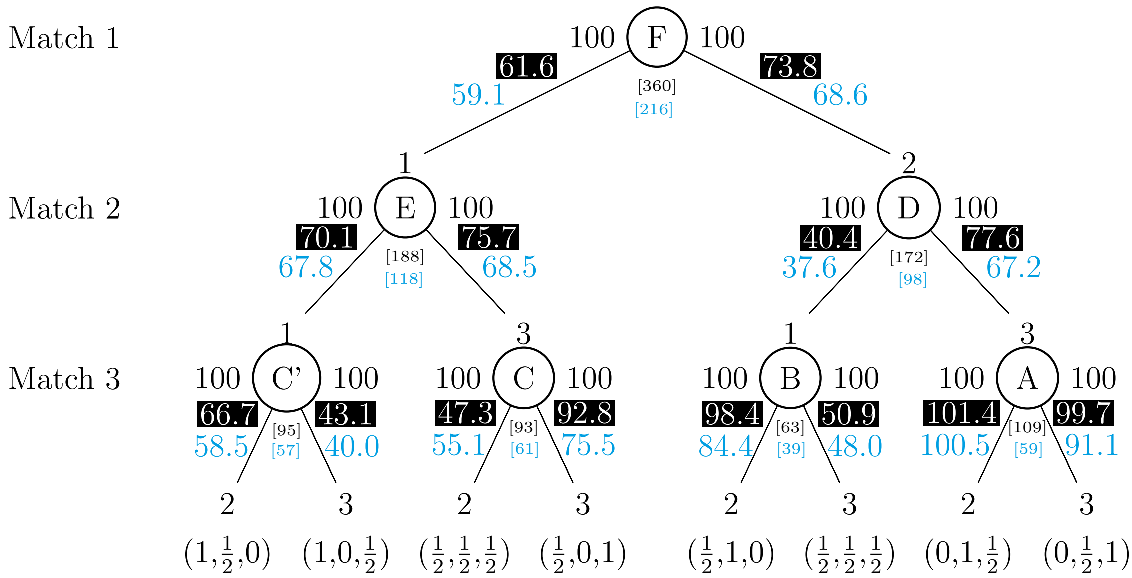
Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

Figure 4.20: 50%-treatment: win/lose mean effort in L12



Note: black $\triangleq$ equilibrium predictions; black box $\triangleq$ A20; light blue $\triangleq$ L12.

Figure 4.21: 50%-treatment: mean effort



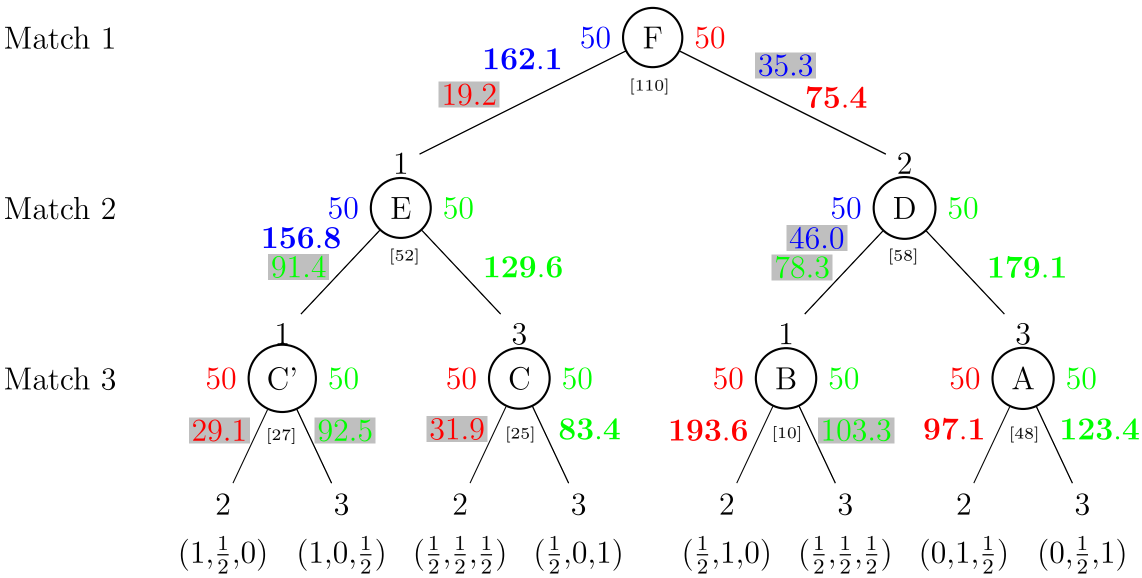
Note: black $\triangleq$ equilibrium predictions; black box $\triangleq$ A20; light blue $\triangleq$ L12.

Figure 4.22: 50%-treatment: mean effort (adjusted)

Table 4.37: Panel Estimations for Effort Dynamics for 0%-treatment without Player 3: L12

Dep. Variable	effort_2 - effort_1			
Sample	Unadjusted		Adjusted	
	Coef.	SE	Coef.	SE
Constant	-2.29	(5.62)	-2.39	(6.30)
Winner_1	118.77***	(11.24)	121.28***	(13.16)
Observations	552		528	
Subjects	46		44	
R^2	0.125		0.129	

Note: Robust standard errors clustered at the session level in parentheses. All Models include a subject-specific random effects error structure. Significance level: *** (1%), ** (5%), * (10%)



Note: blue ≙ player 1 mean effort; red ≙ player 2 mean effort; green ≙ player 3 mean effort; **bold** ≙ mean effort of subjects who **win** their first match; gray box ≙ mean effort of subjects who lose their first match; normal ≙ equilibrium predictions; bracketed ≙ absolute frequency.

Figure 4.23: LC-50%-treatment: win/lose effort

4.B English Translation of the Experimental Instructions

4.B.1 General Instructions

See Chapter 3.B.1.

4.B.2 Instructions for Part 1

See Chapter 3.B.2.

4.B.3 Instructions for Part 2

In the second part of the experiment you participate as a player in 20 independent tournaments. For each single tournament you are randomly allocated into groups of three. On that point you are randomly assigned a player from 1 to 3, i.e. you are either player 1, player 2, or player 3. Your player number remains the same across all 20 tournaments. Within a tournament you sequentially interact with every other player (opponent)in your group. The sequence is the following:

MATCH 1:	Player 1 vs. Player 2
MATCH 2:	Player 1 vs. Player 3
MATCH 3:	Player 2 vs. Player 3

In those matches the participating players make decisions, for each decision you have 35 seconds. In the meantime the non-participating player pauses but has to confirm with a click on 'OK'. Once all matches in a group are completed and consequently the tournament is finished, a new tournament, independent from the previous tournament, starts. For that, you are again randomly assigned with your player number into new composed groups of three. The players interact all over again in the illustrated sequence.

Your decisions in each tournament

You and your opponent compete in every tournament for prizes that you achieve as a 1st ranked and a 2nd ranked. The prize for the 1st rank equals

50%-treatment: $\boxed{R(1) = 400 \text{ Punkte.}}$

100%-treatment: $\boxed{R(1) = 300 \text{ Punkte.}}$

The prize for the 2nd rank equals

50%-treatment: $\boxed{R(2) = 200 \text{ Punkte.}}$

100%-treatment: $\boxed{R(2) = 300 \text{ Punkte.}}$

At the beginning of each tournament, that means after each group allocation, each of you receives, independently of the outcomes of previous tournaments, an *initial endowment* of

$$\boxed{I = 600 \text{ Punkten.}}$$

You can use this endowment to submit it in matches with your opponent. For this purpose you can submit any number of positive integer points Q_1 between 0 and your initial endowment of 600. In your second match you can submit any number of positive integer points Q_2 between 0 and your remaining endowment of $600 - Q_1$.

The winner...

...of a match: After you and your opponent have made your decisions the winner will determined in the as follows. You win the match, if you submitted a larger number of points than your opponent. In case you and your opponent have submitted an identical number of points, a computerized fair coin toss determines the winner.

...of a tournament: Tournament winner is the player who has the most wins in all matches, i.e. overall 2 wins. In case that all players in one group have won the same number of matches (1 win per player), the computer randomly draws a tournament winner. That means, each player's probability to win the tournament is identical.

Your final score at the end of a tournament

The points you have submitted in match are deducted from your endowment irrespective of the outcome of the match and of the tournament. You keep the remaining endowment. Your final score at the end of a tournament therefore equals

$$\boxed{finalscore = \begin{cases} I - Q_1 - Q_2 + R(1), & \text{if you are ranked 1st.} \\ I - Q_1 - Q_2 + R(2), & \text{if you are ranked 2nd.} \\ I - Q_1 - Q_2, & \text{if you are ranked 3rd.} \end{cases}}$$

At the end of each match, as a *participating* player your are informed about (i) the number of points you and your opponent submitted, (ii) the winner of the match, (iii) your total number of wins in matches,(iv) your current endowment. As a *non-particiapting* player you are only informed which of the participating player has won the match. Generally, the display is updated such that you are always informed about (i) your current endowment, (ii) the outcomes in previous matches, (iii) the standings. Exemplary, Figure 4.24 shows a input-screen and Figure 4.25 (Figure 4.26) shows an outcome-screen. At the end of each tournament you are informed if you have won the tournament and about your final score.

Example

The following tables illustrate the decision situation in the second part of the experiment with the help of a fictitious example.

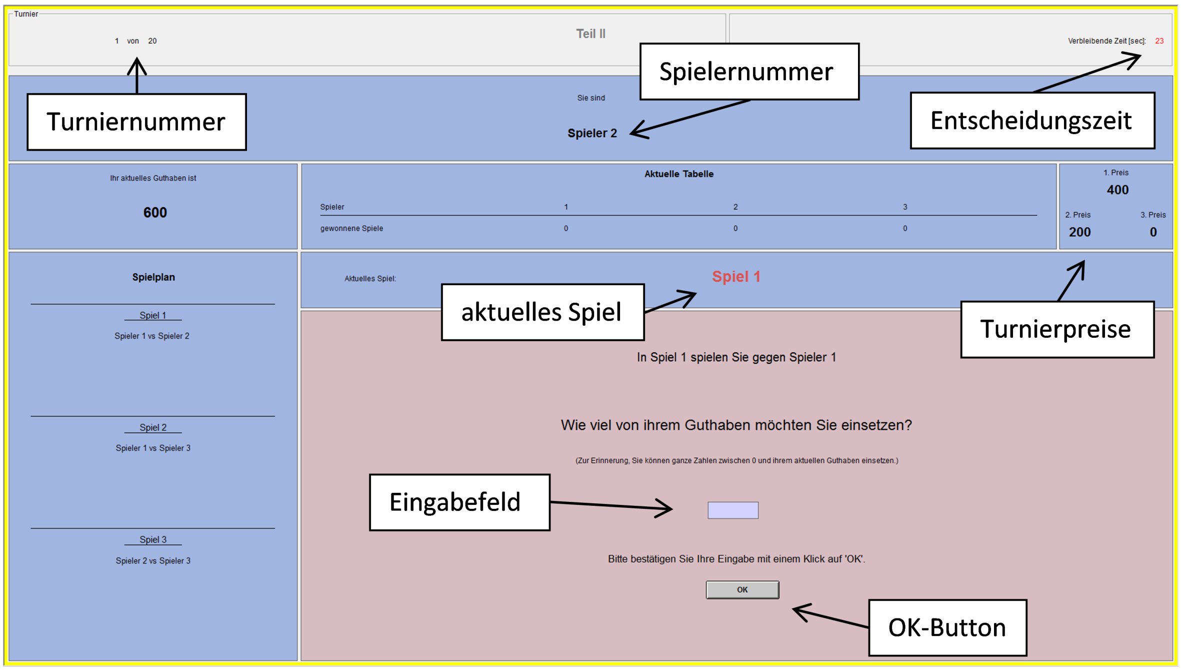


Figure 4.24: 50%-treatment Input-screen

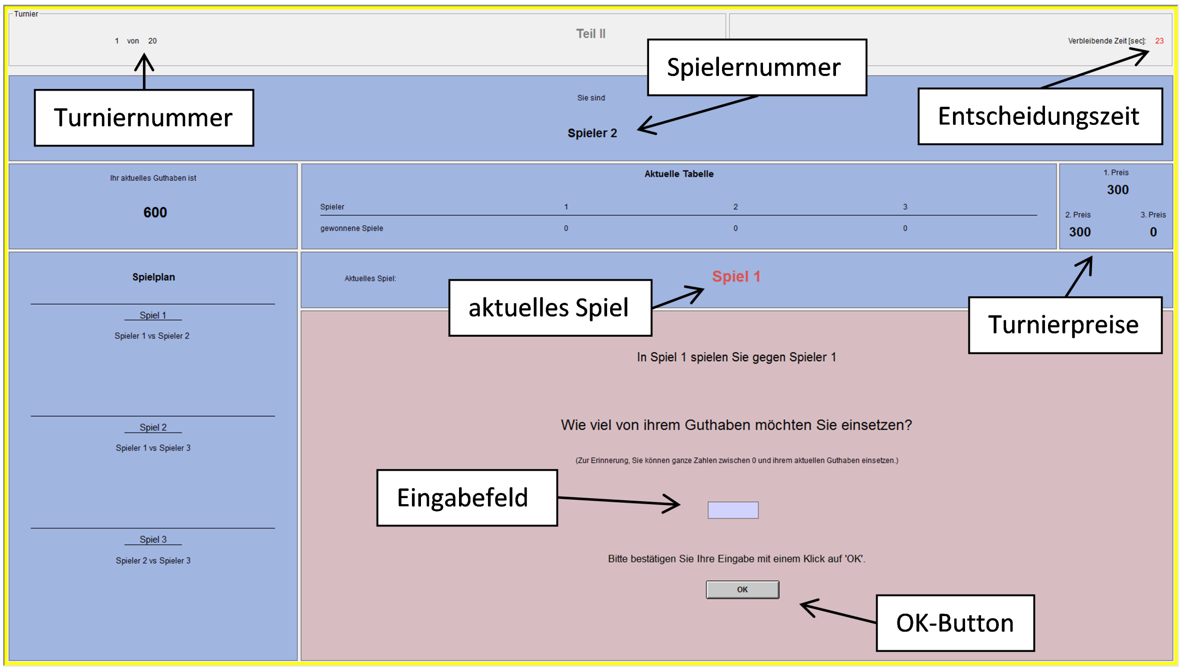


Figure 4.25: 100%-treatment Input-screen

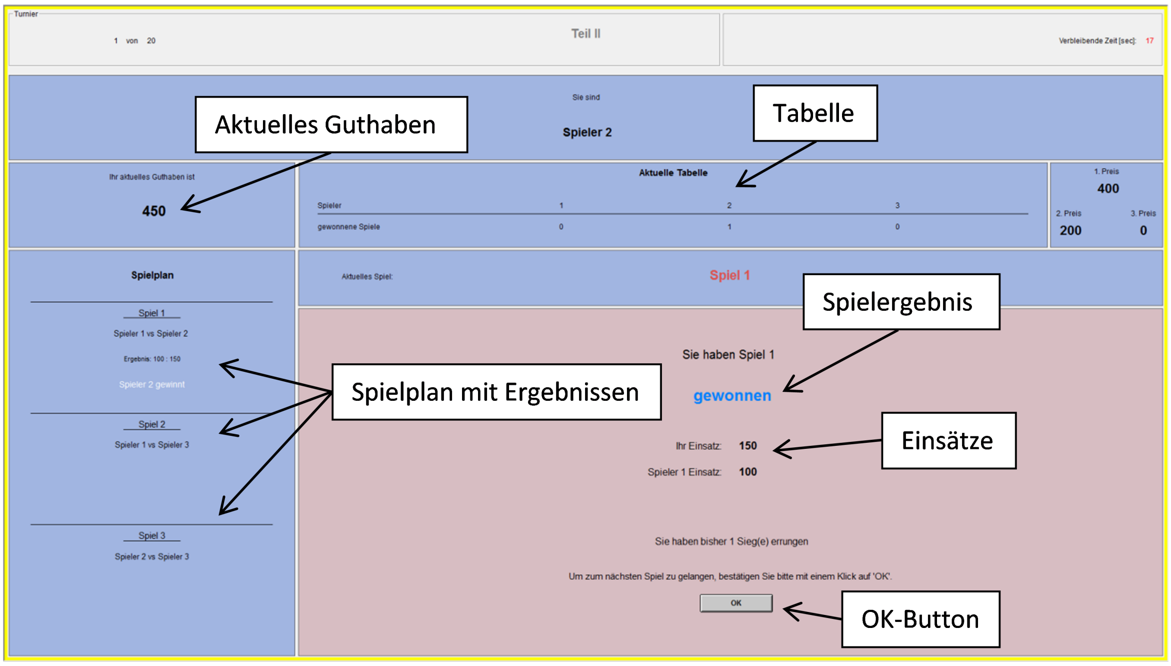


Figure 4.26: 50%-treatment Outcome-screen

Match	Player	Submitted Points	Winner	Current Endowment
1	Player 1	100	Player 2	$600 - 100 = 500$
	Player 2	150		$600 - 150 = 450$
2	Player 1	200	Player 1	$500 - 200 = 300$
	Player 3	100		$600 - 100 = 500$
3	Player 2	200	Player 2	$450 - 200 = 250$
	Player 3	100		$500 - 100 = 400$

Consequently the standings, with each player's final score and player 2 as a first ranked due to two wins in matches, player 1 as second ranked as the only one with one win in matches, yield as follows.

50%-treatment

Player	Wins	Rank	Finalscore
1	1	2nd	$600 - 100 - 200 + 200 = 500$
2	2	1st	$600 - 150 - 200 + 400 = 650$
3	0	3rd	$600 - 100 - 100 = 400$

100%-treatment

Player	Wins	Rank	Finalscore
1	1	2nd	$600 - 100 - 200 + 300 = 600$
2	2	1st	$600 - 150 - 200 + 300 = 550$
3	0	3rd	$600 - 100 - 100 = 400$

Your earnings in the second part

At the end of the 20 tournaments one tournament will be selected. Only the finalscore of this tournament determines your earnings in the second part of the experiment. To deterimene this tournament, one randomly selected participant will throw a 20-sided dice once. The score of this throw determines the tournament relevant for the earnings.

4.B.4 Control Questions

1. Which of the following statements is true?
   ☐ For every tournament you are the same player and you play in the same group.
   ☐ For every tournament you are the same player and you are newly drawn to a random group.
   ☐ For every tournament you are a newly drawn a random player number and you play in the same group.
   ☐ For every tournament you are a newly drawn a random player number and you are newly drawn to a random group.

2. What is your likelihood of winning a match, if you submit exactly half as many points as your opponent?
   ☐ 0 ☐ 1/2 ☐ 1/3

3. Is it possible to rank first in a tournament with one win in matches?
   ☐ Yes. ☐ No.

4. What is your final score, if you submit your entire endowment in a tournament and you do not rank first or second in the tournament?
   ☐ 0 points. ☐ 600 points.

5. If you ranked first in a tournament and you have submitted 300 points. How large is your endowment in the next tournament?
   ☐ 1200 points. ☐ 900 points. ☐ 600 points.

6. And if you ranked third with a submission of 300 points?
   ☐ 600 points. ☐ 300 points. ☐ 0 points.

4.C Experiment

4.C.1 Part 3

See Chapter 3.C.1.

4.C.2 Questionnaire [extract of Experiment related questions]

See Chapter 3.C.2,

Chapter 5

Mergers and Innovation Effort in R&D-Contests

Abstract

I examine the effects of a horizontal merger between two firms on the incentives to innovate and on welfare in an oligopolistic market. I develop a new dynamic model with innovation competition that shapes subsequent Cournot competition on the product market. I use a lottery contest with a contest success function that allows for a draw to model the innovation competition as a R&D-Contest with a difficulty to successfully innovate that possibly prevents an innovational breakthrough. In the presence (absence) of this difficulty, a successful innovation is uncertain (certain) and innovation effort is considered as (un-)productive. I show that there is a robust domain where mergers enhance the efficiency of R&D activity and, thus, total welfare. When effort is unproductive, a merger can reduce undesired duplicative R&D expenses. When effort is productive, a merger with sufficiently large R&D synergies in triopolistic markets provides additional incentives to innovate and increases the probability of a successful innovation.

Keywords: Horizontal Mergers; Lottery Contests with draw; Dynamic oligopoly; Competition Policy; R&D Synergies; Process Innovation

JEL classification: D43, D72, G34, L13, L41, O30

5.1 Introduction

Innovation is a key factor for economic growth. Firms compete in research and development (R&D) to be the first to invent new products and processes that enhance not only their own productivity, but ideally benefit the public. Therefore, it is essential to provide the appropriate environment to foster innovation effort.

In particular, market structures and the form of competition have a strong impact on the incentives to innovate. According to Shapiro (2011), the question how competition affects innovation is one of the most controversial in industrial organization. While Schumpeter (1942) argues that firms with more market power spur innovation, Arrow (1963) counters that more competition spurs innovation.

Mergers are one specific way to alter competition and thereby influence the innovation incentives. The more generally accepted view is in line with Arrow (1963): a merger negatively affects innovation effort as pressure to innovate is reduced due to weaker competition. For example, the *Economist* (2012)[1] claims that mergers in the North American pharmaceutical industry have been responsible for a substantial reduction in the sectoral R&D spending growth rates between 1988 and 2012.

However, an overall reduction in innovation effort may not necessarily reduce welfare. When many firms in a market try to innovate, incentives might be undesirably directed such that socially wasteful duplicative innovation effort occurs. As Loury (1979) already clarifies, "whether or not greater investment in innovation, that less competition [through a merger] might bring forth, is actually in the social interest is an unanswered question". In other words, it depends on the type of innovation competition whether more innovation effort is productive or unproductive.

Moreover, one must not disregard the synergies a merger possibly entails in the R&D process. In the sense of Schumpeter (1942), a merger might be desirable as the synergies might lead to more efficient R&D activities. Competition authorities (CA) therefore pay close attention on how mergers directly and indirectly affect innovation effort.

For instance, the US Merger guidelines quote that a CA "may consider whether a merger is likely to diminish innovation competition by encouraging the merged firm to curtail its innovation efforts below the level that prevail in the absence of the merger [and] [...] also consider whether the merger is likely to enable innovation that would not otherwise take place, by bringing together complementary capabilities that cannot be otherwise combined or for some other merger-specific reason" (section 6.4, p. 23 f). It further states that a CA has to take into account synergies and evaluate the merger on "the ability of the merged firm to conduct research or development more effectively" (section 10, p. 31). The European Commission (EC) Merger guidelines emphasize also that a merger might have (positive) effects on the incentives to innovate not only of the merged firm but also of the non-merged rival firms (par. 38).

To sum up, whether more innovation effort is desired in terms of welfare and how a merger affects innovation incentives, is generally ambiguous. Although, many merger cases[2] have been assessed by CA for innovation concerns, there is only little theoretical research on mergers and innovation compared to market concentration and innovation

[1] Accessed at https://www.economist.com/finance-and-economics/2012/08/25/arrested-development on 03/24/2021.

[2] For example, GE/Alstom, Dow/DuPont, Bayer/Monsanto.

in general.[3]

In this paper I explicitly analyze how a horizontal merger between two firms affects the efficiency of R&D activity and, consequently, welfare in an oligopolistic product market. For that purpose, I examine the incentives to innovate of all firms', merged and non-merged, in a R&D-Contest. To capture the type of innovation effort, I distinguish between two scenarios: firstly, a R&D-Contest in which the success of an innovation is certain, i.e. innovation effort is unproductive. Secondly, a R&D-Contest in which the success of an innovation is uncertain but more innovation effort increases the probability of a successful innovation, i.e. innovation effort is productive.

Beside the R&D-Contest, the product market plays a determining role as firms conduct innovation effort because they intend to gain a product market advantage due to lower marginal cost of production. Hence, I propose a dynamic model with innovation competition that shapes subsequent product market competition. When product market rivals merge to a single decision unit, it usually alters the market conditions in two ways: first, the number of competitors is reduced in both, the R&D-Contest and the product market. Second, asymmetries arise due to synergies between the merged firms. As the focus of this paper is on innovative effort, I only consider synergies in the R&D process that enhance innovation capabilities by combining complementary assets.[4]

I develop a new model for mergers and innovation that combines several theoretical elements. In line with Dasgupta and Stiglitz (1980a) and Delbono and Denicolo (1991), I design a dynamic model with two stages. In the first stage, in a simultaneous R&D-Contest, the winner is awarded a prize referred to as a patent that reduces marginal costs of production in the second stage in which firms encounter Cournot competition on the product market.[5] As in the classic patent race game by Loury (1979) there will be (at most) one successful innovator who exclusively gains the patent, irrespective whether innovation effort is productive or not. The remaining firms are left empty-handed despite their innovation effort. Within this model framework I use an approach which is unique in two dimensions.

Firstly, I employ a lottery contest, which is a special case of the Tullock (1980) rent-seeking contest, to model the innovation process.[6] It captures the important feature of a patent race that a successful innovation does not only depend on a firm's own choice of innovation effort but also on the competitors' choices of innovation effort. This is in contrast to the stochastic innovation model with two stages recently introduced to the literature on mergers and innovation by Federico et al. (2017, 2018) which neglects the

[3]T. Valetti recognized in 2017, then EC's Chief Competition Economist, at a conference at Northwestern University: "There's a huge body of [theoretical] research on the relationship between concentration and innovation[...], but when it comes specifically to mergers and innovation, there's much less."

[4]For instance, according to Shapiro (2011, p.389), in the pharmaceutical industry one requires "an effective R&D program to identify and develop promising new compounds, the skills necessary to navigate the long and complex FDA testing and approval process, possibly demanding manufacturing capabilities, and effective marketing and distribution".

[5]For example, currently in the automotive industry, e-car producers are situated in a patent race for a process innovation. To decrease their e-car production costs and gain a competitive advantage, they invest in R&D to reduce the cost-intense share of cobalt in the batteries. For that they vertically integrate supply chains to produce batteries in-house. At the moment Tesla requires less cobalt than its rivals.

[6]Jost and van der Velden (2006) analyze mergers in a lottery contest setting without possibility of a draw and Tu (2019) studies mergers in asymmetric all-pay auctions. However, both limit their work to the innovation competition and omit the product market.

impact of competitors on the own innovation success.

Secondly, I consider a generalized form of a lottery contest with a contest success function (CSF) that allows for a draw.[7] A draw denotes that some difficulty to successfully innovate exists that possibly prevents an innovational breakthrough despite innovation effort. Hence, in the presence (absence) of this difficulty, a successful innovation is uncertain (certain) which implies that innovation effort is (un-)productive.

A merger entails synergies in the R&D process and affects innovation effort through three channels. Directly via the R&D-Contest: firstly, due to reduced competition and, secondly, due to the asymmetry that arises as a result of the merged firms' R&D pooling. Indirectly via the product market as reduced competition changes profits of each single decision unit and, thus, the incentives to invest in R&D to gain the patent. Throughout the paper I analyze the effects of two-firm mergers with synergies that enhance the effectiveness of R&D investment by comparing market scenarios pre-merger, i.e. the market without merger, and post-merger, i.e. the same market with merger.

I show that a robust domain exists where a horizontal merger of two firms increases the efficiency of R&D activity and total welfare. This result is not trivial as innovation effort entails costs and a trade-off occurs between investing more to increase the probability of a successful innovation and investing less to save R&D expenses. In particular, the CA's goal of welfare maximizing and each firm's goal of profit maximization might be in conflict.

In R&D-Contests with unproductive innovation effort, R&D investment is considered as excessive expense that a CA wants to minimize to increase welfare. A marginal effort of a single firm would be socially optimal and the CA is indifferent which firm gains the patent. In contrast, each firm has incentives to increase innovation effort up to a certain level to maximize the probability that it is the firm which successfully innovates and gains the exclusive reduction in productions costs. A merger can thus reduce undesired duplicative R&D expenses.

When innovation effort is unproductive, I show that in triopolistic markets a merger always decreases total R&D expenses. In larger oligopolisitc markets, total R&D expenses are decreased if the R&D ability of the merged firm is sufficiently large or when the cost reduction (innovation size henceforth) due to a potential successful innovation is large enough. I find that the profitability of a merger is a sufficient condition for a reduction in total R&D expenses in a market if the innovation size is large enough. This, in turn, also implies that the merger increases total welfare. In contrast, consumer surplus (CS) is always reduced post-merger as the product market is more concentrated.

In R&D-Contests with productive innovation effort, both, the firms and the CA, pursue the goal of investment in R&D up to a certain level to increase the probability of a successful innovation. However, the optimal level of innovation effort might be different for a CA compared to firms. When the difficulty to successfully innovate is high (low), from the perspective of a CA (firms), there might be underprovision (overprovision) of innovation effort. A merger can thus provide additional incentives to innovate.

When innovation effort is productive, I analyze triopolistic markets and mergers with synergies that double the R&D ability of the merged entity relative to a non-merged firm. I show that when the difficulty to successfully innovate exceeds a certain level,

[7] Literature on the application of contests with draws is scarce. A notable exception are Bimpikis et al. (2016) who use a CSF with the possibility of a draw where two firms compete in marketing.

firms stop to engage in R&D. Eventually, a merged entity conducts innovation effort as it is more likely to overcome the difficulty to successfully innovate due to its increased ability in R&D. However, such a merger might be total welfare decreasing as the gains of a successful innovation do not compensate for the loss in CS due to reduced competition on the product market. Moreover, I prove that any merger (weakly) increases the probability of a successful innovation, irrespective of the difficulty to successfully innovate.

The remainder of this paper is organized as follows. Section 5.2 provides a short literature review. In Section 5.3 a formal model of dynamic R&D contests is introduced and solved. Section 5.4 analyzes the model for unproductive innovation effort. In Section 5.5, the model with productive innovation effort under some reasonable assumptions is examined. Section 5.6 concludes.

5.2 Literature Review

Mainly two approaches have been established to analyze the effects of innovation effort. The first branch considers non-stochastic models which assume that each investor in R&D certainly becomes a successful innovator, e.g. Dasgupta and Stiglitz (1980a). The second branch, to which this article belongs, considers stochastic models which assume that greater investment in R&D increases the firms probability of becoming a successful innovator. These comprise the seminal static patent race models by Loury (1979) and its reformulation by Lee and Wilde (1980), and Dasgupta and Stiglitz (1980b).[8]

Similar to my model, Loury (1979) employs a non-cooperative game where players simultaneously invest in R&D with the goal to innovate first. Only the firm that firstly innovates is awarded a prize in the form of a patent whereas the other firms gain nothing. However, as he omits the product market, the prize is exogenously given and independent of the number of firms. This is in contrast to my model where a process innovation reduces marginal costs of production on the product market and, thus, changes profits dependent on the number of competitors on the product market.

Contrary to Loury (1979), I exploit as a stochastic innovation model a lottery contest that allows for a draw which is a special generalization of the Tullock (1980) rent-seeking contest. Konrad (2009, Chapter 2.3) surveys the investment incentives in lottery contests without draw. Total investment in equilibrium decreases in the number of contestants whereas increases in the value of the prize. Stein (2002) shows that total equilibrium investment decreases as the difference in contestants' ability increases.

In lottery contests that allow for draws, Nti (1997) shows that for symmetric contestants the individual equilibrium investment increases in the number of contestants, decreases in the size of the draw parameter (here the difficulty to successfully innovate), and increases in the value of the prize. For asymmetric contestants, Deng et al. (2018) show that the introduction of a draw decreases total investment but increases the winner's investment in equilibrium if the inequality in the valuation of the prize is sufficiently dispersed. However, in contrast to my model, both authorships simplify their models as they neglect the difference in prizes between the outcome of a draw and the outcome of a loss. They equally value the prize of both outcomes with zero whereas I assume that the prize of a draw is valued less than the prize of a win but more than the prize of a loss. An optimization problem which incorporates such a prize structure

[8]See Reinganum (1984, 1989) for surveys on classic patent race models.

is considered by Dasgupta and Nti (1998) but only to maximize aggregate effort for symmetric players.

As shown by Baye and Hoppe (2003), the classic patent race games pioneered by Loury (1979) and Dasgupta and Stiglitz (1980b) are strategically equivalent to a Tullock (1980) contest with a discriminatory power small enough which includes the lottery contest. When the discount rate in the classic patent race is larger than zero, it is strategically equivalent to a lottery contest with the possibility of a draw but without a distinct prize for a draw that differs from the prize of a loss equal to zero.

Dynamic innovation models with two stages that capture the role of the product market where quantity-setting symmetric firms compete à la Cournot subsequent of a non-stochastic innovation competition are neatly introduced by Dasgupta and Stiglitz (1980a). They show that an increase in the number of firms decreases each individual firm's investment in R&D, but increases the total investment in R&D.

In a model close to mine, Delbono and Denicolo (1991) extend the one-stage patent race model by Lee and Wilde (1980) which differs from my model and the model of Loury (1979) in the costs that R&D investments incur. They alike assume that symmetric oligopolists compete for an exclusive cost-reducing innovation previous to Cournot competition on the product market. Hence, the prize of the winner depends on the number of firms and firms make positive profits even in the absence of a successful innovation. In a simple model, they show that an increase in the R&D ability, an increase in the cost advantage, or a decrease in the discount rate increases total innovation effort in equilibrium. Moreover, if the number of firms is above some critical value, which depends on the innovation size, and for a low enough relation of ability to discount rate, an increase in the number of firms will reduce each firm's effort. In contrast to my model, they forfeit to consider how heterogeneity due to a merger affects the incentives to conduct R&D.

In line with my model, Matsushima and Yamazaki (2016) use a dynamic two-stage innovation competition model where they combine a Tullock contest to model a R&D-Contest and Cournot competition to model product market competition. They alike consider the case that one firm has more ability in R&D than the remaining equally able firms. However, in contrast to my model, their CSF incorporates no possibility of a draw and they particularly analyze the probability of obtaining an exclusive cost reduction under entry instead of mergers.

Recently, Marshall and Parra (2019) develop a sequential leader/follower extension of the classic patent race game by Loury (1979) and highlight the importance of product market profits on innovation. They find scenarios where weaker competition, e.g. by a merger, increase total investment in R&D and consumer surplus. In comparison to my model, they incorporate no possibility of a draw.

How horizontal mergers, that do not result in a monopoly, affect incentives to innovate, is sparsely investigated by theorists (for a survey see Jullien and Lefouili, 2018). Primarily, the issue of how competition in general affects innovation incentives is covered. In most of the literature, contrarily to my reasonable assumptions, the innovation process is non-stochastic .

Early literature that analyzes mergers in non-stochastic two-stage games focuses on impacts on merged and non-merged firms separately: Davidson and Ferrett (2007) consider two-firm mergers that exploit R&D complementarities to invest for a process innovation that lowers marginal costs of production in the following Cournot competition. They find that changes in innovation effort depend on the level of synergies. Due to a market power effect merged (non-merged) firms are less (more) aggressive after the merger. By contrast, a R&D pooling effect makes the merged firms more aggressive. Matsushima et al. (2013) analyze heterogeneous firms and find that a merger of more able firms always increases the investment in R&D of each firm in the market. Kleer (2012) finds that if firms are asymmetric after a two-firm merger in a triopolistic market, merged firms invest more and the non-merged firms less in R&D.

Later works with non-stochastic models analyze how a merger affects total market innovation effort: in their working paper Motta and Tarantino (2017) analyze R&D investment in process innovation that lowers marginal costs of production in a subsequent market with price competition and differentiated products. They conclude that a merger without synergies will reduce total investment in R&D and harm consumers. However, if synergies are large enough, total R&D investment might increase such that it positively affects consumer welfare. Haucap et al. (2019) consider triopolisitc markets where two efficient and one inefficient firm engage in product innovation and compete à la Cournot. They find that a merger of an efficient and inefficient firm reduces innovation effort of the merged and the non-merged firm in industries with high research intensity. Etro (2019) studies mergers of firms producing complementarities. He shows that the merged incumbents increase total R&D investment but also prevent potential innovation effort through entry deterrence.

An exception for a stochastic model is Federico et al. (2017) who use costly investment in R&D that determines the probability of successful product innovations where all firms are potential coincident innovators. They find that mergers, where the involved firms spread their innovation effort evenly among their R&D departments, reduce total innovation effort and, thus, the probability of a successful innovation. In an answer to that work, Denicolò and Polo (2018) consider two-firm mergers to monopolies where firms can reallocate their innovation effort among their departments. They discover that the merger spurs innovation as it prevents the duplication of innovation effort. In a generalization to their analytical solutions, Federico et al. (2018) extend their model by a product market with price competition and differentiated products. Additionally, they incorporate merger synergies that either increase ability in innovation or lower costs of R&D investment. With numerical simulations they show that in the absence of ability gains total investment in R&D and consumer surplus is reduced. However, sufficient R&D pooling of complementarities increases total investment in R&D and consumer surplus. Their model differs from the classic patent races and my model as it misses to incorporate the impact of competitors innovation effort on the own probability of a successful innovation.

While empirical literature mainly focuses on how mergers affect innovation effort of the firms directly involved (see, e.g., Ornaghi, 2009; Szücs, 2014), evidence on how mergers affect innovation effort of competitors is scarce and mixed. Valentini (2016) observes that in the pharmaceutical industry post-merger the non-merged firms broaden and deepen their R&D activity. Uhlenbruck et al. (2017) notice that the non-merged firms adjust their innovation strategy and introduce new products and product improvements. Haucap et al. (2019) find evidence that in the pharmaceutical industry post-merger R&D activity and patenting of all, the merged and the non-merged firms, significantly declines.

5.3 Model

I consider a two-stage game played by $\eta \in \{n, n-1\}$ firms with $n \geq 3$: In the first stage, firms are engaged in a winner-takes all R&D-Contest with a difficulty to successfully innovate that possibly prevents an innovational breakthrough. It features the characteristic of a classic patent race by Loury (1979) where at most only one firm becomes an innovator and gains a prize referred to as a patent. The patent entails a non-drastic process innovation that exclusively reduces the winner's marginal costs in the production of a homogeneous good by y. The remaining firms are left empty-handed despite their innovation effort and it is assumed that patent licensing is not possible. Innovation effort is strategic and affects the second stage where firms strategically compete à la Cournot on the product market for that homogeneous good. I compare that two-stage game with $n \geq 3$ firms and no merger (i.e. pre-merger) to the two-stage game with $n-1 \geq 2$ firms in which a single merger of two firms occurs previous to the R&D-Contest (i.e. post-merger).

Pre-merger there are n independent firms i whereas post-merger there are $n-2$ non-integrated firms NI. Post-merger, the two merged firms form one integrated firm I that makes decisions as one entity in the R&D-Ccontest and on the product market. If two firms merge they will have no synergies in the production stage. However, the integrated firm has synergies that increase the ability in R&D by the factor $\theta > 1$ due to combining complementary assets in their R&D department in the R&D-Contest. The remaining $n-2$ non-integrated firms NI are not affected in their R&D ability. While all firms are active on the product market, to conduct effort in the R&D-Contest is not obligatory.

R&D-Contest

The R&D-Contest is modeled as a lottery contest between risk-neutral firms with linear costs of effort. It is a special case of the Tullock (1980) rent-seeking contest where the CSF defines a distribution over the contestants' probabilities of being the winner of the patent as a function of their innovation efforts. This implies that innovation effort is strategic such that the probability that a firm is a successful innovator and gains the exclusive cost reduction protected by a patent, depends not only on a firm's own innovation effort but also on the competitors' innovation effort.

Additionally, the form of the lottery contest is a generalized one where the CSF allows for a draw. To be more precise, a CSF is defined in a way, that the sum of winning probabilities adds up to one. In the standard lottery contest that does not allow for a draw, this implies only two outcomes for a given contestant: the probability that he wins the contest prize and the complementary probability that he does not win the contest prize but rather some other contestant does. In contrast, a lottery contest that allows for a draw implies three outcomes as it also defines a positive probability with which none of the contestants wins against all other contestants and therefore the chance to win the prize is forfeited. In other words, all contestants lose and a draw occurs. As described by Jia et al. (2013, p.215), one can imagine a third party called "Nature" that has a constant effort and a draw occurs when it wins.

Transferred to innovation competition: when innovation effort is productive, an increase in innovation effort increases the probability of a successful innovation. However, despite firms conduct innovation effort, with a positive probability no firm successfully innovates because a certain difficulty to successfully innovate exists that possibly prevents an innovational breakthrough. In contrast, when innovation effort is unproductive, the difficulty to successfully innovate plays no role and one of the firms always comes up with a successful innovation.

Pre-merger More specifically, n symmetric firms compete and the probability that an independent firm i successfully innovates and wins the patent is

$$p_i = \frac{x_i}{x_i + x_{-i} + d},$$

where $x_i \in [0, \infty)$ denotes the effort of firm $i \in \{1, ..., n\}$, $x_{-i} = \sum_{j \neq i} x_j$ denotes the effort of all firms except i and $d \geq 0$ is a constant draw parameter that represents the difficulty to successfully innovate. For $d = 0$ the CSF is referred to as standard lottery contest (without draw) with an axiomatic foundation given by Skaperdas (1996). For $d = 1$ an axiomatic foundation is provided by Blavatskyy (2010). In Loury (1979), d corresponds to the interest rate whereas Jia et al. (2013) call d a constant effort that is supplied by “nature” and a draw occurs if it wins. As parameter d describes how difficult it is to successfully innovate, $d = 0$ implies that any investment results in the success of some firm, whereas the probability of a successful innovation decreases as d increases. Hence, for $d = 0$ innovation effort is unproductive whereas for $d > 0$ innovation effort is productive.

More precisely, the probability that no firm successfully innovates, i.e. the R&D-Contest ends without successful innovation, is

$$p_D = \frac{d}{x_i + x_{-i} + d},$$

which implies that the probability of no successful innovation decreases as total market investment in R&D increases. That is a plausible assumption as in markets with higher research intensity, measured as total market investment in R&D, it is more likely that a successful innovation occurs.

An independent firm i chooses x_i in order to maximize its expected payoffs

$$E_i = p_i V + p_D D + \pi_l^* - x_i, \tag{5.1}$$

where $V := V(n, y) = \pi_v^* - \pi_l^*$ denotes the expected value of winning the patent and $D := D(n, y) = \pi_{dl}^* - \pi_l^*$ denotes the expected value of a draw, i.e. ending up in a product market without successful innovation. $\pi_l^* := \pi_l^*(n, y)$ denotes the equilibrium profits from losing when another firm wins the patent, $\pi_v^* := \pi_v^*(n, y)$ denotes the equilibrium profits of winning the patent, and $\pi_{dl}^* := \pi_{dl}^*(n, y)$ denotes the equilibrium profits when no firm wins the patent.

Post-merger $n - 1$ not fully symmetric firms compete. That means there is heterogeneity among firms as the integrated firm has synergies in their R&D department and, thus, has more ability relative to non-integrated firms in the innovation process. However, that does not mean that the merged firm has necessarily more ability in the subsequent production process on the product market. An increase in productivity on the product market is only realized by the single winner of the R&D-Contest. The probability that the integrated firm I successfully innovates and wins the patent is

$$p_I = \frac{\theta x_I}{\theta x_I + x_{-I} + d},$$

where $x_I \in [0, \infty)$ denotes the effort of the integrated firm I and $x_{-I} = \sum_{j \notin I} x_j$ the effort of all other firms. The level of synergies is expressed by $\theta > 1$.[9] The probability that a non-integrated firm $i \notin I$ successfully innovates and wins the patent is

$$p_i = \frac{x_i}{\theta x_I + \sum_{j \notin I} x_j + d}.$$

The probability that no firm successfully innovates is

$$p_D^M = \frac{d}{\theta x_I + \sum_{j \notin I} x_j + d}.$$

The integrated firm I chooses x_I in order to maximize its expected payoffs

$$E_I = p_I V^M + p_D^M D^M + \pi_l^{M^*} - x_I, \tag{5.2}$$

and a non-integrated firm NI chooses $x_{NI} \in [0, \infty)$ in order to maximize its expected payoffs

$$E_{NI} = p_{NI} V^M + p_D^M D^M + \pi_l^{M^*} - x_{NI}, \tag{5.3}$$

where, analogously to the pre-merger case, $V^M := V^M(n-1, y) = \pi_v^{M^*} - \pi_l^{M^*}$ denotes the value of winning the patent, $D^M := D^M(n-1, y) = \pi_{dl}^{M^*} - \pi_l^{M^*}$ denotes the value of a draw. $\pi_l^{M^*} := \pi_l^{M^*}(n-1, y)$ denotes the equilibrium profits from losing when another firm wins the patent, $\pi_v^{M^*} := \pi_v^{M^*}(n-1, y)$ denotes the equilibrium profits of winning the patent, and $\pi_{dl}^{M^*} := \pi_{dl}^{M^*}(n-1, y)$ denotes the equilibrium profits when no firm wins the patent.

Product Market

In the second stage, dependent on the type of innovation, a single firm is confronted on the product market with two (three) different market compositions when innovation effort is unproductive (productive). In the former, a firm either, when it successfully innovates, is the single firm with a cost advantage compared to all other still symmetric firms, or, when it does not successfully innovate, is one of the symmetric firms with a cost disadvantage compared to the successful firm. In the latter, additionally the outcome is possible when none of the firms successfully innovates such that a firm is one among only symmetric firms that all have the same costs.

The product market competition is modeled as Cournot quantity competition for a homogeneous good with $\eta \in \{n, n-1\}$ firms where n is the number of pre-merger firms and $n-1$ is the number of post-merger firms. In case there is a successful innovation in the market (i.e. no draw), the output of a non-innovative firm (non-innovator) l is denoted by q_l and of the innovative firm (innovator) v is denoted by q_v where I add a superscript M to denote post-merger. In case there is no successful innovation in the market (i.e. draw), the output of a (i.e. drawn) non-innovator dl is denoted by q_{dl}.

Each non-innovative firm has constant marginal costs $C_j(q_j) = cq_j$ with $c > 0$,

[9] Level of synergies meausures the 'intellectual closeness', the lower θ the higher the intellectual closeness. Hence, the less is the spectrum of ideas and creativity pooled.

irrespective whether there is a successful innovation in the market or not. An innovative firm gains an exclusive reduction $0 < y < 1$ in constant marginal costs such that $C_v(q_v) = (c - y)q_v$. Market inverse demand is assumed to be linear and given by $p = a - Q$ where Q denotes aggregate supply and $a > c$. I define the market size $A := a - c$ and normalize it to 1. Hence, a reduction in constant marginal costs can be interpreted relative to the market size. As $0 < \frac{y}{A} < 1$, it is ensured that the process innovation is non-drastic and that all non-innovative firms remain in the product market.

In a market with successful innovation and η firms, the innovator chooses q_v in order to maximize its product market profits

$$\pi_v = [1 + y - q_v - q_{-v}] \cdot q_v \tag{5.4}$$

where $q_{-v} = \sum_{j \neq v} q_j$ denotes the output of all non-innovators on the product market. A non-innovator chooses q_l in order to maximize its product market profits

$$\pi_l = [1 - q_l - q_{-l}] \cdot q_l \tag{5.5}$$

where $q_{-l} = q_v + \sum_{j \neq l,v} q_j$ denotes the output of the remaining non-innovators on the product market.

In a market without successful innovation there is standard symmetric Cournot competition and a firm chooses q_{dl} in order to maximize its product market profits

$$\pi_{dl} = [1 - q_{dl} - q_{-dl}] \cdot q_{dl} \tag{5.6}$$

where $q_{-dl} = \sum_{j \neq dl} q_j$ denotes the output of the remaining firms on the product market.

5.3.1 Equilibrium Solution

I will solve the two-stage game by backward induction for the subgame perfect equilibrium. Therefore, I firstly determine the Nash equilibrium on the product market. Eventually, the equilibrium profits of the product market are considered to specify the subgame perfect equilibrium.

2nd stage: Product Market

In the second stage on the product market, the equilibrium production levels can be derived from the first order conditions followed by the maximization of Equation (5.4) and (5.5) for a market with successful innovation and of Equation (5.6) for a market without successful innovation.

For a market with successful innovation, symmetry in equilibrium implies $q_{-v} = (\eta - 1)q_l$ and $q_{-l} = (\eta - 2)q_l + q_v$ which yields the innovator's output reaction function

$$q_v = \frac{1 + y - (\eta - 1)q_l}{2},$$

and a non-innovator's output reaction function

$$q_l = \frac{1 - q_v}{\eta}.$$

In equilibrium product market outputs and profits are

$$q_v^* = \frac{1+\eta y}{\eta+1}, \qquad Q^* = \frac{\eta+y}{\eta+1}, \qquad \pi_v^* = \left(\frac{1+\eta y}{\eta+1}\right)^2,$$
$$q_l^* = \frac{1-y}{\eta+1}, \qquad \pi_l^* = \left(\frac{1-y}{\eta+1}\right)^2.$$

In a market without successful innovation symmetry in equilibrium implies $q_{-dl} = (\eta-1)q_{dl}$ and equilibrium product market outputs and profits are

$$q_{dl}^* = \frac{1}{\eta+1}, \qquad Q^* = \frac{\eta}{\eta+1}, \qquad \pi_{dl}^* = \left(\frac{1}{\eta+1}\right)^2.$$

1st Stage: R&D-Contest

In the first stage in the R&D-Contest, the expected value of winning V is defined by

$$V := V(\eta, y) = \pi_v^* - \pi_l^*, \tag{5.7}$$

which is the difference in profits on the product market of being an innovator and of being a non-innovator. Analogously, the expected value of drawing is defined by

$$D := D(\eta, y) = \pi_{dl}^* - \pi_l^*, \tag{5.8}$$

which is the difference of profits on the product market of being a non-innovator in the market without successful innovation and being a non-innovator in the market with successful innovation (see Appendix 5.A for the precise calculations).

Pre-Merger For $D = 0$, Baye and Hoppe (2003) proof that for Equation (5.1) the classic patent race game by Loury (1979) is strategically equivalent if $V \leq \frac{n^2}{n-1}$ which is always true for $0 < y < 1$ and $n \geq 3$. Nti (1997) shows that there exists a unique positive equilibrium solution if V is large enough. For $D > 0$, the equilibrium investment levels can be derived from the necessary conditions of Equation (5.1)

$$\frac{\partial E_i}{\partial x_i} = \frac{x_{-i}V + d(V-D)}{(x_i + x_{-i} + d)^2} - 1 \stackrel{!}{=} 0$$

where the symmetry in equilibrium of n independent firms i implies $x_{-i} = (n-1)x_i$ yielding

$$x_i^* = \begin{cases} \frac{((n-1)V - 2nd) + \sqrt{4nd(V-nD) + (n-1)^2V^2}}{2n^2} & \text{if} \quad d \leq \frac{ny(2+ny)}{(1+n)^2} =: \hat{d}_i, \\ 0 & \text{otherwise.} \end{cases} \tag{5.9}$$

Hence, for $d \leq \hat{d}_i$, i.e. low enough difficulty to successfully innovate, all independent firms i conduct innovation effort.

Post-Merger Considering Equations (5.2) and (5.3), for $d = 0$, Cornes and Hartley (2005) show the uniqueness of the Nash equilibrium for lottery contests with two or more than two heterogeneous firms. For $d > 0$ and $D^M = 0$, Deng et al. (2018)

provide an equilibrium analysis for two or more than two players with heterogeneity. For $D^M > 0$, the equilibrium investment levels can be derived from the necessary conditions of Equation (5.2)

$$\frac{\partial E_I}{\partial x_I} = \frac{\theta(x_{-I}V^M + d(V^M - D^M))}{(\theta x_I + x_{-I} + d)^2} - 1 \overset{!}{=} 0.$$

where the symmetry in equilibrium of $n-2$ non-integrated firms NI implies $x_{-I} = (n-2)x_{NI}$ yielding

$$x_I = \frac{-(d + (n-2)x_{NI}) + \sqrt{\theta\left(d(V^M - D^M) + V^M(n-2)x_{NI}\right)}}{\theta}, \tag{5.10}$$

and of Equation (5.3)

$$\frac{\partial E_{NI}}{\partial x_{NI}} = \frac{x_{-NI}V^M + d(V^M - D^M)}{(x_{NI} + x_{-NI} + d)^2} - 1 \overset{!}{=} 0$$

where the symmetry in equilibrium of $n-2$ non-integrated firms NI implies $x_{-NI} = (n-3)x_{NI} + \theta x_I$ yielding

$$\begin{aligned} x_{NI} =& \frac{1}{2(n-2)^2}((n-3)V^M - (n-2)2d - 2\theta(n-2)x_I \pm \\ & \sqrt{-4d(n-2)\left((n-2)D^M - V^M\right) + V^M\left((n-3)^2V^M + 4\theta(n-2)x_I\right)}). \end{aligned} \tag{5.11}$$

Solving the system of equations (5.10) and (5.11) yields several solutions, only one of which fulfills the constraints. The unique equilibrium investment levels are given by

$$\begin{aligned} x_I^* =& \frac{-2d(1+\theta(n-2))V^M((\theta-1)(n-2)D^M + V^M)}{2\theta(V^M + \theta(n-2)V^M)^2} + \\ & \frac{\theta}{2\theta(V^M + \theta(n-2)V^M)^2}[(3 + \theta(n-2) - n)(n-2)(V^M)^3 + \\ & \sqrt{\frac{(3+\theta(n-2)-n)^2(V^M)^4}{\theta}} \cdot \\ & \sqrt{(-4d(1+\theta(n-2))((n-1)D^M - V^M) + \theta(n-2)^2(V^M)^2)}] \end{aligned} \tag{5.12}$$

and

$$\begin{aligned} x_{NI}^* =& \frac{\theta(n-2)(V^M)^3 - 2d(1+\theta(n-2))V^M(D^M + \theta(V^M - D^M))}{2(V^M + \theta(n-2)V^M)^2} + \\ & \frac{\sqrt{\theta(V^M)^4(-4d(1+\theta(n-2))((n-1)D^M - V^M) + \theta(n-2)(V^M)^2)}}{2(V^M + \theta(n-2)V^M)^2}. \end{aligned} \tag{5.13}$$

One easily verifies that E_I and E_{NI} reach indeed their maxima at x_I^* and x_{NI}^*, respectively, as

$$\left.\frac{\partial^2 E_I(x_I)}{\partial x_I^2}\right|_{x_I = x_I^*} < 0 \quad \text{and} \quad \left.\frac{\partial^2 E_{NI}(x_{NI})}{\partial x_{NI}^2}\right|_{x_{NI} = x_{NI}^*} < 0$$

are always satisfied for $x_{NI} \geq 0$ and $x_I \geq 0$ under the constraints.[10]

These are the unique equilibrium levels only if all firms are active in the R&D-Contest and invest in R&D which is the case if $d < \frac{\theta(n-1)y(2+(n-2)y)^2(2+(n-1)y)}{(2-y+\theta(n-1)(2+(n-1)y))^2} =: \hat{d}_{NI}$ since $x^*_{NI} > 0$ and $x^*_I > 0$. In contrast for $d > \frac{\theta(n-1)y(2+(n-1)y)}{n^2} =: \hat{d}_I$ no firm will invest in R&D because even the integrated firm has no incentive to invest in R&D as $x^*_I(x^*_{NI} = 0) \leq 0$. For the range $d \in [\hat{d}_{NI}, \hat{d}_I]$ there might be several equilibria. Since, with increasing d, one non-integrated firm after another will refrain from investing in R&D until at some point where solely the integrated firm is still active in the R&D-Contest.[11]

5.3.2 Pre-merger vs. post-merger

For the further analysis, I consider three criteria that are relevant when evaluating a merger from the perspective of a CA and/or a profit maximizing firm. First, the profitability of a merger which is defined by

$$\begin{aligned} E^*_I &\geq 2E^*_i \\ \Leftrightarrow p^*_I V^M + p^{M^*}_D D^M + \pi^{M^*}_l - x^*_I &\geq 2\left(p^*_i V + p^*_D D + \pi^*_l - x^*_i\right) \\ \Leftrightarrow \frac{\theta x^*_I V^M + dD^M}{\theta x^*_I + (n-2)x^*_{NI} + d} + \pi^{M^*}_l - x^*_I &\geq 2\left(\frac{x^*_i V + dD}{nx^*_i + d} + \pi^*_l - x^*_i\right). \end{aligned}$$

Thus, a merger is profitable if the post-merger profit of the integrated firm exceeds the joint pre-merger profit of the merged firms.

Second, the innovation effort in the market, i.e. total investment in R&D. Here I have to distinguish between productive and unproductive R&D investment. The objective of a competition authority is to increase the former and to decrease the latter. Hence, a CA reviews pre- and post-merger R&D investment

$$\begin{aligned} X^{Post} &\gtreqless X^{Pre} \\ \Leftrightarrow x^*_I + (n-2)x^*_{NI} &\gtreqless nx^*_i. \end{aligned}$$

Lastly, a CA reassesses how a merger affects welfare. On the one hand, consumer

[10] In this article, all lengthy analytical computations and graphical solutions are completed with the help of the software *Mathematica* by Wolfram Research, Inc. (2018) and the files are available upon request.

[11] I speculate that a complete description of an equilibrium has the following properties:

range of d	R&D active firms ρ	Equilibrium
$d \in [0, \hat{d}_{NI}]$	$n-1$	$(x^*_I(\rho), x^*_{NI}(\rho), ..., x^*_{NI}(\rho), x^*_{NI}(\rho), x^*_{NI}(\rho))$
$d \in [\hat{d}_{NI}, \hat{d}^{n-3}_{NI}]$	$n-2$	$(x^*_I(\rho), x^*_{NI}(\rho), ..., x^*_{NI}(\rho), x^*_{NI}(\rho), 0)$
$d \in [\hat{d}^{n-3}_{NI}, \hat{d}^{n-4}_{NI}]$	$n-3$	$(x^*_I(\rho), x^*_{NI}(\rho), ..., x^*_{NI}(\rho), 0, 0)$
	$\vdots$	
$d \in [\hat{d}^1_{NI}, \hat{d}_I]$	1	$(x^*_I(\rho), 0, ..., 0, 0, 0)$
$d \in [\hat{d}_I, \infty)$	0	$(0, 0, ..., 0, 0, 0)$

That means one firm after another restrains from investing in R&D in equilibrium. For example, for $n = 3$ it applies $\hat{d}^{n-3}_{NI} = \hat{d}_{NI}$ with an equilibrium $(x^*_I(1), 0)$ where only the integrated firm is active.

surplus is a criterion which is measured by

$$\begin{aligned} CS^{Post} &\geq CS^{Pre} \\ \Leftrightarrow \frac{((1-p_D^M)(q_v^{M^*}+(n-2)q_l^{M^*})+p_D^M(n-1)q_{ld}^{M^*})^2}{2} &\geq \frac{((1-p_D)(q_v^*+(n-1)q_l^*)+p_D n q_{ld}^*)^2}{2} \end{aligned}$$

and on the other hand, total Welfare given by

$$\begin{aligned} W^{Post} &\geq W^{Pre} \\ \Leftrightarrow E_I^* + (n-2)E_{NI}^* + CS^{Post} &\geq nE_i^* + CS^{Pre}. \end{aligned}$$

5.4 R&D-Contest with certain success

As a benchmark, I first consider a R&D-Contest where the difficulty to successfully innovate plays no role. Hence, there is no possibility of a draw and $d = 0$. In this case innovation is certain which means that R&D investment will definitely be successful and results in an implemented innovation. Therefore, innovation effort is unproductive and there is socially wasteful duplication of innovation effort in the market as a marginal R&D investment of a single firm would result in a successful innovation. Due to that, in this model, innovation efforts are excessive *R&D expenses* that need to be minimized from the perspective of a social planner.

5.4.1 Equilibrium

Pre-Merger: Equation (5.1) simplifies to

$$E_i = \frac{x_i}{x_i + x_{-i}} V + \pi_l^* - x_i,$$

and an independent firm i chooses x_i in order to maximize E_i. Szidarovszky and Okuguchi (1997) prove that the Nash equilibrium of the standard lottery contest without the possibility of a draw is unique and in pure strategies given by

$$x_i^* = \frac{n-1}{n^2} V.$$

Post-merger: Equation (5.2) is reduced to

$$E_I = \frac{\theta x_I}{\theta x_I + x_{-I}} V^M + \pi_l^{M^*} - x_I.$$

and the integrated firm I chooses x_I in order to maximize its expected payoffs. Equation (5.3) is reduced to

$$E_{NI} = \frac{x_{NI}}{x_{NI} + x_{-NI}} V^M + \pi_l^{M^*} - x_{NI},$$

and a non-integrated firm NI chooses x_{NI} in order to maximize E_{NI}. This yields the following reaction functions:

$$x_I = \frac{1}{\theta}\left(\sqrt{V^M \theta (n-2) x_{NI}} - (n-2)x_{NI}\right)$$

and

$$x_{NI} = \frac{(n-3)V^M - 2\theta(n-2)x_I + \sqrt{V^M\left((n-3)^2V^M + 4\theta(n-2)x_I\right)}}{2(n-2)^2}.$$

Equilibrium levels are given by

$$x_I^* = \frac{(n-2)V^M}{\left(\theta(n-2)+1\right)^2} \cdot \left(\theta(n-2) - (n-3)\right)$$

and

$$x_{NI}^* = \frac{(n-2)V^M}{\left(\theta(n-2)+1\right)^2} \cdot \theta.$$

Where $x_I^* \geq x_{NI}^*$ for all $\theta > 1$.

5.4.2 Post-merger vs. pre-merger

Profitability of a merger

A merger only occurs if it is profitable. As shown in Appendix 5.C.1, a merger is profitable if

$$\theta \geq \begin{cases} \hat{\theta}^P(3,y) & \text{if} \quad n=3 \quad \wedge \quad 0.0236 \approx \frac{2\sqrt{\frac{38}{3}}-7}{5} < y < 1-\sqrt{\frac{2}{3}} \approx 0.1835, \\ 1 & \text{if} \quad n=3 \quad \wedge \quad y \geq 1-\sqrt{\frac{2}{3}} \approx 0.1835, \\ \hat{\theta}^P(n,y) & \text{if} \quad n \geq 4 \quad \wedge \quad y > \hat{y}_P(n). \end{cases}$$

If the process innovation size is small enough, i.e., the cost reduction y is small enough, a merger is never profitable. For large y, a merger is profitable if the level of synergies θ is larger then some critical value $\hat{\theta}^P$. In triopolistic markets, even mergers without synergies are profitable for $y \geq 1-\sqrt{\frac{2}{3}} \approx 0.1835$.

Intuitively, the profitability of a merger depends on two opposing forces. On the one hand, competition on the product market and in the R&D-Contest is reduced. On the product market the merged entity reduces its total output and, thereby, increases its profits. In the R&D-Contest it increases the innovation effort and, thus, the probability to gain the patent. Hence, it increases its expected profit in the product market. On the other hand, as non-integrated firms react on the product market by increasing their output, the (expected) profit of the merged entity is reduced.

At $\hat{\theta}^P(n,y)$, the synergies of the merger are just large enough to exactly balance the two opposite effects. However, synergies large enough alone is not sufficient for a merger to be profitable. Additionally, the size of the innovation must be large enough. For instance, consider the extreme scenario when the innovation size is zero. Then the synergies in the R&D department of the merged entity have no effect at all. Likewise, for an innovation size close enough to zero, the ability gains due to the synergies of the merger still cannot be sufficiently exploited. From that point on, where synergies can be exploited, the desired effect of the merger becomes more dominant as the size of innovation increases.

In triopolistic markets, for an innovation size large enough the desired effect of reduced competition dominates even without synergies. This is because only one non-integrated firm remains in the market. Then the probability to win the patent and, therefore, the expected profits drastically increase. Even without gaining the patent, profits are relatively high due to that relatively large decline in competition after the merger.

R&D expenses

The objective of a CA is to minimize excessive total R&D expenses. As shown in Appendix 5.C.2, R&D expenses are lower post-merger than pre-merger, if

$$\theta \geq \begin{cases} \hat{\theta}^X(n,y) & \text{if} \quad n \geq 4 \quad \wedge \quad y < \frac{2n-6}{3n-5} =: \hat{y}_X(n), \\ 1 & \text{otherwise.} \end{cases}$$

Hence, a profitable merger in a triopolistic market always decreases excessive R&D expenses. Figure 5.1 illustrates for all combinations (n, y) on a grid over $[4, 20] \times [0, 1]$ the critical level of synergies $\hat{\theta}^X$ above which a merger reduces total R&D expenses. If $y \leq \hat{y}_P(n)$, marked by the dashed line, a merger is not profitable. Ceteris paribus, the lower the cost reduction y by the process innovation, the higher the level of synergies of the integrated firm has to be to reduce total expenses in R&D. In the white area, restricted by the solid line, for a cost reduction $y \geq \hat{y}_X(n)$ even mergers without synergies reduce total R&D expenses. Remarkable is that the two merged firms need not to double the R&D ability relative non-merged firm to reduce total markets expenses as $\hat{\theta}^X < 2$.

Proposition 5.1. *In R&D-Contests with certain success, a single merger of two firms reduces total R&D expenses in*

- *triopolistc markets,*
- *oligopolistic markets with more than three firms if*
  - *the cost reduction is $y \geq \hat{y}_X$ (white area) or*
  - *the cost reduction is $y < \hat{y}_X$ and synergies are $\theta > \hat{\theta}^X$ (colored area).*

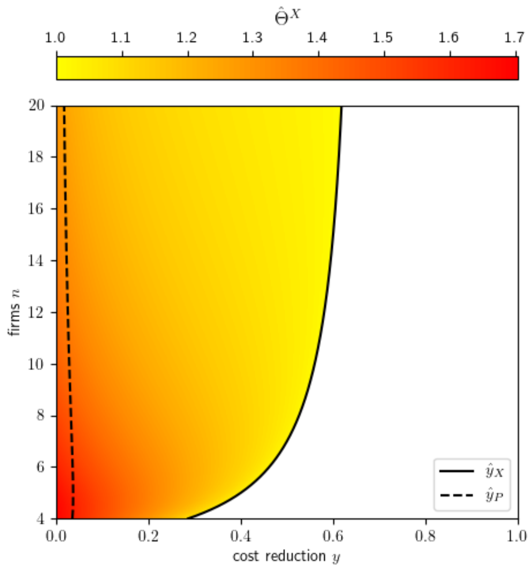


Figure 5.1: Critical $\hat{\theta}^X$ above which a merger reduces total R&D expenses

Here again the R&D expenses depend on two opposing forces. On the one hand, a desired effect, that decreases total R&D expenses due to reduced competition after the merger. On the other hand, an undesired effect, that increases total R&D expenses due to higher expected profits resulting from weaker competition on the product market. The ability gains of the merged entity support the desired effect, as an increased heterogeneity among the firms reduces total R&D expenses.

At $\hat{\theta}^X$, the synergies are just large enough to exactly balance the two opposing forces. The larger the innovation size, the more dominant is the desired effect and, consequently, already smaller levels of synergies are enough to balance the two effects. If the innovation size is large enough, the desired effect of reduced innovation competition will dominate the undesired effect even without the additional desired effect that is induced by synergies.

R&D expenses and Profitability

A CA is interested if the necessary condition that a merger is profitable has implications on the change of total R&D expenses. Hence, I compare the critical levels of synergies that make a merger reduce total R&D expenses $\hat{\theta}^X$ and that make a merger profitable $\hat{\theta}^P$. Figure 5.2 illustrates again combinations (n, y) but this time for which the critical level of synergies that makes a merger profitable is greater than the critical level of synergies that makes a merger reduce total R&D expenses.

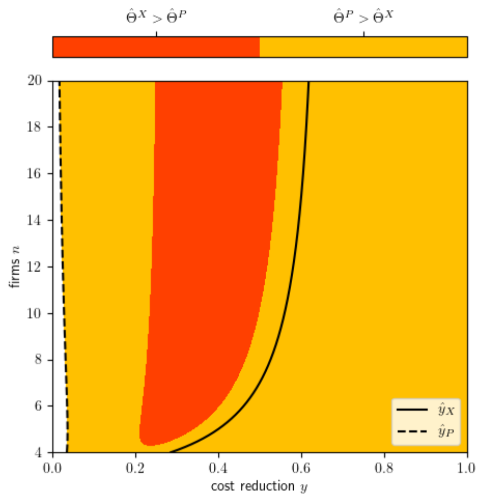


Figure 5.2: $\hat{\theta}^X$ compared to $\hat{\theta}^P$

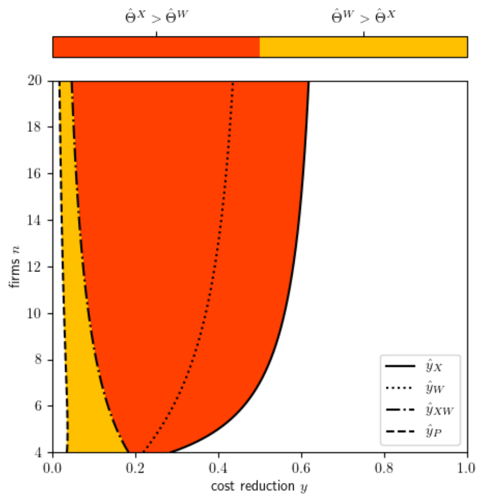


Figure 5.3: $\hat{\theta}^X$ compared to $\hat{\theta}^W$

In the red area, if $y \in [\underline{y}_{XP}, \overline{y}_{XP}]$, a profitable merger might increase total R&D expenses as $\hat{\theta}^X > \hat{\theta}^P$. By contrast in the yellow area, which includes any quadropolistic market irrespective of the size of innovation, a profitable merger definitely reduces total R&D expenses.

Proposition 5.2. *In R&D-Contests with certain success, the profitability of a single merger of two firms is a sufficient condition for a reduction of total R&D expenses in*

- *triopolistic and quadropolistic markets,*
- *markets with more than four firms if the cost reduction is either low or high enough, respectively, i.e.* $y \notin [\underline{y}_{XP}, \overline{y}_{XP}]$ *(yellow area).*

The reason behind this result is that the level of synergies that balances the two opposing forces, for both profitability and total R&D expenses, decreases in the innovation size. However, depending on the size of innovation, the desired effect due to internalized and reduced competition, respectively, has a different impact on profitability than on total R&D expenses. For innovation sizes that are either small or large enough, the desired effect has a greater impact on total R&D expenses than on profitability. Hence, to balance the two opposing forces, a lower level of synergies is required in terms of a reduction of total R&D expenses compared to profitability. In contrast, for intermediate innovation sizes the desired effect has a greater impact on profitability and, thus, to balance the two opposing forces, a lower level of synergies is required in terms of profitability.

Intuitively, for innovation sizes low enough, the ability gains due to the merger cannot be sufficiently exploited and, thus, the synergies have to be very large to make a merger profitable. In contrast, total innovation effort already drops for lower levels of synergies as competition is reduced. At a certain range of intermediate innovation sizes, already lower levels of synergies have a major impact on the profits of the integrated firm as its expected profits on the product market are higher. However, concurrently due to that increase in expected profits on the product market, all firms have incentives not to decrease innovation effort too much. For innovation sizes sufficiently large, the effect of lower competition becomes more dominant such that a very low or even a zero level of synergies reduce total innovation effort while the integrated firm still requires a certain level of synergies to maintain its advantage and be profitable.

Welfare

A social planner may also be interested in maximizing welfare. Notice that a profitable merger never increases consumer surplus (CS) since

$$CS^{Post} := \frac{1}{2}\left(q_v^{M^*} + (n-2)q_l^{M^*}\right)^2 < \frac{1}{2}\left(q_v^* + (n-1)q_l^*\right)^2 =: CS^{Pre}.$$

As shown in Appendix 5.C.3, a profitable merger increases total welfare if

$$\theta \geq \begin{cases} \hat{\theta}^W(3,y) & \text{if} \quad n=3 \quad \wedge \quad 0.0510 \approx \frac{4\sqrt{309}-67}{65} < y < \frac{4\sqrt{30}-19}{17} \approx 0.1711, \\ 1 & \text{if} \quad n=3 \quad \wedge \quad y \geq \frac{4\sqrt{30}-19}{17} \approx 0.1711, \\ \hat{\theta}^W(n,y) & \text{if} \quad n \geq 4 \quad \wedge \quad \hat{y}_P(n) < y < \hat{y}_W(n), \\ 1 & \text{if} \quad n \geq 4 \quad \wedge \quad y \geq \hat{y}_W(n). \end{cases}$$

Hence, a profitable merger decreases total welfare for small enough cost reductions y. For intermediate values of y, a merger increases total welfare if the level of synergies θ is above some critical value and, for large enough y, a profitable merger increases total welfare irrespective of the level of synergies.

Welfare depends on three forces. On the one hand, an undesired effect, since a merger reduces CS due to a reduction in total output as a result of reduced competition. On the other hand, a desired effect, since total profit in the product market is increased. Additionally, the R&D expenses affect welfare either desirably or undesirably depending on the level of synergies. At $\hat{\theta}^W$ synergies are just large enough to balance the desired and undesired effects. The larger the innovation size, the more dominant is the desired effect.

R&D expenses and Welfare

Similarly, a competition authority is interested whether a reduction in R&D expenses increases welfare. Hence, I compare the critical level of synergies that make a merger reduce total R&D expenses and the critical levels of synergies that make a merger increase total welfare. Figure 5.3 illustrates the combinations (n, y) for which the critical level of synergies that makes a merger reduce total R&D expenses $\hat{\theta}^X$ in comparison to the critical level of synergies that make a merger increase total welfare $\hat{\theta}^W$.

In the yellow area a merger that reduces total R&D expenses might decrease total welfare. By contrast, on the right of the dotted dashed line $\hat{y}_{XW}$ in the red area a merger that reduces total R&D expenses always increases total welfare. The dashed line $\hat{y}_W$ shows the critical level of cost reduction above which a merger definitely increases total welfare irrespective of the level of synergies. Right of the solid line $\hat{y}_X$, the white area still reflects that any merger irrespective of level of synergies reduces total R&D expenses.

Proposition 5.3. *In R&D-Contests with certain success, a single merger of two firms that reduces R&D expenses is a sufficient condition for an increase in total welfare for a cost reduction* $y > \frac{1+2n}{1+4n+2n^2} := \hat{y}_{XW}$ *(red and white area).*

Notice that the change in total R&D expenses is also captured in the total welfare analysis. Hence, it is enough to consider only the two opposing forces that affect total welfare in general. Intuitively, for small innovation sizes the undesired effect of reduced CS dominates the desired effect of increased total market profit. Therefore, a smaller level of synergies is required to decrease total R&D expenses than total welfare. For innovation sizes large enough, this dominance is reversed and a smaller level of synergies is required to increase total welfare than to decrease total R&D expenses.

R&D expenses, Profitability and Welfare

From the previous Propositions 5.1 to 5.3, despite ambiguous effects for sufficiently small y, the following Corollary about the relation of total R&D expenses, profitability and total Welfare for sufficiently large y becomes evident.

Corollary 5.1. *In oligopolistic R&D-Contests with certain success, the profitability of a single merger of two firms is a sufficient condition for a reduction of total R&D expenses which implies an increase in total welfare if* $y \geq \frac{4\sqrt{30}-19}{17} \approx 0.1711$.

The reason is, the larger the innovation size, the more dominant becomes the desired effect of reduced competition in terms of total R&D expenses and total welfare. It becomes so dominant that, even without synergies, total R&D expenses (total welfare) are (is) reduced (increased). In contrast, to make a merger profitable, still synergies are necessary to offset the undesired effect of the increased output by non-integrated firms. Hence, any profitable merger is socially desirable for large innovation sizes.

5.5 Triopolistic R&D-Contest with uncertain success

Now I turn to a R&D-Contest with uncertain success where $d > 0$. Hence, this reflects a situation in which R&D investment does not certainly lead to an innovation by some firm, i.e. innovation effort is productive. For the sake of computability, I make two assumptions: firstly, I only analyze triopolistic markets with $n = 3$ firms where post-merger two independent firms remain. In accordance to Shapiro (2011, p.390) this is a reasonable assumption as "merger enforcement only takes place in moderate or highly concentrated markets". It is the smallest possible number of firms that does not result in a monopoly post-merger. Additionally, the advantage is that post-merger equilibria at $d \in [\hat{d}_{NI}, \hat{d}_I]$, where some non-integrated firms may be active and some may not, do not occur. Secondly, I consider mergers of firms that double R&D ability relative to non-merged firms such that $\theta = 2$. That is justified as it is obviously more likely for two independent firms to merge if they complement each other to generate a high level of synergies.

5.5.1 Equilibrium

Pre-merger With $n = 3$ and $\theta = 2$, equilibrium investment levels given by equation (5.9) reduce to

$$x_i^* = \begin{cases} \frac{1}{36}\left(2y(1+y) - 12d + \sqrt{y\left(4y(1+y)^2 + d(6+33y)\right)}\right) & \text{if} \quad d \leq \hat{d}_i = \frac{6y+9y^2}{16}, \\ 0 & \text{otherwise.} \end{cases}$$

Post-merger Equilibrium investment levels given by equations (5.12) and (5.13) reduce to

$$x_I^* = \begin{cases} -\frac{d(4+y)}{9(2+y)} + \frac{\Phi}{27} & \text{if} \quad d < \hat{d}_{NI} = \frac{8y(4+8y+5y^2+y^3)}{(10+7y)^2}, \\ \frac{1}{6}\left(-3d + \sqrt{8dy(1+y)}\right) & \text{if} \quad \hat{d}_{NI} \leq d < \hat{d}_I = \frac{8y(1+y)}{9}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$x_{NI}^* = \begin{cases} -\frac{d(10+7y)}{9(2+y)} + \frac{\Phi}{27} & \text{if} \quad d < \hat{d}_{NI}, \\ 0 & \text{otherwise} \end{cases}$$

with $\Phi = y(2+y) + \sqrt{y\left(y(2+y)^2 + 6d(2+5y)\right)}$. Thus, the difficulty to successfully innovate has a major impact on the number of firms that conduct innovation effort. For $d < \hat{d}_{NI}$ all firms are active. In the range $d \in [\hat{d}_{NI}, \hat{d}_I)$ only the integrated firm engages in R&D whereas if $d \geq \hat{d}_I$ no firm invests in R&D.

Comparative Statics Pre-merger, the total equilibrium effort of the all independent firms decreases (increases) as d (y) increases.

$$\frac{\partial x_i^*}{\partial d} < 0 \quad \text{and} \quad \frac{\partial x_i^*}{\partial y} > 0.$$

$$\frac{\partial x_I^*}{\partial d} = \begin{cases} > 0 & \text{if} \quad d < \frac{-8y+22y^3+18y^4+4y^5}{32+96y+42y^2+5y^3} < \hat{d}_{NI} \wedge y > \frac{1}{2}, \\ < 0 & \text{otherwise,} \end{cases} \quad \frac{\partial x_{NI}^*}{\partial d} < 0, \quad \frac{\partial\left(x_I^* + x_{NI}^*\right)}{\partial d} < 0.$$

Post-merger, the equilibrium effort of the non-integrated firm decreases in d whereas the equilibrium effort of the integrated firm increases in d when it is d is small enough and y is large enough. Overall, total market investment in equilibrium decreases in d.[12] Contrarily, in equilibrium R&D investment increases in the innovation size y as

$$\frac{\partial x_I^*}{\partial y} > 0 \quad \text{and} \quad \frac{\partial x_{NI}^*}{\partial y} > 0.$$

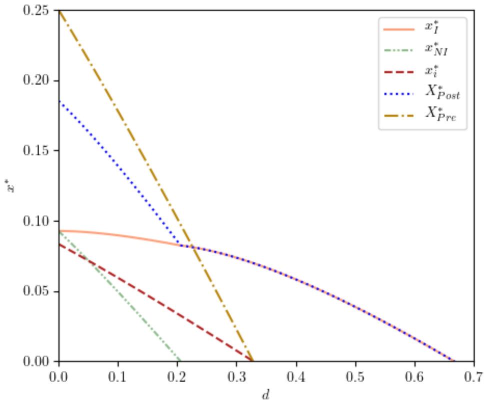


Figure 5.4: Equilibrium innovation effort for $y = 0.5$

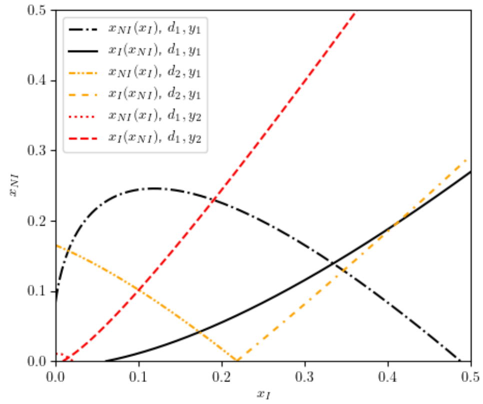


Figure 5.5: Reaction functions with combinations of $d_1 = 0.01$, $y_1 = 0.99$, $d_2 = 0.49$, $y_2 = 0.08$

Example

Figure 5.4 graphically illustrates the equilibrium effort for independent, integrated, and non-integrated firms as well as total equilibrium effort pre- and post-merger as a function of the difficulty to innovate for a fixed innovation size $y = 0.5$.

Post-merger, Figure 5.5 illustrates the reaction functions for combinations of selected values $d_1 = 0.01$, $y_1 = 0.99$, $d_2 = 0.49$ and $y_2 = 0.08$ in the range where both firms, integrated and non-integrated, are active $d < \hat{d}_{NI}$. Intersections of same colored lines reveal the equilibrium investment levels of the integrated and the non-integrated firm. It becomes obvious that for both, small y or high d, equilibrium investment of the non-integrated firm is close to zero, whereas equilibrium investment of the integrated firm is low only for small y.

5.5.2 Post-merger vs. pre-merger

Innovation effort

Figure 5.6 graphically shows how a merger affects innovation effort for combinations of (d, y) on a grid over $[0, 1.8] \times [0, 1]$ where X^{Pre} and X^{Post} are compared. In the yellow (red) area post-merger total R&D investment is increased (reduced) compared to pre-merger. Below the dotted line $\hat{d}_{NI}$ post- and pre-merger all firms conduct innovation effort. Between the dotted line $\hat{d}_{NI}$ and the dashed line $\hat{d}_i$ pre-merger all firms are active in R&D whereas post-merger only the integrated firm is active. Between the dashed line $\hat{d}_i$ and the solid line $\hat{d}_I$ post-merger only the integrated firm engages in R&D whereas pre-merger no firm is engaged. Above the sold line $\hat{d}_I$ in the white area no firm is active in R&D. Hence, a merger is never profitable as

[12] That means d can also be interpreted as a measure of research intensity with a high research intensity for d close to zero. This is similar to Haucap et al. (2019) where in a different model the parameter 'k' is a measure for the research intensity. However, in my model, if d exceeds a critical level, there is no R&D effort and therefore a research intensity of zero irrespective of the value of d.

ability gains due to synergies are only realized when innovation effort is conducted and a two firm merger without synergies is never profitable under Cournot competition.

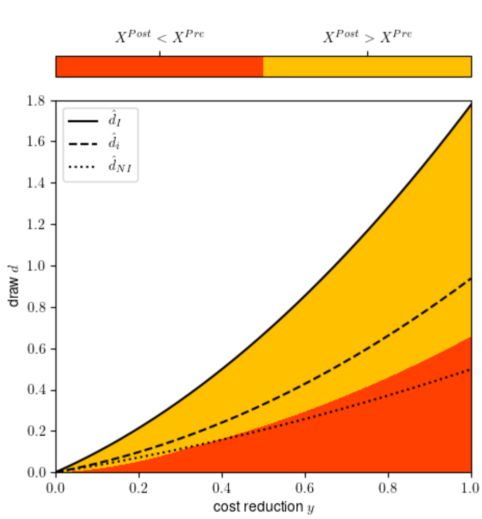


Figure 5.6: Innovation effort

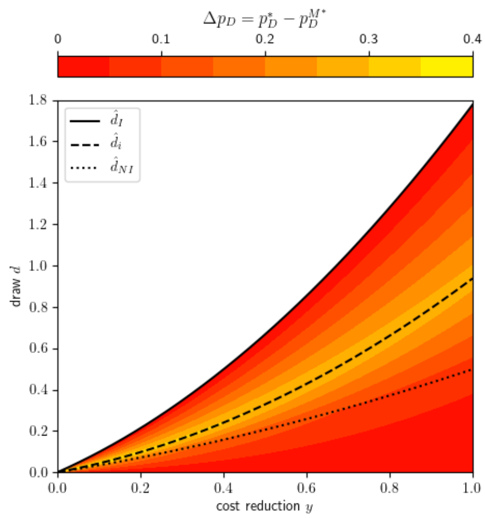


Figure 5.7: Successful innovation

For a difficulty to successfully innovate d low enough, a merger decreases total R&D investment. Within the yellow area between the dashed line $\hat{d}_i$ and the solid line $\hat{d}_I$, obviously only the integrated firm is active in R&D post-merger whereas pre-merger no firm is active which results in the following proposition.

Proposition 5.4. *In a triopolistic market where a merger doubles the R&D ability, for $\hat{d}_i < d < \hat{d}_I$ there is innovation effort in the market if and only if two firms merge.*

The intuition behind this result is that pre-merger between three equally low-pro-ductive firms there is innovation competition until a certain level of difficulty to successfully innovate. However, at some level of difficulty, a low-productive firm is not productive enough to overcome this difficulty on its own while additionally having the pressure to outperform rival firms. To maintain investment in R&D to overcome this difficulty either a reduction of competition or an enhancement in productivity is necessary.

The merger of two firms combines both, reduction of competition through internalization and enhancement in productivity through R&D complementary. The asymmetry in productivity in turn reduces competition post-merger already for a low level of difficulty to successfully innovate. This is because the low-productive firm is not competitive anymore and, eventually, stops to invest in R&D and leaves the R&D process to the high-productive integrated firm.

Another aspect worth considering for a CA is the effect of the merger on the probability of a successful innovation. Therefore I compare the difference between pre- and the post-merger probability of a draw. A merger (weakly) increases the probability of a successful innovation if

$$\Delta p_D := p_D^* - p_D^{M^*} \geq 0,$$

which is satisfied for all combinations of (d, y). Figure 5.7 again uses the grid $[0, 1.8] \times [0, 1]$ to illustrate Δp_D. By and large it reveals the largest increase in the probability to successfully innovate from pre- to post-merger around $\hat{d}_i$ with about 25 percentage points up to 40 percentage points.

Proposition 5.5. *In a triopolistic market, a two-firm merger that doubles the R&D ability (weakly) increases the probability of a successful innovation.*

This can be explained by the fact that the probability of successful innovation is the larger, the larger the total innovation effort of all firms is. The main reason for an increase in the probability of a successful innovation post-merger is the increased ability in R&D of the integrated firm. Even if total R&D spending shrinks post-merger, the increased ability in R&D of the integrated firm doubles the productivity of this investment and, thus, more than compensates the reduction in R&D spending.

Profitability and Welfare

In Figure 5.8 in the orange (red) area the merger is (not) profitable as $E_I > 2E_i$ ($E_I < 2E_i$) whereas in Figure 5.9 in the orange (red) area the merger is (not) increasing total welfare as $W^{Post} > W^{Pre}$ ($W^{Post} < W^{Pre}$).[13] I focus on the area $d \in [\hat{d}_i, \hat{d}_I]$ where post-merger only the integrated firm is active in R&D and pre-merger is no R&D activity. In this area a merger is profitable below the double-dotted dashed line and welfare increasing below the dotted dashed line. A merger is profitable if

$$d \leq \frac{1}{36}(1 + 32y(1+y) - 8\sqrt{2y(1+y)}) := \hat{d}_P \wedge y \geq 0.2462.$$

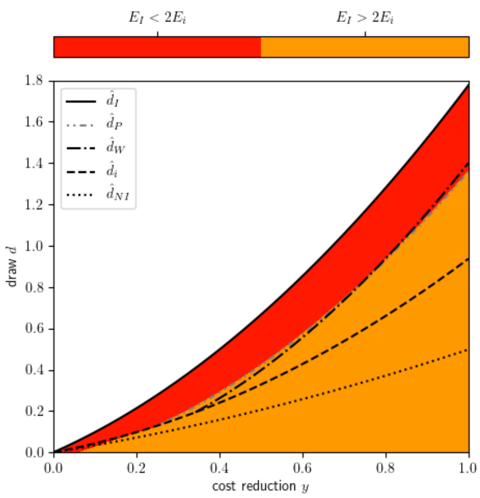


Figure 5.8: Profitability

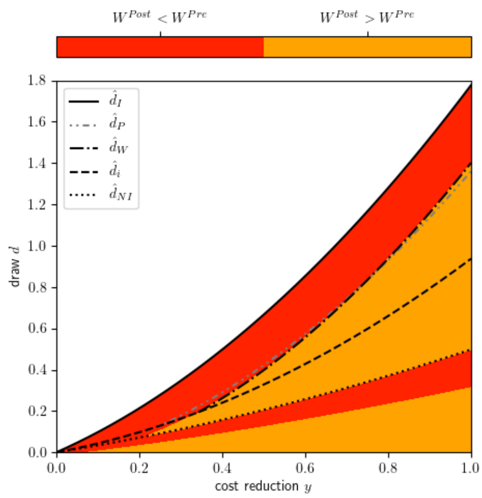


Figure 5.9: Total Welfare

The intuition behind this result is again based on two opposing forces. On the one hand, there are two desired effects that increase profitability. Firstly, by the internalization of competition and secondly, as the integrated firm engages in R&D it has a positive probability of a successful innovation. Both positively affect profits of the integrated firm. On the other hand, firstly, the non-integrated firm increases its product market output and, secondly, innovation becomes more costly as the difficulty to successfully innovate increases. Both negatively affect profits of the integrated firm. At $\hat{d}_P$ the two forces are balanced.

A merger increases total Welfare if

$$\begin{aligned} d \leq & \frac{(1+y)\left(y\left(16y(101y+160)+1087\right)+56\right)}{18(9y+8)^2} - \\ & \frac{16}{18}\sqrt{\frac{y(5y^2+9y+4)^2(16y^3+63y+56)}{(9y+8)^4}} := \hat{d}_W \wedge y \geq 0.3420. \end{aligned}$$

[13]For certain combinations of (d, y) even consumer surplus is increased. For a graphically illustration see Appendix 5.B.

Here again, this stems from opposing forces. On the one hand, a merger decreases total welfare as consumer surplus (if the difficulty to successfully innovate is large enough, see Appendix 5.B Figure 5.10) is reduced and the expenses for R&D are increased. On the other hand, a merger also increases total profits in the product market as competition is reduced and a possible successful innovation reduces production costs for the integrated firm. However, a successful innovation becomes less likely the larger the difficulty to successfully innovate. At $\hat{d}_W$ these two effects are balanced.

Additionally, in Figure 5.9, it becomes apparent that in the red area between $\hat{d}_P$ and $\hat{d}_W$ a merger is profitable but decreases total welfare. However, this area is quite small. Hence, a profitable merger that in the first place results in R&D activity is very likely to increase welfare.

5.6 Conclusion

I have examined the effect of a horizontal two-firm merger with synergies on the incentives to innovate and welfare in oligopolistic markets. I show that a robust domain exists where a merger increases the efficiency of a R&D activity in the market and, consequently, total welfare. To this end, I model a two-stage game with an initial R&D-Contest followed by a product market competition à la Cournot. I consider an unique approach that, firstly, employs a lottery contest to capture innovation competition and, secondly, allows for the possibility of a draw in the contest success function.

Whether more or less R&D investment is socially desired depends on the type of the innovation competition. That is why I distinguish between two scenarios: a R&D-Contest with certain success where innovation effort is unproductive and a R&D-Contest with uncertain success where innovation effort is productive. In the former more effort is redundant as a successful innovation certainly occurs. In the latter effort is desired as some difficulty to successfully innovate exists that possibly prevents an innovational breakthrough and, thus, the success of an innovation is uncertain, despite investment in R&D.

When innovation effort is unproductive, the competing firms' investment in R&D is beyond the optimal level because they do not take into account the parallelism of their efforts. As a CA is indifferent which firm gains the patent, a marginal effort of a single firm would be socially optimal.

The approval of a merger can decrease the duplication of R&D expenses either when the innovation size is high enough or the innovation size is low but the synergies of the merged firms are high. If the innovation size is high enough a reduction of R&D expenses implies an increase in total welfare. Moreover, since for large enough innovation sizes, the profitability of a merger is a sufficient condition for a reduction of R&D expenses, a CA can apply a rough guide for merger enforcement: it can promptly permit a bilateral merger in markets with high innovation potential to improve R&D performance.

When innovation effort is productive, it increases the probability of a successful innovation and there might be no investment in R&D when the difficulty to successfully innovate is too high. In a triopolistic market, eventually, the approval of a two-firm merger that doubles the R&D ability provides additional innovation incentives. However, for an excessive difficulty to successfully innovate such a merger would be welfare detrimental. If a CA's main objective is to spur innovation, a rough rule for triopolistic markets is to permit any merger that doubles the R&D ability as it increases the probability of a successful innovation.

The results are related to the ones of Motta and Tarantino (2017) and Federico et al. (2018) who find an increase in total R&D investment for large enough ability gains in the merged entity. The theoretical insight and empirical observation by Haucap et al. (2019) that in research intense industries a two-firm merger reduces total R&D investment validate a side result. In general my findings are in line with the synergy principle proclaimed by Shapiro (2011, p.365) that a combination of "complementary assets enhances innovation capabilities and thus spurs innovation". Hence, the enforcement of mergers with synergies are a policy that can positively affect innovation effort.

This paper contributes to the growing theoretical literature on mergers and innovation with a different approach based on R&D-Contests with a contest success function that allows for a draw. It serves as a starting point and several dimensions of further development are conceivable. Obviously is the extension of the model with uncertain success to finitely many firms and varying levels of synergies. Another adaptation is the introduction of multiple patent winners or the possibility of patent licensing.

Appendix

5.A R&D-Contest gains

From Equations (5.7) and (5.8) the values of winning and drawing are:

$$V(\eta, y) = \pi_v^* - \pi_l^* = \left(\frac{1+\eta y}{\eta+1}\right)^2 - \left(\frac{1-y}{\eta+1}\right)^2 = \frac{y[2(\eta+1)+y(\eta^2-1)]}{(\eta+1)^2}$$

$$D(\eta, y) = \pi_{dl}^* - \pi_l^* = \left(\frac{1}{\eta+1}\right)^2 - \left(\frac{1-y}{\eta+1}\right)^2 = \frac{y[2-y]}{(\eta+1)^2}$$

5.B Consumer Surplus: R&D-Contest with uncertain success

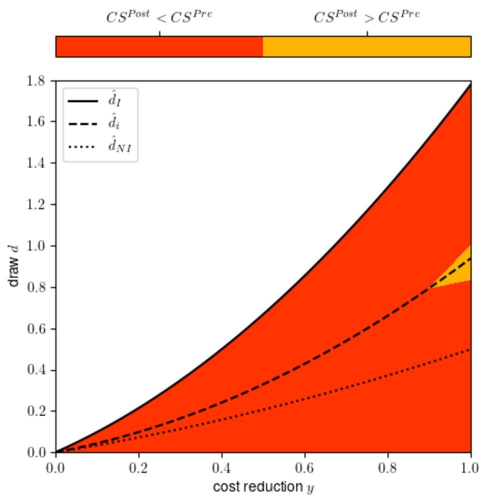


Figure 5.10: Consumer Surplus

5.C Derivations in R&D-Contest with certain success

The following lengthy computations are conducted with the help of the software *Mathematica* by Wolfram Research, Inc. (2018).

5.C.1 Profitability

$$E_I^* \geq 2E_i^*$$

$$\Leftrightarrow \frac{(\theta(n-2)-(n-3))}{(\theta(n-2)-(n-3))+(n-2)} V^M + \pi_l^{M*} - \frac{(n-2)V^M}{(\theta(n-2)+1)^2} \cdot (\theta(n-2)-(n-3)) \geq 2\left(\frac{V}{n^2} + \pi_l^*\right)$$

$$\Leftrightarrow \frac{(\theta(n-2)-(n-3))}{(\theta(n-2)-(n-3))+(n-2)} \cdot \frac{y[2n+y((n-1)^2-1)]}{n^2} + \left(\frac{1-y}{n}\right)^2 - \frac{(n-2)}{(\theta(n-2)+1)^2} \cdot \frac{y[2n+y((n-1)^2-1)]}{n^2} \cdot (\theta(n-2)-(n-3))$$
$$\geq 2\left(\frac{1}{n^2} \cdot \frac{y[2(n+1)+y(n^2-1)]}{(n+1)^2} + \left(\frac{1-y}{n+1}\right)^2\right)$$

- For $n=3$ and $\frac{2\sqrt{\frac{38}{3}}-7}{5} < y < 1-\sqrt{\frac{2}{3}}$

$$\Leftrightarrow \theta \geq \frac{\sqrt{192y+1248y^2+2304y^3+864y^4}-2-12y-18y^2}{2-84y-30y^2} =: \hat{\theta}^P(3,y)$$

- For $n \geq 4$ and $\hat{y}_P(n) := \frac{\sqrt{2}\sqrt{3+6n-6n^3-2n^4+2n^5+n^6}+3+3n-3n^2-n^3}{3-6n^2+n^4} < y < 1$

$$\Leftrightarrow \theta \geq \frac{1+2n+n^2-6y-2ny+12n^2y+2n^3y-2n^4y+3y^2-4ny^2-10n^2y^2+3n^3y^2+3n^4y^2-n^5y^2}{(n-2)(-1-2n+n^2+6y+6ny-6n^2y-2n^3y-3y^2+6n^2y^2-n^4y^2)} +$$
$$\sqrt{\frac{-2ny-8n^2y-8n^3y+2n^5y+14ny^2+47n^2y^2+44n^3y^2+4n^4y^2-6n^5y^2+n^6y^2-18ny^3-50n^2y^3-28n^3y^3+20n^4y^3+14n^5y^3-2n^6y^3+6ny^4+13n^2y^4-12n^4y^4-2n^5y^4+3n^6y^4}{(-1-2n+n^2+6y+6ny-6n^2y-2n^3y-3y^2+6n^2y^2-n^4y^2)^2}} =: \hat{\theta}^P(n,y)$$

5.C.2 R&D expenses

$$X^{Post} := x_I^* + (n-2)x_{NI}^* \leq nx_i^* =: X^{Pre}$$

$$\Leftrightarrow \frac{(n-2)V^M}{(\theta(n-2)+1)^2} \cdot (\theta(n-2)-(n-3)) + (n-2)\frac{(n-2)V^M}{(\theta(n-2)+1)^2} \cdot \theta \leq n \cdot \frac{n-1}{n^2}V$$

$$\Leftrightarrow \frac{(n-2)V^M}{(\theta(n-2)+1)^2} \cdot (2\theta(n-2)-(n-3)) \leq \frac{n-1}{n}V$$

$$\Leftrightarrow \frac{(n-2)}{(\theta(n-2)+1)^2} \cdot \frac{y[2n+y((n-1)^2-1)]}{n^2} \cdot (2\theta(n-2)-(n-3)) \leq \frac{n-1}{n} \cdot \frac{y[2(n+1)+y(n^2-1)]}{(n+1)^2}$$

- For $n = 3$ and $0 < y < 1$

$$\Leftrightarrow \theta \geq 1$$

- For $n \geq 4$ and $0 < y < \frac{2n-6}{3n-5} =: \hat{y}_X(n)$

$$\Leftrightarrow \theta \geq \frac{-2-4n+2n^2+3y+2ny-4n^2y+n^3y}{(n-2)(n-1)(2-y-ny)} + \sqrt{\frac{-12-8n+4n^2+22y+6ny-14n^2y+2n^3y-10y^2+ny^2+8n^2y^2-3n^3y^2}{(n-2)(n-1)^2(2-y-ny)^2}} =: \hat{\theta}^X(n,y)$$

5.C.3 Welfare

$$W^{Post} := E_I^* + (n-2)E_{NI}^* + CS^{Post} \geq nE_i^* + CS^{Pre} =: W^{Pre}$$

$$
\begin{aligned}
\Leftrightarrow & \frac{(\theta(n-2)-(n-3))}{(\theta(n-2)-(n-3))+(n-2)} V^M + \pi_l^{M^*} - \frac{(n-2)V^M}{(\theta(n-2)+1)^2}\cdot(\theta(n-2)-(n-3)) \\
& +(n-2)\left(\frac{V^M}{\theta(n-2)+1} + \pi_l^{M^*} - \frac{(n-2)V^M}{(\theta(n-2)+1)^2}\theta\right) + \frac{1}{2}\left(\frac{1+(n-1)y}{n} + (n-2)\frac{1-y}{n}\right)^2 \\
& \geq \left(\frac{V}{n}+\pi_l^*\right) + \frac{1}{2}\left(\frac{1+ny}{n+1}+(n-1)\frac{1-y}{n+1}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow & \frac{(\theta(n-2)-(n-3))}{(\theta(n-2)-(n-3))+(n-2)} \cdot \frac{y[2(n)+y((n-1)^2-1)]}{n^2} + \left(\frac{1-y}{n}\right)^2 - \frac{n-2}{(\theta(n-2)+1)^2}\frac{y[2n+y((n-1)^2-1)]}{n^2}\cdot(\theta(n-2)-(n-3)) \\
& +(n-2)\left(\frac{1}{\theta(n-2)+1}\cdot\frac{y[2n+y((n-1)^2-1)]}{n^2} + \left(\frac{1-y}{n}\right)^2 - \frac{n-2}{(\theta(n-2)+1)\cdot\frac{y[2n+y((n-1)^2-1)]}{n^2}{}^2}\theta\right) + \frac{1}{2}\left(\frac{1+(n-1)y}{n}+(n-2)\frac{1-y}{n}\right)^2 \\
& \geq \left(\frac{1}{n}\cdot\frac{y[2(n+1)+y(n^2-1)]}{(n+1)^2} + \left(\frac{1-y}{n+1}\right)^2\right) + \frac{1}{2}\left(\frac{1+ny}{n+1}+(n-1)\frac{1-y}{n+1}\right)^2
\end{aligned}
$$

- For $n=3$ and $\frac{4\sqrt{309}-67}{65} < y < \frac{4\sqrt{30}-19}{17}$

$$
\Leftrightarrow \theta \geq \frac{\sqrt{2}\sqrt{1344y-6624y^2-6912y^3-1632y^4}-7-58y-31y^2}{7-134y-65y^2} =: \hat{\theta}^W(3,y)
$$

- For $n \geq 4$ and $\hat{y}_P < y < \frac{\sqrt{-2n+19n^2+38n^3+12n^4-4n^5+n^6}+1-4n-6n^2+n^3}{1-7n-6n^2+4n^3} =: \hat{y}_W$

$$
\begin{aligned}
\Leftrightarrow \theta \geq & \frac{\left(2n\left((n-3)n(n^2-2)+5\right)+1\right)y^2+2\left(n\left(n\left(2(n-1)n-7\right)-5\right)-1\right)y+2n+1}{\left(n\left(2(n-3)n^3+6n+3\right)+2\right)y^2-2n^2+2(n-2)(2n^3+n^2+n+1)y+3n+2} + \\
& \sqrt{2}\sqrt{-\frac{n(n+1)^2y((n-2)y+2)(2n^3y(2y-1)-2n^2(3(y-2)y+1)+ny(8-7y)+n+(y-1)^2)}{(n-2)((2n(n(-n^2+n+2)+1)+1)y^2-2(2n^3+n^2+n+1)y+2n+1)^2}} =: \hat{\theta}^W(n,y)
\end{aligned}
$$

Appendix

Statistical Methods

To test the central propensity of effort choices in Chapter 3, we use a Wilcoxon signed rank test and a Kolmogorov-Smirnov test (see, e.g., Moffatt, 2015, p. 56 f, 67). The Wilcoxon signed rank test is a non-parametric test that compares the median of effort choices against the hypothetical median of 0. With the Kolmogorov-Smirnov test we compare the entire distributions of effort choices in first periods and the last periods.

The gathered data in both Chapters, 3 and 4, has a panel structure as each subject's effort choices in his two matches of a tournament are repeatedly measured across 20 periods (see, e.g., Wooldridge, 2016). For the examination of individual behavior, we thus control for dependency of multiple decisions made by a subject with a panel estimator whenever reasonable. To account for that, we primarily use fixed-effects linear regression models (PEF) which control for the individualistic, unobserved heterogeneity and assume that it is constant over time for each subject. That means, the PEF includes an individual-specific time constant error term and allows the error term to potentially correlate with the independent variables. Implicitly, the PEF controls for all time invariant independent variables and provides an unbiased estimation (see Wooldridge, 2016, p. 435 f).

However, with a PEF it is not possible to explicitly estimate the effects of time invariant variables such as the predefined player type in which we are especially interested (see, e.g., Moffatt, 2015, p. 90). As these variables are assumed to be correlated with other independent variables which vary over time, they are eliminated in the estimation. For that purpose, a random-effects linear regression model (PER) provides an efficient estimation as it additionally assumes that the individual-specific error term is uncorrelated with the dependent variables . Hence, when we explicitly want to measure the influence of time invariant variables, we use the PER instead of a PEF and assume that the control variables are not correlated with the individual-specific error term (see Wooldridge, 2016, p .441). In general, we report cluster-robust standard errors in regressions where session is the cluster variable. Thereby, we consider that naturally our sampling method is not completely randomly as subjects self-select into sessions (e.g. only early birds register for morning sessions) and that the information of participants' actions influences behavior within a session (e.g. the frequent occurrence of focal bids such as zero). Due to these session effects, recently, it has become more common to adjust standard errors by session in regressions (see, e.g., March and Sahm, 2018; Mago and Razzolini, 2019).

In case the dependent variable is discrete and takes on only a small number of values, several drawbacks might occur when estimating with a linear model (Wooldridge, 2010, p. 452; Wooldridge, 2016, pp. 226, 524 f). Even though the use of the linear model is not inappropriate, we complement our analysis on the binary (ternary) dependent variable WQP and additionally estimate a binary (ordered) logit model with the dependent variable tournament win (rank) (Wooldridge, 2010, pp. 457 f, 504 ff). It is based on the nonlinear standard logistic cumulative distribution function (CDF) and ensures that predicted values are bounded between zero and one (Wooldridge, 2016, p. 525 f). As it turns out, in our results, the drawbacks of unrealistic predictions of negative values or values greater than one are not relevant. We thus restrict our presentation of estimations results on the linear model due to its straightforward interpretation of probabilities. We only quote if direction and p values of the estimated coefficients differ in both models and provide the estimations upon request.

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