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Can misinformation be tolerated? Analyzing the influence of disinformation on financial market fluctuations

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ABSTRACT

In this study, we explore the dynamics of price fluctuations, caused by predatory trading, in the context of misinformation. Key findings include: a diminishing effect of misinformation on price fluctuations over time, undermining long-term market manipulation; a baseline fluctuation size anchored in factual market data, regardless of misinformation; and the inability of misinformation to reduce fluctuations, only to inflate them. A novel contribution of this research is quantifying a threshold of misinformation that markets can tolerate without any additional fluctuations to be expected, with a higher tolerance observed in more liquid stocks and markets with lower information asymmetry. Interestingly, widespread misinformation can sometimes lead to less severe fluctuations, suggesting a complex market response to distorted information. Additionally, we examine the neutralizing effect of contradictory misinformation, presenting challenges in using market data to identify misinformation. The study further highlights the dynamic nature of markets' tolerance to misinformation, influenced by ongoing information updates, offering strategic implications for market interventions and stability.

1. Introduction

In the dynamic arena of financial markets, the understanding of market price fluctuations represents a pivotal area of interest, merging academic inquiry with practical significance. This theoretical paper delves into the intricate landscape of stock market fluctuations through the lens of a predatory trading game, focusing on the impact of misinformation on market dynamics. We adopt a microstructural approach to explore the effects of a predatory trading game on price stability. Diverging from existing literature, we permit the information received by each participant to be tainted, thereby centering our investigation on the impacts of such information distortions, with particular emphasis on the resulting price fluctuations. The distortions or misrepresentations under our scrutiny range from minor errors, such as estimations or inaccuracies in information transmission, to outright fake news. This spectrum effectively mirrors the real information landscape within which market participants navigate.

To embed misinformation into the game, we propose the approach that each player is convinced of the accuracy of their information. This assumption, initially appearing quite strong, offers the crucial advantage of enabling the application of existing theoretical foundations from games under correct information when solving such systems under misinformation. Furthermore, we soften the constraints of this assumption by allowing information updates throughout the course of the game.

A pivotal revelation of this research is the quantification of a threshold of misinformation that a system can withstand without exacerbating fluctuation severity. This tolerance level is notably higher for more liquid stocks and markets with lower information asymmetry. The spread of misinformation is found to generally increase market volatility; however, in certain circumstances, it can lead to less severe fluctuations, indicating a complex market response to information distortions.

The research findings presented herein can be used to advance our comprehension of stock market fluctuations. Firstly, the study reveals that the impact of misinformation on the volatility diminishes over time, illustrating the ineffectiveness of long-term market manipulation through misinformation. Secondly, it identifies a baseline size for expected fluctuations, dictated by underlying factual market data, irrespective of misinformation. Thirdly, it is demonstrated that misinformation can inflate the size of expected fluctuations but never diminish them.

Moreover, the research explores the ramifications of misinformation on traders' profit expectations, uncovering both advantageous and detrimental impacts. It also discusses how contradictory misinformation can neutralize each other, highlighting the challenges in using market data to deduce the presence of misinformation.

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Finally, the study examines the effects of information updates during the game, shedding light on how the system's tolerance threshold dynamically shifts over time, providing critical insights for the design and timing of effective market interventions and information updates. This aspect is particularly relevant for managing market stability and preventing exacerbated fluctuation scenarios.

Our work is related to several strands in the literature. First, our approach extends the existing research on predatory trading, which has predominantly been centered around trading based on veracious information and its ensuing impacts. In their foundational work, Brunnermeier and Pedersen (2005) introduce a deterministic model to analyze the spillover effects inherent in such trading scenarios. Building upon this, Carlin et al. (2007) propose a stochastic framework, thereby broadening the scope of investigation to include the dynamics of cooperative trading. Furthering this line of inquiry, Carmona and Yang (2011) endeavor to generalize the stochastic model, aiming to delineate a closed-loop equilibrium by employing a game-theoretic adaptation of the Hamilton–Jacobi-Bellman approach.[1] Our research contribution enhances this domain by incorporating the element of games predicated on falsified information, whereas the cited references focus on a setting with complete information.

Second, our research touches on the broad area of herding in financial markets. Welch (2022) focuses on the case of Robinhood traders, dealing with the phenomenon of many tiny investors actively participating on the capital market in large scale. More general, Blasco et al. (2012) demonstrate in their study that there is a relationship between herding behavior and market volatility.[2] Our work proposes a theoretical model for such a relationship in the context of predatory trading, showing that the threshold of misinformation a system can tolerate without expecting additional fluctuations decreases as the size of the herd pursuing the victim increases. Thus, our theoretical findings are in line with empirical studies.

Third, we contribute to the field of information-driven asset price dynamics by highlighting how misinformation shapes the formation and persistence of market fluctuations. Recent research emphasizes that volatility is not solely caused by fundamental shocks but also by the way information is interpreted and processed. Veldkamp (2006) shows that information frictions and asymmetries can amplify price variability, while (Andrei & Hasler, 2015) provides empirical evidence that heterogeneous beliefs and selective attention to news are key drivers of excess volatility. We contribute to this line of literature by demonstrating that the magnitude of price fluctuations depends both on misinformation, which can be interpreted as information asymmetry, and on the underlying true information. This interplay offers a series of new insights. For instance, correct information sets a kind of minimum level for volatility, which can only be exacerbated by misinformation. Furthermore, we derive a series of general results regarding the impacts of information updates during the course of the game. This level of abstraction also allows for an alignment with feedback trading, especially when players adjust their updates based on observed market prices. The impact of feedback trading has been investigated in Barber et al. (2009), Bhattacharya et al. (2009) or (Shiller, 2002). We complement this perspective by highlighting that our model does not inherently explain fluctuations through feedback trading, but rather demonstrates how feedback can influence information asymmetry (created by misinformation) and, in turn, affect the market volatility.

The paper is organized as follows. Section 2 describes the model set-up. Section 3 presents the effects of falsified information on price fluctuations and discusses how much misinformation a system can tolerate. In Section 4 we allow for players to receive new information during the game. This notably includes the possibility for players to become aware of their mistakes and correct them. Section 5 concludes. In order not to impair the flow of reading, all proofs can be found in the appendix.

2. Model

We investigate potential impacts of distorted or falsified information on predatory trading. For this purpose, we adopt the general microstructural approach[3] as previously utilized by Carlin et al. (2007) and Carmona and Yang (2011).[4] Thus, for a detailed discussion of this approach as well as a comprehensive description of all parameters, please refer to the cited literature.

2.1. Base model

A market is examined where both a risk-free asset and a risky asset are traded, with the risk-free interest rate being set to zero without loss of generality.[5] We consider a market participant, referred to as the victim, who is compelled to adjust their positions in the risky asset within the time period $[0, T]$, and at least one other player who becomes aware of the victim's predicament and attempts to exploit this for his own benefit. These latter players are termed predators. We enumerate the players as $1, \dots, N$, with $N \in \mathbb{N}$, assuming that the first player is the victim without loss of generality. Denoting the total quantity of the risky asset held by player n at time t by X_t^n it follows that

$$X^n(t) = X^n(0) + \int_0^t \alpha^n(s)\mathrm{d}s \tag{1}$$

holds, where α_t^n denotes the trading rate of the nth player. The processes α_t cannot be arbitrarily chosen by the players but are subject to certain restrictions. Initially, it must be ensured that the trading strategies are always selected such that the necessary portfolio adjustments are achieved. This is traditionally accomplished through the requirement

$$\alpha_t^n = (\alpha_t^n)_{0\le t\le T} \in \mathbb{A}^n = \left\{ \alpha_t^n \middle| \mathcal{H}^2_{[0,T]} \text{ and } X_T^n = 0 \right\} \tag{2}$$

with $X_0^n = -x_0^n$, where x_0^n denotes the trading constraint of the nth player. The strategies α_t^n must be chosen from the set

$$\mathcal{H}^2_{[0,T]} = \left\{ \alpha_t^n \middle| \text{ adapted and } \mathbb{E}\int_0^T |\alpha_t^n|\,\mathrm{d}t < \infty \right\}, \tag{3}$$

where this condition also guarantees the existence of a solution. Furthermore, the required structure of the process α_t captures the information sources that a player can utilize to establish his trading strategy. Since it seems unrealistic to assume that each player knows the exact portfolio positions of all players at any given time, we restrict our self to open loop strategies, this is $\alpha_t^n = \phi\left(t, X_0\right)$[6] with some deterministic function ϕ. This means that players set their strategy at the beginning and either follow it until the end of the game (Section 3) or adjust it at specific points when they receive new information (Section 4).

The transaction price P_t, that is, the price at which the transaction takes place, may differ from the mid-price X_t. The discrepancy $P_t - X_t^0$ is known as the temporary price impact. This factor reflects how rapidly market participants eat up the bid–ask spread in the limit-order book

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[1] Further studies that deal with predatory trading include, for example, Brunnermeier and Oehmke (2014), which specifically address the case of short selling, or (van Kervel & Menkveld, 2019), which empirically test theoretical model results.

[2] In the different context of social media groups, also (Umar et al., 2021) finds that investing groups can create inefficiency in the market.

[3] The microstructure approach studies how trading mechanisms affect the price formation process and is one of the main sources of illiquidity investigated in the literature, see e.g. Eesley et al. (1996), O'Hara (1995).

[4] Others employing this framework with variations in design include Brunnermeier and Pedersen (2005), Schoeneborn and Schied (2009), and Schied and Schoeneborn (2008), among others.

[5] See Carmona and Yang (2011), Carlin et al. (2007) and Brunnermeier and Pedersen (2005).

[6] X_0 denotes the initial state of the game, this is $X_0 = (X_0^0, \dots, X_0^n)$.

at given trading volumes. Consequently, the temporary price impact is a function dependent on the rate of trade, expressed as[7]

$$P_t - X_t^0 = \lambda \sum_{i=1}^{N} \alpha_t^i. \tag{4}$$

Here, the parameter λ denotes the elasticity factor, which characterizes the depth of the limit-order book.[8] Substantiated by a multitude of empirical investigations,[9] there is also a permanent price impact, which represents the enduring effect of trading rates on the asset's mid-price, given by

$$\mathrm{d}X_t^0 = \gamma \sum_{i=1}^{N} \alpha_t^i \mathrm{d}t + \sigma \mathrm{d}W_t. \tag{5}$$

Note that due to the brevity of time spans pertinent to predatory trading, a conventional drift term $\mu(t)$ can be neglected in (5).[10] The coefficient γ stands for the plasticity factor of the market and is intrinsically linked to the asymmetry of information prevalent in the market concerning the risky asset. The martingale diffusion process with volatility parameter σ in the price dynamics (5) results form the presence of noise traders. These uninformed participants, playing the role of a clearing market, submit limit or market orders and thus influence the price movement over time while the average effect on the temporary price slippage is zero.[11]

The objective of all players participating in the game is to maximize their profit

$$-\int_0^T P_t \,\mathrm{d}X_t^n. \tag{6}$$

However, since this profit depends not only on their own trading strategy but also on the transaction price and thus indirectly on the chosen trading strategies of the other players, the described situation can be formulated as a stochastic differential game. The dynamics of the $(N+1)$-dimensional system are captured by the following:

$$\begin{cases} \mathrm{d}X_t^0 = \gamma \sum_{i=1}^{N} \alpha_t^i \mathrm{d}t + \sigma \mathrm{d}W_t, \\ \mathrm{d}X_t^n = \alpha_t^n \mathrm{d}t \quad \forall n \in 1, \dots, N. \end{cases} \tag{7}$$

The associated revenue functional for the respective players are derived by substituting (1) and (4) into (6) as

$$J^n(\boldsymbol{\alpha}) = \mathbb{E}\left(\int_0^T \alpha^n \left(X_t^0 + \lambda \sum_{i=1}^{N} \alpha_t^i\right) \mathrm{d}t\right). \tag{8}$$

2.2. Disinformation

Information Distortion: After the general system has been established, the influence of distorted information needs to be specified. To this end, let x_0^1 represent the actual quantity that the victim is compelled to buy or sell. While this quantity is naturally known to the victim, all other players receive a random distortion of this information $\tilde{x}_{0,n}^1 = x_0^1 + \epsilon_n$. The hitherto unspecified random variables ϵ_n are realized before the start of the game and reflect the individual distortion of the original information. Thus, ϵ_n might represent whether information is received from official announcements such as disclosed financial information or trading reports or other sources like public news might and social media.

Given that embedding misinformation into game-theoretic models is an exceedingly challenging task even with basic models,[12] we propose the following approach[13]: Each player m selects their trading strategy α_t^n in such a way that $\boldsymbol{\alpha}_{(m)} = (\boldsymbol{\alpha}_{(m)}^{\mathbf{1}}, \dots, \boldsymbol{\alpha}_{(m)}^{\boldsymbol{N}})$ with

$$J^n(\boldsymbol{\alpha}_{(m)}) \leq J^n(\boldsymbol{\alpha}_{(m)}^{-\boldsymbol{n}}, \boldsymbol{\alpha}^{\boldsymbol{n}}), \quad \forall n \in \{1, \dots, N\}, \quad \forall \boldsymbol{\alpha}^{\boldsymbol{n}} \in \mathbb{A}^n \tag{9}$$

holds true under the assumption

$$\tilde{x}_{0,i}^1 = \tilde{x}_{0,m}^1 \qquad \forall\, i \in \{1, \dots, N\}. \tag{10}$$

This method presupposes the initial assumption that each player is convinced of the veracity of their information. This is, information distributions cause information asymmetry in the market, leading to deviations from equilibrium. Thus the question appears: What are the consequences of theses shifts and can they be tolerated? The approach further offers the crucial advantage of transferring the unique solvability of the problem without misinformation directly to the current case. Additionally, in Section 4, we discuss potential approaches and their impacts, aimed at weakening the assumption concerning the unconditional belief in possibly distorted information by allowing updates.

Misinformation vs. Uncertainty: It is crucial to distinguish between a game under uncertainty and a game under misinformation, as introduced here. Under uncertainty, players receive a (randomly) distorted signal, of which they are aware. This means that a player not only knows that the information is distorted but also understands the nature of the distortion. Thus, the available observations can be used to estimate the opponent's actions throughout the game. Subsequently, a relationship must be established between the played strategies and the original (unbiased) information, which typically relies on specific assumptions about player behavior. Consequently, uncertainty problems are closely linked to (partial) updating problems. If solvable (and uniquely so), such an approach typically require numerical solution methods, which can lead to difficulties in larger player settings and further analyses.[14]

In the case of misinformation, the player is unaware that the information has been distorted and therefore perceives it as correct. Consequently, this approach does not involve a continuous updating problem, as there is no need to filter out the correct information when one believes to already possess it. Such an approach can be justified both by the typically short duration of a predatory trading game and by psychological findings on the belief perseverance bias. The latter one describes 'the tendency to cling to one's initial belief even after receiving new information that contradicts or disconfirms the basis of that belief', see Anderson (2007), and has been examined in numerous studies, such as Anderson et al. (1980).[15] Thus, it is reasonable that players adhere to their information (at least for a while), even if observable variables exist that could challenge it. The updating problem in this context, therefore, refers – unlike in the case of uncertainty –

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[7] The linear structure in both, the permanent and the temporary price impact is in line with the literature. For more detailed discussions on this topic, see Carmona and Yang (2011).

[8] For a detailed explanation of the parameters λ (and the soon-to-be introduced γ), see Carlin et al. (2007), Carmona and Yang (2011), and Schoeneborn and Schied (2009).

[9] Refer to, for instance, Kraus and Stoll (1972), Holthausen et al. (1990), Chan and Lakonishok (1995), Madhavan and Cheng (1997), Sadka (2006), and Meng et al. (2020).

[10] This attempt is discussed by Huberman and Stanzl (2004) and in line with preceding literature as Carlin et al. (2007), Carmona and Yang (2011), and Schoeneborn and Schied (2009).

[11] The resulting influence of trading rates on asset prices can be found in a similar way in both empirical and theoretical studies like, Almgren and Chriss (1999), Kaul et al. (2000), Almgren and Chriss (2001) and Huberman and Stanzl (2004).

[12] See e.g. Fudenberg and Levine (1993) for the case of self-confirming equilibria in a sequential two-step game.

[13] In the present case, it should be noted that the misinformation affects the set of permissible control strategies $\mathbb{A}$ rather than the dynamics of the game, as is more common in optimal control problems.

[14] Numerical simulations of a one-sided (partial) updating problem for the two-player case are presented in Teguia (2015).

[15] See also Anderson (1983) for the connection with causal reasoning and Siebert and Siebert (2023) for strategies to mitigate the belief perseverance bias.

to the modification of beliefs. For instance, in Section 4, it is examined when such a misconception must be eliminated at the latest in order to avoid additional price fluctuations.

3. Influence of falsified information

3.1. Two competing information

We begin our study by examining a situation where the market features only two different types of information. Specifically, there is a group of N_r correctly informed players, each identified by $\epsilon_i = 0 \; \forall \; i \in \{1, \dots, N_r\}$. In sharp contrast, there exists a group of N_w participants. Each of these participants operates under the same falsified information, represented by $\tilde{x}^1_{0,i} = \tilde{x} \neq x^1_0 \; \forall \; i \in \{N_r + 1, \dots, N\}$. This seemingly simple setup actually provides an important context to understand the significant effects of misinformation. Our main focus is to analyze how this misinformation impacts price movements, which we will explore in the following theorem:

Theorem 3.1. *Without loss of generality $\tilde{x}^1_0$ is the falsely assumed buy or sell target. Consider the stochastic differential game , where N_w misinformed and N_r correctly informed players try to maximize their personal profit (8) under the constraint (2). Then the price process is given by*

$$X^0_t = X^0(0) - \frac{1 - e^{-\frac{N-1}{N+1}\frac{t\gamma}{\lambda}}}{1 - e^{-\frac{N-1}{N+1}\frac{T\gamma}{\lambda}}}\gamma\left(\sum_{i=1}^{N} x^i_0 + \tilde{\nu}\right) + \frac{e^{\frac{\gamma t}{\lambda}} - 1}{e^{\frac{\gamma T}{\lambda}} - 1}\gamma\tilde{\nu} + \sigma\left(W_t - W_0\right), \tag{11}$$

where we defined the error factor $\tilde{\nu} \in \mathbb{R}$ as

$$\tilde{\nu} := \frac{N_w}{N}\left(\tilde{x}^1_0 - x^1_0\right). \tag{12}$$

The formula presented above requires a detailed explanation. Initially, particularly for small values of t, it is the second term on the right-hand side of Eq. (11) that mainly influences the price fluctuations. As the game progresses over time, the third term on the right-hand side of (11) becomes more significant. Due to the opposite signs of these terms, a contrasting price movement emerges, different from the initial trend. Importantly, the impact of this latter term is directly proportional to the extent of misinformation and becomes negligible when $\tilde{\nu} = 0$. This implies that in the absence of misinformation, there is no opposing price movement. In such cases, Eq. (11) is consistent with price trends commonly recognized in the existing literature, as discussed in Carlin et al. (2007). This highlights that the introduction of misinformation leads to the loss of the characteristic monotonicity in expected price paths, a property that arises in Carlin et al. (2007) under full information transparency. This constitutes a fundamental difference, the implications of which will be explored in greater detail in the subsequent analysis. Additionally, in the situation where $x^1_0 = 0$ and all predators strictly follow a hands-clean scheme, the price development described by (11) represents a market influenced only by false information or fake news. This specific case is analyzed in Herzing and Muck (2024).

Error factor: Theorem 3.1 highlights a strong connection between the price trajectory and the associated error factor. This relationship is clearly depicted in Panel 1(a).[16] Insights from this panel emphasize the critical role of the error factor in influencing key characteristics like the monotonicity and direction of the expected price trend. Due to its significance, this factor deserves detailed examination.

Two main elements significantly affect the error factor. The first, represented as $\frac{N_w}{N} \in [0, 1)$, reflects the ratio of misinformed participants to the total number of players. Therefore, it acts as an indicator of the extent of misinformation relative to the overall spread of information.[17] An increase in misinformation leads to a corresponding rise in the error factor. The second element, denoted by $\left(\tilde{x}^1_0 - x^1_0\right) \in \mathbb{R}$, measures the difference between the incorrect and correct information. It essentially indicates the extent of deviation of the circulating false data from the truth, or the level of distortion affecting the original information. A significant distortion of the primary information substantially impacts the error factor.

The error factor, shaped by the combination of the misinformation's magnitude, $\tilde{x}^1_0 - x^1_0$, and its spread, $N_w N^{-1}$, is represented by the isoquants $\mathcal{L}_{\tilde{\nu}=K}$ at a particular level $K \in \mathbb{R}$ in the two-dimensional space

$$\begin{aligned}\mathcal{L}_{\tilde{\nu}=K} &:= \left\{ \begin{pmatrix} N_w N^{-1} \\ \tilde{x}^1_0 - x^1_0 \end{pmatrix} \middle| \tilde{x}^1_0 - x^1_0 = \frac{K}{N_w N^{-1}} \right\} \\ &= \begin{pmatrix} y \\ \frac{K}{y} \end{pmatrix} \in \mathbb{R}^2, \; y \in (0, 1) \quad \forall K \in \mathbb{R}. \end{aligned} \tag{13}$$

Thus, a high error factor is linked to significant misinformation, especially when its spread is limited. Conversely, in cases with widespread misinformation, even small deviations from the original information can lead to major inaccuracies. This interaction is vividly illustrated in Panel 1(b).

Initial conclusions: We now embark on a detailed analysis of how the error factor affects the predicted price trajectory and the potential for additional fluctuation. To assist in this analysis, we introduce the following notation:

Definition 3.1. For the game with error factor ν we define the maximum expected price fluctuation during the time period $[t_*, t^*]$ as

$$MPF_\nu(t_*, t^*) := \max_{t_1, t_2 \in [t_*, t^*]} \left| \mathbb{E}\left(X^0_{t_1} - X^0_{t_2}\right) \right|. \tag{14}$$

The metric MPF measures the largest expected fluctuation in the equilibrium (mid) price during a certain period of time. We will use the MPF as a measure of the intensity of additional volatility caused by the game. This seems reasonable, as the changes in the equilibrium price are solely attributed to the (supposed) trading constraints of a victim and the intentions of other market participants to exploit this for their own benefit. Note that an increase in this metric is associated with larger expected fluctuation sizes, while a decrease is associated with smaller expected fluctuation sizes.

Suppose the deterministic part of Eq. (11) is denoted by $f(t)$. By using the martingale property of Brownian motion, we can deduce that the conditional expectation of the price evolution is given by:

$$\mathbb{E}\left(X^0_t | \mathcal{F}_s\right) = X^0_0 - f(t) + \sigma W_s \sim \mathcal{N}\left(X^0_0 - f(t), \sigma^2 s\right), \quad \forall t \in [s, T]. \tag{15}$$

This indicates that the expected price at time t, based on the information available at time s (represented by the filtration $\mathcal{F}_s$), is primarily determined by the function $f(t)$. To elaborate further:

$$\mathbb{E}\left(\mathbb{E}\left(X^0_t | \mathcal{F}_s\right)\right) = X^0_0 - f(t), \quad \forall t \in [s, T]. \tag{16}$$

Since $f(t)$ is independent of the error term $\tilde{\nu}$ at the end of the game T, this leads us directly to the following lemma:

Lemma 3.1. *Assume the situation as in Theorem 3.1, then:*

$$\mathbb{E}\left(\mathbb{E}\left(X^0_T | \mathcal{F}_s\right)\right) = X_0(0) - \gamma \sum_{i=1}^{N} x^i_0. \tag{17}$$

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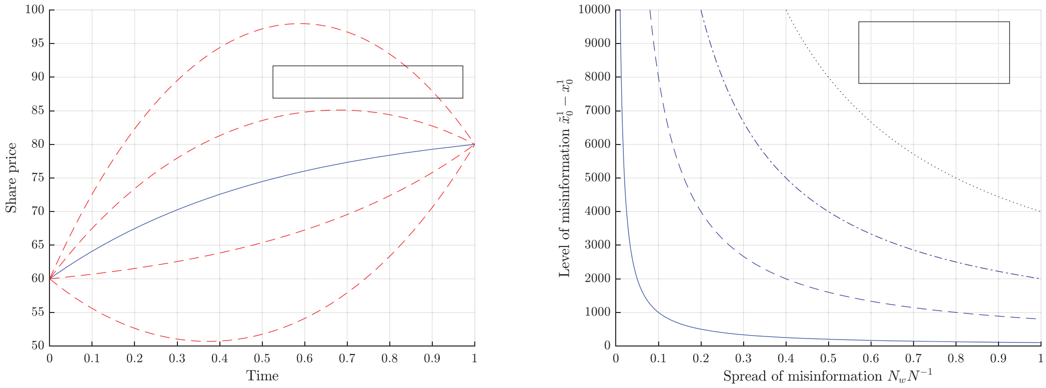
[16] These and the following figures serve to visualize the corresponding explicit solutions. The parameters used were therefore chosen for illustrative purposes, with a focus on so-called plastic markets (see, for instance, Carmona and Yang (2011) and Carlin et al. (2007)). The parameter values fall within similar ranges used in related literature, such as Carmona (2016).

[17] Herzing and Muck (2024) argue that the number of market participants is a reasonable measure for information distribution. They also show that distinguishing between 'major' and 'minor' players is unnecessary in this context.



(a) Expected price dynamics for different error factors.

(b) Level curves for different error factors.

Fig. 1. Fig. 1(a) time the expected price trend for a game with the parameters $N = 50,\ \gamma = 0.1,\ \lambda = 0.5\gamma,\ T = 1,\ \sigma = 3.9,\ \sum x_0 = -200$. The middle solid line shows the case without misinformation. The dashed lines below and above show the case with rising positive and rising negative error factors, respectively. Fig. 1(b) shows the level curves of the error factor in dependence of the spread of the misinformation and the level of the misinformation.

Lemma 3.1 suggests that the expected final price is unaffected by the error factor. Therefore, although the error factor may impact the expected fluctuations during the game, misinformation cannot sustain an ongoing impact on price fluctuations over an extended period. Regarding the expected maximum price deviation, it is concluded that for a game with arbitrary given error factor $\tilde{v}^*$

$$MPF_{\tilde{v}^*}(0,T) \geq \min_{\tilde{v}\in\mathbb{R}} MPF_{\tilde{v}}(0,T) = \gamma \sum_{i=1}^{N} x_0^i \tag{18}$$

holds. This equation indicates that the fluctuation size should be equal to or greater than the value given in (18), regardless of how widespread the misinformation is. This implies a lower limit on the expected size of the fluctuation.

Compatibility sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$: To understand the effects of misinformation on the price fluctuation more clearly, we propose the following:

Definition 3.2. Assume the situation as in Theorem 3.1 with all parameters except the error factor $\tilde{v}$ fixed. We define the two sets

$$\mathcal{V} := \left\{ \tilde{v} \in \mathbb{R} \middle| MPF_{\tilde{v}}(0,T) = \min_{\tilde{v}\in\mathbb{R}} MPF_{\tilde{v}}(0,T) \right\}; \quad \tilde{\mathcal{V}} := \left\{ \tilde{v} \in \mathbb{R} \middle| \tilde{v} \notin \mathcal{V} \right\}. \tag{19}$$

Within the set $\mathcal{V}$, we collect error factors that lead to the smallest expected fluctuation size. In contrast, $\tilde{\mathcal{V}}$ includes those error factors that are associated with larger fluctuations. Due to the consistent increase of $|f(t)|$ when there is no misinformation, it follows from (16) that 0 is included in $\mathcal{V}$. Therefore, the smallest fluctuation is expected in a scenario where there is no misinformation. The following lemma outlines a significant implication of this finding:

Lemma 3.2. *Assume the situation as in Theorem 3.1 with all parameters except $\tilde{v}$ fixed, then:*

$$MPF_0(0,T) \leq MPF_{\tilde{v}}(0,T) \quad \forall \tilde{v} \in \mathbb{R}. \tag{20}$$

Lemma 3.2 emphasizes that incorrect information can only worsen expected price fluctuations by adding additional volatility and never improve them. However, this does not mean that every type of misinformation will always lead to a larger expected fluctuation. Therefore, a critical question is identifying the thresholds of misinformation that the system can withstand before causing harmful price fluctuations. The following theorem provides an answer to this:

Theorem 3.2. *Assume the situation as in Theorem 3.1. Then $\tilde{v} \in \mathcal{V}$, iff*

$$\begin{cases} b_2 \leq \tilde{v} \leq b_1 & \textit{if} \quad \sum_{i=1}^{N} x_0^i \geq 0 \\ b_1 \leq \tilde{v} \leq b_2 & \textit{if} \quad \sum_{i=1}^{N} x_0^i < 0 \end{cases} \tag{21}$$

and $\tilde{v} \in \tilde{\mathcal{V}}$ otherwise. Hereby, the constants $b_1, b_2 \in \mathbb{R}$ are given by

$$b_1 = \frac{\sum_{i=1}^{N} x_0^i}{d_1 - 1}; \quad b_2 = \frac{\sum_{i=1}^{N} x_0^i}{d_2 - 1}; \quad d_1 = \frac{N+1}{N-1} \frac{e^{\frac{2N}{N+1}\frac{T\gamma}{\lambda}} - e^{\frac{T\gamma}{\lambda}}}{e^{\frac{T\gamma}{\lambda}} - 1};$$

$$d_2 = \frac{N+1}{N-1} \frac{1 - e^{\frac{1-N}{1+N}\frac{T\gamma}{\lambda}}}{e^{\frac{T\gamma}{\lambda}} - 1}.$$

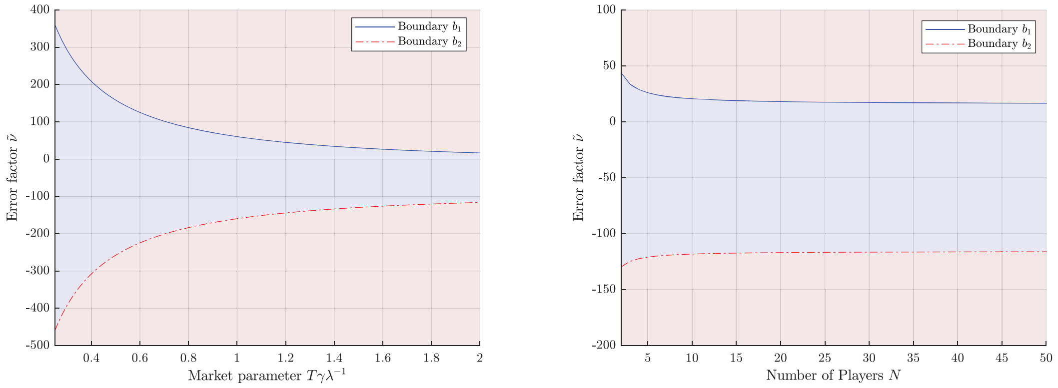
The mentioned theorem establishes explicit boundaries, labeled b_1 and b_2. If the error factor falls within this specified range, it is included in the set $\mathcal{V}$. This suggests that, even with misinformation present, it may not negatively impact the expected fluctuation. However, exceeding these limits in either direction implies reaching a level of misinformation at which more significant price distortions are likely. Therefore, the boundary values b_1 and b_2 become key factors in understanding the relationship between misinformation and price fluctuations.

Boundary parameter b_1 and b_2: The boundary values, b_1 and b_2, depend on various system parameters. The subsequent discussion briefly elaborates on the impact of these parameters. We begin by discussing the market parameter, symbolized by $\lambda\gamma^{-1}$, which combines elasticity and plasticity factors. If $\sum x_0 > 0$, then the boundary d_1 shows a steady increase in relation to the market parameter. On the other hand, if $\sum x_0 < 0$, there is a consistent decrease. Conversely, the effect on the boundary d_2 is reversed: it increases if $\sum x_0 > 0$ and decreases if $\sum x_0 < 0$. According to Theorem 3.2, the set $\mathcal{V}$ becomes smaller as the market parameter grows. To phrase it differently, an amplified market parameter precipitates more pronounced fluctuations for a specified error factor.

The game's duration, represented by T, exhibits similar monotonic behavior as the market parameter. Thus, as the game lasts longer, a specific error factor, $\tilde{v}$, is more likely to fall outside of $\mathcal{V}$, suggesting an increased chance of larger fluctuations. For extended durations where $T \to \infty$, we have:

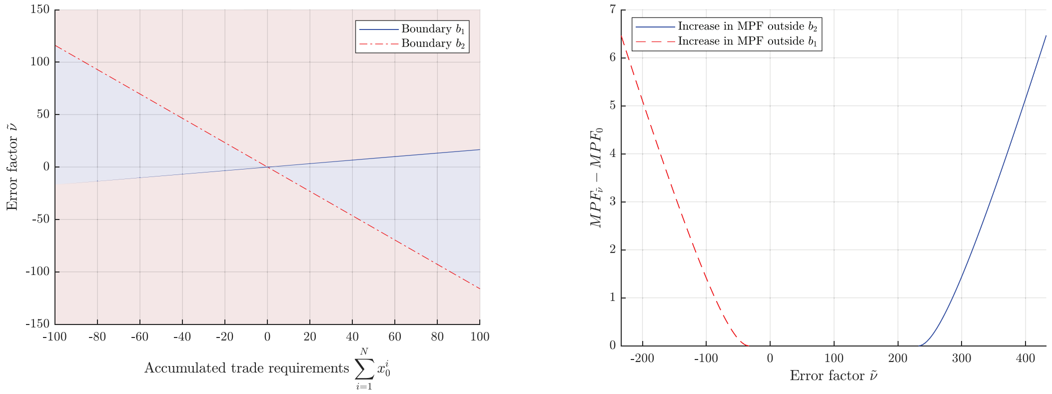
$$\mathcal{V} = \left\{ \tilde{v} \in \mathbb{R} \middle| \tilde{v} \in \begin{cases} \left[-\sum x_0, 0\right] & \text{if} \quad \sum x_0 \geq 0 \\ \left[0, -\sum x_0\right] & \text{if} \quad \sum x_0 < 0 \end{cases} \right\} \tag{22}$$

This shows that, regardless of game duration, there is a permissible error threshold within the system. During longer games, errors are tolerated only in one direction. For example, if a participant needs to liquidate positions, overestimating this amount leads to greater fluctuations, whereas underestimations are acceptable to a certain extent. For



(a) The sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$ in dependence of the market parameters.

(b) The sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$ in dependence of the number of players.



(c) The sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$ in dependence of the accumulated trade requirements.

(d) Increase of the MPF outside the set $\tilde{\mathcal{V}}$ in dependence of $\tilde{\nu}$.

Fig. 2. The figure shows the sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$ in dependence of the market parameter 2(a) the number of players 2(b) and the accumulated trade requirements 2(c). The set $\mathcal{V}$ is given by the (blue) area between the solid line and the dashed line. The (red) areas above and below the solid and dashed lines represent the set $\tilde{\mathcal{V}}$. 2(d) shows the increase of the MPF outside the set $\tilde{\mathcal{V}}$ with $\sum x_0 = -200$. If not set as variable or mentioned otherwise, the corresponding parameters are $\sum x_0 = 100,\ N = 50,\ T\gamma\lambda^{-1} = 2$.

very short games where $T \to 0$, any error factor falls within $\mathcal{V}$, meaning that no increase in distortions is expected at any misinformation level if the game is short enough. Therefore, in short-duration scenarios, misinformation does not have enough time to cause harm. However, as the duration increases, this temporal window becomes available and the potential for negative impacts from misinformation grows. The relationship between T and the market parameter is shown in Panel 2(a).

The cumulative trade requirement, $\sum x_0$, sums up all trading objectives. The boundary d_1 increases linearly with the cumulative trade requirement, whereas d_2 decreases linearly. Notably, any form of misinformation leads to expected higher fluctuations when cumulative trade requirements are neutralized. This effect is more pronounced with unfounded rumors. The influence of the cumulative trade requirement is depicted in Panel 2(c).

The player count, N, reflects the overall information spread. Typically, d_1 strictly increases with N, while d_2 strictly decreases. To understand the impact on b_1 and b_2, we need to consider that the player count also affects the cumulative trade requirement. Assuming predators follow a hands-clean approach,[18] b_1 strictly decreases and b_2 strictly increases with N, given $x_0^1 > 0$. The opposite is true if $x_0^1 < 0$. Therefore, even with a constant misinformation-to-informed-player ratio, an increase in total players, N, could cause $\tilde{\nu}$ to deviate from $\mathcal{V}$. Elucidating further, from a price stability perspective, it is generally more detrimental when 500 out of 1000 players are misinformed, in comparison to 5 out of 10 players receiving inaccurate information. The impact of widespread information is illustrated in Panel 2(b).

A detailed overview of the influential parameters and their respective impacts on the boundaries b_1 and b_2 is encapsulated in Table 1.

Additional fluctuations in $\tilde{\mathcal{V}}$*:* When $\tilde{\nu}$ resides within $\mathcal{V}$, the maximum price fluctuation remains invariant. But what happens when an increasing error factor enters $\tilde{\mathcal{V}}$? To address this query, we introduce the following definition:

Definition 3.3. For fixed game parameters and error term υ the distance to die boundaries b_1 and b_2 is given by

$$D_{\tilde{\nu},d_1} := \begin{cases} \mid \tilde{\nu} - d_1 \mid & \text{if } Sign\ d_1 = Sign\ \tilde{\nu} \\ 0 & \text{else,} \end{cases}$$
$$D_{\tilde{\nu},d_2} := \begin{cases} \mid \tilde{\nu} - d_2 \mid & \text{if } Sign\ d_2 = Sign\ \tilde{\nu} \\ 0 & \text{else.} \end{cases} \tag{23}$$

Definition 3.3 measures how far the error term $\tilde{\nu}$ is from the set $\mathcal{V}$. It is important to note that as the level of misinformation increases, this distance also grows proportionally. The following lemma outlines the relationship between the distances $D_{\tilde{\nu},d_1}$, $D_{\tilde{\nu},d_2}$ and the maximum price fluctuation.

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[18] A hands-clean scheme implies predators do not need to adjust their portfolios and aim to exploit others' needs. Technically, this means: $x_0^i = x_T^i\ \forall\ i \in \{2, \ldots, N\}$.

Table 1
The table shows the influence of the single factors on the boundaries b_1 and b_2. Only the case $\sum x_0 > 0$ is shown. The case $\sum x_0 < 0$ results directly by changing each increasing to decreasing and vice versa. Furthermore, all limits which result in $\pm\infty$ must be multiplied by -1. Note further that all the statements about the monotony are strict. To describe the parameter influence of player numbers, a hands clean scheme was adopted.

Parameter	Effect on b_1			Effect on b_2		
	monotonic	limit		monotonic	limit	
		$\to 0$	$\to \infty$		$\to 0$	$\to \infty$
$\frac{T\gamma}{\lambda}$	decreasing	$+\infty$	0	increasing	$-\infty$	$-\sum x_0$
T	decreasing	$+\infty$	0	increasing	$-\infty$	$-\sum x_0$
λ	increasing	0	$+\infty$	decreasing	$-\sum x_0$	$-\infty$
γ	decreasing	$+\infty$	0	increasing	$-\infty$	$-\sum x_0$
$\sum x_0$	increasing	0	$+\infty$	decreasing	0	$-\infty$
N	decreasing	–	$\frac{x_0^1}{e^{\frac{T\gamma}{\lambda}}-1}$	increasing	–	$\frac{x_0^1}{e^{-\frac{T\gamma}{\lambda}}-1}$

Lemma 3.3. *Let all game parameters be fixed and assume $\tilde{v}_1,\ \tilde{v}_2 \in \tilde{\mathcal{V}}$. Assume further that $D_{\tilde{v}_1,d_1} < D_{\tilde{v}_2,d_1}$ and $D_{\tilde{v}_1,d_2}, D_{\tilde{v}_2,d_2} = 0$, respectively $D_{\tilde{v}_1,d_2} < D_{\tilde{v}_2,d_2}$ and $D_{\tilde{v}_1,d_1}, D_{\tilde{v}_2,d_1} = 0$ holds. Then:*

$$MPF_{\tilde{v}_1}(0,T) < MPF_{\tilde{v}_2}(0,T). \tag{24}$$

Lemma 3.3 fundamentally conveys that the maximum price fluctuation on both sides of the boundaries b_1 and b_2 strict monotonically increases with $|\tilde{v}|$. This signifies that beyond $\mathcal{V}$, an amplification in misinformation invariably corresponds to heightened anticipated fluctuations. This phenomenon is graphically represented in Panel 2(d). The accentuation of the price fluctuation becomes evident when traversing the b_2 boundary to the right and b_1 to the left.

Profit and loss analysis: In the subsequent discussion, we elucidate the degree to which misinformation shapes the profit expectations of players. The pivotal role of the error factor is underscored by the ensuing theorem:

Theorem 3.3. *Consider the situation as in Theorem 3.1 with the players following a hands clean scheme, then:*

1. *The players expected gain is of the form*

$$J^n(\boldsymbol{\alpha}) = K^1_{\mathit{right/false}} + vK^2_{\mathit{right/false}}, \tag{25}$$

where the functions $K^1_{\mathit{right/false}}$ and $K^2_{\mathit{right/false}}$ result from the proof of the theorem and are independent of the spread of information $N_w N^{-1}$.

2. *The right informed predators earn an expected gain during the game if*

$$\begin{cases} \tilde{v} < \frac{x_0^1}{N} & \text{if} \quad x_0^1 > 0, \\ \tilde{v} > \frac{x_0^1}{N} & \text{if} \quad x_0^1 < 0 \end{cases} \tag{26}$$

and an expected loss otherwise.

3. *The false informed predators earn an expected gain during the game if*

$$\begin{cases} \tilde{v} < \frac{x_0^1}{N} & \text{if} \quad \tilde{x}_0^1 > 0 \quad \text{and} \quad x_0^1 > 0, \\ \tilde{v} > \frac{x_0^1}{N} & \text{if} \quad \tilde{x}_0^1 < 0 \quad \text{and} \quad x_0^1 < 0 \end{cases} \tag{27}$$

and an expected loss otherwise.

The first part of the mentioned theorem plays a crucial role in understanding how the spread of information affects expected profits. By keeping all game parameters the same except for N_w, Eq. (25) becomes a linear function of $\tilde{v}$, influenced by the number of misinformed players. The impact of information spread is directly represented by the slope $K^2_{\text{right/false}}$, which can be either positive or negative. Specifically, a positive slope K^2_{right} means that as more players possess incorrect information, those with correct information potentially gain more. In contrast, a negative K^2_{right} indicates the reverse situation. Here, a large number of misinformed players can negatively impact those with accurate information. Utilizing the linearity and the profit constraints described in the theorem, we can infer that the beneficial effect occurs when misinformed players underestimate the correct value x_0^1. On the other hand, overestimation leads to the negative consequences mentioned earlier.[19] Both scenarios are illustrated in Panel 3.

The latter part of the theorem sets limits on the expected profits of players and conveys three key observations. Firstly, even players privy to accurate information can incur losses if an overwhelming majority of players heavily relies on severe misinformation. Secondly, misinformed players might anticipate gains, provided their misinformation does not deviate excessively from the truth. Thirdly, no circumstances exist wherein misinformed players anticipate gains to the exclusion of their accurately informed counterparts. Finally, it is important to note that the likelihood of anticipated profit or loss is independent of the game parameters T, γ, and λ. This constitutes a crucial distinction to the boundary factors b_1 and b_2 for additional expected volatility. As demonstrated in Table 1, this leads to the scenario where, for large market parameters, correctly informed players can anticipate profits, even though $\tilde{v} \in \tilde{\mathcal{V}}$ already applies, this is the prevailing misinformation has reached a level such that additional fluctuations are to be expected. Similarly, it may occur that the error factor is already so large that even correctly informed market participants must fear losses, but due to the small market parameter, the misinformation has not yet affected the fluctuations, thus $\tilde{v} \in \mathcal{V}$. The same applies to players with distorted information, showing that it indeed depends on the considered market and the duration of the game whether losses for individual players are associated with additional volatility or if the opposite is the case.

3.2. General spread of information

We consider the general situation where each player acts according to his personal, potentially falsified information. Thus, for each player n, it applies that $\tilde{x}^1_{0,n} = x_0^1 + \epsilon_n$, where ϵ_n denotes the personal information distortion of the nth player.

3.2.1. Generalized error factor

In line with Section 3.1, we assume that all ϵ_n are realized before the game begins. We show that in this more general case, the findings from the previous section remain valid if the error factor is adjusted accordingly. Specifically, the implications of this generalization for the expected price trajectory are given in the following theorem:

Theorem 3.4. *Let $\mathcal{X}_w := \left\{x_0^1, \tilde{x}^1_{0,w_1}, \dots, \tilde{x}^1_{0,w_k}\right\}$, $k \leq N-1$ denote the variously assumed target positions and let $\mathcal{N}_w := \left\{N_r, N_{w_1}, \dots, N_{w_k}\right\}$ with $\sum_{l=1}^{k} N_{w_l} + N_r = N$ be the number of players following $\mathcal{X}_w$. This is N_{w_1} players believe that $\tilde{x}^1_{0,w_1}$ is the right target position and so on. Consider the stochastic differential game , where all players try to maximize their personal profit (8) under the constraint (2). Then the price process is given as in Theorem 3.1, but the error factor generalizes to*

$$v := \frac{1}{N} \sum_{l=1}^{k} N_{w_l} \left(\tilde{x}^1_{0,w_l} - x_0^1\right). \tag{28}$$

The above theorem shows that the generalization of the spread of information is reflected only in the composition of the error factor. Here the generalized error factor v behaves basically like the special error factor $\tilde{v}$. For each misinformation $\tilde{x}^1_{0,w_l}$, $l \in [1,k]$, the degree of falsification $\left(\tilde{x}^1_{0,w_l} - x_0^1\right)$ is decisive on the one hand and on the other

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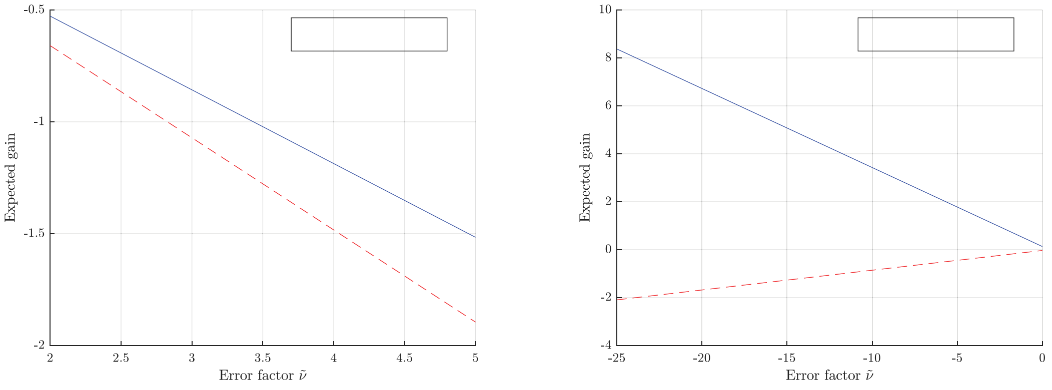
[19] This applies to the situation where $x_0^1 > 0$. Similar conclusions are drawn for the case $x_0^1 < 0$.



(a) Game with misinformation $\tilde{x}_0^1 = 25$.

(b) Game with misinformation $\tilde{x}_0^1 = -5$.

Fig. 3. Shows the expected profit and loss for both, the right and misinformed players, in dependence of the number of misinformed players N_w. The corresponding parameters are $\sum x_0 = x_0^1 = 20,\ N = 50,\ T\gamma\lambda^{-1} = 2$. 3(a) shows a situation in which a higher absolute error factor $|\tilde{\nu}|$ has a positive effect on the correctly informed players profit. Contrary, 3(b) shows a situation in which a higher absolute error factor $|\tilde{\nu}|$ has a negative effect on the correctly informed players profit.

hand the propagation of this information $\frac{N_{w_l}}{N}$ is relevant. The product of these two factors gives the influence of the misinformation $\tilde{x}_{0,w_l}^1$ on the error term ν. Summing up the influences of all misinformation finally states ν itself.

As a consequence of Theorem 3.4 the results already presented concerning the error term are preserved when $\tilde{\nu}$ is generalized by ν. In particular, with respect to the regions $\mathcal{V}$ and $\tilde{\mathcal{V}}$, it thus holds:

Theorem 3.5. *Assume the situation as in Theorem 3.4. Then $\nu \in \mathcal{V}$, iff*

$$\begin{cases} b_2 \le \nu \le b_1 & \text{if} \quad \sum_{i=1}^{N} x_0^i \ge 0 \\ b_1 \le \nu \le b_2 & \text{if} \quad \sum_{i=1}^{N} x_0^i < 0 \end{cases} \tag{29}$$

and $\nu \in \tilde{\mathcal{V}}$ otherwise. Hereby, the constants b_1, b_2 are as given in Theorem 3.2.

Theorem 3.5 extends the statements of Theorem 3.2 to the case of arbitrary information scattering. Since the boundary values b_1 and b_2 remain unchanged, all results from the previous section can be taken over one by one. Nevertheless, due to the now richer composition of the error term, it is also necessary to investigate the influence of ν itself. Because of the additive structure in (28), it can be seen that various information distortions can mitigate each other. This is the case when information circulates which overestimates the actual quantity x_0^1 as well as information which underestimates x_0^1. In the context of Theorem 3.5, this means that even in the case of massive dissemination of strong misinformation, stronger price fluctuations are not necessarily to be expected. In fact, in the extreme case, it is even possible that existing misinformation is completely shortened, this is $\nu = 0$. In such a case, according to Theorem 3.4, it is not possible to draw conclusions about the existence of misinformation from the observed price trend, even with knowledge of the white noise. This insight is stated in the following lemma.

Lemma 3.4. *It is not possible to draw conclusions about the individual information of the players from the observed price trend. In particular, a price trend that behaves as under perfect information does not mean that there is no misinformation in the market.*

Even if, according to Theorems 3.4 and 3.5, the error factor is the decisive variable for the expected price trend and the maximum price fluctuation, Lemma 3.4 shows that only very limited conclusions about the information distribution can be drawn from this variable. It therefore seems appropriate to use the distribution of the random numbers ϵ_n directly as an explanatory variable.

3.2.2. Random distortion

We move away from the assumption of knowing the received information of individual players. Consequently, it can no longer be predetermined whether $\nu \in \mathcal{V}$ applies or not. Nevertheless, under certain distributional assumptions on the distortions ϵ_n, the probability of such an event can be estimated:

Theorem 3.6. *Without loss of generality assume $b_2 < b_1$. Let $\epsilon_i \in \mathcal{L}^2(P),\ i \in \{1, N\}$ be uncorrelated and set $\bar{\epsilon} := N^{-1}\sum_{i=1}^{N} \epsilon_i$, then $\forall \epsilon > 0$ and $\forall N \in \mathbb{N}$*

$$P\left(\nu \in \tilde{\mathcal{V}}\right) \le \frac{V}{b^2 N}, \qquad P(\nu \in \mathcal{V}) \ge 1 - \frac{V}{b^2 N} \qquad \text{if} \quad \mathbb{E}(\bar{\epsilon}) \in \left[b_2, b_1\right], \tag{30}$$

$$P(\nu \in \mathcal{V}) \le \frac{V}{b^2 N}, \qquad P\left(\nu \in \tilde{\mathcal{V}}\right) \ge 1 - \frac{V}{b^2 N} \qquad \text{if} \quad \mathbb{E}(\bar{\epsilon}) \notin \left[b_2, b_1\right], \tag{31}$$

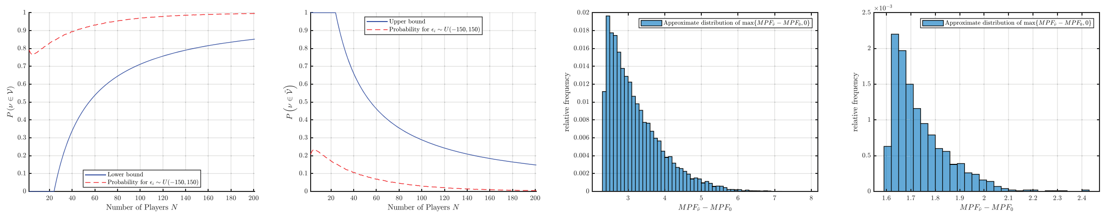
holds with $b := \min_{x \in \{b_1, b_2\}} |\mathbb{E}(\bar{\epsilon}) - x|$ and $V := \max_{i \in [1,N]} Var(\epsilon_i)$.

The above Theorem 3.6 provides an estimation of the likelihood of more severe price fluctuations caused by random information distortions. These estimators are significantly dependent on the number of players, i.e., the spread of information itself, leading to the following result: In cases where misinformation is not substantially erroneous, a broad distribution might, in fact, be advantageous. Conversely, when the misinformation gravitates towards high inaccuracy, its restricted dissemination is more favorable.

A critical determinant in this context is the direction and magnitude of the misinformation's deviation. This expected (total) deviation is quantified by the metric $\mathbb{E}(\bar{\epsilon})$, which represents the expected mean of all individual deviations.

Beneficial information spread: When $\mathbb{E}(\bar{\epsilon})$ resides within the boundaries b_2 and b_1, Theorem 3.6 suggests that as the spread of information increases, the system's stability is expected to be maintained or even enhanced (i.e., $\nu \in \mathcal{V}$). Notably, this may not hold true for limited misinformation spread. From a financial stability perspective, in such scenarios, extensive misinformation distribution is indeed more beneficial. It is imperative to note that this is especially true for centered misinformation distortions, where $\mathbb{E}(\bar{\epsilon}) = 0$. Such a case is illustrated in Panel 4(a), where the distortions are modeled by a uniform centered distribution. It is clearly observable how $P(\nu \in \mathcal{V})$ increases with the growing dissemination of information, or conversely, how $P\left(\nu \in \tilde{\mathcal{V}}\right)$ decreases when considering Panel 4(b) instead. Further the distribution of $MPF_\nu - MPF_0$, which states the additional increase in the fluctuation size or the additional volatility added by the presence of misinformation, is approximated in the Panels 4(c) and 4(d). This demonstrates a clear decline in the expected additional volatility with increased dissemination of information.

Adverse information spread: On the other hand, if the misinformation is so concentrated and skewed that $\mathbb{E}(\bar{\epsilon}) \notin [b_2, b_1]$, a minimal spread of

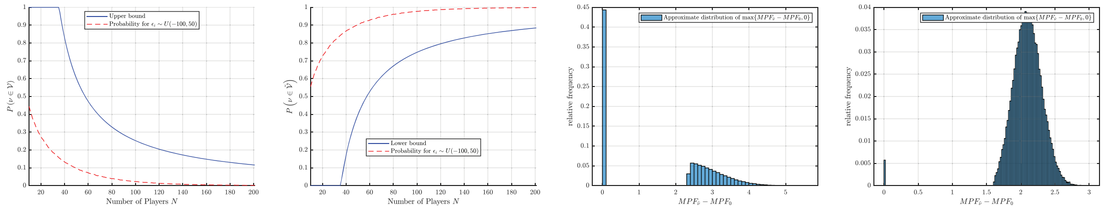


(a) Lower bound of theorem 3.6 and the exact probabilities $P(v \in \mathcal{V})$ for $\epsilon_i \sim U(-150,150)$.

(b) Upper bound of theorem 3.6 and the exact probabilities $P(v \in \tilde{\mathcal{V}})$ for $\epsilon_i \sim U(-150,150)$.

(c) Distribution of additional volatility with 10 Players and $\epsilon_i \sim U(-150,150)$.

(d) Distribution of additional volatility with 150 Players and $\epsilon_i \sim U(-150,150)$.

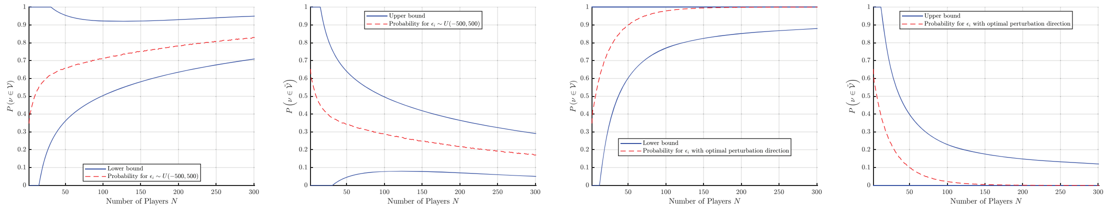


(e) Upper bound of theorem 3.6 and the exact probabilities $P(v \in \mathcal{V})$ for $\epsilon_i \sim U(-100,50)$.

(f) Lower bound of theorem 3.6 and the exact probabilities $P(v \in \tilde{\mathcal{V}})$ for $\epsilon_i \sim U(-100,50)$.

(g) Distribution of additional volatility with 10 Players and $\epsilon_i \sim U(-100,50)$.

(h) Distribution of additional volatility with 150 Players and $\epsilon_i \sim U(-100,50)$.

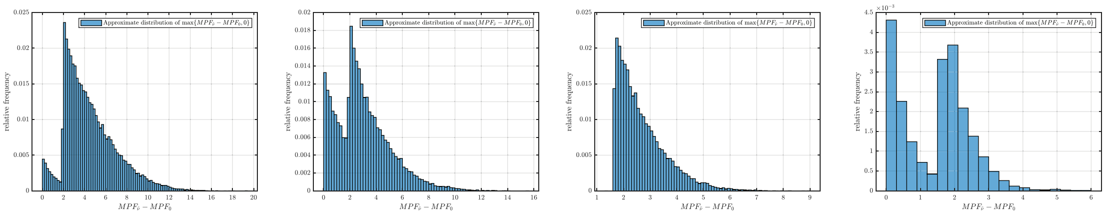


(i) Bounds of theorem 3.7 and the exact probabilities $P(v \in \mathcal{V})$ for $\epsilon_i \sim U(-500,500)$.

(j) Bounds of theorem 3.7 and the exact probabilities $P(v \in \tilde{\mathcal{V}})$ for $\epsilon_i \sim U(-500,500)$.

(k) Bounds of theorem 3.7 and the exact probabilities $P(v \in \mathcal{V})$ for ϵ_i optimal according to (34).

(l) Bounds of theorem 3.7 and the exact probabilities $P(v \in \tilde{\mathcal{V}})$ for ϵ_i optimal according to (34).



(m) Distribution of additional volatility with 20 Players and $\epsilon_i \sim U(-500,500)$.

(n) Distribution of additional volatility with 20 Players and ϵ_i optimal according to (34).

(o) Distribution of additional volatility with 100 Players and $\epsilon_i \sim U(-500,500)$.

(p) Distribution of additional volatility with 100 Players and ϵ_i optimal according to (34).

Fig. 4. Demonstration of Theorems 3.6 and 3.7 in a hands clean case with parameters $x_0^1 = -100$, $T\gamma\lambda^{-1} = 2$. Figs. 4(a) and 4(b) demonstrate compliance with the limits in Theorem 3.6 for $\mathbb{E}(\bar{\epsilon}) \in [b_2, b_1]$. Figs. 4(e) and 4(f) illustrate the same for $\mathbb{E}(\bar{\epsilon}) \notin [b_2, b_1]$. Figs. 4(c), 4(d), 4(g), and 4(h) approximate the distribution of additional volatility $MPF_v - MPF_0$ for 10 and 150 players, respectively, for the corresponding error terms ϵ_i. Figs. 4(i)–4(l) demonstrates the bound from Theorem 3.7. Here 4(i), 4(j) belong to the case of a centered disturbance. Figs. 4(k) and 4(l) demonstrate the shift of these boundaries while maintaining the distribution family as well as the variance of the ϵ_i in case of a distortion directed towards (34). Figs. 4(m)–4(p) show the changes in the distribution of additional volatility caused by this shift. For illustrative purposes, in all histograms except 4(g) and 4(h), the bar corresponding to 0 has been omitted.

misinformation is decidedly preferable. In such cases, the risks associated with expansive dissemination become too substantial to overlook. This fact is illustrated in Panel 4(e) to 4(h).

Further estimates: The estimations made in Theorem 3.6 can be further specified if the information of all players is distorted by the same source of disturbance. This is the case, for example, but not exclusively, with a standard normal distribution distortion, this is $\epsilon_i \sim \mathcal{N}_{0,1} \;\; \forall \; i \in \{2, \dots, N\}$.

Theorem 3.7. *Without loss of generality assume $b_2 < b_1$. Let ϵ_i, $i \in \{2, N\}$ be independent and identically distributed with $\mathbb{E}(\epsilon_2) = \mu$, $Var(\epsilon_2) = \sigma^2$ and $\gamma := \mathbb{E}\left(|\epsilon_2 - \mu|^3\right) < \infty$. Further Φ_{μ,σ^2} denotes the distribution function of a $\mathcal{N}\left(\mu, \sigma^2\right)$ normally distributed random variable. Then for every $N \geq 2$ and every $\zeta > 0$*

$$\Phi_{\mu,\sigma_*^2}\left(b_1\right) - \Phi_{\mu,\sigma_*^2}\left(b_2\right) - k \leq P\left(v \in \mathcal{V}\right) \leq \Phi_{\mu,\sigma_*^2}\left(b_1\right) - \Phi_{\mu,\sigma_*^2}\left(b_2 - \zeta\right) + k, \tag{32}$$

$$\Phi_{\mu,\sigma_*^2}\left(b_2 - \zeta\right) - \Phi_{\mu,\sigma_*^2}\left(b_1\right) + 1 - k \leq P\left(v \in \tilde{\mathcal{V}}\right) \leq \Phi_{\mu,\sigma_*^2}\left(b_2\right) - \Phi_{\mu,\sigma_*^2}\left(b_1\right) + 1 + k, \tag{33}$$

holds, where $\sigma_* := \frac{\sqrt{N-1}}{N}\sigma$ and $k := \frac{1.6\gamma}{\sigma^3\sqrt{N-1}}$. If further $P\left(v = b_2\right) = 0$, which is especially the case if ϵ_2 is continuously distributed, then the above estimates hold true with $\zeta = 0$.

The presented theorem enables the probabilistic estimation of $\nu \in \mathcal{V}$ as well as $\nu \in \tilde{\mathcal{V}}$ from both sides using the Gaussian distribution. Significantly, this approximation holds even when the error terms ϵ_i are not themselves normally distributed. This refines and augments the insights of the preceding theorem, offering a deeper understanding of how certain misinformation can potentially jeopardize price stability.

Can information distortion be beneficial? Theorem 3.7 unveils a counterintuitive insight. For large values of N, there emerges an approximate relationship $P(\nu \in \mathcal{V}) \approx \Phi_{\mu,\sigma_*^2}(b_1) - \Phi_{\mu,\sigma_*^2}(b_2)$, from which

$$\max_{\mu\in\mathbb{R}} P(\nu \in \mathcal{V}) \approx \frac{b_1+b_2}{2} \tag{34}$$

directly follows.[20] This demonstrates that the probability of not expecting worse price fluctuations is highest when the true information is on average distorted towards $(b_1 + b_2)/2$. Since this value is generally non-zero if the aggregated trading objectives do not cancel out, it follows: In scenarios of extensive information spread, the presence of a certain weighted misinformation can, paradoxically, be more beneficial than a neutral dispersion around the actual value. This challenges conventional intuitions and underscores the nuanced dynamics that need consideration when analyzing information distortions in the system.

Panels 4(i)–4(l) illustrate such a scenario using a uniform distribution of misinformation. For instance, Panel 4(i) displays the boundaries from Theorem 3.7 along with the actual realized probabilities $P(\nu \in \mathcal{V})$ in the centered case, i.e., $\mathbb{E}(\epsilon_2) = 0$. In Panel 4(k), the dispersion of information is additionally directed in the false direction (34), while maintaining both the uniform distribution and the variance. By comparing these two figures, one can observe that in this case, the probability of not expecting worse fluctuations significantly increases faster with the spread of misinformation. Additionally, examining the approximated distribution of the additional volatility in Panels 4(m)–4(p), it becomes evident that the probability of high additional price fluctuations also decreases, provided that $\nu \in \tilde{\mathcal{V}}$ holds. Thus, a directed misinformation can be in deed more beneficial in terms of price stability than a undirected one.

However, it is important to note that, as demonstrated in Theorem 3.3, the question of expected profits for individual players does not coincide with the quest for minimal volatility. In specific cases, outsiders, who have the ability to influence the general direction of information dispersion, must therefore weigh the benefits of reduced fluctuations against the risk that their intervention may push players from the profit zone into the loss zone.

4. New information flows

The processing of misinformation, as per (9) and (10), is accompanied by the belief of each player in the accuracy of his information. While the approach of constant information updates does not necessarily align with the initial assumption, it is conceivable that individual players may unexpectedly acquire new information regarding the actual trading quantity x_0^1 during the game.[21] To remain consistent with the previous assumptions, it is important to mention that the occurrence of such an information update is not inherently expected by the players from the outset. The exact nature of such an information impulse is irrelevant in this context.

We do not restrict ourselves to the assumption that a player receiving an information update actually learns the truth; however, we do assume that the new information is believed to be true and not dismissed due to belief perseverance. Furthermore, the player can assume that all other players also possess the new information. Consequently, the trading strategy adjusted by the information update can be determined consistently within the initial model assumptions.

4.1. General influence of new information

We assume that some players receive new information at the time $\tau \in [0, T]$. The following theorem describes the impact of the information update on the price dynamics:

Theorem 4.1. *Let $\tilde{x}_{0,n}^1$ denote the original information of the players, and let $\tilde{x}_{\tau,n}^1$ represent the adjusted information at time τ. Hereby $\tilde{x}_{\tau,n}^1 = \tilde{x}_{0,n}^1 + \int_0^\tau \tilde{\alpha}_n^1(t)\mathrm{d}t$ with $\tilde{\alpha}_n^1$ being the optimal control strategy player n assumes for player 1 according to (10) holds if player i has not received any new information. In this case, the price dynamics are described by*

$$\begin{cases} X_t^0 &= X^0(0) - \dfrac{1-e^{-\frac{N-1}{N+1}\frac{t\gamma}{\lambda}}}{1-e^{-\frac{N-1}{N+1}\frac{T\gamma}{\lambda}}}\gamma\left(\sum_{i=1}^N x_0^i + \nu\right) + \dfrac{e^{\frac{\gamma t}{\lambda}}-1}{e^{\frac{\gamma T}{\lambda}}-1}\gamma\nu + \sigma W_t, \quad t \in [0,\tau], \\ X_t^0 &= X^0(\tau) - \dfrac{1-e^{-\frac{N-1}{N+1}\frac{(t-\tau)\gamma}{\lambda}}}{1-e^{-\frac{N-1}{N+1}\frac{(T-\tau)\gamma}{\lambda}}}\gamma\left(\sum_{i=1}^N x_\tau^i + \nu_u\right) \\ &\qquad + \dfrac{e^{\frac{\gamma(t-\tau)}{\lambda}}-1}{e^{\frac{\gamma(T-\tau)}{\lambda}}-1}\gamma\nu_u + \sigma W_{t-\tau}, \ t \in [\tau, T] \end{cases} \tag{35}$$

with the initial and updated error factor as well as the modified initial conditions

$$\nu := \frac{1}{N}\sum_{n=1}^N \left(\tilde{x}_{0,n}^1 - x_0^1\right), \quad \nu_u := \frac{1}{N}\sum_{n=1}^N \left(\tilde{x}_{\tau,n}^1 - x_\tau^1\right), \quad x_\tau^n := x_0^n + \int_0^\tau \alpha_t^n \mathrm{d}t. \tag{36}$$

The above theorem demonstrates that the effects of an information update can be described by updating the error term. Therefore, by replacing the initial error term ν with the updated error term ν_u at time τ, the price process continues to adhere to the previously known structure. This fact is intuitively understandable since the newly acquired information precisely affects the originally distorted information, and in the best-case scenario, it may even eliminate any misinformation altogether. Furthermore, it should be noted that Theorem 4.1 can be directly extended to the case of multiple consecutive information arrivals. In this scenario, one only needs to repeatedly adjust the initial conditions and the error term as described above. Finally one might note that, with slight abuse of notation,

$$X_0^T = X_0^0 + \gamma\int_0^\tau \sum_{i=1}^N \alpha_t^i \mathrm{d}t + \sigma W_\tau - \gamma\sum_{i=1}^N x_\tau^i + \sigma W_{T-\tau} = X_0^0 - \gamma\sum_{i=1}^N x_0^i + \sigma W_T \tag{37}$$

is valid. This demonstrates that not only the misinformation itself, but also the updating of it, has no impact on the final price. This reinforces the previously made statements that misinformation, over an extended period, cannot further inflate nor artificially suppress price fluctuation.

Compatibility of new information: In the short term, within the period [0, T], updating information however may indeed influence the expected fluctuation. To gain a better understanding of when and how this occurs, the definitions of the Sets $\mathcal{V}$ and $\tilde{\mathcal{V}}$ need to be adapted to the possibility of new information flows:

Definition 4.1. For a game with one information updates we define the sets

$$\mathcal{V}_{\text{up}} := \left\{ \nu \in \mathbb{R}^2 \middle| MPF_\nu(0,T) = \min_{\nu\in\mathbb{R}^2} MPF_\nu(0,T) \right\}; \quad \tilde{\mathcal{V}}_{\text{up}} := \left\{ \nu \in \mathbb{R}^2 \middle| \nu \notin \mathcal{V}_{\text{up}} \right\}. \tag{38}$$

Hereby the vector $\nu := (\nu_0, \upsilon_1)$ collects the initial error term ν_0 and all the error terms υ_1 associated with the information update.

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[20] This can be corroborated using the approach $\frac{dP}{d\mu} = \frac{d}{d\mu}\left(\Phi_{\mu,\sigma_*^2}(b_1) - \Phi_{\mu,\sigma_*^2}(b_2)\right) \overset{!}{=} 0$.

[21] It should be noted that in the present case, the information update affects the set $\mathbb{A}$ of permissible control processes, rather than the dynamics of the game, as is often the case in optimal control problems under uncertainty.

The set $\mathcal{V}_{\text{up}}$ now collects all error vectors v under which no additional fluctuations due to misinformation are expected. The error vector itself now consists of the already known initial error term v_0 and additionally the updated error term v_1. The latter describes the new amount of error within the system, which arises from the acquisition of new information. Note that both $\mathcal{V}_{\text{up}}$ and $\tilde{\mathcal{V}}_{\text{up}}$ coincide with $\mathcal{V}$ and $\tilde{\mathcal{V}}$, respectively, if no new information is obtained during the game. Furthermore, it is still always the case that $\vec{0} = (0,0) \in \mathcal{V}_{\text{up}}$. The following theorem sheds more light on the structure of the set $\mathcal{V}_{\text{up}}$:

Theorem 4.2. *Consider a game with one information update at time $\tau \in (0,T)$. Further v_0, v_1 denote the initial and corresponding error term caused by the update.*

1. *For arbitrary $v = \left(v_0, v_1\right) \in \mathbb{R}^2$ and $\mathcal{V}_\tau$ being as in Definition 3.2 but with the time interval $[\tau, T]$ instead of $[0,T]$*

$$\mathcal{V}_{up} \supset \left\{ v \,\middle|\, v_1 \in \mathcal{V}_\tau \ \wedge \ v_0 \in \mathcal{V} \right\}, \tag{39}$$

   where the subset is true.
2. *Let $v_1 = v_1^{no} := \frac{1}{N}\sum_{n=1}^{N}\left(\tilde{x}^1_{\tau,n} - x^1_\tau\right)$ with $x^n_\tau := x^n_0 + \int_0^\tau \alpha^n_t \mathrm{d}t$ and $\tilde{x}^1_{\tau,n} = \tilde{x}^1_{0,n} + \int_0^\tau \tilde{\alpha}^1_n(t)\mathrm{d}t$ as described in Theorem 4.1 $\forall n \in [1,\dots,N]$. Then the vector $v^{no} := \left(v_0, v_1^{no}\right)$ is equivalent to the game with initial error v_0 and no information update, consequently*

$$v^{no} \in \mathcal{V}_{up} \quad \textit{iff} \quad v_0 \in \mathcal{V} \qquad \textit{and} \qquad v_1^{no} \in \mathcal{V}_\tau \quad \textit{iff} \quad v_0 \in \mathcal{V}. \tag{40}$$

The above theorem introduces the set $\mathcal{V}_\tau$. This set collects all information updates whose associated error term does not lead to additional fluctuations during the remaining time period. If there is no expectation of additional fluctuations for both, the subgame on $[\tau, T]$ with error term v_1 and the original game on [0, T] under v_0, then the same holds true for the game that receives an information update from v_0 to v_1 during its runtime.

It should be further noted that $\mathcal{V}_\tau$ is identical to $\mathcal{V}$ for a game with initial conditions $\hat{x}^n_0 := x^n_0 + \int_0^\tau \alpha^n_t \mathrm{d}t$ and total duration $T - \tau$. Thus, we can directly refer to the already known limits from Theorem 3.5 for verification. Similarly, in the case of random updates, Theorems 3.6 and 3.7 can be used to estimate $P\left(v \in \mathcal{V}_{\text{up}}\right)$.

Furthermore, Theorem 4.2 demonstrates that any updates, which do not bring new information, have no impact on expected price fluctuations. This initially very intuitive statement becomes intriguing when one considers that for all $v_0 \in \mathcal{V}$, the corresponding error term v_1^{no} lies in $\mathcal{V}_\tau$. Therefore, we know that the acquisition of new information, even if it exacerbates the error term present at time τ, is not expected to further inflate the fluctuation, provided that this new error term is associated with an original error under which no further anomalies were anticipated from the outset. With this insight, the question naturally arises whether every element $v_1 \in \mathcal{V}_\tau$ can be expressed in terms of v_1^{no} with a corresponding $v_0 \in \mathcal{V}$. This question can be negated by a direct counterexample, as shown in Fig. 5, thereby proving the following lemma:

Lemma 4.1. *It is possible that the described system can tolerate a larger amount of misinformation through information updates over the course of the game than would have originally been the case.*

Lemma 4.1, expressed in other terms, states that the amount of error a system can tolerate dynamically changes over time, particularly it can increase, while due to $v_1^{\text{no}} \in \mathcal{V}_\tau \ \forall \ \tau \in (0,T)$ the original misinformation, adjusted to the current time point, is tolerated at any time. Under such a temporal adjustment, a reduction in the tolerated amount of error over time is therefore excluded.

Indeed, this statement aligns with the insights already gained. As established, misinformation requires a certain time span to unfold its potentially harmful effect. The later players receive new information, the shorter the remaining time they have to incorporate it into the game. Thus, even if the newly acquired information is devastating misinformation, the remaining time might simply be insufficient for it to unfold its effect.

Fig. 5(a) shows the dynamic development of the boundary values b_1 and b_2 for the set $\mathcal{V}_\tau$ in dependence on the time τ, which indicates when the information update is received. Particularly considering the development of the boundary b_1, it becomes clear that the error factor a system can withstand may increase over time. Further, note that the decreasing boundary b_2 in this example is consistent with the above statements, as the initial error converges towards zero over time, thus always remaining within the specified boundaries. This associated convergence of the initial sizes is illustrated in Fig. 5(b).

Ultimately, it may also be noted that Theorem 4.2 speaks of a proper subset. Therefore, the additional amount of error compatible with new information is even larger than that already represented by $\mathcal{V}_\tau$. Since this fact does not alter the previously elaborated results but rather reinforces them, it will not be further discussed in the following.

When must an update occur? The question arises as to the latest point in time by which misinformation, under which more severe price fluctuations are to be feared, must be eliminated at the latest. Therefore, we highlight a further characterization of the set $\mathcal{V}_{\text{up}}$:

Theorem 4.3. *With the definitions as before and the stopping times*

$$T_{v_0} := \inf\left\{t \geq 0 : X^0_t \notin \left[X^0_0, \mathbb{E}\left(X^0_T\right)\right], T\right\}, \tag{41}$$

$$T_{\mathbb{E},v_0} := \inf\left\{t \geq 0 : \mathbb{E}\left(X^0_t\right) \notin \left[X^0_0, \mathbb{E}\left(X^0_T\right)\right], T\right\} \tag{42}$$

the following statements hold:

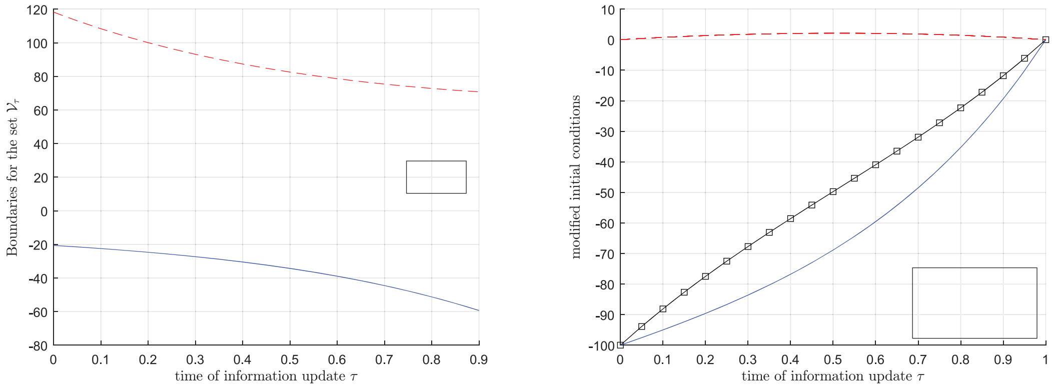
1. *Assume $\sum_{n=1}^{N} x^n_0 < 0$, then for $v_0 \in \tilde{\mathcal{V}}$ with $v_0 > b_2$ we have $T_{\mathbb{E},v_0} = 0$. For $v_{0,1}, v_{0,2} \in \tilde{\mathcal{V}}$ with $b_1 > v_{0,1} > v_{0,2}$ we have $T_{\mathbb{E},v_{0,2}} < T_{\mathbb{E},v_{0,1}} \in (0,T)$. Finally for every $v_0 \in \mathcal{V}$ $T_{\mathbb{E},v_0} = T$ is valid. If $\sum_{n=1}^{N} x^n_0 > 0$ holds, the greater-than relationships regarding the boundaries b_1 and b_2 must be replaced with less-than signs.*
2. *If $\tau \in (0,T) \leq T_{\mathbb{E},v_0}$, then $v \in \mathcal{V}_{up}$ if $v_1 \in \mathcal{V}_\tau$. Otherwise, if $\tau > T_{\mathbb{E},v_0}$, $v \in \tilde{\mathcal{V}}_{up}$ for all $v_1 \in \mathbb{R}$.*
3. *For arbitrary initial error factor v_0 up to time T_{v_0} an information update is possible, such that no worse price fluctuations are to be expected.*

While Theorem 4.2 answered the question of how much additional error a system can tolerate through an update if the initial error is acceptable, a different question now takes center stage. Theorem 4.3 provides clarification on whether, and if so, how long it is possible to avoid additional price fluctuations that are expected due to the original misinformation.

The answer to this question strongly depends on the amount of misinformation already present. If it is so minor that no additional fluctuations were expected from the beginning, i.e., $v_0 \in \mathcal{V}$, then it is always possible to find an information update under which no worse fluctuations are to be expected. In fact, this reflects the situation already described by Theorem 4.2.

If the error is so significant that it causes players to believe, on average, that the victim is forced to sell shares, even though it needs to acquire them, or vice versa, then an additional expansion of the fluctuation is to be expected at any time. There is, therefore, no point during the game when a correction of the misinformation would still be timely to prevent additional volatility.

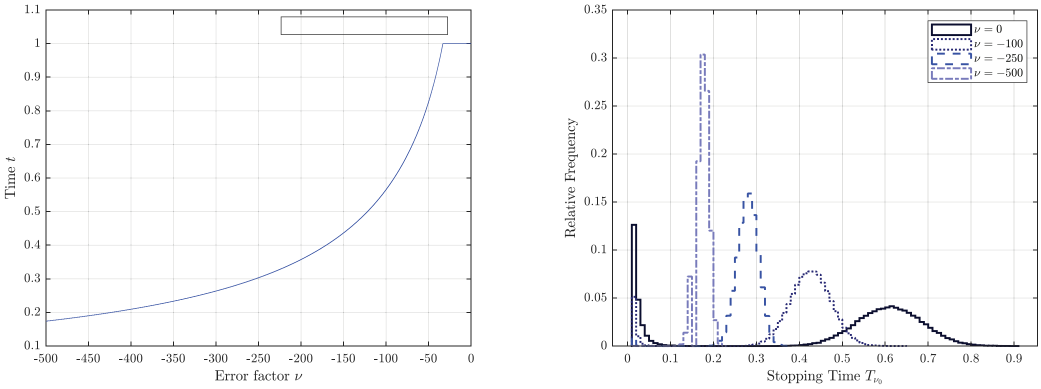
If the prevailing misinformation is severe but essentially points in the right direction, as is the case, for example, if the amount to be sold is significantly overestimated, then it is possible, with new information, to avoid an expected additional inflation of the fluctuation until the time $T_{\mathbb{E},v_0}$. Note that this time is strictly greater than zero, which means that in this case, additional expected price fluctuations can always be avoided if new information is received quickly enough. Furthermore, the theorem shows that a correction must occur the sooner, the larger the original error factor. This relationship is depicted in Panel 6(a).



(a) Boundary values of the set $\mathcal{V}_\tau$ in dependence of τ.

(b) Development of initial conditions over time.

Fig. 5. Fig. 5(a) shows the dynamic development of the set $\mathcal{V}_\tau$ in time. Fig. 5(b) demonstrates the corresponding development of the initial conditions of the game. For demonstration a hands clean case with parameters $N = 10,\ \gamma = 0.1,\ \lambda = 0.5\gamma,\ T = 1,\ \sigma = 3.9,\ x_0^1 = -100,\ \tilde{x}_0^1 = -50$ was chosen.



(a) Last update time $T_{\mathbb{E},\nu_0}$ in dependence of the initial error factor ν.

(b) Approximate distribution of T_{ν_0} for different error factors.

Fig. 6. Fig. 6(a) shows the last update time $T_{\mathbb{E},\nu_0}$ in dependence of the initial error factor ν. Fig. 6(b) approximates the distribution of T_{ν_0} for different error factors, showing that the higher the initial error is, the less likely a possible correction becomes. For demonstration a hands clean case with parameters $N = 50,\ \gamma = 0.1,\ \lambda = 0.5\gamma,\ T = 1,\ \sigma = 3.9,\ \sum x_0 = -200$ was chosen.

Furthermore, it is shown what kind of information is suitable for correction. Certainly, this can again be guaranteed for those in the set $\mathcal{V}_\tau$. This shows that not only a complete correction, i.e., the elimination of all misinformation, is suitable for preventing the additional inflation of the fluctuation, but this can also already be achieved by reducing the original error factor. In other words, not all players need to become aware of their misinformation; it may already be sufficient if a few players receive the information that their original assumption was distorted.

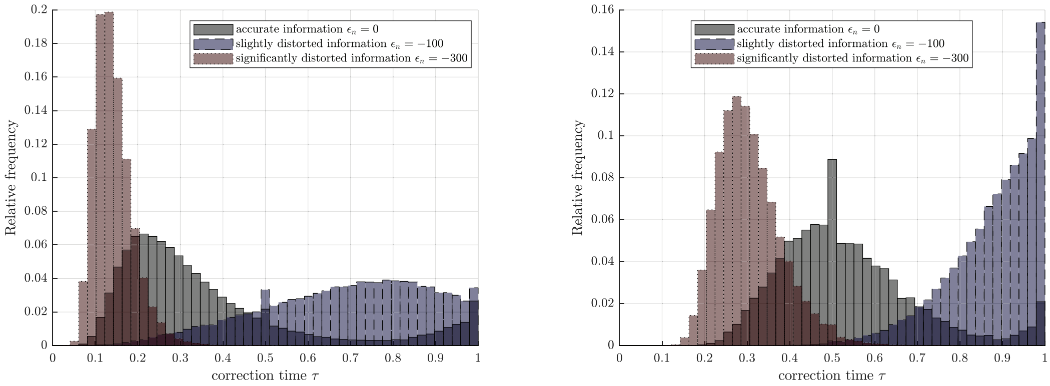
Finally, Theorem 4.3 introduces the latest possible correction time T_{ν_0}. In contrast to $T_{\mathbb{E},\nu_0}$, this represents the last possible moment, under a pathwise consideration, up to which corrections are possible so that no additional distortions are expected from that point on. It is important to note that such a time T_{ν_0} exists for any initial error factor, because even if it is so significant that additional fluctuations are immediately expected (as measured by $T_{\mathbb{E},\nu_0}$), this may not be the case in the actual realization, since the price movement is a stochastic process, which, as is well known, can deviate from its expectations. Thus, it also becomes clear that T_{ν_0}, in contrast to $T_{\mathbb{E},\nu_0}$, is a real stopping time, i.e., a random variable, whose realization can only be observed during the course of the game. The distributions, on the other hand, can be estimated from the outset. Panel 6(b) exemplifies this for various error factors. It is evident here that, once again, the larger the initial error factor, the greater the probability that it needs to be corrected quickly.

4.2. Information update through price observation

Lemma 3.4 has already demonstrated that observing the stock price development is only partially suitable for making statements about the quantity of existing misinformation in the system for outside observers. For the players themselves, the stock price is an available source of information that they can use to assess the plausibility of their information. Indeed, it is conceivable that a player may doubt the correctness of his information if the realized stock price starts to exhibit a completely different trajectory than what he would have expected based on his information. Let us assume that in the event of such an occurrence, the player suddenly realizes that his original information is incorrect. In other words, there are price thresholds whose breach is so shocking for the player that it overcomes belief perseverance. In such a case, his information may be adjusted through further research, without necessarily resulting in the exact information.

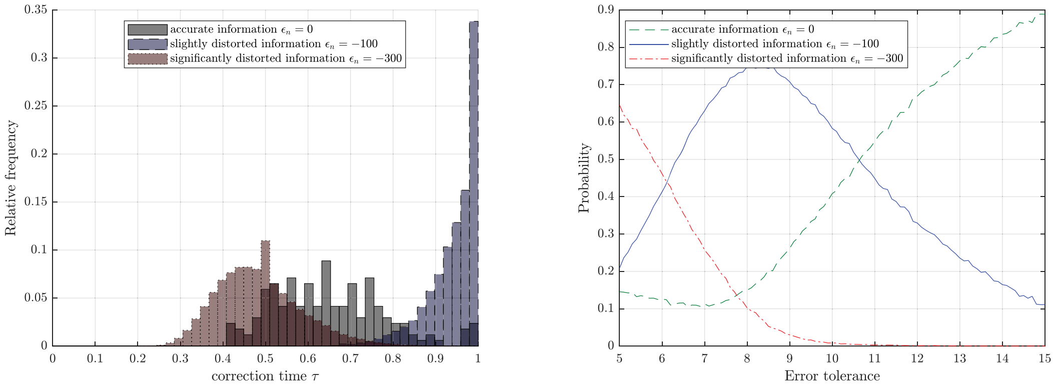
It should be noted that the special case described here fits into the above general information update schema with random update time τ and random information update ν_τ. The already known results are therefore applicable to the present situation. Nevertheless, we want to use this exemplary assumption for an information update to highlight some peculiarities associated with the stochastic component of the system.

Let $[-\iota_n, \kappa_n]$ with $\iota_n, \kappa_n \in \mathbb{R}^+$ be the tolerance threshold of the nth player. This means that the player doubts his information if the actual



(a) Approximate distribution of the update time τ for error tolerance $[-5, 5]$ for three players with varying levels of misinformation.

(b) Approximate distribution of the update time τ for error tolerance $[-10, 10]$ for three players with varying levels of misinformation.



(c) Approximate distribution of the update time τ for error tolerance $[-15, 15]$ for three players with varying levels of misinformation.

(d) The probability that one of the three differently informed player types first mistrusts their information, depending on the error tolerance.

Fig. 7. The Figs. 7(a)–7(c) depict the approximate distribution of update times τ for three different players under varying error tolerances. The players differ in the degree of distortion in their information, specifically, we have $\epsilon = 0, ; \epsilon = -100$, or $\epsilon = -300$. Fig. 7(d) provides the probability that one of the three player types receives their information update before the other players. A hands clean case with parameters $N = 10$, $\gamma = 0.1$, $\lambda = 0.5\gamma$, $T = 1$, $\sigma = 3.9$, $\sum x_0 = -200$ was chosen.

price deviates from his expected price by at least ι_n below or by κ_n above. This is

$$\tau_n = \inf\left\{t \geq 0 : X_t^0 - \mathbb{E}_n\left(X_t^0\right) \in \{-\iota_n, \tau_n\}\right\}, \tag{43}$$

where $\mathbb{E}_n\left(X_t^0\right)$ denotes the expected price trend by player n. The following lemma illustrates an important special case:

Lemma 4.2. *Let the initial error be such that $\nu = 0$, then the probability that right informed predator doubts his information by time t is given by*

$$P(\tau \leq t) = 1 - \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}\exp\left(-\frac{(2k+1)^2\pi^2 t}{2(\iota+\kappa)^2}\right)\sin\left(\frac{(2k+1)\pi\iota}{\iota+\kappa}\right). \tag{44}$$

Here we assumed without loss of generality $\iota_n, \kappa_n = \iota, \kappa$ for all right informed players.

The above lemma provides two key insights. Firstly, it demonstrates that due to the stochastic component, even correctly informed players may doubt their information, and this can occur even when the most favorable scenario of mutually offsetting misinformation arises. Subsequently, such a player may be tempted to follow a seemingly false information based on the realized price trajectory. This illustrates that the stochastic influence can lead to an increase in the error factor during the game, possibly even turning an initially benign situation into the opposite. It is noteworthy that this can occur even when originally all players trade correctly, meaning there is no misinformation present. In summary, when the actual price trajectory is used to assess the plausibility of existing information, as described above, the stochastic influence can lead to correct information being regarded as incorrect.

Secondly, Lemma 4.2 demonstrates that such a false correction becomes more likely as the game continues and as the players' error tolerances become smaller. The latter point is particularly intriguing as, initially, each player assumes to possess the correct information, whether it is accurate or not. Therefore, a higher error tolerance would be advisable. However, if the original information happens to be distorted, it is most likely to be detected with a small error tolerance. This dilemma highlights the challenge of choosing an appropriate tolerance threshold and an associated difficulty with misinformation in general.

In the general case, τ_n can only be determined numerically. Panel 7 illustrates this for a game with three different pieces of information: the truth, which states that the victim must acquire 200 units of a stock, a slight distortion claiming that 300 units must be acquired, and a significant misinformation stating that 500 units are to be acquired. The information is evenly distributed in this case, meaning that one-third of the predators follow each of the three pieces of information.

Panels 7(a)–7(c) show the estimated distribution of correction times τ_n for the three different types of players. As previously mentioned, it is observed that the greater the error tolerance of the players, the later a questioning of the original information is expected. Panel 7(d) illustrates the interplay of the three different correction times. Here, the probability is estimated, which of the three types of players will first doubt their original information. As can be clearly seen, this question depends on the error tolerance of the players. If the error tolerance is low, in the given case, it is most likely that those players who have received the most distorted misinformation will be the first to doubt it. However, this is not always the case. For instance, if one assumes more error-tolerant players, it becomes apparent that now the correctly informed predators are most likely to be the first to (erroneously) consider their information implausible.

The above example illustrates that the intuition that players subjected to the worst misinformation will be the first to doubt it is not correct. This can be explained as follows: Assume that, except for a few correctly informed players, all players follow the same strong misinformation. In this case, one would also expect that initially, the price development, driven by the (false) trading strategies of the numerous misinformed players, behaves as they expect. However, since this clearly differs from the expectations of the remaining correctly informed players, these will doubt their information much earlier than the actually misinformed players.

In fact, the question of which player is most likely to first notice misinformation largely depends on the specific situation considered and must be simulated separately from case to case. Further, it should also be noted that the question of which player first corrects a mistake also influences which players might notice their mistake second, and so on. One might imagine a game scenario where the most misinformed players first recognize their mistake. Suppose they then receive the correct information and continue the game with it. In this case, the remaining error factor is reduced, making it less likely that the less misinformed players will even notice their mistake. Conversely, it could also happen that the correctly informed players first doubt their information and then adopt a faulty assumption. This would drive the price more strongly in the direction that the misinformed players expect, making it more difficult for them to recognize their mistake.

The discussion in this chapter shows that updating information in a stochastic system comes with numerous difficulties and does not always have a positive effect on the expected fluctuation size. Despite this complexity, Theorems 4.2 and 4.3 can still be used to determine how long a correction is possible to avoid worse fluctuations and to show which kind of new information will be needed.

5. Conclusion

In conclusion, our research focuses on understanding the dynamics of stock market fluctuations, focusing on the effects of misinformation in a predatory trading game scenario. Our approach, diverging from traditional models, allowed for the integration of tainted information, mirroring the actual information environment of market participants.

Key findings demonstrate the diminishing impact of misinformation on fluctuations over time, challenging the effectiveness of long-term market manipulation. We established a baseline fluctuation size dictated by factual market data, unaltered by misinformation, and found that misinformation inflates but does not shrink fluctuation sizes. This contributes to a more nuanced understanding of market dynamics under information distortions.

A significant revelation is the quantification of a misinformation threshold that markets can absorb without exacerbating fluctuation severity, with higher tolerance in more liquid stocks and less asymmetric markets. The research shows that misinformation generally tends to increases market volatility but a increasing dissemination of misinformation can occasionally increase the probability of less severe fluctuations, indicating a complex and adaptive market response.

The investigation into the effects of misinformation on traders' profit expectations and the neutralizing potential of contradictory misinformation underscores the intricacies in utilizing market data for detecting misinformation.

Importantly, our study highlights the dynamic nature of the market's tolerance to misinformation, influenced by ongoing information updates. These insights are crucial for designing effective market interventions and timing information updates, particularly for maintaining market stability and mitigating extreme fluctuation scenarios.

This research not only deepens the academic discourse on price fluctuations and misinformation but also offers practical insights for market practitioners and policymakers in developing strategies to manage market stability in the face of evolving information landscapes.

CRediT authorship contribution statement

Tobias J. Herzing: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Visualization, Project administration. **Matthias Muck:** Conceptualization, Resources, Writing – original draft, Writing – review & editing, Supervision, Project administration.

Declaration of funding

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proofs

A.1. Proof of Theorems 3.1 and 3.4

Proof of Theorem 3.4. As shown in Carlin et al. (2007) the optimal trading strategy[22] for the nth player is given by

$$\alpha_t^n = ae^{-\frac{N-1}{N+1}\frac{\gamma}{\lambda}t} + b_n e^{\frac{\gamma}{\lambda}t}, \tag{45}$$

where the coefficients $a \in \mathbb{R}$ and $b_n \in \mathbb{R}$, $n \in \{1, \dots, N\}$, are determined by

$$a = -\frac{N-1}{N+1}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{N-1}{N+1}\frac{\gamma}{\lambda}T}\right)^{-1}\sum_{i=1}^{N}\frac{x_0^i}{N}, \quad b_n = \frac{\gamma}{\lambda}\left(e^{\frac{\gamma}{\lambda}T} - 1\right)^{-1}\left(\sum_{i=1}^{N}\frac{x_0^i}{N} - x_0^n\right). \tag{46}$$

Using Ito's Lemma, one verifies that the price process described by is given by

$$X_t^0 = X^0(0) + \gamma\int_0^t\sum_{i=1}^{N}\alpha_s^i \mathrm{d}s + \sigma\left(W(t) - W(0)\right) \quad \forall t \in [0, T]. \tag{47}$$

Inserting (45) with

$$a = -\frac{N-1}{N+1}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{N-1}{N+1}\frac{\gamma}{\lambda}T}\right)^{-1}\left(\frac{\tilde{x}_{0,w_l}^1}{N} + \sum_{i=2}^{N}\frac{x_0^i}{N}\right), \tag{48}$$

$$b_n = \frac{\gamma}{\lambda}\left(e^{\frac{\gamma}{\lambda}T} - 1\right)^{-1}\left(\frac{\tilde{x}_{0,w_l}^1}{N} + \sum_{i=2}^{N}\frac{x_0^i}{N} - x_0^n\right) \tag{49}$$

for $n \in \left\{N_r + \sum_{l=1}^{k-1} N_{w_l} + 1, \dots, N_r + \sum_{l=1}^{k} N_{w_l}\right\}$, $l \in \{1, \dots, k\}$ and

$$a = -\frac{N-1}{N+1}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{N-1}{N+1}\frac{\gamma}{\lambda}T}\right)^{-1}\sum_{i=1}^{N}\frac{x_0^i}{N}, \quad b_n = \frac{\gamma}{\lambda}\left(e^{\frac{\gamma}{\lambda}T} - 1\right)^{-1}\left(\sum_{i=1}^{N}\frac{x_0^i}{N} - x_0^n\right) \tag{50}$$

for $n \in \{1, \dots, N_r\}$ into (47) and evaluating the integrals states the result. □

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[22] To be more precise, the authors restricted themselves to deterministic strategy profiles. However, by means of a subspace argument, it can be shown that the solutions found also represent the optimal open loop strategies for the larger space (2). See for instance (Carmona, 2016).

Proof of Theorem 3.1. This is a direct consequence of Theorem 3.4. ☐

A.2. *Proof of Theorems 3.2 and 3.5*

Proof of Theorem 3.5. We show that $\mathbb{E}\left(X_t^0\right)$ has either one or no extremum within the interval $[0,T]$ depending on ν. In the case where there is no extremum, ν must belong to $\mathcal{V}$ since the expected price is then always monotonically decreasing or increasing. Conversely, if there exists one (and only one) extremum, then, due to the fixed initial price and the independence of the final price from the error factor, ν must belong to $\tilde{\mathcal{V}}$. Thus we have to demonstrate under which conditions there exists an extremum and that there is only one in such cases. From the first order condition $\frac{\partial \mathbb{E}(X^0(t))}{\partial t} = 0$ follows

$$t = \frac{(N+1)\lambda}{2N\gamma}\log\left(\frac{e^{\frac{NT\gamma}{\lambda(N+1)}}\left(1-e^{\frac{T\gamma}{\lambda}}\right)(N-1)\left(\sum_{i=1}^N x_0^i+\nu\right)}{\left(e^{\frac{T\gamma}{(N+1)\lambda}}-e^{\frac{NT\gamma}{(N+1)\lambda}}\right)(N+1)\nu}\right). \tag{51}$$

Note that (51) is real if $Sign\ \nu = Sign\left(\sum_{i=1}^N x_0^i+\nu\right)$. However, since this follows immediately from (29), we will assume this to be satisfied in the following. The sufficient condition $\frac{\partial \mathbb{E}(X^0(t))}{\partial t^2} \neq 0$ is satisfied because of

$$\frac{\partial \mathbb{E}(X^0(t))}{\partial t^2}(t) = \frac{\gamma^3}{\lambda^2}\left(\frac{e^{\frac{\gamma t}{\lambda}}\nu}{e^{\frac{\gamma T}{\lambda}}-1} + \frac{(N-1)^2 e^{\frac{1-N}{1+N}\frac{\gamma t}{\lambda}}\left(\sum_{i=1}^N x_0^i+\nu\right)}{(N+1)^2\left(1-e^{\frac{1-N}{1+N}\frac{\gamma T}{\lambda}}\right)}\right) \neq 0. \tag{52}$$

Hereby we used that, because of $N \geq 2$, every factor in the above fractions is positive except ν and $\sum_{i=1}^N x_0^i+\nu$. The latter two have the same sign according to the above assumption, making both summands either negative or positive. Thus the existence of a maximum/minimum is shown. To proof that the extreme value occurs during the game, we further need to show $0 <$ (51) $< T$. For the sake of clarity, we substitute $\frac{\gamma T}{\lambda}$ by x and obtain for $Sign\ \nu = \pm$ on the one hand the condition

$$\log\left(\frac{e^{\frac{N}{N+1}x}\left(1-e^x\right)(N-1)\left(\sum_{i=1}^N x_0^i+\nu\right)}{\left(e^{\frac{1}{N+1}x}-e^{\frac{N}{N+1}x}\right)(N+1)\nu}\right) < x\frac{2N}{N+1}, \tag{53}$$

from which after algebraic simplifications

$$\sum_{i=1}^N x_0^i \lesseqgtr \left(\frac{N+1}{N-1}\frac{e^{\frac{2N}{N+1}x}-e^x}{e^x-1}-1\right)\nu \tag{54}$$

follows and on the other hand

$$\log\left(\frac{e^{\frac{N}{N+1}x}\left(1-e^x\right)(N-1)\left(\sum_{i=1}^N x_0^i+\nu\right)}{\left(e^{\frac{1}{N+1}x}-e^{\frac{N}{N+1}x}\right)(N+1)\nu}\right) > 0, \tag{55}$$

what guarantees the existence of (51) and from what

$$\sum_{i=1}^N x_0^i \gtreqless \left(\frac{N+1}{N-1}\frac{1-e^{\frac{1-N}{1+N}x}}{e^x-1}-1\right)\nu \tag{56}$$

follows. Next, note that

$$\frac{N+1}{N-1}\frac{e^{\frac{2N}{N+1}x}-e^x}{e^x-1} > 1 \quad \text{and} \quad \frac{N+1}{N-1}\frac{1-e^{\frac{1-N}{1+N}x}}{e^x-1} < 1 \quad \forall N \geq 2,\ x > 0 \tag{57}$$

is valid. So combining (54) with (56) and reformulating the result in terms of the Sets $\mathcal{V}$, $\tilde{\mathcal{V}}$ ends the proof. ☐

Proof of Theorem 3.2. This is a direct consequence of Theorem 3.5. ☐

A.3. *Proof of Lemma 3.3*

Proof. For $\tilde{\nu}_1, \tilde{\nu}_2 \in \tilde{\mathcal{V}}$ we have

$$X_t^0\left(\tilde{\nu}_1\right) - X_t^0\left(\tilde{\nu}_2\right) = \gamma\left(\tilde{\nu}_1-\tilde{\nu}_2\right)\left(-\frac{1-e^{-\frac{N-1}{N+1}\frac{t\gamma}{\lambda}}}{1-e^{-\frac{N-1}{N+1}\frac{T\gamma}{\lambda}}} + \frac{e^{\frac{\gamma t}{\lambda}}-1}{e^{\frac{\gamma T}{\lambda}}-1}\right) \tag{58}$$

according to (11), where the last factor on the right hand side is less or equal zero for all valid game parameter and $\forall t \in [0,T]$. Further (58) is only zero iff $t \in \{0,T\}$. Next note, that according to the proof of theorem 3.2 (11) is guaranteed to have one and only one extreme point in (0,T). Thus MPF_ν is given by the distance from this extreme point to either the initial price or the final price. As the latter two are independent of $\tilde{\nu}$, MPF_ν is uniquely described by the extreme value. It follows from the assumptions that either $0 < \tilde{\nu}_1 < \tilde{\nu}_2$ or $0 > \tilde{\nu}_1 > \tilde{\nu}_2$ is valid. As seen in the proof of theorem 3.2 we have a minimum in the first case and as $X_t^0\left(\tilde{\nu}_2\right) < X_t^0\left(\tilde{\nu}_1\right)$ is valid $\forall t \in (0,T)$ according to (58), the minimum value associated with $\tilde{\nu}_2$ needs to be smaller than the minimum value associated with $\tilde{\nu}_1$ and thus $MPF_{\tilde{\nu}_2} > MPF_{\tilde{\nu}_1}$. The same argument applied to the second case $0 > \tilde{\nu}_1 > \tilde{\nu}_2$ finishes the proof. ☐

A.4. *Proof of Theorem 3.3*

Proof. We restrict our self to show the claim for the right informed players. The proof for the misinformed players follows the exactly same scheme. Inserting α_t^n as defined in the proof of theorem 3.1 into (8) and applying some further algebraic simplifications leads to

$$J^n(\boldsymbol{\alpha}) = \frac{\left(-e^x+e^{x(1+2N)}-e^{x(2+N)}N+e^{xN}N\right)\gamma x_0^1\left(x_0^1-\nu N\right)}{\left(e^{x(N+1)}-1\right)\left(e^x-e^{Nx}\right)(N+1)N} \tag{59}$$

for $n \in \{2,\ldots,N-f(N)\}$, where the error factor ν is as defined in Theorem 3.1 and $x := \frac{T\gamma}{\lambda(N+1)} > 0$. Since $N \geq 2$ holds, the denominator in the above fraction is always negative. Further, a direct estimation shows that the first factor in numerator is always positive. Thus given the case (26) $J^n(\boldsymbol{\alpha})$ is negative and positive otherwise. Noting that this result needs to be corrected by the factor -1 due to the change of a maximization problem to a minimization finishes the proof. ☐

A.5. *Proof ot Theorem 3.6*

Proof. Without loss of generality we assume $b_2 < b_1$. For $\mathbb{E}\left(\bar{\epsilon}\right) \in [b_2, b_1]$ one can estimate

$$P\left(\nu \in \tilde{\mathcal{V}}\right) = P\left(\nu \notin \left[b_2, b_1\right]\right) < P\left(\nu \notin [\mathbb{E}(\nu)-b, \mathbb{E}(\nu)+b]\right) \tag{60}$$

$$= P\left(|\nu-\mathbb{E}(\nu)| > b\right) \leq \frac{V}{b^2 N}. \tag{61}$$

Hereby we used the fact that $\nu = \bar{\epsilon}$ because of $\tilde{x}_i = x + \epsilon_i$ as well as Bienaymés formula and Chebyshevs inequality in the last estimate. From the above estimate follows directly:

$$P\left(\nu \in \mathcal{V}\right) = 1 - P\left(\nu \in \tilde{\mathcal{V}}\right) \geq 1 - \frac{V}{b^2 N}. \tag{62}$$

Using the same arguments as above in the case $\mathbb{E}\left(\bar{\epsilon}\right) \notin [b_2, b_1]$ we get

$$P\left(\nu \in \mathcal{V}\right) < P\left(|\nu-\mathbb{E}(\nu)| > b\right) \leq \frac{V}{b^2 N} \quad \text{and} \quad P\left(\nu \in \tilde{\mathcal{V}}\right) \geq 1 - \frac{V}{b^2 N}. \tag{63}$$

☐

A.6. *Proof of Theorem 3.7*

Proof. Setting $\bar{x} := N\sqrt{N-1}^{-1}\sigma^{-1}(x-\mu)$ and noting that $\nu = \frac{1}{N}\sum_{i=2}^N \epsilon_i = \frac{1}{N}\sum_{i=1}^{N-1}\epsilon_i$ because of $\epsilon_1 = 0$ and rearranging we get

$$\sup_{x\in\mathbb{R}}\left|P\left(\nu \leq x\right) - \Phi_{0,1}\left(\bar{x}\right)\right| = \sup_{x\in\mathbb{R}}\left|P\left(\frac{1}{N}\sum_{i=1}^{N-1}\epsilon_i \leq x\right) - \Phi_{0,1}\left(\bar{x}\right)\right|$$

$$= \sup_{\bar{x}\in\mathbb{R}}\left|P\left(\frac{1}{\sqrt{N-1}\sigma}\sum_{i=1}^{N-1}\left(\epsilon_i-\mu\right) \leq \bar{x}\right) - \Phi_{0,1}\left(\bar{x}\right)\right| \leq \frac{0.8\gamma}{\sigma^3\sqrt{N-1}}, \tag{64}$$

where we used Berry–Esseen's theorem to get the last inequality. By substitution it is straight forward to verify that $\Phi_{0,1}\left(\bar{x}\right) = \Phi_{\mu,\sigma_*^2}\left(x\right)$ holds, thus

$$P\left(v \in \mathcal{V}\right) \leq P\left(v \leq b_1\right) - P\left(v \leq b_2 - \zeta\right) \leq \Phi_{\mu,\sigma_*^2}\left(b_1\right) - \Phi_{\mu,\sigma_*^2}\left(b_2 - \zeta\right) + \frac{1.6\gamma}{\sigma^3\sqrt{N-1}},$$
$$P\left(v \in \mathcal{V}\right) \geq P\left(v \leq b_1\right) - P\left(v \leq b_2\right) \geq \Phi_{\mu,\sigma_*^2}\left(b_1\right) - \Phi_{\mu,\sigma_*^2}\left(b_2\right) - \frac{1.6\gamma}{\sigma^3\sqrt{N-1}}. \tag{65}$$

Using $P\left(v \in \tilde{\mathcal{V}}\right) = 1 - P\left(v \in \mathcal{V}\right)$ to get the remaining inequalities finishes the proof. □

A.7. *Proof of Theorem 4.1*

Proof. Note that the optimal strategies given by (45) are weak time consistent, this is

$$\alpha^n_{[0,T]} = \alpha^n_{[\tau,T]} \quad \forall\, t \in [\tau, T],\ \forall n \in \{1, \dots, N\} \tag{66}$$

were $\alpha^n_{[0,T]}$ denotes the optimal strategy of the game starting at time 0 with initial conditions $X_0^n = x_0^n$ and $\alpha^n_{[\tau,T]}$ denotes the optimal strategy of the game starting at time $\tau \in [0,T]$ with initial conditions $X_0^n = x_0^n + \int_0^\tau \alpha^n_{[0,T]}(t)\mathrm{d}t$. When combined with the Markovian structure of the system, utilizing this approach allows for the termination of the game at time τ, and a new game can commence over the interval $[\tau, T]$ with the information $\tilde{x}^1_{\tau,n}$. The price process then directly follows from Theorem 3.4. □

A.8. *Proof of Theorem 4.2*

Proof. The first claim can be demonstrated as follows: For arbitrary $v_0 \in \mathcal{V}$ and arbitrary $\tau \in (0,T)$ we have $\mathbb{E}\left(X_0(t)\right) \in \left[X_0^0, \mathbb{E}\left(X_0^T\right)\right]$ for every $t \in [0,\tau]$ per definition. Further for every $v_1 \in \mathcal{V}_\tau$ we have $\mathbb{E}\left(X_0(t)\right) \in \left[\mathbb{E}\left(X_0^\tau\right), \mathbb{E}\left(X_0^T\right)\right]$ for every $t \in [\tau, T]$ per definition. Thus in total $\mathbb{E}\left(X_0(t)\right) \in \left[X_0^0, \mathbb{E}\left(X_0^T\right)\right]$ for every $t \in [0,T]$ which shows that $v := \left(v_0, v_1\right) \in \mathcal{V}_{\text{up}}$. Furthermore, there exist v_1 with $\mathbb{E}\left(X_0(t)\right) \in \left[X_0^0, \mathbb{E}\left(X_0^T\right)\right]$ for every $t \in [\tau, T]$, such that $\left(v_0, v_1\right) \in \mathcal{V}_{\text{up}}$ holds as well. The fact that such a v_1 is not in $\mathcal{V}_\tau$ proof the fact of a real subset.

The second claim is a direct consequence of the weak time consistence (see the proof of Theorem 4.1). □

A.9. *Proof of Theorem 4.3*

Proof. The first claim can be demonstrated as follows: Assume $\sum_{n=1}^N x_0^n < 0$ and $v_0 > b_2$. Then for arbitrary $0 < \epsilon < t^*$ we have $\mathbb{E}\left(X_0(\epsilon)\right) < X_0^0$ according to Theorem 3.4 with t^* according to (51). Thus $T_{\mathbb{E},v_0} = 0$. However if $b_1 > v_{0,1} > v_{0,2}$ applies, we have that $\mathbb{E}_{v_{0,1}} X_0(t) < \mathbb{E}_{v_{0,2}} X_0(t)$ for all $t \in [0,T]$ because of (58). Thus $T_{\mathbb{E},v_{0,2}} < T_{\mathbb{E},v_{0,1}}$ applies. The fact that those times are in the interval $(0,T)$ is a direct consequence of the proof of Theorem 3.5. Finally it follows from the definition of the set $\mathcal{V}$, that for every $v_0 \in \mathcal{V}$ $T_{\mathbb{E},v_0} = T$. In the case $\sum_{n=1}^N x_0^n > 0$ the same arguments can be applied.

Claims 2 and 3 follow the same arguments as outlined in the proof of Theorem 4.2. □

A.10. *Proof of Lemma 4.2*

Proof. Following the assumptions of Lemma 4.2, we have

$$\tau = \inf\left\{t \geq 0 : X_t^0 - \mathbb{E}_n\left(X_t^0\right) \in \{-\iota, \tau\}\right\} = \inf\left\{t \geq 0 : W_t \in \{-\iota, \tau\}\right\} \tag{67}$$
$$= \inf\left\{t \geq 0 : W_t^\iota \in \{0, \tau + \iota\}\right\}, \tag{68}$$

where W_t^ι denotes a (standard) Brownian Motion starting in ι. From the known results on first-passage times (see, for example, Klenke (2014)), the assertion follows. □

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