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Dealing with information-uncertainty Insights from a predatory trading game

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ABSTRACT

We develop a predatory trading model with fixed beliefs about the permissible strategy sets of market participants. These sets reflect individual trading constraints and capture the inherently uncertain informational environment of financial markets. Uncertainty arises when the trading volume of a distressed player is unknown and must be inferred from limited information, allowing us to analyze the costs of informational frictions, such as misinformation or fake news. We show how actual and expected trading volumes shape these costs, and that a player's own uncertainty may sometimes be beneficial rather than harmful. This highlights the difficulty of identifying winners and losers in uncertain markets. In the special case of pure rumors, however, the outcome is unambiguous: informed players consistently profit, while the uninformed bear the costs.

1. Introduction

Advancements in information technology have accelerated information flows while simultaneously exposing market participants to noisy and sometimes contradictory signals. This raises a central question: how does informational uncertainty shape strategic interaction in markets? To address this, models are required that explicitly incorporate imperfect or asymmetric information and capture its impact on strategic behavior and payoffs.

In this theoretical letter, a predatory trading game[1] model that explicitly incorporates uncertainty regarding permissible strategy sets is developed. Since these sets describe the trading needs of the players, we consider a game in which market participants are uncertain whether another player is subject to trading compulsions, and if so, how severe these constraints are. This method allows us to refine the theoretical model to better reflect real-world trading environments, where participants often operate under imperfect information.

Methodologically, the model of Carlin et al. (2007) constitutes the starting point of our investigation, which we extend by incorporating uncertainties – formally modeled through fixed beliefs[2] – drawing on the results of Herzing and Muck (2025). Such beliefs may, for example, be attributable either to a short game duration or to the belief perseverance bias; see, for instance, Anderson (2007). The focus of this study lies specifically on the additional costs that arise for players due to prevailing uncertainty. In doing so, we distinguish in particular between the effects of one's own uncertainty and the effects of another player's uncertainty on oneself.

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[1] Predatory trading describes the scenario in which a market participant is compelled to trade large quantities of an asset, and other players attempt to exploit this compulsion to their advantage. A listing of such known past incidents can be found in Brunnermeier and Pedersen (2005).

[2] A short discussion of belief updating and the challenges associated with it is relegated to Appendix B. Further a brief overview of fixed beliefs and related concepts can be found in Appendix C.

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Uncertainty about information in predatory trading games does not always have a straightforward effect on the trading gains of participants. While an individual's information uncertainty often impacts him negatively and benefits his opponent, there are also instances where the opposite occurs: the less informed player may actually gain an advantage from his uncertainty, whereas the fully informed player may experience adverse effects. This is intriguing for two reasons: it demonstrates how informational ambiguity in simultaneous games can produce results similar to those typically associated with sequential games, and it underscores the challenges in identifying the beneficiaries and those disadvantaged by uncertainty and potential associated regulatory interventions.

Likewise, we show that in the special case of pure trading rumors, the situation becomes considerably more clear-cut. In this case, one's own uncertainties invariably lead to losses, indicating that it cannot be expected to profit from a rumor merely by chance. Conversely, however, one can always benefit if an opponent succumbs to a rumor. In other words, in the case of pure rumors, it is always the informed player who profits, while the uninformed player bears the costs.

Trading large blocks is a well-known issue in financial economics, and optimal strategies in this regard are discussed in Bertsimas and Lo (1998). The inclusion of additional market participants leads to the expansion into the predatory trading game. Brunnermeier and Pedersen (2005) introduce a deterministic model to investigate this extension[3] and highlight the importance of such incidents by demonstrating how price fluctuations of individual stocks can lead to contagious effects for the financial system.[4]

Carlin et al. (2007) propose a stochastic framework in continuous time and set up the trading problem as a stochastic differential game. This model serves as the foundation for numerous economically driven extensions and also represents a popular application example of game-theoretic insights. In this context, Carmona and Yang (2011) demonstrate the use of a game-theoretic adaptation of the Hamilton–Jacobi-Bellman approach to delineate a closed-loop equilibrium. Furthermore, it is used in Carmona and Delarue (2013) to demonstrate a mean-field approach, while Schoeneborn and Schied (2009) explore the removal of identical trading intervals. In Herzing and Muck (2024, 2026), it serves as a foundation for modeling fake news, where the model presented there is based on a perceived equilibrium, whereas the model employed here derives an actual equilibrium. We extend the investigation into the impact of uncertainty about trading volume. Our approach mimics a market environment under information uncertainty and provides insight into the impact of the information structure on players' payoffs.

It is well-established in game theory that the classical Nash equilibrium, see Nash (1950), does not mimic the actual environment in which the actors operate.[5] For this, there are a variety of solution approaches, such as considering incomplete information or beliefs, most notably the theory of self-confirming equilibria.[6] The definition of new, more general equilibrium concepts, as presented in Goeree and Louis (2021) or the introductions of learning agents, as considered by McKenzie et al. (2024), are also conceivable. A downside of these equilibrium proposals, in addition to the question of existence in more complex setups, is the significant computational effort required, even for static or two-step sequential games. We therefore make use of an adopted equilibrium definition as presented in Herzing and Muck (2025). This approach allows us to derive a closed-form solution to the problem.

The letter is organized as follows. Section 2 describes the model setup, introduces uncertainty through fixed beliefs, and finally presents the equilibrium strategies in closed form. Section 3 discusses the costs arising from uncertainty, distinguishing between one's own uncertainty and that of the opponent. Section 4 concludes. To avoid the disturbance of the reading flow, all proofs are presented in Appendix A.

2. Model

In this section, we introduce the two player predatory trading model under consideration, with a focus on incorporating uncertainty expressed through fixed beliefs regarding the actual trading volume. Our procedure is as follows: first, we establish the underlying model; next, we formalize the uncertainty by means of fixed beliefs; finally, we provide the optimal strategies – referred to interchangeably as control or trading strategies – in explicit form.

2.1. The underlying model

For the modeling of predatory trading, a stochastic differential game is employed, as initially introduced by Carlin et al. (2007) and since then established in the literature.[7] The focus will be on the key cornerstones, for detailed explanations and empirical validations of the model, please refer to the cited literature. A graphical illustration of the model is shown in Fig. 1.

We consider a market where both a risk-free asset and a risky asset are traded, with the risk-free interest rate being set to zero without loss of generality.[8] Denoting the total quantity of the risky asset held by player $n \in \{1, 2\}$ at time t by X_t^n it holds that

$$X^n(t) = X^n(0) + \int_0^t \alpha^n(s)\,\mathrm{d}s, \tag{1}$$

[3] Further studies that deal with predatory trading include, for example, Brunnermeier and Oehmke (2014), which specifically address the case of short selling, or van Kervel and Menkveld (2019), which empirically test theoretical model results.

[4] These effects are frequently referred to in the literature as rational bubbles. They are tested and explained by Abreu and Brunnermeier (2003), Asako et al. (2020), Moinas and Pouget (2013), Brunnermeier and Morgan (2010), Brunnermeier (2001) in a different model set-up. A general overview of bubbles is provided in Scherbina and Schlusche (2014).

[5] Often, the strategies actually chosen do not align with the corresponding Nash strategies. See, for example, Goeree and Holt (2001), Crawford et al. (2013), Goeree et al. (2016), Dhami (2016).

[6] See Fudenberg and Levine (1993), Pahlke (2018), Battigalli et al. (2019), Hanany et al. (2020)

[7] See, for instance, Schoeneborn and Schied (2009), Carmona and Yang (2011), Carmona (2016), and Herzing and Muck (2024).

[8] See Carmona and Yang (2011), Carlin et al. (2007) and Brunnermeier and Pedersen (2005).

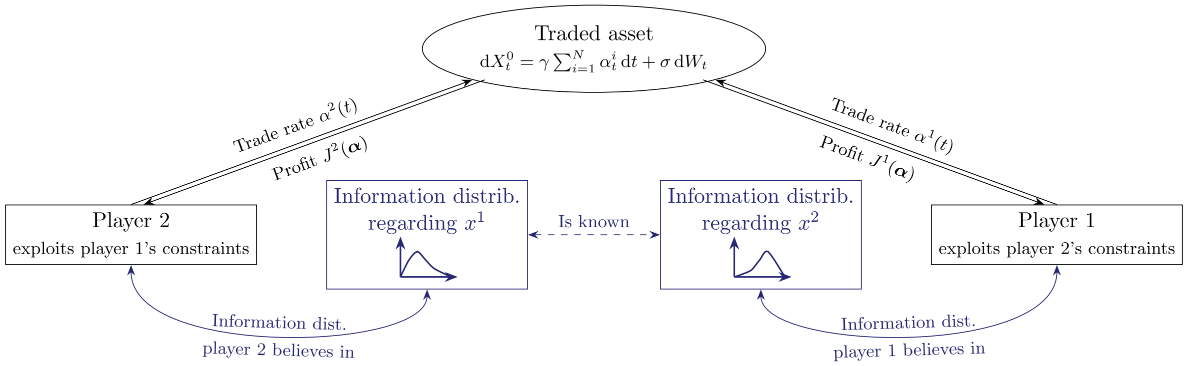


Fig. 1. The figure outlines the model used. The general dynamics of the game are depicted in black, while the model's information structure is highlighted in blue.

where α_t^n denotes the trading rate of the nth player. The processes $\alpha_t = \alpha(t)$ is assumed to be in open loop form and needs to be chosen from the player's admissible strategy set

$$\alpha_t^n = (\alpha_t^n)_{0\le t\le T} \in \mathbb{A}_x^n := \left\{ \alpha_t^n \middle| \, \mathcal{H}^2_{[0,T]} \text{ and } X_T^n - X_0^n = x_n \right\}, \tag{2}$$

which ensures that every player trades the quantity $x_n \in \mathbb{R}$ during the game.[9,10] Here, $T \in \mathbb{R}^+$ denotes the total duration of the game, that is, the time within which the players must execute the required positions. The mid-price dynamics of the risky asset X^0 are given by

$$\mathrm{d}X_t^0 = \gamma \sum_{i=1}^{2} \alpha_t^i \,\mathrm{d}t + \sigma \,\mathrm{d}W_t, \tag{3}$$

where, due to the brevity of time spans pertinent to predatory trading, a conventional drift term $\mu(t)$ can be neglected.[11] The actual transaction price P_t, that is, the price at which the transaction takes place, may differ from the mid-price. This discrepancy $P_t - X_t^0$ describes the thinning out of the limit-order book and is given by

$$P_t - X_t^0 = \lambda \sum_{i=1}^{2} \alpha_t^i. \tag{4}$$

The coefficients $\gamma \in \mathbb{R}^+$ and $\lambda \in \mathbb{R}^+$ stem from the microstructural approach[12] of the model, substantiated by a multitude of empirical investigations,[13] and indicate the ongoing and temporary influence of the cumulated trade rates on the price.[14] Here, the parameter λ captures the elasticity of prices and thus reflects the depth of the limit-order book. A broad body of empirical work[15] also documents a permanent component of price impact. This effect, represented by the parameter γ, captures the persistent shift in the asset's mid-price that arises from executed trading flows.[16] In summary, we obtain an 3 dimensional dynamic system

$$\begin{cases} \mathrm{d}X_t^0 = \gamma \sum_{i=1}^2 \alpha_t^i \,\mathrm{d}t + \sigma \,\mathrm{d}W_t, \\ \mathrm{d}X_t^1 = \alpha_t^1 \,\mathrm{d}t, \\ \mathrm{d}X_t^2 = \alpha_t^2 \,\mathrm{d}t, \end{cases} \tag{5}$$

9 The common notation $\mathcal{H}^2_{[0,T]} := \left\{ \alpha_t^n \middle| \text{ adapted and } \mathbb{E} \int_0^T \left(\alpha_t^n\right)^2 \mathrm{d}t < \infty \right\}$ is used. Further, since the resulting model depends solely on the changes in inventories, we proceed without loss of generality by implementing the normalization $X_T^n = 0$, $X_0^n = -x_n$.

10 Since uncertainty may naturally lead agents to delay trading, we note for completeness that such behavior is already included in the admissible strategy set: a player may choose a trading rate that is equal to zero on an initial subinterval $[0,t) \subset [0,T]$ and execute the required volume on $[t,T]$. Thus, temporary inaction ("wait–then–trade") patterns are fully captured by the given formulation.

11 This attempt is discussed by Huberman and Stanzl (2004) and in line with preceding literature as Carlin et al. (2007), Carmona and Yang (2011), and Schoeneborn and Schied (2009).

12 The microstructure approach studies how trading mechanisms affect the price formation process and is one of the main sources of illiquidity investigated in the literature, see e.g. Eesley et al. (1996), O'Hara (1995).

13 Refer to Kraus and Stoll (1972), Holthausen et al. (1990), Chan and Lakonishok (1995), Madhavan and Cheng (1997), Sadka (2006), and Meng et al. (2020).

14 The resulting influence of trading rates on asset prices can be found in a similar way in both empirical and theoretical studies like, Almgren and Chriss (1999), Kaul et al. (2000), Almgren and Chriss (2001) and Huberman and Stanzl (2004).

15 See footnote .

16 For a comprehensive discussion of the parameters λ and γ see Carlin et al. (2007), Carmona and Yang (2011), and Schoeneborn and Schied (2009).

under which each player, considering the trading impact of all others, seeks to maximize their trading profit, or equivalently, minimize their personal trading costs

$$\tilde{J}^n(\boldsymbol{\alpha}) = \mathbb{E}\left(\int_0^T P_t \,\mathrm{d}X_t^n\right) = \mathbb{E}\left(\int_0^T \alpha_t^n \left(X_t^0 + \lambda \sum_{i=1}^{2} \alpha_t^i\right) \mathrm{d}t\right), \tag{6}$$

where we employ the well-known notations $\boldsymbol{\alpha} := \left(\alpha^1, \alpha^2\right)^\top$ for the hole strategy profile.

2.2. Modeling uncertainty

We present the incorporation of static uncertainties into the predatory trading model. In doing so, we follow Herzing and Muck (2025) and would like to refer the reader to this reference for further details of our deliberately concise introduction.

Uncertainty: Each player is uncertain about the constraints of the other player. In detail, for $n \in \{1,2\}$, $i \in \{1,\dots,M_n\}$ let $p_i^n \in [0,1]$ denote the probability with which player n believes that the other player m is subject to the constraints $x_{m,i}^n \in \mathbb{R}$. This is, due to the constraints, equivalent to assuming that player m is required to play a strategy $\alpha^m_{x^n_{m,i}} \in \mathbb{A}^m_{x^n_{m,i}}$. Hereby $M_n \in \mathbb{N}^+$ denotes the number of realizations that player n considers possible. As usual, we require $\sum_{i=1}^{M_n} p_i^n = 1$.

Notation. To simplify the notation, we summarize all strategies that player n considers possible for player m as follows. For $n \in \{1,2\}$ and $m \neq n \in \{1,2\}$, we define:

$$\vec{\alpha}_n^p := \left(\alpha^m_{x^n_{m,1}}, \dots, \alpha^m_{x^n_{m,M_n}}\right) \in \mathbb{A}_n^p := \mathbb{A}^m_{x^n_{m,1}} \times \dots \times \mathbb{A}^m_{x^n_{m,M_n}}. \tag{7}$$

This allows us to define the expected trading costs under uncertainty. This approach reflects risk-neutral behavior and is thus consistent with the original focus on the expected value in $\tilde{J}(\cdot)$.

Definition 2.1 (*Trading Profit Under Static Uncertainty*). With $\tilde{J}$ from (6), we define the objective functional under fixed beliefs J^n for all players $n \in \{1,2\}$ by

$$J^n\left(\alpha^n, \vec{\alpha}_n^p\right) := \mathbb{E}\left(\tilde{J}^n\left(\alpha^n, \vec{\alpha}_n^p\right)\right) := \sum_{i=1}^{M_n} p_i^n \tilde{J}^n\left(\alpha^n, \alpha^m_{x^n_{m,i}}\right). \tag{8}$$

Finally, this allows us to define the equilibrium under static uncertainty. It differs from the classical Nash equilibrium in that all strategies considered possible are explicitly taken into account.

Definition 2.2 (*Equilibrium with static uncertainty*). We call the strategy profile $\hat{\boldsymbol{\alpha}} := \left(\hat{\vec{\alpha}}_1^p, \hat{\vec{\alpha}}_2^p\right) \in \mathbb{A}_1^p \times \mathbb{A}_2^p$ a Nash Equilibrium (with static uncertainty) if

$$\begin{cases} J^1\left(\hat{\alpha}^1_{x^2_{1,i}}, \hat{\vec{\alpha}}_1^p\right) < J^1\left(\alpha^1_{x^2_{1,i}}, \hat{\vec{\alpha}}_1^p\right) & \forall \alpha^1_{x^2_{1,i}} \in \mathbb{A}^1_{x^2_{1,i}},\ \forall i \in [1,\dots,M_2], \\ J^2\left(\hat{\alpha}^2_{x^1_{2,i}}, \hat{\vec{\alpha}}_2^p\right) < J^2\left(\alpha^2_{x^1_{2,i}}, \hat{\vec{\alpha}}_2^p\right) & \forall \alpha^2_{x^1_{2,i}} \in \mathbb{A}^2_{x^1_{2,i}},\ \forall i \in [1,\dots,M_1]. \end{cases} \tag{9}$$

2.3. Equilibrium trading strategies

The following theorem presents the optimal equilibrium trading strategies and forms the foundation for the subsequent analysis.

Theorem 2.1. *Define the expected constraints* $\mathbb{E}\left(x_m^n\right)$ *of player* $n \in \{1,2\}$ *regarding player* $m \neq n \in \{1,2\}$ *by* $\mathbb{E}\left(x_m^n\right) := \sum_{i=1}^{M_n} p_i^n x_{m,i}^n$, *the equilibrium solution* $\hat{\alpha}$ *can be explicitly computed and is given by*

$$\hat{\alpha}^1_{x^2_{1,i}} = \hat{\alpha}^1_{\mathbb{E}\left(x^2_1\right)} + \left(x^2_{1,i} - \mathbb{E}\left(x^2_1\right)\right) T^{-1}, \quad \hat{\alpha}^2_{x^1_{2,i}} = \hat{\alpha}^2_{\mathbb{E}\left(x^1_2\right)} + \left(x^1_{2,i} - \mathbb{E}\left(x^1_2\right)\right) T^{-1}, \tag{10}$$

with

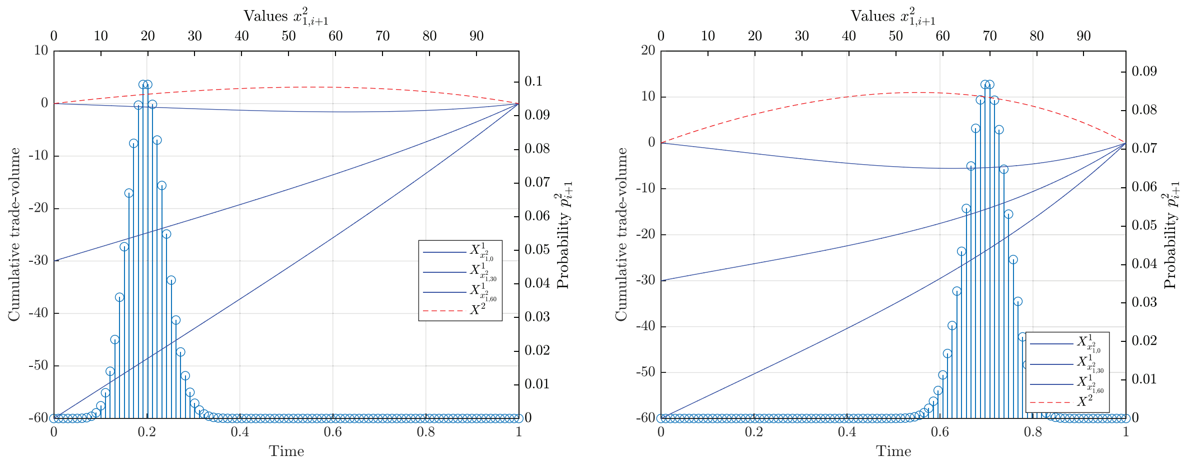
$$\hat{\alpha}^1_{\mathbb{E}(x^2_1)} = a e^{-\frac{1}{3}\frac{\gamma}{\lambda} t} + b_1 e^{\frac{\gamma}{\lambda} t}, \quad \hat{\alpha}^2_{\mathbb{E}(x^1_2)} = a e^{-\frac{1}{3}\frac{\gamma}{\lambda} t} + b_2 e^{\frac{\gamma}{\lambda} t}, \tag{11}$$

where the coefficients $a \in \mathbb{R}$ *and* $b_n \in \mathbb{R}$, $n \in \{1,2\}$, *are determined by*

$$a = -\frac{1}{3}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{1}{3}\frac{\gamma}{\lambda} T}\right)^{-1} \frac{x_0^1 + x_0^2}{2}, \quad b_n = \frac{\gamma}{\lambda}\left(e^{\frac{\gamma}{\lambda} T} - 1\right)^{-1}\left(\frac{x_0^1 + x_0^2}{2} - x_0^n\right), \tag{12}$$

with $x_0^1 = -\mathbb{E}\left(x_1^2\right)$ *and* $x_0^2 = -\mathbb{E}\left(x_2^1\right)$.

Panel 2 illustrates Theorem 2.1 by exemplifying how the optimal trading (control) strategies respond to two different probability distributions of the beliefs.



(a) Demonstrates Theorem 2.1 with $p = 0.2$. (b) Demonstrates Theorem 2.1 with $p = 0.7$.

Fig. 2. Demonstrates exemplary the portfolio holdings of the predator as well as selected portfolio holdings of the victim in the case where the victim must actually acquire $0, 30, 60$ units of the risky good according to Theorem 2.1. Additionally, the associated distribution is given, which reflects the uncertainty of the predator and thus influences the optimal strategies. For demonstration, it is assumed that the predator follows a hands-clean scheme, and the game parameters are chosen as $\gamma = 0.1, \lambda = 0.5\gamma, T = 1, M_2 = 100$ for a binomial distribution, this is $p^2_{i+1} = \binom{M_2-1}{i} p^i (1-p)^{M_2-i-1}$, $\forall\, i \in \{0, \dots, M_2 - 1\}$. Here, $X^1_{x^2_{1,i}}$ denotes the portfolio holdings associated with $\hat{\alpha}^1_{x^2_{1,i}}$.

2.4. Empirical verification

Although the focus of this letter is theoretical, we briefly outline empirical findings that could be used for an empirical verification of the model. Several existing datasets and identification strategies naturally lend themselves to testing the qualitative predictions of our model. Empirical work on large block trades and institutional order execution (e.g., Kraus and Stoll (1972), Holthausen et al. (1990), Chan and Lakonishok (1995), van Kervel and Menkveld (2019)) as well as recent studies of the GameStop episode (Hasso et al., 2022; Umar et al., 2021; Klein, 2022) provide settings in which trading pressure, liquidity stress, and the strategic responses of other market participants can be observed in high frequency.

To connect this evidence to our belief-based framework, one could combine such data with established methods for inferring market participants' beliefs or expectations from observable variables. Order-flow–based inference of private information (Eesley et al., 1996; O'Hara, 1995), sentiment- and attention-driven belief measures (Barber et al., 2009; Shiller, 2002), or option-implied expectation proxies provide plausible ways to identify whether traders behave as if they hold persistent beliefs about an opponent's liquidity needs.

3. Costs of uncertainty

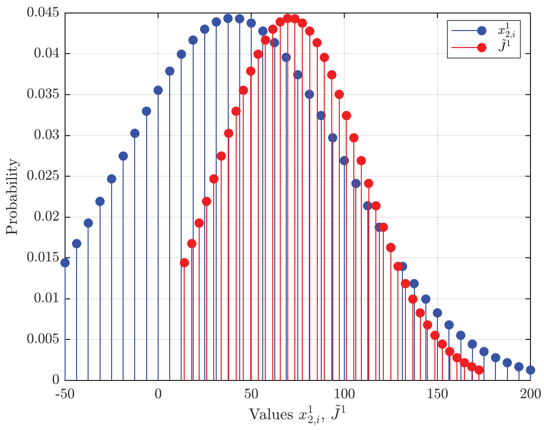
In the following, we provide a detailed analysis of the impact of uncertainty in the predatory trading game. We distinguish between the effect of own uncertainty, i.e., the uncertainty a player has with respect to the other player, and the effect of external uncertainty, i.e., the uncertainty the other player has with respect to oneself. Without loss of generality, we adopt the perspective of the first player.

3.1. Own uncertainty

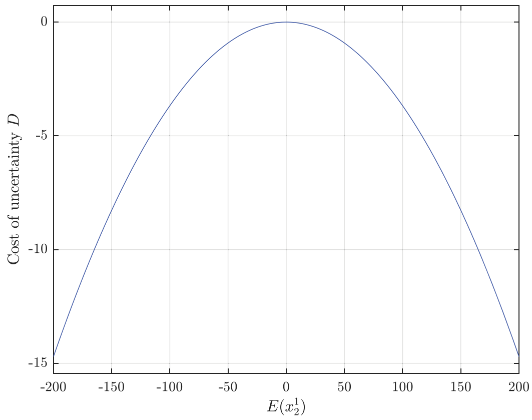
We begin with the case in which the first player is uncertain about the actual trading volumes of the second player, whereas the second player is fully informed about all actual trading volumes and thus does not face any uncertainty.

3.1.1. Cost distribution

The main characteristic of own uncertainty is that, in contrast to the case without uncertainty, the player is confronted with a profit/cost distribution that depends on the actual trading volume of the opponent. Depending on the realized trading volume, the actual outcome may therefore deviate to varying degrees from the expected profit (8). This general result will be illustrated in Panel 3(a).



(a) Distribution of the assumed possible trading costs $\tilde{J}^1(\cdot)$ depending on the underlying information distribution with $x_1 = 65$.



(b) Demonstrates the losses from Theorem 3.2 depending on the expected trading volume of the opponent.

Fig. 3. The figure depicts the general cost distribution in the presence of own uncertainty (Fig. 3(a)). Shown in blue is the first player's uncertainty regarding the second player's trade quantity, which follows the illustrated probability distribution. Depending on the realized value, the resulting costs are shown in red; that is, the cost distribution follows directly from the distribution of the underlying uncertainty. Further the expected losses according to Theorem 3.2 (Fig. 3(b)) are illustrated. It becomes clearly evident that the losses increase the further the expectations deviate from the actual trade quantity $x_2 = 0$. The game parameters are chosen as $\gamma = 0.1, \lambda = 0.5\gamma, T = 1$.

3.1.2. General costs

We now consider the costs incurred by the first player due to his informational uncertainty. To this end, we fix arbitrary trading volumes x_1, x_2 for the two players and compare the profit of the first player that would have arisen in the case without uncertainty with the profit that arises under uncertainty, expressed through $\mathbb{E}(x_2^1)$. Formally, we consider the difference

$$D := \tilde{J}^1\left(\tilde{\alpha}^1, \tilde{\alpha}^2\right) - \tilde{J}^1\left(\hat{\alpha}^1, \hat{\alpha}^2\right), \tag{13}$$

where $\left(\tilde{\alpha}^1, \tilde{\alpha}^2\right) \in \mathbb{A}^1_{x_1} \times \mathbb{A}^2_{x_2}$ denote the classical equilibrium strategies without uncertainty, and $\left(\hat{\alpha}^1, \hat{\alpha}^2\right)$ denote the equilibrium strategies that arise under uncertainty according to Definition 2.2 for the respective realizations. The following theorem summarizes the fundamental properties:

Theorem 3.1. *The quantity D is a quadratic function in x_2 as well as in $\mathbb{E}(x_2^1)$, and linear in x_1. In particular, own uncertainty may affect the first player either positively or negatively.*

Theorem 3.1 essentially reveals two fundamental insights. First, D is a linear function in the trading volume of the first player. In other words, a change in the known trading volume x_1 has only a constant impact on the costs, whereas the unknown trading volume x_2 and its estimate $\mathbb{E}(x_2^1)$ exert a quadratic influence on the trading costs. Hence, for larger realizations, the unknown trading volume carries significantly more weight than the known one. Second, the theorem demonstrates that uncertainty is by no means always detrimental for the respective player. Indeed, situations may arise in which an uncertain player incurs lower costs than if the trading volume had been known from the outset. Such cases may be motivated, for instance, by the property of *forced reaction*, which we briefly outline at the end of this section. Furthermore, Panel 4 provides a graphical illustration of the theorem.

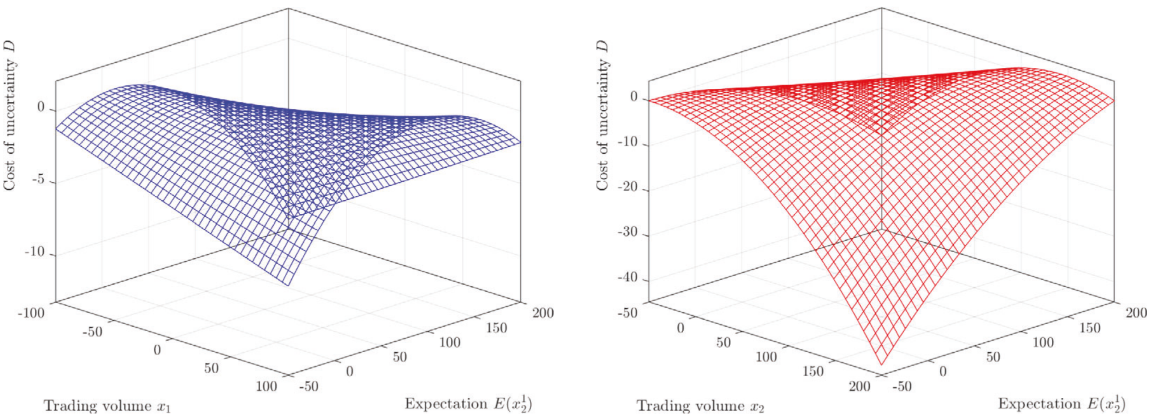
3.1.3. Pure rumors

We now focus on the special case in which neither player is actually forced to trade, that is, $x_1 = 0$, $x_2 = 0$, which we refer to as the *pure rumor case*. In this situation, all trading arises solely from rumors, i.e., from the uncertain information of the first player rather than from actual trading needs. This isolates the informational channel in its simplest form and allows us to analyze situations in which gains or losses emerge purely from belief-driven misperceptions.

Theorem 3.2. *Under the assumptions made in this subsection, own uncertainty never has a positive effect on the player himself. For all possible realizations with $\mathbb{E}(x_2^1) \neq 0$ it is negative, i.e., associated with higher expected costs, and it has no effect in the case $\mathbb{E}(x_2^1) = 0$. More precisely, with the notation introduced above, we obtain*

$$D = -\frac{\mathbb{E}(x_2^1)^2}{3T}\left(T\gamma \coth\left(\frac{T\gamma}{6\lambda}\right) - 6\lambda\right). \tag{14}$$

Theorem 3.2 shows that the cost difference induced by own uncertainty is quadratic in $\mathbb{E}(x_2^1)$. Hence, the larger the actual trading blocks of the second player, the more pronounced the impact of uncertainty becomes. Moreover, the theorem establishes that in the case of pure rumors this impact is always negative, i.e., it is associated with losses. Thus, one cannot expect profits to arise



(a) Demonstrates the linear effect of x_1 described in Theorem 3.1 for fixed $x_2 = 40$.

(b) Demonstrates the quadratic effect of x_2 described in Theorem 3.1 for fixed $x_1 = 40$.

Fig. 4. The figure illustrates the linear (Fig. 4(a)) and quadratic (Fig. 4(b)) effects described in Theorem 3.1 as a function of different expected trading volumes. If one fixes, for example, a value x_1 in Fig. 4(a), the quadratic shape of the cost function (as a function of the information state $E(x_2)$) becomes immediately apparent. In particular, it becomes evident that there are regions above the zero line. These correspond to cases in which the first player benefits from his own uncertainty — i.e., situations in which having imperfect information is more profitable for him than having perfect information. The game parameters are chosen as $\gamma = 0.1, \lambda = 0.5\gamma, T = 1$.

from pure uncertainty, which constitutes a crucial difference to the general case considered above. A graphical demonstration of the relationship between the expected trading volume and the expected loss is provided in Panel 3(b).

3.2. External uncertainty

We now consider the case in which the first player knows the exact trading volumes, while the second player faces uncertainty regarding the actual trading volume of the first player. In contrast to the case of own uncertainty, the first player does not encounter a profit distribution that depends on the actual trading volumes (since these are known to him from the outset). Therefore, we may directly proceed with the analysis of the general costs.

3.2.1. General costs

We consider the costs D arising from uncertainty as defined in (13) and are interested in the general structure of these costs.

Theorem 3.3. *The quantity D is a quadratic function in x_1 as well as in $\mathbb{E}(x_1^2)$, and linear in x_2. In particular, external uncertainty may affect the first player either positively or negatively.*

Theorem 3.3 shows that, for larger trading volumes, the own trading volume x_1 as well as the belief of the second player $\mathbb{E}(x_1^2)$ constitute the relevant quantities. In particular, together with the insights from Theorem 3.1, one may conclude that it is always the actual trading volume of the player who is not subject to uncertainty, along with the expectations of the uncertain player, that exert a decisive influence on the trading profits. By contrast, the actual trading constraints of the uncertain player themselves have a less prominent impact. The effects described in the theorem are visualized in Panel 5(a).

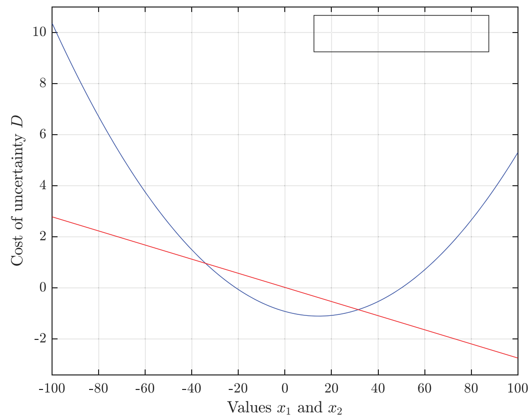
3.2.2. Pure rumors

We return to the special case in which the game is not induced by actual trading constraints but arises purely from uncertainty, that is, $x_1 = 0, x_2 = 0$. As already discussed in Section 3.1.3, all trading activity in this setting is driven entirely by informational ambiguity; however, unlike in the previous case, it is now the opponent's uncertainty that is the primary focus. The following theorem summarizes the result:

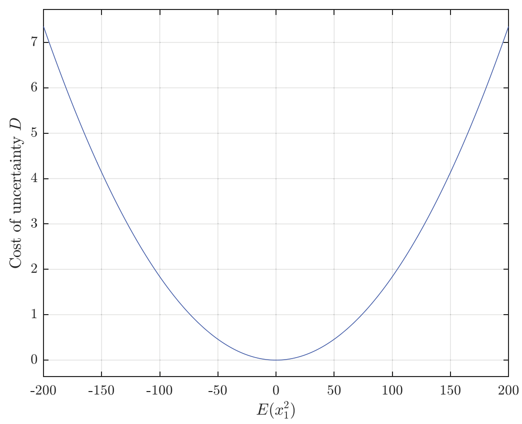
Theorem 3.4. *Under the assumptions made in this subsection, external uncertainty never has a negative effect on the player. For all possible realizations with $\mathbb{E}(x_1^2) \neq 0$ it is even positive, and in the case $\mathbb{E}(x_1^2) = 0$ it equals zero. More precisely, with the notation introduced above, we obtain*

$$D = \frac{1}{6}\mathbb{E}(x_1^2)^2 \left(\gamma \coth\left(\frac{\gamma T}{6\lambda}\right) - \frac{6\lambda}{T} \right). \tag{15}$$

Theorem 3.4 demonstrates that a player without actual trading constraints can always exploit uncertainties regarding his own trading constraints to his advantage, with profitability increasing (at a quadratic rate) the further the erroneously assumed trading values deviate from zero. Taken together with Theorem 3.2, one may thus conclude that in the case of pure rumors own uncertainty is always exploited by the other player and leads to losses, whereas external uncertainty can always be exploited profitably. The results of the above theorem are depicted in Panel 5(b).



(a) Illustrates the linear effect of x_2 and the quadratic effect of x_1 on trading costs, as described in Theorem 3.3.



(b) Illustrates the profit that can be gained from the opponent's uncertainty, as stated in Theorem 3.4.

Fig. 5. Illustrates the general costs of uncertainty as described in Theorem 3.3, together with the specific costs in the case of pure rumors as stated in Theorem 3.4. The quadratic (blue) line in Fig. 5(a) illustrates, for example, that the second player's uncertainty can be either profitable or costly for the first player, depending on the latter's actual trade quantity. In contrast, in Fig. 5(b) the line never falls below zero, indicating that the first player benefits from the opponent's uncertainty in all cases. The game parameters are chosen as $\gamma = 0.1, \lambda = 0.5\gamma, T = 1$.

Table 1
Comparative statics of D with respect to market parameters under own and external uncertainty. We use $a := T\gamma/(6\lambda)$, $E_1 := \mathbb{E}(x_2^1)^2$, and $E_2 := \mathbb{E}(x_1^2)^2$. The signs are derived for $\gamma, \lambda, T > 0$ using standard inequalities for hyperbolic functions (in particular $\sinh(x) > x$ for $x > 0$).

Quantity	Uncertainty	Parameter	Partial derivative	Sign/monotonicity
		γ	$-\frac{E_1}{3}\coth(a) + \frac{E_1 \gamma T}{18\,\lambda\,\sinh^2(a)}$	< 0 (decreasing)
D	own	λ	$-\frac{E_1 T \gamma^2}{18\,\lambda^2\,\sinh^2(a)} + \frac{2E_1}{T}$	> 0 (increasing)
		T	$\frac{E_1 \gamma^2}{18\,\lambda\,\sinh^2(a)} - \frac{2E_1\lambda}{T^2}$	< 0 (decreasing)
		γ	$\frac{E_2}{6}\coth(a) - \frac{E_2 \gamma T}{36\,\lambda\,\sinh^2(a)}$	> 0 (increasing)
D	external	λ	$\frac{E_2 T \gamma^2}{36\,\lambda^2\,\sinh^2(a)} - \frac{E_2}{T}$	< 0 (decreasing)
		T	$-\frac{E_2 \gamma^2}{36\,\lambda\,\sinh^2(a)} + \frac{E_2\lambda}{T^2}$	> 0 (increasing)

3.3. Influence of market parameters

Although the qualitative results of the model do not depend on specific parameter values, the magnitude of the effects identified in the analysis naturally varies with the underlying market environment. It is therefore informative to examine how changes in these parameters shape the strength of the predicted effects. For the purpose of a transparent and analytically precise demonstration, we restrict this sensitivity discussion to the pure rumor case. In this setting, the closed-form expressions permit a direct comparison across different market regimes. Table 1 highlights how variations in γ, λ or in the time horizon T amplify or attenuate the quantitative effects, while leaving the qualitative structure of the equilibrium unchanged. It is observable that, under own uncertainty, increases in γ and T reduce the cost of uncertainty D, while a larger market depth λ raises it. In contrast, under external uncertainty, the parameters exhibit exactly the opposite behavior: γ and T amplify D, whereas λ exerts a mitigating effect.

3.4. Economic interpretation of uncertainty costs

Although the model does not impose explicit penalties for maintaining strategic ambiguity, the endogenous cost of uncertainty can be related to real-world institutional mechanisms. Regulatory measures that reduce informational uncertainty – such as disclosure requirements or enhanced transparency standards – would eliminate both the losses borne by disadvantaged agents and the potential gains enjoyed by those who profit from uncertainty. In this sense, the cost term D can also be viewed as capturing the economic consequences that would arise if such uncertainty were removed through regulation.

3.5. Forced reaction

Although the model assumes a simultaneous game, the information (equilibrium) structure can place players in a reactive position. In such cases, a player is allowed and required to respond to the strategy of another player. For better understanding, consider the case with two possible trading quantities $x^2_{1,1}$ with $p^2_1 = 0$ and $x^2_{1,2}$ with $p^2_2 = 1$ assumed by player 2. Let $x^2_{1,1}$ be the actual quantity traded by player 1. According to Definition 2.2, it follows that

$$\min_{\alpha^1_{x^2_{1,1}} \in \mathbb{A}^1_{x^2_{1,1}}} \tilde{J}^1\left(\alpha^1_{x^2_{1,1}}, \alpha^2\right), \min_{\alpha^1_{x^2_{1,2}} \in \mathbb{A}^1_{x^2_{1,2}}} \tilde{J}^1\left(\alpha^1_{x^2_{1,2}}, \alpha^2\right), \min_{\alpha^2 \in \mathbb{A}^2} \tilde{J}^2\left(\alpha^1_{x^2_{1,2}}, \alpha^2\right), \tag{16}$$

holds in the equilibrium. Thus, player 1 recognizes that player 2 will respond optimally to a misperceived strategy $\alpha^1_{x^2_{1,2}}$ ((16) (conditions 2,3)). Consequently, the first player is left with no choice but to respond in (16) (condition 1) as effectively as possible to this strategy. Although the reactive position of player 1 may initially resemble non-simultaneous games, it must be clearly distinguished in this case, as player 2 is unaware that its actions compel the first player to react. The described effect, therefore, fully and exclusively arises from the prevailing uncertainty in the information structure.

4. Conclusion

In this paper, we introduced a predatory trading model that integrates fixed beliefs to capture informational uncertainty in financial markets. Building on existing approaches, we derived closed-form equilibrium strategies and analyzed how uncertainty alters players' outcomes. Our analysis shows that the effects of uncertainty are not uniform: while it often imposes losses on the uncertain player, it can in some cases turn into an advantage, thereby complicating the allocation of gains and losses across participants. It is further demonstrated that ambiguity in simultaneous games can generate dynamics that are usually associated with sequential interactions.

A clearer picture emerges when the environment is driven solely by rumors. In this setting, uncertainty unequivocally disadvantages the uninformed, whereas informed players consistently benefit. This asymmetry highlights the importance of distinguishing between different forms of informational frictions when assessing market outcomes. Future work could explore richer belief structures or dynamic updating mechanisms to further understand how uncertainty shapes trading behavior and market efficiency.

Appendix A. Proofs

Proof of Theorem 2.1. The described problem falls into the general structure of Herzing and Muck (2025), so that (after an adjustment of notation) we directly obtain

$$\hat{\alpha}^1_{x^2_{1,i}} = \hat{\alpha}^1_{\mathbb{E}\left(x^2_1\right)} + \left(x^2_{1,i} - \mathbb{E}\left(x^2_1\right)\right) T^{-1}, \quad \hat{\alpha}^2_{x^1_{2,i}} = \hat{\alpha}^2_{\mathbb{E}\left(x^1_2\right)} + \left(x^1_{2,i} - \mathbb{E}\left(x^1_2\right)\right) T^{-1}, \tag{17}$$

where $\left(\hat{\alpha}^1_{\mathbb{E}\left(x^2_1\right)}, \hat{\alpha}^2_{\mathbb{E}\left(x^1_2\right)}\right)$ denotes the equilibrium of the identical game without uncertainty, provided that the trading volumes $x_1 = \mathbb{E}\left(x^2_1\right)$ and $x_2 = \mathbb{E}\left(x^1_2\right)$ must be traded. This equilibrium, in turn, is known from Carlin et al. (2007) and explicitly determined. In combination, the claim follows directly. □

Proof of Theorem 3.1. To show the first claim, note that the optimal trading strategies $\tilde{\alpha}^1, \tilde{\alpha}^2, \hat{\alpha}^1, \hat{\alpha}^2$ are directly given by Theorem 2.1, where the strategies $\tilde{\alpha}^1, \tilde{\alpha}^2$ without uncertainty are given by $\mathbb{E}(x^2_1) = x_1$ and $\mathbb{E}(x^1_2) = x_2$. Plugging into (6) and evaluating the expression leads directly to the quadratic structure in $x_2, \mathbb{E}(x^1_2)$ and linear structure in x_1. To prove the second claim, it suffices to consider the special case $x_1 = 0$. Thus D is given by the function

$$f\left(\mathbb{E}(x^1_2), x_2\right) := \left(\mathbb{E}(x^1_2) - x_2\right) \cdot \left(\mathbb{E}(x^1_2) \cdot \alpha + x_2 \cdot \beta\right),$$

where the coefficients α and β are defined as

$$\alpha := 12\lambda - 2T\gamma \coth\left(\frac{T\gamma}{6\lambda}\right), \qquad \beta := \frac{2T\gamma \sinh\left(\frac{T\gamma}{3\lambda}\right)}{1 + 2\cosh\left(\frac{T\gamma}{3\lambda}\right)},$$

with parameters $T, \gamma, \lambda > 0$. We now prove by contradiction that the expression $f\left(\mathbb{E}(x^1_2), x_2\right)$ is not strictly negative for all $\mathbb{E}(x^1_2) \neq x_2$. Assume, for the sake of contradiction, that $f < 0$ holds for all such inputs. Then, for every $\mathbb{E}(x^1_2) \neq x_2$, the two factors in the expression must have opposite signs. In particular, the sign of the linear function

$$\psi\left(\mathbb{E}(x^1_2)\right) := \mathbb{E}(x^1_2) \cdot \alpha + x_2 \cdot \beta$$

must be opposite to the sign of $\mathbb{E}(x^1_2) - x_2$. This can only happen if $\psi\left(\mathbb{E}(x^1_2)\right)$ changes sign precisely at $\mathbb{E}(x^1_2) = x_2$, i.e., if

$$\psi(x_2) = x_2 \cdot \alpha + x_2 \cdot \beta = x_2(\alpha + \beta) = 0.$$

This implies that $\alpha + \beta = 0$ must hold. However, the expressions for α and β defined above do not satisfy $\alpha + \beta = 0$ in general. Therefore, the condition required for the sign argument to hold fails in general. This contradiction shows that the original claim – namely that $f < 0$ for all $\mathbb{E}(x_2^1) \neq x_2$ and all $T, \gamma, \lambda > 0$ – cannot be true. □

Proof of Theorem 3.2. We consider the expression

$$f = -\frac{\mathbb{E}(x_2^1)^2}{3T}\left(-6\lambda + T\gamma \coth\left(\frac{T\gamma}{6\lambda}\right)\right)$$

and aim to show that it is strictly negative for all $\mathbb{E}(x_2^1) \neq 0$ and all parameters $T, \gamma, \lambda > 0$. Since the prefactor $-\frac{1}{3T}$ is negative and the squared expectation $\mathbb{E}(x_2^1)^2$ is strictly positive, the sign of the expression is determined by the term in parentheses. Substituting $x := \frac{T\gamma}{6\lambda}$, we rewrite this term as

$$-6\lambda + T\gamma \coth\left(\frac{T\gamma}{6\lambda}\right) = T\gamma\left(\coth(x) - \frac{1}{x}\right).$$

The inequality $\coth(x) > \frac{1}{x}$ for all $x > 0$ is well known from real analysis. It follows that the entire expression in parentheses is strictly positive, which implies that f is strictly negative for all $\mathbb{E}(x_2^1) \neq 0$ and all positive parameters. □

Proof of Theorem 3.3. By the procedure described in Theorem 3.1, the quantity D can be computed explicitly. This reveals a quadratic dependence on x_1 and $\mathbb{E}(x_1^2)$, as well as a linear dependence on x_2. To establish that D attains both positive and negative values, it is sufficient to focus on the case $x_2 = 0$, since finding two admissible parameter choices with opposite signs already proves the claim in the general setting. For $x_2 = 0$, introduce the abbreviations

$$A := \coth\left(\tfrac{\gamma T}{6\lambda}\right), \qquad B := \operatorname{csch}\left(\tfrac{\gamma T}{2\lambda}\right), \qquad C := 2\cosh\left(\tfrac{\gamma T}{2\lambda}\right) + \cosh\left(\tfrac{\gamma T}{6\lambda}\right),$$

so that

$$D = \tfrac{1}{6}\left(\mathbb{E}(x_1^2) - x_1\right)\left[\gamma\,\mathbb{E}(x_1^2)\,A + \tfrac{6\lambda}{T}\left(x_1 - \mathbb{E}(x_1^2)\right) - \gamma x_1\, C\, B\right] =: \tfrac{1}{6} f_1 f_2,$$

with $f_1 := \mathbb{E}(x_1^2) - x_1$ and f_2 the bracketed term. Both f_1 and f_2 are continuous for $\gamma, T, \lambda > 0$. If D were to have the same sign for all admissible parameters, then both factors would have to change sign at exactly the same points. In particular, since both are continuous, they would have to have their zeros at the same locations. The first factor f_1 is zero precisely when $x_1 = \mathbb{E}(x_1^2)$. Evaluating f_2 at this point yields

$$f_2\Big|_{x_1=\mathbb{E}(x_1^2)} = -\frac{2\gamma x_1\,\sinh\left(\frac{\gamma T}{3\lambda}\right)}{2\cosh\left(\frac{\gamma T}{3\lambda}\right) + 1},$$

where $x_1 = \mathbb{E}(x_1^2)$ is understood. For $\gamma > 0$, $T > 0$, and $\lambda > 0$, the denominator is strictly positive and $\sinh\left(\frac{\gamma T}{3\lambda}\right) > 0$, so $f_2\Big|_{x_1=\mathbb{E}(x_1^2)} \neq 0$ in the generic case $x_1 \neq 0$. Hence the zero sets of f_1 and f_2 do not coincide. Therefore, as x_1 crosses $\mathbb{E}(x_1^2)$ while f_2 stays nonzero, the product $D = \frac{1}{6} f_1 f_2$ changes sign. Consequently, D assumes both positive and negative values. □

Proof of Theorem 3.4. Let $T > 0$, $\gamma > 0$, $\lambda > 0$, and $\mathbb{E}(x_1^2) \in \mathbb{R}$. Consider

$$\frac{1}{6}\,\mathbb{E}(x_1^2)^2\left(\gamma\,\coth\left(\frac{\gamma T}{6\lambda}\right) - \frac{6\lambda}{T}\right).$$

Setting $x := \frac{\gamma T}{6\lambda} > 0$ gives

$$\gamma\,\coth(x) - \frac{6\lambda}{T} = \gamma\left(\coth(x) - \frac{1}{x}\right).$$

By the well-known inequality $\coth(x) > \frac{1}{x}$ for $x > 0$, the bracket is strictly positive. Since $\frac{1}{6}\mathbb{E}(x_1^2)^2 \geq 0$ and vanishes only if $\mathbb{E}(x_1^2) = 0$, the entire expression is strictly positive for $\mathbb{E}(x_1^2) \neq 0$ and equal to zero exactly when $\mathbb{E}(x_1^2) = 0$. □

Appendix B. Updating beliefs

We briefly discuss two arguments that support the use of fixed beliefs and highlight the considerable difficulties that arise in a belief-updating framework. First, if one assumes that players do not extract information from variables generated within the game, then beliefs remain fixed simply because the information set does not change during the interaction. Under this interpretation, the question of dynamic updating does not arise in the first place.

Second, if one instead allows players to use observable quantities to update their beliefs, a nontrivial game-theoretic issue emerges. Any updating rule must specify how observed variables are mapped into inferences about the opponent's strategy and underlying objectives. If a player does not know the opponent's updating rule, he must form beliefs about how the opponent forms beliefs, shifting the specification problem one level down. If, at that lower level, one again permits dynamic belief updating, the same issue reappears, ultimately generating an infinite regress of higher-order belief hierarchies without ever resolving the specification problem. If, alternatively, the updating rule were common knowledge, the opponent could exploit it strategically (classic signal jamming) by choosing actions that deliberately distort the inferred beliefs. Anticipating such manipulation, a rational player would again have to question whether learning based on this rule is optimal, meaning that learning is strategically undermined.

Thus, introducing belief updating leads either to unresolved belief hierarchies or to strategic manipulation of the learning mechanism itself. Adopting fixed beliefs avoids these issues and yields a consistent and tractable information structure for the analysis.

A concrete attempt to introduce dynamic belief updating was made by Teguia (2015). In this approach, updating at the first level merely pushes the belief problem one level down. To avoid the resulting belief regression, it had to be assumed that only one of the two players updates at all, while the second player neither updates himself nor reacts to the opponent's updating rule, even though he is assumed to know exactly how the first player updates.

Appendix C. Fixed beliefs and related concepts

Fixed beliefs have been studied in a variety of broader game-theoretic contexts. In classical Bayesian models of incomplete information, players are endowed with fixed priors about the types of their opponents, which remain constant whenever no new information is observed. This structure is explicit in Harsanyi's type-space formulation (Harsanyi, 1967, 1968a,b), and forms the basis of standard static Bayesian games as presented in Mas-Colell et al. (1995). Kreps (1990) further emphasizes that fixed hierarchies of beliefs are fundamental to Bayesian analysis whenever no informative feedback is available during strategic interaction.

Beyond these classical foundations, several modern equilibrium concepts also rely on fixed or non-updated belief specifications. The self-confirming equilibrium of Fudenberg and Levine (1993) allows players to hold beliefs that need not be correct off the equilibrium path, provided they are consistent with observed play. Analogy-based expectation equilibria as introduced by Jehiel (2005), Jehiel and Koessler (2018) likewise replace full Bayesian updating with coarse, stable belief partitions, while Goeree and Louis (2021) propose the M-equilibrium, in which beliefs must satisfy simplicity and consistency requirements but are not tied to Bayes' rule. These frameworks illustrate that equilibrium concepts with fixed or non-Bayesian beliefs play a substantial role in the broader game-theoretic literature.

At the same time, both Bayesian updating and self-confirming learning can face substantial technical challenges even in simple two-step repeated games (see, e.g., Battigalli and Guaitoli (1997), Battigalli and Siniscalchi (2002)), and their application in fully dynamic stochastic environments is often limited from a practical perspective. A short discussion of these difficulties is provided in Appendix B.

Finally, the concept of piecewise fixed beliefs should be mentioned. In such approaches, updating is permitted only at certain discrete points in time, while beliefs remain fixed between these points. It should be noted, however, that depending on the specific updating rule, the issues discussed in Appendix B may still arise at those discrete updating times. An approach of this type is illustrated in Herzing and Muck (2026).

Data availability

No data was used for the research described in the article.

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