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Competition With Limited Attention to Quality Differences

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ABSTRACT

I analyze the implication of consumers' limited attention to quality differences on market outcomes and welfare. I model this limited attention with a perception threshold, that is, consumers only perceive quality differences between goods that exceed their perception threshold. The model allows for equilibria with distinguishable and with indistinguishable qualities. If firms are sufficiently horizontally differentiated, firms produce goods with indistinguishable qualities. Then, limited attention harms consumers and benefits firms. If firms are not sufficiently horizontally differentiated, firms produce goods with distinguishable qualities. Then, limited attention has no effect on consumers' welfare or firms' profits.

1 | Introduction

Increasing evidence documents that consumers pay only limited attention to the goods in the market.[1] As a consequence, consumers often do not notice quality differences between the goods in the market. Empirical and experimental research shows that consumers, for example, do not notice differences in fuel costs of cars [7], differences in energy costs of lightbulbs [8], differences in coffee quality [9], or differences in restaurant hygiene [10].

The objective of this article is to analyze the implications of such limited attention to quality differences on market outcomes and welfare. In particular, I explore whether consumers' limited attention influences the quality distribution in the market and whether consumers are harmed by imperfectly attending to quality differences between goods. Limited attention to quality differences harms consumers, for example, if consumers do not notice existing quality differences and buy a good with lower quality than expected or if firms' investments in quality decrease as a consequence of limited attention. I also analyze under which circumstances firms have an incentive to exploit this limited attention of consumers to quality differences and under which circumstances firms have an incentive to ensure consumers' attention to quality differences.

Firms have an incentive to exploit consumers' limited attention to quality differences between goods by reducing their quality compared to their competitor unnoticeably to save costs. Examples include firms that sell diluted goods, for example, diluted olive oil [11] or diluted honey [12], or firms that replace expensive high-quality ingredients with cheap low-quality ingredients, for example, cream with coconut oil [13]. In contrast, firms have an incentive to increase quality compared to a competitor in a way that ensures that consumers notice the quality difference to increase the demand for their goods and to be able to raise prices. For example Bordalo et al. [14], argue that when Starbucks entered the coffee market it offered specialty coffee with noticeably higher quality than existing competitors.

To analyze firms' incentives and the resulting equilibrium qualities, I construct a model where two firms compete for consumers who pay limited attention to quality differences. I model limited attention to quality differences with a perception threshold (see, e.g., [15–20]): If the quality difference exceeds the consumers' perception threshold, consumers notice the quality difference

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and perceive the qualities perfectly. If the quality difference is below the perception threshold, consumers do not notice the quality difference and perceive the qualities as identical.

Such a perception threshold captures, for example, situations, where large differences in the quality dimension are salient and draw consumers' attention. Consumers then investigate the quality dimension of the goods in detail and perceive the true qualities of the goods perfectly. In contrast, small differences do not draw consumers' attention and consumers will not investigate the quality dimension. In the consumption decision, the quality difference then no longer comes to mind and the decision is dominated by the price. Modeling limited attention as a perception threshold also captures situations where the quality of goods is complex (i.e., consisting of different dimensions, for example, service quality and longevity). Then, inferring the overall quality of the goods and, consequently, comparing goods is difficult. Nevertheless, if the quality difference increases, comparing the overall qualities becomes easier. In particular, if the quality difference is so large that one good performs best in all dimensions, inferring which good has the overall highest quality is comparatively easy.

If consumers pay only limited attention to quality differences, two types of equilibria can occur: In equilibria with distinguishable qualities, firms choose qualities such that consumers perceive the quality difference between the goods perfectly and choose the utility-maximizing good. In equilibria with indistinguishable (but potentially different) qualities, firms choose qualities such that consumers do not perceive the quality difference between the goods; instead, consumers perceive the goods as having identical quality and, therefore, some consumers might be harmed by buying a good with worse quality than expected.

In line with the anecdotal evidence that in some markets firms reduce quality unnoticeably, for example, by diluting olive oil, and in some markets firms prominently increase quality, for example, in the coffee market, I find both types of equilibria and contribute by clarifying which conditions facilitate which equilibrium. In particular, I highlight the role of horizontal product differentiation by deriving the results as a function of the degree of horizontal product differentiation. Allowing for different degrees of horizontal product differentiation also captures the comparison of markets where consumers pay different degrees of attention to the horizontal characteristics of the goods.

Being horizontally differentiated gives firms market power to set prices above marginal costs for otherwise identical goods. In contrast, without horizontal product differentiation, price competition for identical goods is intense and firms choose prices equal to marginal cost. Therefore, the presence/absence of market power due to horizontal product differentiation affects firms' incentives to differentiate on the quality dimension. Firms produce goods with distinguishable qualities strategically to increase market power. If firms have sufficient market power even with indistinguishable qualities because they are sufficiently horizontally differentiated, producing distinguishable qualities is not necessary.

I assume that firms incur fixed costs for quality. This captures situations where investing in quality is independent of the quantity that a firm sells; for example, investing in research and development, updating software and hardware, buying more efficient machines, or training employees to increase service quality. In the main model, I assume that firms choose quality sequentially. This captures situations, where firms are asymmetric with one firm leading in the innovation decision; for example, due to an information advantage. In addition, this assumption ensures that a pure-strategy subgame-perfect equilibrium always exists.

I show that, if firms are sufficiently horizontally differentiated, firms produce goods with indistinguishable qualities in equilibrium: Both firms want to avoid being the firm with noticeably lower quality as this leads to lower prices and less demand. To make the quality difference noticeable, the second-mover would always have to set a quality that exceeds the quality of the first-mover by the perception threshold. To avoid being the firm with noticeably lower quality, the first-mover has to choose its quality high enough to discourage the second-mover from setting a noticeably higher quality. In the subgame-perfect equilibrium, the second-mover sets the lowest possible quality that is still indistinguishable from the quality of the first-mover. That means, the second-mover free rides: The second-mover benefits from the limited attention of the consumers, who think that the second-mover offers the good at the same quality as the first-mover, without incurring the same quality cost as the first-mover. With decreasing attention, that is, with increasing perception thresholds, the quality that discourages the second-mover from producing goods with noticeably higher quality decreases. Thus with decreasing attention, both firms reduce their quality until both firms produce zero quality.

The horizontal product differentiation ensures that firms can set prices above marginal cost and make positive profits even if they produce goods with indistinguishable qualities. As firms produce goods with indistinguishable qualities, firms choose the same prices, split the market equally, and receive the same revenue. As the first-mover produces higher quality, it incurs higher cost. Consequently, the first-mover makes less profit and has a *first-mover disadvantage*. As with decreasing attention firms choose (weakly) lower quality, quality cost are (weakly) decreasing. Thus as revenues are constant, firms' profits (weakly) increase with decreasing attention. In contrast, as firms sell at the same price and consumers are harmed by lower quality, consumer surplus decreases with decreasing attention. Overall, the increasing profits of firms do not balance the decreasing surplus of consumers such that overall welfare decreases with decreasing attention.

If the firms are not sufficiently horizontally differentiated, firms produce goods with distinguishable qualities in equilibrium. If firms would produce goods with indistinguishable qualities, consumers would perceive the goods as almost identical and the intense price competition would reduce equilibrium prices. Consequently, firms have an incentive to produce goods with distinguishable qualities. By setting a sufficiently high quality in the first stage, the first-mover can ensure that it is the firm with the noticeably higher quality and realize a higher profit. Then, the second-mover responds with a noticeably lower quality. In the subgame-perfect equilibrium, firms maximally differentiate in the quality dimension. The first-mover thus has an advantage if firms are not sufficiently horizontally differentiated. Furthermore, in contrast to sufficiently horizontally differentiated firms, as firms always maximally differentiate, firms' qualities are independent of the perception threshold. That means, independent of how attentive consumers are, firms always choose the same qualities and prices. The attentiveness of consumers thus also has no effect on producer surplus, consumer surplus, and welfare. Consequently, a low degree of horizontal product differentiation changes firms' incentives: Firms need to differentiate noticeably in the quality dimension to create market power and realize positive profits.

Consumers' limited attention affects the range of horizontal product differentiation for which each type of equilibrium occurs. If limited attention increases, firms produce goods with indistinguishable qualities for a larger range of horizontal product differentiation.

If firms choose qualities sequentially, a subgame-perfect equilibrium in pure strategies always exists. In an extension, I explore the implications of simultaneous quality choice. The existence of the equilibrium with distinguishable qualities is robust to this extension. In the simultaneous game, two asymmetric equilibria exist where one firm produces goods with high quality and the other firm produces goods with low quality. In the sequential game, only one equilibrium exists: The first-mover produces goods with high-quality and the second-mover produces goods with low quality. In contrast, the equilibrium with indistinguishable qualities is not robust to changing the timing of the quality choice. This equilibrium only exists for full attention and full inattention. For intermediate levels of attention, this equilibrium no longer exists.

The remainder of this article is structured as follows: Section 2 describes the contribution to the related literature. Section 3 introduces the model. Section 4 derives the results and Section 5 provides the welfare analysis. In Section 6, I discuss a model with simultaneous quality choice. Section 7 concludes.

2 | Related Literature

A growing literature analyzes the implications of consumers' limited attention on market outcomes and welfare.[2] I model limited attention to quality differences with a perception threshold such that consumers notice large quality differences that exceed the perception threshold, but do not notice small quality differences that are below the perception threshold.[3] Therefore, I contribute primarily to models of market interactions where consumers only notice differences between goods that exceed their perception threshold [15–18, 20].

Allen and Thisse [15] and Bachi [16] analyze price competition in duopolies where consumers' perception of prices is subject to a perception threshold. If the prices are too similar, consumers perceive them as identical. Both models show that consumers' perception thresholds lead to prices above marginal cost and positive profits. In other words, the imperfect perception of consumers allows firms to overcome the Bertrand paradox. Balart [17] analyzes the consequences of a perception threshold in a model of horizontal product differentiation. Balart [17] shows that firms may have more incentives to differentiate under limited than under full attention and that more inattention to the horizontal characteristics of the goods might lead to higher profits for the firms. Chung et al. [18] analyze situations where consumers cannot detect small utility differences between two options. Then, adding a third option that is noticeably different from one but not both options helps consumers to infer the better deal.

Webb [20] analyzes firms' strategic interactions when consumers have a relative perception threshold about quality differences. Although, I also study a model with limited attention to quality differences, the models differ significantly in the setup as well as in the results. Webb [20] focuses on a relative perception threshold. In contrast, I focus on an absolute perception threshold. A relative perception threshold is often justified by Weber's Law.[4] Yet, a relative perception threshold is only sensible for quality values that are sufficiently large. With one firm producing zero quality, even if the rival firm produces goods with extremely low quality, the relative difference is infinite. Thus all consumers notice the quality difference between the goods—even if they have a high perception threshold. This logic explains why Weber's law does not hold at the extremes (see, e.g., [45]). Consequently, I focus on the implications of an absolute perception threshold (see also [16–18]). Thereby, I capture new insights into the incentives of firms and allow for equilibria with indistinguishable but low qualities. In contrast, in Webb [20], firms always produce goods with distinguishable qualities. In addition, in Webb [20], profits and consumer surplus depend on the perception threshold, whereas, I show that qualities, profits, and consumer surplus do not always depend on the perception threshold.

Overall, I contribute to the literature by showing that equilibria with indistinguishable qualities can occur. In addition, I provide a better understanding of the conditions under which equilibria with indistinguishable qualities and with distinguishable qualities exist. In particular, I highlight the role of market power by showing how the results depend on the degree of horizontal product differentiation. Firms produce distinguishable qualities strategically to increase market power. If firms have sufficient market power even with indistinguishable qualities, producing distinguishable qualities is not necessary.

3 | Model

Consider two horizontally differentiated firms, Firm A and Firm B, that compete in qualities and prices for a unit mass of consumers. I follow Hotelling [46] in modeling horizontal product differentiation as a real line $[0, 1]$ with Firm A located at 0 and Firm B located at 1.

Consumers are uniformly distributed on $[0, 1]$. The position $x \in [0, 1]$ of a consumer denotes the consumer's ideal version of the good. Consumers buy exactly one unit of the good. By buying that unit from firm $i \in \{A, B\}$, the consumer at position x receives utility

$$u_x(i) = v + q_i - p_i - h(x - y_i)^2 \tag{1}$$

where $v \gg 0$ is the gross utility of the good, $q_i \in [0, 1]$ is the quality, $p_i \in \mathbb{R}_0^+$ is the price, and y_i is the location of firm i in the product space. The parameter $h \in [0, 1]$ represents consumers' transportation costs and thus the degree of horizontal product differentiation. I assume that v is sufficiently large: $v > 3h$. I follow d'Aspremont et al. [47] in modeling disutility from consuming a non-ideal good as quadratic: $(x - y_i)^2$.[5]

Consumers are constrained in their perception of quality. Consumers only perceive quality differences between the goods if the quality difference is sufficiently large. The perceived quality $\hat{q}_i$ is thus

$$\hat{q}_i = \begin{cases} q_i & \text{if } |q_A - q_B| > \tau \\ q(q_A, q_B) & \text{if } |q_A - q_B| \leq \tau \end{cases} \tag{2}$$

where $q(q_A, q_B) \in [0, 1]$ and the perception threshold is $\tau \in [0, 1]$. The perception threshold τ is identical for all consumers. The perception threshold captures the consumers' limited attention to quality differences. The larger the perception threshold, the more inattentive consumers are. The model includes perfect perception as the limiting case $\tau = 0$. In that case, if qualities are tied, I assume $q(q_A, q_B) = q_A = q_B$. If the quality difference is larger than the perception threshold τ, consumers perceive the quality of each firm perfectly. If the quality difference is smaller than the perception threshold τ, the consumers perceive the quality of firm i as $\hat{q}_i = q(q_A, q_B)$ for both $i \in \{A, B\}$. In other words, the consumers perceive the quality of Firm A and the quality of Firm B as identical. For instance, $q(q_A, q_B)$ could be the average of q_A and q_B. Yet, for the analysis it is not necessary to specify $q(q_A, q_B)$. This implies that consumers are naive. Consumers believe that no quality difference exists if they do not notice a quality difference.[6]

For the consumption decision, consumers use the perceived utilities $\hat{u}_x(A)$ and $\hat{u}_x(B)$ instead of the true utilities $u_x(A)$ and $u_x(B)$ given in (1). The perceived utilities differ from the true utilities only in the quality dimension: Instead of the true qualities, consumers take the perceived qualities given in (2) into account. A consumer is indifferent between buying from Firm A and Firm B if

$$\hat{u}_x(A) = \hat{u}_x(B) \Leftrightarrow v + \hat{q}_A - p_A - hx^2 = v + \hat{q}_B - p_B - h(1 - x)^2$$

Denote the indifferent consumer by

$$\overline{x} \equiv \frac{h + \hat{q}_A - \hat{q}_B + p_B - p_A}{2h}$$

Then, all consumers $x \leq \overline{x}$ buy from Firm A and all consumers $x > \overline{x}$ buy from Firm B. As long as $\overline{x} \in (0, 1)$, both firms capture some demand. However, if firms choose sufficiently different prices and/or qualities, one firm may capture the demand of all consumers.

Firms play a three-stage game, depicted in Figure 1: In the first stage, firms observe the perception threshold and, then, Firm A chooses its quality q_A. In the second stage, Firm B observes the quality of Firm A and chooses its quality q_B. In the third stage, Firm A observes the quality of Firm B and both firms, independently and simultaneously, set prices. Subsequently, consumers perceive the goods and buy either from Firm A or from Firm B. I assume that, taking into account the best reply of Firm B, if Firm A is indifferent between a quality that is noticeably lower than the quality of Firm B and a quality that is indistinguishable to the quality of Firm B, Firm A chooses the indistinguishable quality. In addition, I assume that if Firm B is indifferent between a quality that is indistinguishable to the quality of Firm A and a quality

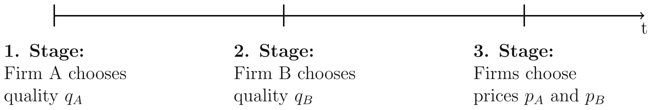

FIGURE 1 | Timeline.

that is distinguishable form the quality of Firm A, Firm B chooses the indistinguishable quality.[7]

As Firm B chooses its quality after observing the quality of Firm A, Firm B decides whether the quality difference is noticeable. That means, if Firm B wants to make its quality indistinguishable from the quality of Firm A, Firm B has to choose a quality $q_B \in [q_A - \tau, q_A + \tau]$. In contrast, if Firm B wants to make its quality distinguishable from the quality of Firm A, Firm B has to choose a quality $q_B \in [0, q_A - \tau) \cup (q_A + \tau, 1]$.

I assume identical cost functions $C(q_i) = cq_i^2$ with $c \in (0, 1/12)$ for both firms.[8] That means, the cost for quality do not depend on the quantity that the firms sell.[9]

4 | Results

I solve the game by backward induction looking for the pure-strategy subgame-perfect equilibria. In the price-setting stage, firms simultaneously set prices to maximize the following profits, which depend on the qualities set in the first stage and in the second stage[10]

$$\Pi_A(p_A, p_B, q_A, q_B) = \begin{cases} p_A - cq_A^2 & \text{if } p_A < p_B + \hat{q}_A - \hat{q}_B - h \\ p_A\overline{x} - cq_A^2 & \text{if } p_B + \hat{q}_A - \hat{q}_B - h \leq p_A \\ & \quad \leq p_B + \hat{q}_A - \hat{q}_B + h \\ -cq_A^2 & \text{if } p_A > p_B + \hat{q}_A - \hat{q}_B + h \end{cases}$$

$$\Pi_B(p_A, p_B, q_A, q_B) = \begin{cases} p_B - cq_B^2 & \text{if } p_B < p_A + \hat{q}_B - \hat{q}_A - h \\ p_B(1 - \overline{x}) - cq_B^2 & \text{if } p_A + \hat{q}_B - \hat{q}_A - h \leq p_B \\ & \quad \leq p_A + \hat{q}_B - \hat{q}_A + h \\ -cq_B^2 & \text{if } p_B > p_A + \hat{q}_B - \hat{q}_A + h \end{cases}$$

As all consumers $x \leq \overline{x}$ buy from Firm A and all consumers $x > \overline{x}$ buy from Firm B, as long as $0 \leq \overline{x} \leq 1 \Leftrightarrow p_B + \hat{q}_A - \hat{q}_B - h \leq p_A \leq p_B + \hat{q}_A - \hat{q}_B + h$ (respectively, $p_A + \hat{q}_B - \hat{q}_A - h \leq p_B \leq p_A + \hat{q}_B - \hat{q}_A + h$) both firms receive some demand. The demand of Firm A is $\overline{x}$, the demand of Firm B is $1 - \overline{x}$. In contrast, if $\overline{x} \notin [0, 1]$, one firm captures all consumers. If $p_A < p_B + \hat{q}_A - \hat{q}_B - h \Leftrightarrow p_B > p_A + \hat{q}_B - \hat{q}_A + h$, $\overline{x} > 1$ and Firm A captures the demand of all consumers, that is, the demand of Firm A is 1 and the demand of Firm B is 0. If $p_A > p_B + \hat{q}_A - \hat{q}_B + h \Leftrightarrow p_B < p_A + \hat{q}_B - \hat{q}_A - h, \overline{x} < 0$ and Firm B captures the demand of all consumers, that is, the demand of Firm A is 0 and the demand of Firm B is 1. Nevertheless, firms always have costs cq_i^2.

The equilibrium prices are summarized in Lemma 1. The derivation is included in Appendix A.

Lemma 1. *The equilibrium prices depend on whether qualities are indistinguishable or distinguishable.*

i. **Indistinguishable qualities**: *If qualities are indistinguishable, equilibrium prices are* $p_A^* = p_B^* = h$.

ii. **Distinguishable qualities**: *If qualities are distinguishable, equilibrium prices depend on the quality difference. If* $q_A - q_B > 3h$, *equilibrium prices are* $p_A^* = q_A - q_B - h$ *and* $p_B^* = 0$. *If* $-3h \leq q_A - q_B \leq 3h$, *equilibrium prices are* $p_A^* = (q_A - q_B + 3h)/3$ *and* $p_B^* = (q_B - q_A + 3h)/3$. *If* $q_A - q_B < -3h$, *equilibrium prices are* $p_A^* = 0$ *and* $p_B^* = q_B - q_A - h$.

The prices depend on the consumers' perception of quality. Consumers are only willing to pay for a quality difference that they perceive. If $q_B \in [q_A - \tau, q_A + \tau]$, that is, if the qualities are so similar that consumers do not notice the difference, consumers think the firms offer goods with identical quality. Therefore, they are not willing to pay a mark-up for quality. Consequently, in equilibrium, both firms set identical prices. However, as the goods are horizontally differentiated, which reduces price competition, firms can charge prices above marginal cost (here above zero). The resulting equilibrium prices are $p_A^* = h$ and $p_B^* = h$.

If $q_B \in [0, q_A - \tau,) \cup (q_A + \tau, 1]$, that is, if the qualities are sufficiently different so that consumers notice the quality difference, consumer are willing to pay for quality differences and firms can adjust their prices accordingly. If the quality difference is sufficiently small, firms choose prices such that both firms receive some demand in equilibrium $p_A^* = (q_A - q_B + 3h)/3$ and $p_B^* = (q_B - q_A + 3h)/3$. The noticeable quality difference allows the firm with the higher quality to charge a higher price than its competitor. If the quality difference increases, the firm with the lower quality has to reduce its price to offset the quality disadvantage. Consequently, if the quality difference becomes sufficiently large, in equilibrium, the firm with the lower quality sets prices equal to the marginal cost of 0. The firm with the higher quality can set a positive price that accounts for the quality difference and the horizontal product differentiation. Then, in equilibrium, all consumers buy from the firm with the higher quality.

In the second stage, Firm B chooses its quality to maximize its profit taking the prices into account and given the quality choice of Firm A. As the prices depend on the quality difference between the goods, if $\tau \geq 3h$, Firm B's profit is[11]

$$\Pi_B(q_A, q_B) = \begin{cases} q_B - q_A - h - cq_B^2 & \text{if } q_B > q_A + \tau \\ \frac{1}{2}h - cq_B^2 & \text{if } q_A - \tau \leq q_B \leq q_A + \tau \\ -cq_B^2 & \text{if } q_B < q_A - \tau \end{cases} \tag{3}$$

and, if $\tau < 3h$, Firm B's profit is[12]

$$\Pi_B(q_A, q_B) = \begin{cases} q_B - q_A - h - cq_B^2 & \text{if } q_B > q_A + 3h \\ \frac{1}{18h}\left(q_B - q_A + 3h\right)^2 - cq_B^2 & \text{if } q_A + \tau < q_B \leq q_A + 3h \\ \frac{1}{2}h - cq_B^2 & \text{if } q_A - \tau \leq q_B \leq q_A + \tau \\ \frac{1}{18h}\left(q_B - q_A + 3h\right)^2 - cq_B^2 & \text{if } q_A - 3h \leq q_B < q_A - \tau \\ -cq_B^2 & \text{if } q_B < q_A - 3h \end{cases} \tag{4}$$

If Firm B prefers a quality that is indistinguishable from the quality of Firm A, Firm B chooses the lowest quality that is indistinguishable from the quality of Firm A, that is, $q_B = \max\{0, q_A - \tau\}$. In other words, it is impossible that Firm A has an unnoticeably lower quality than Firm B. If Firm B chooses a quality that is indistinguishable from the quality of Firm A, that is, $q_A - \tau \leq q_B \leq q_A + \tau$, no consumer notices a quality difference. As firms then choose identical prices $p_A^* = p_B^* = h$ in the price-setting stage, firms split the demand equally. That means, Firm B makes the same revenue with any $q_B \in [\max\{0, q_A - \tau\}, \min\{q_A + \tau, 1\}]$. But, Firm B's cost are increasing in its quality. Thus Firm B chooses the lowest quality such that consumers do not perceive the quality difference; this quality is $q_B = \max\{0, q_A - \tau\}$.

If Firm B prefers a quality that is distinguishable from the quality of Firm A, Firm B either chooses the highest or the lowest quality that is distinguishable from the quality of Firm A, that is, $q_B \in \{0, 1\}$. If Firm B chooses a quality $q_B < q_A - \tau$ or $q_B > q_A + \tau$, all consumers notice the quality difference. Then, the prices and, consequently, also the profits depend on the quality difference. If $\tau \geq 3h$, the profit of Firm B is increasing for all $q_B \in (q_A + \tau, 1]$ and decreasing for all $q_B \in [0, q_A - \tau)$. If $\tau < 3h$, the profit of Firm B is increasing for all $q_B \in (q_A + \tau, 1]$ and decreasing for all $q_B \in [0, q_A - 3h)$. In addition, for Firm B, choosing the lowest quality that is indistinguishable to the quality of Firm A is preferable to choosing a slightly lower quality that is then distinguishably lower than the quality of Firm A, that is, any $q_B \in [q_A - 3h, q_A - \tau)$: With $q_B = q_A - \tau$, firms split the demand equally and prices are $p_A = p_B = h$. In contrast, with any $q_B \in [q_A - 3h, q_A - \tau)$, due to the noticeable quality difference, Firm B reduces its price to less than h (to compensate for the noticeably lower quality) and receives less than half of the demand in equilibrium. Although Firm B's cost are lower with any $q_B \in [q_A - 3h, q_A - \tau)$ than with $q_B = q_A - \tau$, the lower revenue ensures that the profit is lower with any $q_B \in [q_A - 3h, q_A - \tau)$ than with $q_B = q_A - \tau$. Therefore, if Firm B prefers a quality that is distinguishable from the quality of Firm A, Firm B chooses the highest quality from $(q_A + \tau, 1]$ or the lowest quality from $[0, q_A - \tau)$. Overall, depending on τ, Firm B maximizes over (3) or (4) to derive the best response to the quality of Firm A. For a detailed derivation of the best reply, see Appendix B.

In the first stage, Firm A chooses its quality taking the subsequent decision of Firm B into account. As Firm A can never produce unnoticeably lower quality than Firm B, Firm A is either the firm with noticeably lower, noticeably higher, or unnoticeably higher quality. To avoid being the firm with noticeably lower quality, Firm A has to choose a sufficiently high quality in the first stage. Then, Firm B would have to exceed the quality of Firm A by the perception threshold τ to make the quality difference noticeable. If the quality of Firm A is already high, this is impossible for Firm B.

If goods are sufficiently horizontally differentiated, that is, if $h \geq 2c(1 - \tau)^2$, in the subgame-prefect equilibrium, Firm A provides a quality that is high enough to prevent Firm B from noticeably overbidding Firm A's quality. The necessary quality for this depends on the size of the perception threshold. The lower the perception threshold, the higher the necessary quality. As quality is costly, Firm A prefers the lowest such quality to prevent Firm B from noticeably overbidding its quality. Thus the quality of Firm A decreases with the perception threshold. As a best response, Firm B undercuts the quality of Firm A such that the quality difference is just not noticeable, that is, $q_B^* = \max\{0, q_A^* - \tau\}$. Thus the quality of Firm B (weakly) decreases with increasing perception thresholds. Proposition 1 characterizes the resulting subgame-perfect equilibria.

In contrast, if goods are insufficiently horizontally differentiated, that is, if $h < 2c(1-\tau)^2$, firms benefit from noticeably differentiating in the quality dimension: In the price-setting stage, two types of subgames exist. In subgames with indistinguishable qualities, as the degree of horizontal product differentiation is low, consumers perceive the goods as almost identical. Consequently, price competition drives down prices and ensures low revenues. In subgames with distinguishable qualities, firms set prices that account for the quality difference. This allows at least the firm with the higher quality to charge positive prices and to obtain a positive revenue. Thus firms have an incentive to avoid subgames with indistinguishable qualities.

As the first-mover, Firm A can ensure that it is the firm with the higher profit by setting a sufficiently high quality in the first stage. Then, Firm B responds with a noticeably lower quality. To avoid quality costs, Firm B chooses a quality of zero. Consequently, if goods are insufficiently horizontally differentiated, in the subgame-perfect equilibrium, firms produce goods with distinguishable qualities, Firm B makes zero profit, and Firm A has a first-mover advantage and makes positive profits. Proposition 1 characterizes the resulting subgame-perfect equilibria.

Proposition 1. *Subgame-perfect equilibria dependent on the degree of horizontal product differentiation $h \in [0, 1]$.*

i. **Equilibrium with distinguishable qualities**: *If $h < 2c(1-\tau)^2$, that is, if goods are not sufficiently horizontally differentiated, in the unique subgame-perfect equilibrium, firms produce goods with distinguishable qualities $q_A^* = 1$ and $q_B^* = 0$ and prices $p_A^* = 1 - h$ and $p_B^* = 0$.*

ii. **Equilibrium with indistinguishable qualities**: *If $h \geq 2c(1-\tau)^2$, that is, if goods are sufficiently horizontally differentiated, a pure-strategy subgame-perfect equilibrium exists, where firms produce goods with indistinguishable qualities $q_A^* = 1 - \tau$ and $q_B^* = \max\{0, 1 - 2\tau\}$ and identical prices $p_A^* = p_B^* = h$.*

The proof is in Appendix B. The following considerations explain the threshold that distinguishes the two types of equilibria, that is, $h = 2c(1-\tau)^2$: Firm A is able to maintain the equilibrium with distinguishable qualities, that is, $q_A^* = 1$ and $q_B^* = 0$, as long as Firm B prefers to differentiate noticeably from Firm A with zero quality instead of undercutting Firm A unnoticeably:

$$\Pi_B(q_A = 1, q_B = 0) > \Pi_B(q_A = 1, q_B = q_A - \tau)$$
$$\Leftrightarrow 0 > \frac{1}{2}h - c(1-\tau)^2$$
$$\Leftrightarrow h < 2c(1-\tau)^2$$

Consequently, if $h \geq 2c(1-\tau)^2$ and Firm A chooses $q_A = 1$, Firm B deviates to indistinguishable qualities. As Firm A cannot increase its quality further to make qualities distinguishable, Firm A chooses its quality to maintain the equilibrium with indistinguishable qualities. That is, Firm A chooses the lowest quality that ensures that Firm B undercuts the quality of Firm A unnoticeably instead of providing noticeably higher quality than Firm A.

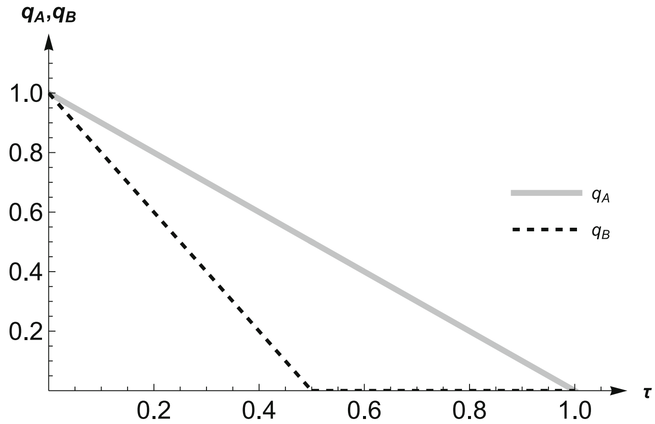

FIGURE 2 | Equilibrium qualities of Firm A (solid) and Firm B (dashed) as a function of the perception threshold τ if $h \geq 2c$.

In the equilibrium with distinguishable qualities, equilibrium qualities are constant in the perception threshold τ. Consequently, in the benchmark case under perfect perception, that is, $\tau = 0$, as well as under imperfect perception, that is, $\tau > 0$, firms always maximally differentiate in the quality dimension and choose the distinguishable qualities $q_A^* = 1$ and $q_B^* = 0$.

In contrast, in the equilibrium with indistinguishable qualities, equilibrium qualities depend on the perception threshold. If $h \geq 2c$, only the equilibrium with indistinguishable qualities occurs. Figure 2 illustrates the equilibrium qualities in the equilibrium with indistinguishable qualities as a function of the perception threshold τ. In the benchmark case under perfect perception, that is, $\tau = 0$, both firms produce goods with identical quality: $q_A^* = q_B^* = 1$. In contrast, for $\tau > 0$, it becomes profitable for Firm B to undercut the quality of Firm A unnoticeably. That is, firms set qualities such that consumers do not notice the quality difference. Then, the firms set identical prices and split the demand equally. Yet, Firm A produces goods with strictly higher quality than Firm B: By setting a sufficiently high quality, Firm A wants to discourage Firm B from producing a noticeably higher quality than Firm A. As a consequence, the best reply of Firm B is to unnoticeably undercut the quality of Firm A. As the perception threshold τ increases, Firm A can reduce its quality without Firm B producing noticeably higher quality. Firm B, which produces $q_B^* = \max\{0, q_A^* - \tau\}$, then also reduces its quality until $q_B^* = 0$. Firm A reduces its quality further with increasing perception threshold τ until, for $\tau = 1$, both firms produce zero quality.

In sum, horizontal product differentiation dampens the price competition between the firms. If firms are sufficiently horizontally differentiated, both firms can still charge prices above marginal cost and make positive profits even if they produce goods with indistinguishable qualities; horizontal product differentiation gives firms market power. Therefore, horizontally differentiated firms produce goods with indistinguishable qualities. In contrast, firms that are not sufficiently horizontally differentiated have no incentive to produce goods with indistinguishable qualities as indistinguishable qualities imply intense price competition and low revenues. Consequently, producing goods with distinguishable qualities becomes more attractive. Limited attention affects the threshold on horizontal product differentiation that determines which type of equilibrium occurs. If limited attention increases, that is, if the perception threshold τ increases, the equilibrium with indistinguishable qualities occurs already for lower degrees of horizontal product differentiation.

5 | Welfare Analysis

In this section, I discuss the welfare within the two equilibria separately before I provide a welfare analysis across the two equilibria. Lemma 2 summarizes the consumer surplus, producer surplus, and welfare results if $h < 2c(1-\tau)^2$.

Lemma 2. *If $h < 2c(1-\tau)^2 \Leftrightarrow \tau < 1 - \sqrt{h/(2c)}$, firms produce goods with distinguishable qualities in equilibrium. Then, neither producer surplus, consumer surplus, nor welfare depends on the perception threshold τ.*

In the equilibrium with distinguishable qualities, that is, if $h < 2c(1-\tau)^2 \Leftrightarrow \tau < 1 - \sqrt{h/(2c)}$, firms' qualities are independent of the perception threshold τ. That means, independent of how attentive consumers are, firms always choose the same qualities and the same prices. The attentiveness of consumers thus has no effect on producer surplus, consumer surplus, and welfare (see Lemma 2). The producer surplus, consumer surplus, and overall welfare are included in Appendix C.

Lemma 3 summarizes the consumer surplus, producer surplus, and welfare results if $h \geq 2c(1-\tau)^2$.

Lemma 3. *If $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1 - \sqrt{h/(2c)}$, firms produce goods with indistinguishable qualities. Then, producer surplus is strictly increasing with consumers' inattention and consumer surplus and welfare are strictly decreasing with consumers' inattention.*

If $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1 - \sqrt{h/(2c)}$, each firm's revenue is $h/2$; the joint revenue is h. In addition, the firms have costs for quality. With increasing perception threshold τ, firms reduce quality. As consumers buy exactly one unit of the good independent of quality, firms' profits are increasing in τ. Consequently, the producer surplus is increasing in τ and the firms reach the highest producer surplus ($PS = h$) when $\tau = 1$.

In contrast, as consumers prefer higher quality to lower quality, consumer surplus is decreasing in the perception threshold τ if $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1 - \sqrt{h/(2c)}$. The firms' qualities are indistinguishable and firms sell the goods at the same price so that all consumers $x \leq \overline{x} = 1/2$ buy from Firm A and all $x > \overline{x} = 1/2$ buy from Firm B. Consequently, the highest consumer surplus is achieved when firms produce the goods with the highest quality. As firms reduce their qualities with increasing τ, firms produce the highest qualities when $\tau = \max\{0, 1 - \sqrt{h/(2c)}\}$ and the consumer surplus reaches its highest value for $\tau = \max\{0, 1 - \sqrt{h/(2c)}\}$.

In sum, if $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1 - \sqrt{h/(2c)}$, as the perception threshold τ increases, the gains of the firms do not balance the losses of the consumers and welfare is decreasing in τ. As the demand is fixed, each firm serves half the market. As prices are a reallocation of welfare from consumers to firms, welfare depends on the qualities. Consumers benefit from higher qualities, firms are harmed by higher qualities. With increasing τ, qualities are decreasing. In sum, welfare is decreasing in τ such that welfare reaches its highest value for $\tau = \max\{0, 1 - \sqrt{h/(2c)}\}$.

Proposition 2 summarizes the welfare results.

Proposition 2. *If $h < 2c$, producer surplus and welfare reach their highest value for any $\tau^* \in [0, 1 - \sqrt{h/(2c)}\,)$; consumer surplus reaches its highest value at $\tau^* = 1 - \sqrt{h/(2c)}$. If $h \geq 2c$, producer surplus reaches its highest value at $\tau^* = 1$ and consumer surplus and welfare reach their highest value at $\tau^* = 0$.*

The proof is in Appendix C. Figure 3 illustrates the consumer surplus, the producer surplus, and the welfare as a function of the perception threshold τ for $h < 2c$.

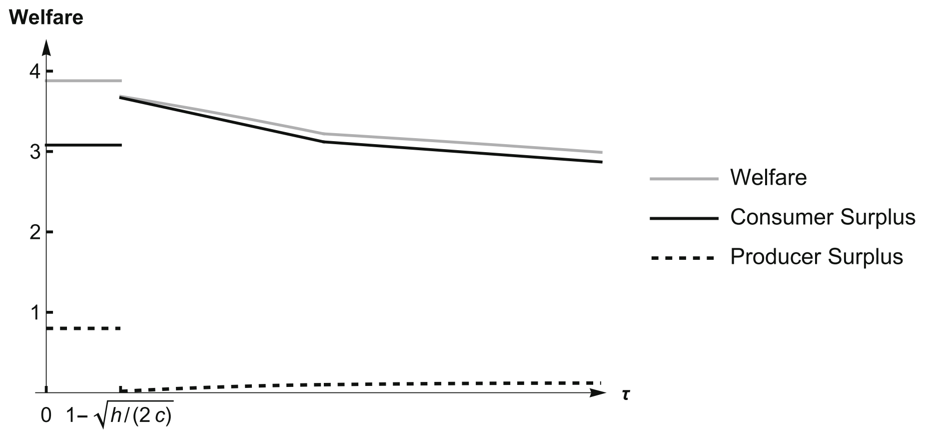

FIGURE 3 | Welfare (solid gray), consumer surplus (solid black), and producer surplus (dashed) as a function of the perception threshold τ for $v = 3$, $h = 0.12$, and $c = 0.08$.

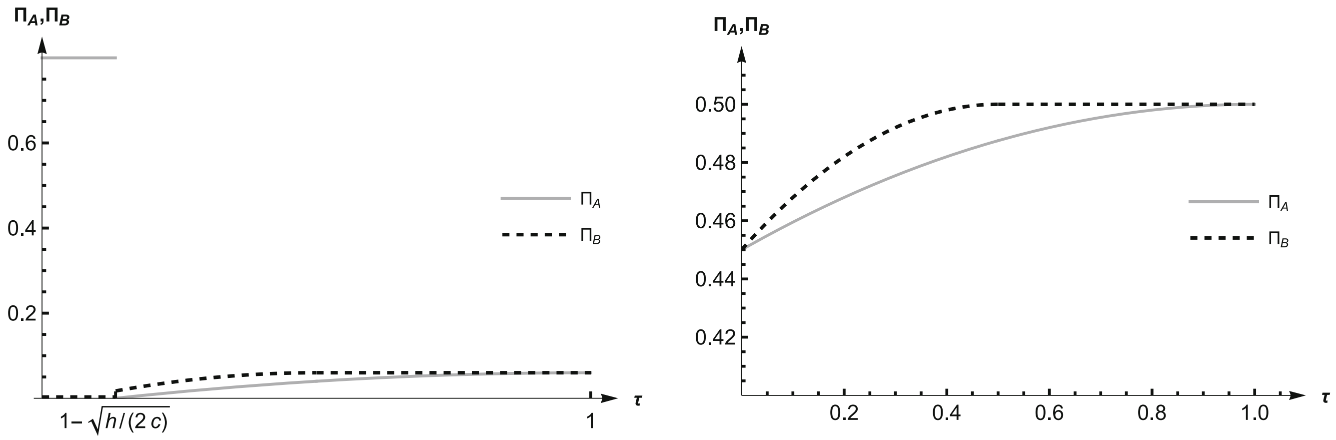

FIGURE 4 | Equilibrium profits of Firm A (solid) and Firm B (dashed) as a function of the perception threshold τ (left panel: $h = 0.12$ and $c = 0.08$; right panel: $h = 1$ and $c = 0.05$).

If $h \geq 2c$, only the equilibrium with indistinguishable qualities exists. Consequently, it follows from Lemma 3 that consumer surplus and welfare are highest under full attention, whereas, firms prefer inattentive consumers. In contrast, if $h < 2c$, for $h < 2c(1-\tau)^2 \Leftrightarrow \tau < 1 - \sqrt{h/(2c)}$, the equilibrium with distinguishable qualities occurs and, for $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1 - \sqrt{h/(2c)}$, the equilibrium with indistinguishable qualities occurs. Therefore, for $h < 2c$, the equilibrium and thus also producer surplus, consumer surplus, and welfare change at $\tau = 1 - \sqrt{h/(2c)}$. Consequently, as producer surplus, consumer surplus, and welfare are constant for $\tau < 1 - \sqrt{h/(2c)}$, producer surplus, consumer surplus, and welfare reach their highest value either for any $\tau \in [0, 1 - \sqrt{h/(2c)}\ )$ or at $\tau = 1$ for producer surplus, respectively, at $\tau = 1 - \sqrt{h/(2c)}$ for consumer surplus and welfare.

Figure 4 illustrates the subgame-perfect equilibrium profits of Firm A and Firm B as a function of the perception threshold τ. If $h < 2c$ (left panel of Figure 4), for $\tau < 1 - \sqrt{h/(2c)}$, the equilibrium with distinguishable qualities occurs; for $\tau \geq 1 - \sqrt{h/(2c)}$, the equilibrium with indistinguishable qualities occurs. If $h \geq 2c$ (right panel of Figure 4), for all $\tau \in [0, 1]$, only the equilibrium with indistinguishable qualities exists. In the equilibrium with distinguishable qualities (compare Lemma 2), firms maximally differentiate in the quality dimension and all consumers buy from Firm A. Consequently, the profit of Firm A is strictly positive and the profit of Firm B is zero. As the qualities do not depend on the perception threshold, the profits and thus also the producer surplus are constant in τ.

In the equilibrium with indistinguishable qualities (compare Lemma 3), firms produce goods with indistinguishable qualities and with increasing perception threshold τ both firms reduce their quality. Firm A makes less profit than Firm B for all $\tau \in [\max\{0, 1 - \sqrt{h/(2c)}\}, 1]$: Because the quality difference is unnoticeable, both firms sell at the same price and split the market equally. Thus both firms receive the same revenue, but Firm A has higher quality cost. Consequently, Firm A makes less profit than Firm B and has a first-mover disadvantage. As τ increases, Firm A and Firm B reduce their quality until at $\tau = 1$ both firms produce zero quality. Then, as the firms receive the same revenue and have zero quality cost, firms make the same profits. In addition, as revenues are constant and cost are decreasing in τ, the profits of both firms and thus also the producer surplus are (weakly) increasing in τ.

In general, a higher consumer surplus can be achieved in the equilibrium with indistinguishable qualities, that is, if horizontal product differentiation is sufficiently high $h \geq 2c(1-\tau)^2$. An increase in consumers' limited attention increases the range of values for h for which this equilibrium with indistinguishable qualities occurs. That means, if consumers are limited attentive, this equilibrium occurs more frequently than if consumers are fully attentive.

6 | Simultaneous Quality Choice

In the main model, I assume that firms play a three-stage game and choose their qualities sequentially. In this section, I assume that firms play a two-stage game: In the first stage, both firms simultaneously choose their qualities. In the second stage, both firms simultaneously choose their prices.

The price-setting stage is identical to the game with sequential quality choice. Consequently, the equilibrium prices are given in Lemma 1. In the quality-setting stage, both firms simultaneously choose their quality to maximize profits. Proposition 3 describes the resulting pure-strategy subgame-perfect equilibria.

Proposition 3. *Let $i, j \in \{A, B\}$ with $i \neq j$.*

i. **Equilibria with distinguishable qualities**: *If $h < 2c(1-\tau)^2$, two asymmetric pure-strategy subgame-perfect equilibria exist where firms choose distinguishable qualities $q_i^* = 1$ and $q_j^* = 0$ and prices $p_i^* = 1 - h$ and $p_j^* = 0$.*

ii. **Equilibrium with indistinguishable qualities**: *If $\tau = 0$ and $h \geq 2c$ or if $\tau = 1$, a pure-strategy subgame-perfect equilibrium exists where firms choose indistinguishable qualities $q_i^* = 1 - \tau$ and $q_j^* = \max\{0, 1 - 2\tau\}$ and prices $p_i^* = p_j^* = h$.*

The proof is in the Supporting Information available on the Journal's editorial web site.

Proposition 3 shows that the equilibrium with distinguishable qualities is robust to the change in timing of the quality choice. In the simultaneous game, if $h < 2c(1-\tau)^2$, two asymmetric subgame-perfect equilibria in pure strategies exist: One in which Firm A and one in which Firm B produces a strictly positive quality. The other firm always produces zero quality. In the sequential game, only one asymmetric equilibrium with distinguishable qualities exists: As the first mover, Firm A can influence which of these two equilibria exists. As the firm with the higher quality receives a higher profit, in the sequential game, the asymmetric equilibrium where Firm A sets a quality $q_A^* = 1$ results.

The equilibrium with indistinguishable qualities is not robust to changing the timing of the game. The equilibrium with indistinguishable qualities exists only if $\tau = 0$ and $h \geq 2c$ or if $\tau = 1$.

Generally, no pure-strategy subgame-perfect equilibrium exists for $\tau \in (0,1)$ and $h \geq 2c(1-\tau)^2$. As the model with sequential quality choice shows, for $h \geq 2c(1-\tau)^2$, firms prefer to undercut the quality of the other firm unnoticeably. This suggests that a mixed-strategy equilibrium might exists. Existence theorems for mixed-strategy Nash equilibria, for example, are provided by Dasgupta and Maskin [52]. However, as the sum of firms' profits is not upper semi-continuous, Theorem 5 of Dasgupta and Maskin [52] does not apply (see the Supporting Information available on the Journal's editorial web site).

7 | Conclusion

In this article, I analyze the implications of consumers' limited attention to quality differences on equilibria and welfare. I capture limited attention with a perception threshold such that consumers do not perceive quality differences between goods that are below their perception threshold. I show that in how far the perception threshold affects investments in quality depends on whether goods are horizontally differentiated. With sufficient horizontal product differentiation, firms produce indistinguishable qualities. Without sufficient horizontal product differentiation, firms produce distinguishable qualities. An increase in limited attention increases the range of horizontal product differentiation for which firms produce indistinguishable qualities.

Understanding under which conditions firms produce goods with distinguishable and under which conditions firms produce goods with indistinguishable qualities is helpful for public policy. When firms produce goods with distinguishable qualities, consumers are fully informed and always choose the utility-maximizing good. In these situations, policy interventions to protect consumers are unnecessary. However, in many situations, firms produce goods with different, but indistinguishable qualities. When firms produce goods with different, but indistinguishable qualities, some consumers buy goods with lower quality than expected. Policy interventions should focus on these situations.

In general, as welfare is weakly increasing in consumers' attention, policies should aim to increase consumers' attention. Possible policy interventions that increase consumer attention include information campaigns and disclosure laws. Information campaigns sensitize consumers to check (certain dimensions of) quality or educate consumers on how to infer and compare qualities from provided information. For example, financial literacy campaigns sensitize consumers to check for hidden costs in contracts or teach consumers how to calculate the total costs of contracts (e.g., how to calculate compound interests). Consequently, information campaigns ensure that consumers are able to make quality comparisons given the provided information.

In contrast, disclosure laws ensure that consumers receive additional information, for example, through labels. Then, if labels are well-designed, all consumers are able to compare the qualities of the goods easily. Such labels include, for example, providing kWh information on electric appliances, nutrition tables, or restaurant hygiene scores.

To keep the model tractable, I make a number of assumptions that limit the scope of the analysis. For example, I assume that all consumers have the same perception threshold. A more realistic assumption would allow for heterogeneity between the consumers. In addition, the perception threshold only allows for pairwise comparison. In markets with more than two firms, an adjustment is necessary. For example, consumers can compare the quality of a good to the average quality (similar to [27]). Furthermore, I assume that consumers pay limited attention to vertical differences but perfectly observe horizontal differences. Abstracting from an additional perception threshold for the horizontal characteristics of the goods keeps the model tractable. Moreover, allowing for different degrees of horizontal product differentiation gives insights how limited attention of consumers to the horizontal characteristic affects equilibrium outcomes. Nevertheless, a detailed analysis of limited attention to horizontal and vertical differences would improve our understanding of firms' strategies further. I leave this to future research.

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Disclosure

The author has nothing to report.

Endnotes

[1] Limited attention of consumers has been documented, for example, with regard to taxes [1], odometer mileage and age of used cars [2–4], add-on costs [5], or house prices [6].

[2] For example, Gabaix and Laibson [21]; Armstrong and Chen [22]; Eliaz and Spiegler [23, 24]; Haan and Moraga-González [25]; Bordalo et al. [14, 26, 27]; Ghosh and Galbreth [28]; Köszegi and Szeidl [29]; de Clippel et al. [30]; Heidhues et al. [31, 32]; Martin [33]; Boyaci and Akçay [34]; Hefti [35]; Manzini and Mariotti [36]; Astorne-Figari et al. [37]; Hefti and Liu [38]; Malmendier and Szeidl [39]; Armstrong and Vickers [40]; Saur et al. [41]; Carroni et al. [42].

[3] The assumption that individuals perceive similar options as identical proves troublesome for the transitivity of the indifference relation. For decision-making models that account for such constraints on similarity perception see, for example, Luce [43]; Rubinstein [44]. Such coarse perception is not necessarily detrimental. Horan et al. [19] identify conditions under which coarser perception in a model with noisy perception might even improve choices.

[4] Weber's Law states that the difference between two stimuli which is just noticeable depends on the overall level of the stimuli [45].

[5] Assuming a linear disutility from consuming a non-ideal good, that is, $|x - y_i|$, would not change the demand of the consumers and thus would have no effect on the subgame-perfect equilibrium prices and qualities. A linear instead of quadratic disutility would shift the consumer surplus and the welfare downward by $h/6$.

[6] In equilibrium, not noticing a quality difference implies that Firm B unnoticeably undercuts the quality of Firm A, that is, $q_B = \max\{q_A - \tau, 0\}$. Consequently, although consumers do not notice a quality difference, a quality difference exists. For evidence on naivety see, for example, Szembrot [48]; Enke [49]; Jin et al. [50]; Sheth [51].

[7] Both assumptions ensure that for $h = 2c(1-\tau)^2$ exactly one subgame-perfect equilibrium exists and that consumer surplus and welfare have a well-defined maximum. $h = 2c(1-\tau)^2$ is a critical set of parameters for the main result.

[8] Limiting c to values smaller than 1/12 simplifies the derivation of best replies. The profit of Firm B consists of different cases. Therefore, to derive Firm B's best reply, it is necessary to maximize over the different cases. Assuming $c < 1/12$, reduces the candidates for best reply in some cases in favor of the boundary.

[9] This captures situations where investing in quality is independent of the quantity that a firm sells; for example, investing in research and development, updating software and hardware, buying more efficient machines, or training employees.

[10] If the competitor chooses sufficiently low prices, not all cases of the profit function exist.

[11] As $q_B \in [0,1]$, if $q_A \geq 1-\tau$, the first case of (3) does not exist; if $q_A \leq \tau$, the third case of (3) does not exist.

[12] As $q_B \in [0,1]$, if $q_A \geq 1-3h$, the first case of (4) does not exist; if $q_A \geq 1-\tau$, the second case of (4) does not exist; if $q_A \leq \tau$, the fourth case of (4) does not exist; if $q_A \leq 3h$, the fifth case of (4) does not exist. The third case of (4) always exists.

[13] To be precise, the profit of firm i is given in (A1) if $h > 0$. If $h = 0$, consumers buy from the firm with the lower price (accounting for quality differences). Then, this intense price competition drives down prices to $p_i^* = \hat{q}_i - \hat{q}_j$ and $p_j = 0$ where firm i is the firm with the higher perceived quality.

[14] Although the best reply to $p_i \leq \hat{q}_i - \hat{q}_j - h$ is $p_j^*(p_i) \in [0, \infty)$, a mutual best reply for $q_i - q_j > 3h$ only exists with $p_j^*(p_i) = 0$.

[15] If $\tau \geq 1/2$, this case does not exist.

[16] By assumption, if Firm B is indifferent between an indistinguishable quality and a distinguishable quality, Firm B chooses the indistinguishable quality. In addition, if $\tau \geq 1/2$, the range on h becomes $0 \leq h < 2c(1-\tau)^2$, respectively $0 \leq h \leq \tau/3$.

[17] By assumption, if Firm B is indifferent between an indistinguishable quality and a distinguishable quality, Firm B chooses the indistinguishable quality.

[18] By assumption, any indifference of Firm A between $q_A = 1-\tau$ and $q_A = 0$ for $h = 2c(1-\tau)^2$ is solved in favor of $q_A = 1-\tau$.

[19] By assumption, any indifference of Firm A between $q_A = 1-\tau$ and $q_A = 0$ for $h = 2c(1-\tau)^2$ is solved in favor of $q_A = 1-\tau$.

[20] By assumption, any indifference of Firm A between $q_A = 1-\tau$ and $q_A = 0$ for $h = 2c(1-\tau)^2$ is solved in favor of $q_A = 1-\tau$.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section. **Data S1.** Supporting Information.

Appendix A

Proof of Lemma 1

The profit of firm i with $i, j \in \{A, B\}$ and $i \neq j$ is[13]

$$\Pi_i(p_i, p_j, q_i, q_j) = \begin{cases} p_i - cq_i^2 & \text{if } p_i < p_j + \hat{q}_i - \hat{q}_j - h \\ p_i \frac{h+\hat{q}_i-\hat{q}_j+p_j-p_i}{2h} - cq_i^2 & \text{if } p_j + \hat{q}_i - \hat{q}_j - h \le p_i \le p_j + \hat{q}_i - \hat{q}_j + h \\ -cq_i^2 & \text{if } p_i > p_j + \hat{q}_i - \hat{q}_j + h \end{cases} \quad \text{(A1)}$$

First, if $p_j < \hat{q}_j - \hat{q}_i - h$, only the third case of (A1) exists. Then, firm i makes the same profit with any $p_i \ge 0$ and thus the best reply of firm i is $p_i^*(p_j) \in [0, \infty)$. Second, if $\hat{q}_j - \hat{q}_i - h \le p_j \le \hat{q}_j - \hat{q}_i + h$, the second and third case of (A1) exist. For $0 \le p_i \le p_j + \hat{q}_i - \hat{q}_j + h$, $\partial\Pi_i(p_i, p_j, q_i, q_j)/(\partial p_i) \ge 0 \Leftrightarrow p_i \le (p_j + \hat{q}_i - \hat{q}_j + h)/2$. The profit with $p_i = (p_j + \hat{q}_i - \hat{q}_j + h)/2$ is strictly higher than with any $p_i > p_j + \hat{q}_i - \hat{q}_j + h$ as long as $p_j > \hat{q}_j - \hat{q}_i - h$ and the profit is identical for $p_j = \hat{q}_j - \hat{q}_i - h$. Therefore, if $\hat{q}_j - \hat{q}_i - h < p_j \le \hat{q}_j - \hat{q}_i + h$, the best reply of firm i is $p_i^*(p_j) = (p_j + \hat{q}_i - \hat{q}_j + h)/2$ and, if $p_j = \hat{q}_j - \hat{q}_i - h$, the best reply of firm i is $p_i^*(p_j) = (p_j + \hat{q}_i - \hat{q}_j + h)/2$ and $p_i^*(p_j) \in [0, \infty)$. Third, if $p_j > \hat{q}_j - \hat{q}_i + h$, all cases of (A1) exist. Then, if $p_j \le \hat{q}_j - \hat{q}_i + 3h$, the best reply of firm i is $p_i^*(p_j) = (p_j + \hat{q}_i - \hat{q}_j + h)/2$ and, if $p_j > \hat{q}_j - \hat{q}_i + 3h$, the best reply of firm i is the boundary $p_i^*(p_j) = p_j + \hat{q}_i - \hat{q}_j - h$.

In sum, the best reply of firm i depends on the price of firm j:

i. If $p_j \le \hat{q}_j - \hat{q}_i - h$, the best reply of firm i is

$$p_i^*(p_j) \in [0, \infty)$$

ii. If $\hat{q}_j - \hat{q}_i - h \le p_j \le \hat{q}_j - \hat{q}_i + 3h$, the best reply of firm i is

$$p_i^*(p_j) = \frac{1}{2}(p_j + \hat{q}_i - \hat{q}_j + h)$$

iii. If $p_j > \hat{q}_j - \hat{q}_i + 3h$, the best reply of firm i is

$$p_i^*(p_j) = p_j + \hat{q}_i - \hat{q}_j - h$$

Consequently, if firms choose indistinguishable qualities in the first stage, that is, $|q_A - q_B| \leq \tau$ and thus $\hat{q}_A = \hat{q}_B$, the equilibrium prices are

$$p_A^* = p_B^* = h$$

If firms choose distinguishable qualities in the first stage, that is, $|q_A - q_B| > \tau$ and thus $\hat{q}_A = q_A$ and $\hat{q}_B = q_B$, equilibrium prices depend on the quality difference. If $-3h \leq q_A - q_B \leq 3h$, an interior equilibrium exists. In equilibrium, firms choose prices

$$p_A^* = \frac{1}{3}(q_A - q_B + 3h)$$
$$p_B^* = \frac{1}{3}(q_B - q_A + 3h)$$

Let firm i be the firm with the higher quality. If the quality difference increases such that $q_i - q_j > 3h$, the interior solution no longer exists, instead a corner solution exists.[14] Consequently, in equilibrium, firms choose prices $p_i^* = q_i - q_j - h$ and $p_j^* = 0$. Therefore, in equilibrium, if $q_A - q_B > 3h$, firms choose prices

$$p_A^* = q_A - q_B - h$$
$$p_B^* = 0$$

and, if $q_A - q_B < -3h$, firms choose prices

$$p_A^* = 0$$
$$p_B^* = q_B - q_A - h$$

Appendix B

Proof of Proposition 1

The proof proceeds in two parts. In the first part, I derive the best reply of Firm B. In the second part, I derive the quality of Firm A.

Second Stage: Best Reply of Firm B

$\tau \geq 3h \Leftrightarrow h \leq \tau/3$

Assume $\tau \geq 3h \Leftrightarrow h \leq \tau/3$, then the profit of Firm B is

$$\Pi_B(q_A, q_B) = \begin{cases} q_B - q_A - h - cq_B^2 & \text{if } q_B > q_A + \tau \\ \frac{1}{2}h - cq_B^2 & \text{if } q_A - \tau \leq q_B \leq q_A + \tau \\ -cq_B^2 & \text{if } q_B < q_A - \tau \end{cases} \tag{B1}$$

As $q_B \in [0,1]$, if $q_A \geq 1-\tau$, the first case of (B1) does not exist; if $q_A \leq \tau$, the third case of (B1) does not exist.

First, as $c < 1/12$, the profit of Firm B is strictly increasing in $q_B \in (q_A + \tau, 1]$. Consequently, if $q_A < 1-\tau$, $q_B(q_A) = 1$ with profit $\Pi_B(q_A, q_B = 1) = 1 - q_A - h - c$ is a candidate for best reply. Second, the profit of Firm B is strictly decreasing in $q_B \in [\max\{0, q_A - \tau\}, \min\{q_A + \tau, 1\}]$. Consequently, if $q_A \leq \tau$, $q_B(q_A) = 0$ with profit $\Pi_B(q_A, q_B = 0) = h/2$ is a candidate for best reply and, if $q_A > \tau$, $q_B(q_A) = q_A - \tau$ with profit $\Pi_B(q_A, q_B = q_A - \tau) = h/2 - c(q_A - \tau)^2$ is a candidate for best reply. Third, the profit of Firm B is strictly decreasing in $q_B \in [0, q_A - \tau)$. Consequently, if $q_A > \tau$, $q_B(q_A) = 0$ with profit $\Pi_B(q_A, q_B = 0) = 0$ is a candidate for best reply.

i. Assume all three cases of (B1) exist, that is, $\tau < q_A < 1-\tau$ and thus $\tau < 1/2$. Then,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = q_A - \tau)$$
$$\Leftrightarrow 1 - q_A - h - c > \frac{1}{2}h - c(q_A - \tau)^2 \tag{B2}$$

which is more difficult to satisfy for large q_A and large h. As $\tau < q_A < 1-\tau$ and $h \leq \tau/3$, substituting $q_A = 1-\tau$ and $h = \tau/3$ into (B2) becomes $1 > 8c(1-\tau)$ which is always satisfied.
In addition,

$$\Pi_B(q_A, q_B = 1) \geq \Pi_B(q_A, q_B = 0) \Leftrightarrow 1 - q_A - h - c \geq 0$$
$$\Leftrightarrow q_A \leq 1 - h - c$$

where $1 - h - c > \tau$ for any $c < 1/12$ and $h \leq \tau/3$ where $\tau < 1/2$. Furthermore, $1 - h - c < 1 - \tau \Leftrightarrow \tau - c < h$. Therefore, if $\tau - c < h \leq \tau/3$, the best reply of Firm B for $\tau < q_A < 1-\tau$ is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 - h - c \\ 0 & \text{if } q_A \geq 1 - h - c \end{cases}$$

Otherwise, the best reply of Firm B for $\tau < q_A < 1-\tau$ is

$$q_B^*(q_A) = 1$$

ii. Assume only case one and two of (B1) exist, that is, $q_A < 1-\tau$ and $q_A \leq \tau$. Then,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0) \Leftrightarrow 1 - q_A - h - c > \frac{1}{2}h$$
$$\Leftrightarrow q_A < 1 - \frac{3}{2}h - c$$

Note that $1 - 3h/2 - c > \min\{\tau, 1-\tau\}$ and thus $\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0)$ is always fulfilled. Consequently, the best reply of Firm B for $q_A < 1-\tau$ and $q_A \leq \tau$ is

$$q_B^*(q_A) = 1$$

iii. Assume only case two and three of (B1) exist, that is, $q_A \geq 1-\tau$ and $q_A > \tau$. Then,

$$\Pi_B(q_A, q_B = q_A - \tau) \geq \Pi_B(q_A, q_B = 0)$$
$$\Leftrightarrow \frac{1}{2}h - c(q_A - \tau)^2 \geq 0$$
$$\Leftrightarrow \tau - \sqrt{\frac{h}{2c}} \leq q_A \leq \tau + \sqrt{\frac{h}{2c}}$$

Note that $q_A \geq \tau - \sqrt{h/(2c)}$ is always fulfilled. In addition, $1 - \tau \leq \tau + \sqrt{h/(2c)} < 1 \Leftrightarrow 2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$. Furthermore,

$$2c(1-2\tau)^2 \leq \frac{\tau}{3} \Leftrightarrow \tau \geq \frac{1 + 24c - \sqrt{1+48c}}{48c} \text{ and}$$
$$2c(1-\tau)^2 \leq \frac{\tau}{3} \Leftrightarrow \tau \geq \frac{1 + 12c - \sqrt{1+24c}}{12c}$$

Therefore, if $\tau < (1 + 24c - \sqrt{1+48c})/(48c)$ or if $(1 + 24c - \sqrt{1+48c})/(48c) \leq \tau < (1 + 12c - \sqrt{1+24c})/(12c)$ and $h < 2c(1-2\tau)^2$ or if $\tau \geq (1 + 12c - \sqrt{1+24c})/(12c)$ and $h < 2c(1-2\tau)^2$, the best reply of Firm B for $q_A \geq 1-\tau$ and $q_A > \tau$ is[15]

$$q_B^*(q_A) = 0$$

If $(1 + 24c - \sqrt{1+48c})/(48c) \leq \tau < (1 + 12c - \sqrt{1+24c})/(12c)$ and $2c(1-2\tau)^2 \leq h \leq \tau/3$ or if $\tau \geq (1 + 12c - \sqrt{1+24c})/(12c)$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$, the best reply of Firm B for $q_A \geq 1-\tau$ and $q_A > \tau$ is[16]

$$q_B^*(q_A) = \begin{cases} q_A - \tau & \text{if } q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ 0 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

If $\tau \geq (1 + 12c - \sqrt{1 + 24c})/(12c)$ and $h \geq 2c(1 - \tau)^2$, the best reply of Firm B for $q_A \geq 1 - \tau$ and $q_A > \tau$ is

$$q_B^*(q_A) = q_A - \tau$$

iv. Assume only case two of (B1) exists, that is, $1 - \tau \leq q_A \leq \tau$ and thus $\tau \geq 1/2$. Then, as the profit of Firm B is decreasing in $q_B \in [0, 1]$, the best reply of Firm B for $1 - \tau \leq q_A \leq \tau$ is

$$q_B^*(q_A) = 0$$

$\tau < 3h \Leftrightarrow \tau/3 < h$

Assume $\tau < 3h \Leftrightarrow \tau/3 < h$, then the profit of Firm B is

$$\Pi_B(q_A, q_B) = \begin{cases} q_B - q_A - h - cq_B^2 & \text{if } q_B > q_A + 3h \\ \frac{1}{18h}\left(q_B - q_A + 3h\right)^2 - cq_B^2 & \text{if } q_A + \tau < q_B \leq q_A + 3h \\ \frac{1}{2}h - cq_B^2 & \text{if } q_A - \tau \leq q_B \leq q_A + \tau \\ \frac{1}{18h}\left(q_B - q_A + 3h\right)^2 - cq_B^2 & \text{if } q_A - 3h \leq q_B < q_A - \tau \\ -cq_B^2 & \text{if } q_B < q_A - 3h \end{cases} \tag{B3}$$

As $q_B \in [0, 1]$, if $q_A \geq 1 - 3h$, the first case of (B3) does not exist; if $q_A \geq 1 - \tau$, the second case of (B3) does not exist; if $q_A \leq \tau$, the fourth case of (B3) does not exist; if $q_A \leq 3h$, the fifth case of (B3) does not exist. The third case of (B3) always exists.

First, as $c < 1/12$, the profit of Firm B is strictly increasing in $q_B \in (q_A + \tau, 1]$. Consequently, if $q_A < 1 - 3h$, $q_B(q_A) = 1$ with profit $\Pi_B(q_A, q_B = 1) = 1 - q_A - h - c$ is a candidate for best reply and if $q_A \geq 1 - 3h$, $q_B(q_A) = 1$ with profit $\Pi_B(q_A, q_B = 1) = (1 - q_A + 3h)^2/(18h) - c$ is a candidate for best reply. Second, the profit of Firm B is strictly decreasing in $q_B \in [\max\{0, q_A - \tau\}, \min\{q_A + \tau, 1\}]$. Consequently, if $q_A \leq \tau$, $q_B(q_A) = 0$ with profit $\Pi_B(q_A, q_B = 0) = h/2$ is a candidate for best reply and, if $q_A > \tau$, $q_B(q_A) = q_A - \tau$ with profit $\Pi_B(q_A, q_B = q_A - \tau) = h/2 - c(q_A - \tau)^2$ is a candidate for best reply. Third, the profit of Firm B is strictly decreasing in $q_B \in [0, q_A - 3h)$. Consequently, if $q_A > 3h$, $q_B(q_A) = 0$ with profit $\Pi_B(q_A, q_B = 0) = 0$ is a candidate for best reply. Fourth, the profit of Firm B for any $q_B \in [q_A - 3h, q_A - \tau)$ is always lower than the profit at $q_B = q_A - \tau$: Let

$$q_B' \equiv \frac{q_A - 3h - \sqrt{\begin{array}{c} 9h^2 - 18ch(q_A - \tau)^2 - 108ch^2 q_A \\ + 18chq_A^2 + 18^2c^2h^2(q_A - \tau)^2 \end{array}}}{1 - 18ch}$$

$$q_B'' \equiv \frac{q_A - 3h + \sqrt{\begin{array}{c} 9h^2 - 18ch(q_A - \tau)^2 - 108ch^2 q_A \\ + 18chq_A^2 + 18^2c^2h^2(q_A - \tau)^2 \end{array}}}{1 - 18ch}$$

If $1 - 18ch > 0$,

$$\Pi_B\left(q_A, q_B = q_A - \tau\right) > \Pi_B\left(q_A, q_B \in [q_A - 3h, q_A - \tau)\right)$$
$$\Leftrightarrow \frac{1}{2}h - c(q_A - \tau)^2 > \frac{1}{18h}\left(q_B - q_A + 3h\right)^2 - cq_B^2$$
$$\Leftrightarrow q_B' < q_B < q_B''$$

if $1 - 18ch = 0$,

$$\Pi_B\left(q_A, q_B = q_A - \tau\right) > \Pi_B\left(q_A, q_B \in [q_A - 3h, q_A - \tau)\right)$$
$$\Leftrightarrow q_B < \frac{6hq_A - q_A^2 - 18ch(q_A - \tau)^2}{2(3h - q_A)}$$

and if $1 - 18ch < 0$,

$$\Pi_B\left(q_A, q_B = q_A - \tau\right) > \Pi_B\left(q_A, q_B \in [q_A - 3h, q_A - \tau)\right)$$
$$\Leftrightarrow q_B < q_B'' \vee q_B > q_B'$$

As $q_B' < q_A - 3h$ and $q_B'' \geq q_A - \tau$ for $1 - 18ch > 0$, $(6hq_A - q_A^2 - 18ch(q_A - \tau)^2)/(2(3h - q_A)) \geq q_A - \tau$ for $1 - 18ch = 0$, and $q_B'' \geq q_A - \tau$ for $1 - 18ch < 0$, no $q_B \in [q_A - 3h, q_A - \tau)$ is ever a best reply.

Fifth, if $\tau < q_A < 1 - \tau$, $q_B = q_A - \tau$ is never a best reply for Firm B: Let

$$\phi \equiv \sqrt{9h^2 + 36ch\tau + 108ch^2\tau - 18ch\tau^2 - 108ch^2 + 324c^2h^2}$$

If $q_A < 1 - 3h$,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = q_A - \tau)$$
$$\Leftrightarrow 1 - q_A - h - c > \frac{1}{2}h - c(q_A - \tau)^2$$
$$\Leftrightarrow q_A < \tau + \frac{1 - \sqrt{1 + 6ch + 4c^2 - 4c + 4c\tau}}{2c} \vee$$
$$q_A > \tau + \frac{1 + \sqrt{1 + 6ch + 4c^2 - 4c + 4c\tau}}{2c}$$

If $q_A \geq 1 - 3h$,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = q_A - \tau)$$
$$\Leftrightarrow \frac{1}{18h}\left(1 - q_A + 3h\right)^2 - c > \frac{1}{2}h - c(q_A - \tau)^2$$
$$\Leftrightarrow q_A < \frac{1 + 3h + 18ch\tau - \phi}{1 + 18ch} \vee q_A > \frac{1 + 3h + 18ch\tau + \phi}{1 + 18ch}$$

As $\tau + (1 - \sqrt{1 + 6ch + 4c^2 - 4c + 4c\tau})/(2c) > 1 - 3h$ and $(1 + 3h + 18ch\tau - \phi)/(1 + 18ch) \geq 1 - \tau$, if $\tau < q_A < 1 - \tau$, $q_B = q_A - \tau$ is never a best reply for Firm B.

i. Assume all cases of (B3) exist, that is, $3h < q_A < 1 - 3h$ (and thus also $\tau < q_A < 1 - \tau$) and thus $h < 1/6$. Then,

$$\Pi_B(q_A, q_B = 1) \geq \Pi_B(q_A, q_B = 0)$$
$$\Leftrightarrow 1 - q_A - h - c \geq 0 \Leftrightarrow q_A \leq 1 - h - c$$

Note that for all $h < 1/6$ and $c < 1/12$, $1 - h - c > 3h$ and

$$1 - h - c < 1 - 3h \Leftrightarrow h < \frac{1}{2}c$$

Therefore, if $h < c/2$, the best reply of Firm B for $3h < q_A < 1 - 3h$ is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 - h - c \\ 0 & \text{if } q_A \geq 1 - h - c \end{cases}$$

and, if $h \geq c/2$, the best reply of Firm B for $3h < q_A < 1 - 3h$ is

$$q_B^*(q_A) = 1$$

ii. Assume only the first, second, third, and fourth case of (B3) exist, that is, $\tau < q_A \leq 3h$ and $q_A < 1 - 3h$ (and thus also $q_A < 1 - \tau$). As the profit of Firm B is strictly increasing in $q_B \in (q_A + \tau, 1]$ and no $q_B \leq q_A + \tau$ can be a best reply, the best reply of Firm B for $\tau < q_A \leq 3h$ and $q_A < 1 - 3h$ (and thus also $q_A < 1 - \tau$) is

$$q_B^*(q_A) = 1$$

iii. Assume only the first, second, and third case of (B3) exist, that is, $q_A < 1 - 3h$ (and thus also $q_A < 1 - \tau$) and $q_A \leq \tau$ (and thus also $q_A \leq 3h$). Then,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0) \Leftrightarrow 1 - q_A - h - c > \frac{1}{2}h$$
$$\Leftrightarrow q_A < 1 - \frac{3}{2}h - c$$

As $1 - 3h/2 - c > \min\{\tau, 1 - 3h\}$, $\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0)$ is always fulfilled. Therefore, the best reply of Firm B for $q_A < 1 - 3h$ (and thus also $q_A < 1 - \tau$) and $q_A \leq \tau$ (and thus also $q_A \leq 3h$) is

$$q_B^*(q_A) = 1$$

iv. Assume only the second, third, fourth, and fifth case of (B3) exist, that is, $q_A \geq 1 - 3h$, $q_A > 3h$ (and thus also $q_A > \tau$), and $q_A < 1 - \tau$. Then,

$$\Pi_B(q_A, q_B = 1) \geq \Pi_B(q_A, q_B = 0)$$
$$\Leftrightarrow \frac{1}{18h}\left(1 - q_A + 3h\right)^2 - c \geq 0$$
$$\Leftrightarrow q_A \leq 1 + 3h - \sqrt{18ch} \vee q_A \geq 1 + 3h + \sqrt{18ch}$$

Note that $1 + 3h + \sqrt{18ch} > 1$ and

$$\max\{3h, 1 - 3h\} \leq 1 + 3h - \sqrt{18ch} < 1 - \tau$$
$$\Leftrightarrow \tau < \frac{3}{2}c \text{ and } \frac{c}{2} \leq h < \frac{3c - \tau}{3} + \sqrt{\frac{3c^2 - 2c\tau}{3}}$$

Therefore, if $\tau < 3c/2$ and $h < c/2$, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A > 3h$ (and thus also $q_A > \tau$), and $q_A < 1 - \tau$ is

$$q_B^*(q_A) = 0$$

If $\tau < 3c/2$ and $c/2 \leq h < (3c - \tau)/3 + \sqrt{(3c^2 - 2c\tau)/3}$, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A > 3h$ (and thus also $q_A > \tau$), and $q_A < 1 - \tau$ is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 + 3h - \sqrt{18ch} \\ 0 & \text{if } q_A \geq 1 + 3h - \sqrt{18ch} \end{cases}$$

Otherwise, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A > 3h$ (and thus also $q_A > \tau$), and $q_A < 1 - \tau$ is

$$q_B^*(q_A) = 1$$

v. Assume only the second, third, and fourth case of (B3) exist, that is, $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A < 1 - \tau$, and $q_A > \tau$. As the profit of Firm B is strictly increasing in $q_B \in (q_A + \tau, 1]$ and no $q_B \leq q_A + \tau$ is a best reply in this case, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A < 1 - \tau$, and $q_A > \tau$ is

$$q_B^*(q_A) = 1$$

vi. Assume only the second and third case of (B3) exist, that is, $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A < 1 - \tau$, and $q_A \leq \tau$. Then,

$$\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0)$$
$$\Leftrightarrow \frac{1}{18h}\left(1 - q_A + 3h\right)^2 - c > \frac{1}{2}h$$
$$\Leftrightarrow q_A < 1 + 3h - \sqrt{9h^2 + 18ch} \vee q_A > 1 + 3h + \sqrt{9h^2 + 18ch}$$

As $1 + 3h - \sqrt{9h^2 + 18ch} > \min\{\tau, 1 - \tau\}$, $\Pi_B(q_A, q_B = 1) > \Pi_B(q_A, q_B = 0)$ is always fulfilled. Therefore, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A < 1 - \tau$, and $q_A \leq \tau$ is

$$q_B^*(q_A) = 1$$

vii. Assume only the third, fourth, and fifth case of (B3) exist, that is, $q_A > 3h$ (and thus also $q_A > \tau$) and $q_A \geq 1 - \tau$ (and thus also $q_A \geq 1 - 3h$). Then,

$$\Pi_B(q_A, q_B = q_A - \tau) \geq \Pi_B(q_A, q_B = 0)$$
$$\Leftrightarrow \frac{1}{2}h - c(q_A - \tau)^2 \geq 0$$
$$\Leftrightarrow \tau - \sqrt{\frac{h}{2c}} \leq q_A \leq \tau + \sqrt{\frac{h}{2c}}$$

Note that $\tau - \sqrt{h/(2c)} < \tau$ and $1 - \tau \leq \tau + \sqrt{h/(2c)} < 1 \Leftrightarrow 2c(1 - 2\tau)^2 \leq h < 2c(1 - \tau)^2$. In addition, if $3h \geq 1 - \tau$, $\tau + \sqrt{h/(2c)} > 1$. Furthermore,

$$\frac{\tau}{3} < 2c(1 - 2\tau)^2 \Leftrightarrow \tau < \frac{1 + 24c - \sqrt{1 + 48c}}{48c} \text{ and}$$
$$\frac{\tau}{3} < 2c(1 - \tau)^2 \Leftrightarrow \tau < \frac{1 + 12c - \sqrt{1 + 24c}}{12c}$$

Therefore, if $\tau < (1 + 24c - \sqrt{1 + 48c})/(48c)$ and $h < 2c(1 - 2\tau)^2$, the best reply of Firm B for $q_A > 3h$ (and thus also $q_A > \tau$) and $q_A \geq 1 - \tau$ (and thus also $q_A \geq 1 - 3h$) is

$$q_B^*(q_A) = 0$$

if $\tau < (1 + 24c - \sqrt{1 + 48c})/(48c)$ and $2c(1 - 2\tau)^2 \leq h < 2c(1 - \tau)^2$ of if $(1 + 24c - \sqrt{1 + 48c})/(48c) \leq \tau < (1 + 12c - \sqrt{1 + 24c})/(12c)$ and $h < 2c(1 - \tau)^2$, the best reply of Firm B for $q_A > 3h$ (and thus also $q_A > \tau$) and $q_A \geq 1 - \tau$ (and thus also $q_A \geq 1 - 3h$) is[17]

$$q_B^*(q_A) = \begin{cases} q_A - \tau & \text{if } q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ 0 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

otherwise, the best reply of Firm B for $q_A > 3h$ (and thus also $q_A > \tau$) and $q_A \geq 1 - \tau$ (and thus also $q_A \geq 1 - 3h$) is

$$q_B^*(q_A) = q_A - \tau$$

viii. Assume only the third and fourth case of (B3) exist, that is, $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A \geq 1 - \tau$, and $q_A > \tau$. Then, as the profit of Firm B is decreasing in $q_B \in [q_A - \tau, 1]$, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A \geq 1 - \tau$, and $q_A > \tau$ is

$$q_B^*(q_A) = q_A - \tau$$

ix. Assume only the third case of (B3) exist, that is, $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A \geq 1 - \tau$, and $q_A \leq \tau$. Then, as the profit of Firm B is decreasing in $q_B \in [0, 1]$, the best reply of Firm B for $q_A \geq 1 - 3h$, $q_A \leq 3h$, $q_A \geq 1 - \tau$, and $q_A \leq \tau$ is

$$q_B^*(q_A) = 0$$

Best Reply of Firm B

The results of the above subsections show that the best reply of Firm B depends on the cost parameter c, the degree of horizontal product differentiation h, and the perception threshold τ. Combining the results of the above subsections yields the following best reply of Firm B:

Case I, that is, if (i) $0 \leq \tau < c$ and $h \leq \tau/3$, (ii) $c \leq \tau < 3c/2$ and $\tau - c < h \leq \tau/3$, or (iii) $\tau < 3c/2$ and $\tau/3 < h < c/2$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 - h - c \\ 0 & \text{if } q_A \geq 1 - h - c \end{cases}$$

Case II, that is, if (i) $c \leq \tau < 3c/2$ and $h \leq \tau - c$, (ii) $\tau < 3c/2$ and $(3c - \tau)/3 + \sqrt{(3c^2 - 2c\tau)/3} \leq h < 2c(1 - 2\tau)^2$, (iii) $3c/2 \leq \tau < (1 + 24c - \sqrt{1 + 48c})/(48c)$ and $h \leq \tau/3$, (iv) $3c/2 \leq \tau < (1 + 24c - \sqrt{1 + 48c})/(48c)$ and $\tau/3 < h < 2c(1 - 2\tau)^2$, (v) $(1 + 24c - \sqrt{1 + 48c})/(48c) \leq \tau < (1 + 12c - \sqrt{1 + 24c})/(12c)$ and $h < 2c(1 - 2\tau)^2$, or (vi) $(1 + 12c - \sqrt{1 + 24c})/(12c) \leq \tau < 1/2$ and $h < 2c(1 - 2\tau)^2$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A < 1 - \tau \\ 0 & \text{if } q_A \geq 1 - \tau \end{cases}$$

Case III, that is, if (i) $\tau < 3c/2$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$, (ii) $3c/2 \leq \tau < (1+24c-\sqrt{1+48c})/(48c)$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$, (iii) $(1+12c-\sqrt{1+24c})/(12c) \leq \tau < 1/2$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$, (iv) $(1+24c-\sqrt{1+48c})/(48c) \leq \tau < (1+12c-\sqrt{1+24c})/(12c)$ and $\tau/3 < h < 2c(1-\tau)^2$, or if (v) $(1+24c-\sqrt{1+48c})/(48c) \leq \tau < (1+12c-\sqrt{1+24c})/(12c)$ and $2c(1-2\tau)^2 \leq h \leq \tau/3$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A < 1-\tau \\ q_A - \tau & \text{if } 1-\tau \leq q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ 0 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

Case IV, that is, if (i) $\tau < 3c/2$ and $2c(1-\tau)^2 \leq h < 1/6$, (ii) $3c/2 \leq \tau < (1+24c-\sqrt{1+48c})/(48c)$ and $2c(1-\tau)^2 \leq h < 1/6$, (iii) $(1+24c-\sqrt{1+48c})/(48c) \leq \tau < (1+12c-\sqrt{1+24c})/(12c)$ and $2c(1-\tau)^2 \leq h < 1/6$, (iv) $(1+12c-\sqrt{1+24c})/(12c) \leq \tau < 1/2$ and $2c(1-\tau)^2 \leq h \leq \tau/3$, (v) $(1+12c-\sqrt{1+24c})/(12c) \leq \tau < 1/2$ and $\tau/3 < h < 1/6$, (vi) $\tau < 1-3h$ and $h \geq 1/6$, or (vii) $1-3h \leq \tau < 1/2$ and $h \geq 1/6$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A < 1-\tau \\ q_A - \tau & \text{if } q_A \geq 1-\tau \end{cases}$$

Case V, that is, if $\tau \geq 1/2$ and $h < 2c(1-\tau)^2$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A < 1-\tau \\ 0 & \text{if } 1-\tau \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ 0 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

Case VI, that is, if (i) $\tau \geq 1/2$ and $2c(1-\tau)^2 \leq h \leq \tau/3$ or (ii) $1/2 \leq \tau < 3h$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A < 1-\tau \\ 0 & \text{if } 1-\tau \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \end{cases}$$

Case VII, that is, if $\tau < 3c/2$ and $c/2 \leq h < (3c-\tau)/3 + \sqrt{(3c^2 - 2c\tau)/3}$, the best reply of Firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 + 3h - \sqrt{18ch} \\ 0 & \text{if } q_A \geq 1 + 3h - \sqrt{18ch} \end{cases}$$

First Stage: Quality Choice of Firm A

In the first stage, Firm A chooses its quality q_A to maximize its profit, taking into account the best reply of Firm B and the equilibrium prices. The cases between which I distinguish in the following correspond to the cases in the above section.

Case I: The profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A \leq 1-h-c \\ q_A - h - cq_A^2 & \text{if } q_A \geq 1-h-c \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-h-c]$ and increasing in $q_A \in [1-h-c, 1]$ and as

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

Case II:

i. If $\tau < 3c/2$ and $(3c-\tau)/3 + \sqrt{(3c^2 - 2c\tau)/3} \leq h < 2c(1-2\tau)^2$ or if $3c/2 \leq \tau < (1+24c-\sqrt{1+48c})/(48c)$ and $\tau/3 < h < 2c(1-2\tau)^2$, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1-3h \\ \frac{1}{18h}(q_A - 1 + 3h)^2 - cq_A^2 & \text{if } 1-3h \leq q_A < 1-\tau \\ q_A - h - cq_A^2 & \text{if } q_A \geq 1-\tau \end{cases}$$

Note that the profit of Firm A is decreasing in $q_A \in [0, 1-3h)$ and increasing in $q_A \in [1-\tau, 1]$. As

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1) > \Pi_A(q_A) \forall q_A \in [1-3h, 1-\tau)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

ii. Otherwise, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1-\tau \\ q_A - h - cq_A^2 & \text{if } q_A \geq 1-\tau \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-\tau)$ and increasing in $q_A \in [1-\tau, 1]$ and as

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

Case III:

i. If $\tau < 3c/2$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$ or if $3c/2 \leq \tau < (1+24c-\sqrt{1+48c})/(48c)$ and $2c(1-2\tau)^2 \leq h < 2c(1-\tau)^2$ or if $(1+24c-\sqrt{1+48c})/(48c) \leq \tau < (1+12c-\sqrt{1+24c})/(12c)$ and $\tau/3 < h < 2c(1-\tau)^2$, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1-3h \\ \frac{1}{18h}(q_A - 1 + 3h)^2 - cq_A^2 & \text{if } 1-3h \leq q_A < 1-\tau \\ \frac{1}{2}h - cq_A^2 & \text{if } 1-\tau \leq q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ q_A - h - cq_A^2 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-3h)$ and in $q_A \in [1-\tau, \tau + \sqrt{h/(2c)}]$ and increasing in $q_A \in (\tau + \sqrt{h/(2c)}, 1]$ and as

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1) = 1 - h - c > \frac{1}{2}h - c(1-\tau)^2 = \Pi_A(q_A = 1-\tau),$$
$$\Pi_A(q_A = 1) > \Pi_A(q_A) \forall q_A \in [1-3h, 1-\tau)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

ii. Otherwise, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1-\tau \\ \frac{1}{2}h - cq_A^2 & \text{if } 1-\tau \leq q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ q_A - h - cq_A^2 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-\tau)$ and in $q_A \in [1-\tau, \tau + \sqrt{h/(2c)}]$ and increasing in $q_A \in (\tau + \sqrt{h/(2c)}, 1]$ and as

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1) = 1 - h - c > \frac{1}{2}h - c(1-\tau)^2 = \Pi_A(q_A = 1-\tau)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

Case IV:

i. If $\tau < 3c/2$ and $2c(1-\tau)^2 \leq h < 1/6$ or if $3c/2 \leq \tau < (1 + 24c - \sqrt{1+48c})/(48c)$ and $2c(1-\tau)^2 \leq h < 1/6$ or if $(1 + 24c - \sqrt{1+48c})/(48c) \leq \tau < (1 + 12c - \sqrt{1+24c})/(12c)$ and $2c(1-\tau)^2 \leq h < 1/6$ or if $(1 + 12c - \sqrt{1+24c})/(12c) \leq \tau < 1/2$ and $\tau/3 < h < 1/6$ or if $\tau < 1 - 3h$ and $h \geq 1/6$ or if $1 - 3h \leq \tau < 1/2$ and $h \geq 1/6$, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - 3h \\ \frac{1}{18h}(q_A - 1 + 3h)^2 - cq_A^2 & \text{if } 1 - 3h \leq q_A < 1 - \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } q_A \geq 1 - \tau \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-3h)$ and in $q_A \in [1-\tau, 1]$ and as

$$\Pi_A(q_A = 1-\tau) = \frac{1}{2}h - c(1-\tau)^2 \geq 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1-\tau) > \Pi_A(q_A)\forall q_A \in [1-3h, 1-\tau)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1 - \tau$.[18] Consequently, Firm B chooses $q_B^* = 1 - 2\tau$. The quality difference is indistinguishable to consumers.

ii. Otherwise, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } q_A \geq 1 - \tau \end{cases}$$

As the profit of Firm A is decreasing in $q_A \in [0, 1-\tau)$ and in $q_A \in [1-\tau, 1]$ and as

$$\Pi_A(q_A = 1-\tau) = \frac{1}{2}h - c(1-\tau)^2 \geq 0 = \Pi_A(q_A = 0)$$

in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1 - \tau$.[19] Consequently, Firm B chooses $q_B^* = 1 - 2\tau$. The quality difference is indistinguishable to consumers.

Case V: The profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } 1 - \tau \leq q_A \leq \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } \tau < q_A \leq \tau + \sqrt{\frac{h}{2c}} \\ q_A - h - cq_A^2 & \text{if } q_A > \tau + \sqrt{\frac{h}{2c}} \end{cases}$$

Note that the profit of Firm A is decreasing in $q_A \in [0, 1-\tau)$ and in $q_A \in [1-\tau, \tau + \sqrt{h/(2c)}]$ and that the profit of Firm A is increasing in $q_A \in [\tau + \sqrt{h/(2c)}, 1]$. In addition,

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1) = 1 - h - c > \frac{1}{2}h - c(1-\tau)^2 = \Pi_A(q_A = 1-\tau)$$

Therefore, in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

Case VI:

i. If $1/2 \leq \tau < 3h$, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - 3h \\ \frac{1}{18h}(q_A - 1 + 3h)^2 - cq_A^2 & \text{if } 1 - 3h \leq q_A < 1 - \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } q_A \geq 1 - \tau \end{cases}$$

Note that the profit of Firm A is decreasing in $q_A \in [0, 1-3h)$ and in $q_A \in [1-\tau, 1]$. In addition,

$$\Pi_A(q_A = 1-\tau) = \frac{1}{2}h - c(1-\tau)^2 > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1-\tau) > \Pi_A(q_A)\forall q_A \in [1-3h, 1-\tau)$$

Therefore, in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1 - \tau$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is indistinguishable to consumers.

ii. Otherwise, the profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - \tau \\ \frac{1}{2}h - cq_A^2 & \text{if } q_A \geq 1 - \tau \end{cases}$$

Note that the profit of Firm A is decreasing in $q_A \in [0, 1-\tau)$ and in $q_A \in [1-\tau, 1]$. In addition,

$$\Pi_A(q_A = 1-\tau) = \frac{1}{2}h - c(1-\tau)^2 \geq 0 = \Pi_A(q_A = 0)$$

Therefore, in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1 - \tau$.[20] Consequently, Firm B chooses $q_B^* = 0$. The quality difference is indistinguishable to consumers.

Case VII: The profit of Firm A is

$$\Pi_A(q_A) = \begin{cases} -cq_A^2 & \text{if } q_A < 1 - 3h \\ \frac{1}{18h}(q_A - 1 + 3h)^2 - cq_A^2 & \text{if } 1 - 3h \leq q_A \\ & \leq 1 + 3h - \sqrt{18ch} \\ q_A - h - cq_A^2 & \text{if } q_A \geq 1 + 3h - \sqrt{18ch} \end{cases}$$

Note that the profit of Firm A is decreasing in $q_A \in [0, 1-3h)$ and increasing in $q_A \in [1 + 3h - \sqrt{18ch}, 1]$. In addition,

$$\Pi_A(q_A = 1) = 1 - h - c > 0 = \Pi_A(q_A = 0),$$
$$\Pi_A(q_A = 1) > \Pi_A(q_A)\forall q_A \in [1-3h, 1 + 3h - \sqrt{18ch}]$$

Therefore, in the subgame-perfect equilibrium, Firm A chooses $q_A^* = 1$. Consequently, Firm B chooses $q_B^* = 0$. The quality difference is distinguishable to consumers.

Summarizing the subgame-perfect equilibria across cases I to VII, yields Proposition 1.

Appendix C

Proof of Proposition 2

i. If $h < 2c(1-\tau)^2 \Leftrightarrow \tau < 1 - \sqrt{h/(2c)}$, in the subgame-perfect equilibrium, firms choose $q_A^* = 1$, $q_B^* = 0$, $p_A^* = 1 - h$, and $p_B^* = 0$. Then, the indifferent consumer is $\bar{x} = 1$, that is, all consumers buy from Firm A. The producer surplus, consumer surplus, and welfare are

$$PS = \Pi_A + \Pi_B = 1 - h - c$$
$$CS = \int_0^1 v + q_A^* - p_A^* - hx^2 \text{dx} = v + \frac{2}{3}h$$
$$W = PS + CS = v + 1 - \frac{1}{3}h - c$$

ii. If $h \geq 2c(1-\tau)^2 \Leftrightarrow \tau \geq 1-\sqrt{h/(2c)}$, in the subgame-perfect equilibrium, firms choose $q_A^* = 1-\tau$, $q_B^* = \max\{0, 1-2\tau\}$, and $p_A^* = p_B^* = h$. Then, the indifferent consumer is $\overline{x} = 1/2$. The producer surplus, consumer surplus, and welfare are

$$
\begin{aligned}
PS &= \Pi_A + \Pi_B = \frac{1}{2}h - c(1-\tau)^2 + \frac{1}{2}h - c(\max\{0, 1-2\tau\})^2 \\
CS &= \int_0^{\overline{x}} v + q_A^* - p_A^* - hx^2 \mathrm{d}x + \int_{\overline{x}}^1 v + q_B^* - p_B^* - h(1-x)^2 \mathrm{d}x \\
&= v + \frac{1}{2}(1-\tau) + \frac{1}{2}\max\{0, 1-2\tau\} - \frac{13}{12}h \\
W &= PS + CS \\
&= v + \frac{1}{2}(1-\tau) + \frac{1}{2}\max\{0, 1-2\tau\} - \frac{1}{12}h - c(1-\tau)^2 \\
&\quad - c(\max\{0, 1-2\tau\})^2
\end{aligned}
$$

$1-\sqrt{h/(2c)} \leq 0 \Leftrightarrow h \geq 2c$

Assume $1-\sqrt{h/(2c)} \leq 0 \Leftrightarrow h \geq 2c$. As

$$
\frac{\partial PS}{\partial \tau} > 0 \ \forall \tau \in [0,1) \text{ and } \frac{\partial PS}{\partial \tau} = 0 \text{ for } \tau = 1,
$$
$$
\frac{\partial CS}{\partial \tau} < 0, \ \frac{\partial W}{\partial \tau} < 0
$$

producer surplus reaches its highest value at $\tau^* = 1$ and consumer surplus and welfare reach their highest value at $\tau^* = 0$.

$1-\sqrt{h/(2c)} > 0 \Leftrightarrow h < 2c$

Assume $1-\sqrt{h/(2c)} > 0 \Leftrightarrow h < 2c$.

If $\tau < 1-\sqrt{h/(2c)}$,

$$
\frac{\partial PS}{\partial \tau} = 0, \ \frac{\partial CS}{\partial \tau} = 0, \ \frac{\partial W}{\partial \tau} = 0
$$

If $\tau \geq 1-\sqrt{h/(2c)}$,

$$
\frac{\partial PS}{\partial \tau} > 0 \ \forall \tau \in [0,1) \text{ and } \frac{\partial PS}{\partial \tau} = 0 \text{ for } \tau = 1,
$$
$$
\frac{\partial CS}{\partial \tau} < 0, \ \frac{\partial W}{\partial \tau} < 0
$$

Producer surplus:

$$
PS\left(\tau \in \left[0, 1-\sqrt{\frac{h}{2c}}\right)\right) > PS(\tau = 1) \Leftrightarrow 1 - h - c > h
$$
$$
\Leftrightarrow h < \frac{1-c}{2}
$$

As $(1-c)/2 > 2c$, $PS\left(\tau \in [0, 1-\sqrt{h/(2c)}\,)\right) > PS(\tau = 1)$ is always fulfilled. Therefore, producer surplus reaches its highest value for any $\tau^* \in [0, 1-\sqrt{h/(2c)}\,)$.

Consumer surplus: First, assume $h > c/2$. Then,

$$
\begin{aligned}
& CS\left(\tau = 1-\sqrt{\frac{h}{2c}}\right) > CS\left(\tau \in \left[0, 1-\sqrt{\frac{h}{2c}}\right)\right) \\
& \Leftrightarrow v + \frac{3}{2}\sqrt{\frac{h}{2c}} - \frac{1}{2} - \frac{13}{12}h > v + \frac{2}{3}h \\
& \Leftrightarrow \frac{9 - 14c - \sqrt{81-252c}}{49c} < h < \frac{9 - 14c + \sqrt{81-252c}}{49c}
\end{aligned}
$$

As $\left(9-14c-\sqrt{81-252c}\right)/(49c) < c/2$ and $\left(9-14c+\sqrt{81-252c}\right)/(49c) > 1$, consumer surplus reaches its highest value for $\tau^* = 1-\sqrt{h/(2c)}$.

Second, assume $h \leq c/2$. Then,

$$
\begin{aligned}
& CS\left(\tau = 1-\sqrt{\frac{h}{2c}}\right) > CS\left(\tau \in \left[0, 1-\sqrt{\frac{h}{2c}}\right)\right) \\
& \Leftrightarrow v + \frac{1}{2}\sqrt{\frac{h}{2c}} - \frac{13}{12}h > v + \frac{2}{3}h \\
& \Leftrightarrow h < \frac{2}{49c}
\end{aligned}
$$

As $2/(49c) > 2c$, consumer surplus reaches its highest value for $\tau^* = 1-\sqrt{h/(2c)}$.

Welfare: First, assume $h > c/2$. Then,

$$
\begin{aligned}
& W\left(\tau \in \left[0, 1-\sqrt{\frac{h}{2c}}\right)\right) > W\left(\tau = 1-\sqrt{\frac{h}{2c}}\right) \\
& \Leftrightarrow v + 1 - \frac{1}{3}h - c > v + \frac{3}{2}\sqrt{\frac{h}{2c}} - \frac{1}{2} - \frac{1}{12}h \\
& \quad - \frac{h}{2} - c\left(2\sqrt{\frac{h}{2c}} - 1\right)^2 \\
& \Leftrightarrow h < \frac{64c^2 - 6c + 9 - \sqrt{4096c^4 - 768c^3 - 1728c^2 - 108c + 81}}{81c} \vee \\
& \quad h > \frac{64c^2 - 6c + 9 + \sqrt{4096c^4 - 768c^3 - 1728c^2 - 108c + 81}}{81c}
\end{aligned}
$$

As $(64c^2 - 6c + 9 - \sqrt{4096c^4 - 768c^3 - 1728c^2 - 108c + 81})/(81c) > 2c$, $W\left(\tau \in [0, 1-\sqrt{h/(2c)}\,)\right) > W\left(\tau = 1-\sqrt{h/(2c)}\right)$ is always fulfilled. Therefore, welfare reaches its highest value for any $\tau^* \in [0, 1-\sqrt{h/(2c)}\,)$.

Second, assume $h \leq c/2$. Then,

$$
\begin{aligned}
& W\left(\tau \in \left[0, 1-\sqrt{\frac{h}{2c}}\right)\right) > W\left(\tau = 1-\sqrt{\frac{h}{2c}}\right) \\
& \Leftrightarrow v + 1 - \frac{1}{3}h - c > v + \frac{1}{2}\sqrt{\frac{h}{2c}} - \frac{1}{12}h - \frac{h}{2} \\
& \Leftrightarrow h < \frac{4c^2 - 4c + 1 - \sqrt{8c^2 - 8c + 1}}{c} \vee \\
& \quad h > \frac{4c^2 - 4c + 1 + \sqrt{8c^2 - 8c + 1}}{c}
\end{aligned}
$$

As $\left(4c^2 - 4c + 1 - \sqrt{8c^2 - 8c + 1}\right)/c > 2c$, $W\left(\tau \in [0, 1-\sqrt{h/(2c)}\right) > W\left(\tau = 1-\sqrt{h/(2c)}\right)$ is always fulfilled. Therefore, welfare reaches its highest value for any $\tau^* \in [0, 1-\sqrt{h/(2c)})$.