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Modeling and Reliability Analysis of a Redundant Transport System in a Markovian Environment

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Abstract. We consider the multipath communication between a client and a server that is established at the transport and session layers of an SDN/NFV protocol stack in a fog computing scenario. We analyze the reliability function of an associated redundant transport system comprising two logical channels that are susceptible to failures. The failure processes of both channels are described by Markov-modulated failure times that are driven by the transitions of a common Markovian environment. We model the error-prone system with repair and independent phase-type distributed repair times by a continuous-time Markov chain. We identify its generator matrix in terms of Kronecker products of the underlying parameter matrices that are determined by the interarrival times driven by Markov-modulated failure processes and the independent phase-type distributed repair times. We show that the steady-state distribution of the restoration model can be effectively calculated by an iterative aggregation-disaggregation method for block matrices and compute the associated reliability function of the transport system by a uniformization method.

Keywords: SDN/NFV · Reliability modeling · Reliability function · Markov-modulated process · Phase-type distributed repair times · Kronecker matrices

1 Introduction

Currently, fog computing is applied to integrate new services based on modern multimedia and machine-to-machine communication in the evolving Internet-of-Things into a cloud computing infrastructure (cf. [1,2,7]). The architecture is based on software-defined networks (SDN) and network function virtualization (NFV) technologies (cf. [6,9,23]). In this context it has been realized that multipath communication established at the transport and session layers of the SDN/NFV protocol stack can provide means to improve the capacity and reliability of the interprocess communication of these new services. Protocols like SCTP, multipath TCP, or multipath QUIC support such concepts and offer a set of redundant transport paths between clients and servers that enable multihoming due to the use of multiple interfaces (see Fig. 1, cf. [3,5,8]). In future wireless networks triggered by the rapidly evolving 5G standard the virtualized server functionality will be shifted closer to the edge of the associated radio access network (RAN) to reduce the end-to-end delay and to improve the transport capacity. Diverse radio technologies may be integrated into the RAN environment that is attached to the SDN core network. The latter will support new cloud computing concepts like fog and mobile edge computing (see Fig. 2, cf. [7,14,20]).

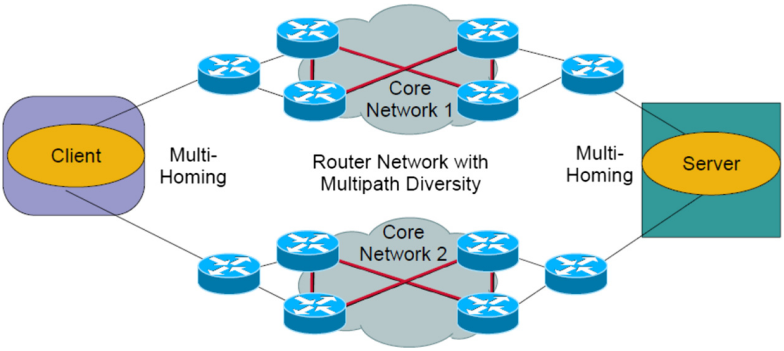

Fig. 1. Multipath diversity in modern SDN networks.

Regarding such a wireless access network, the modeling and analysis of the transport performance of single-path and multipath TCP communication associated with a client-server system that is embedded into a fog computing scenario provides numerous scientific challenges. The analysis of a simple TCP flow-control model has been a topic of our previous research (cf. [19]). In this performance study the related fog computing scenario will be considered from a different perspective.

In this paper we study a basic performance model describing two logical transport channels which provide a redundant communication system at the session layer between a client as sender and a server as receiver of a high-speed interprocess communication path in an SDN/NFV/RAN environment. The two system components modeling the transport channels are subject to errors that can decrease their throughputs below prescribed levels which is considered as a failure of the respective functional block. We model the failure times as Markov-modulated Poisson process (cf. [10]). The entire redundant system is managed by a scalable, virtualized management system applying Docker or Kubernetes container virtualization techniques (cf. [11,12]). It can instantiate one or two repair

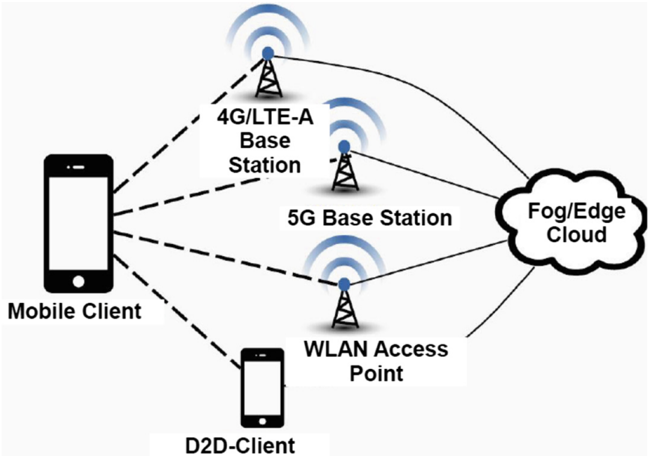

Fig. 2. Evolving radio access networks supporting fog and mobile edge computing.

functions as virtualized network functions and restore the original transport status after a generally distributed, nonnegative restoration period. The latter is approximated by a phase-type distribution (cf. [24]). We develop a Markovian reliability model of this redundant transport system in a random environment and compute its reliability function employing computational solution methods for finite Markov chains.

The paper is organized as follows. In Sect. 2 we describe the redundant transport system with two error-prone channels. In Sect. 3 we derive a finite Markov chain with its generator matrix and calculate its related steady-state vector. In Sect. 4 we compute the reliability function of the transport system. The exposition is finalized by some conclusions and an outlook on extensions of the reliability model.

2 Modeling a Redundant Transport System of Two Error-Prone Channels

We model the considered multipath transport system as a redundant technical system comprising two coupled logical (or physical) channels of identical structure that are able to transfer data packets (or more generally, energy or information) (see Figs. 1 and 2, see also [13,21,22]). The transfer function of each channel is subject to different types of failures that are driven by a common internal or external environment. The random transition behavior of the latter system is described by a Markov chain $\{Y(t), t \geq 0\}$ in continuous time with a finite state space $\Sigma_Y = \{1, \ldots, K\}$ and an irreducible generator matrix $Q \in \mathbb{R}^{K \times K}$. Its associated unique steady-state probability vector is denoted by $p \in \mathbb{R}^K$.

In the following, we formulate all algebraic relations of our model in terms of column-vectors to support a standard implementation as matrix-vector product by numerical methods of linear algebra. Furthermore, we use the order relation $0 \ll x$ for vectors $x \in \mathbb{R}^N$ indicating that all components $x_i > 0$ of a vector $x \in \mathbb{R}^N$ are positive. The order relation $0 < x$ means that $x \in \mathbb{R}^N$ is a nonnegative, non-zero vector, i.e. $0 \leq x_i$ for all $i \in \{1, \dots N\}$ and at least one $0 < x_i$ occurs (cf. [4]).

We assume that for a given state $Y(t) = j \in \Sigma_Y$ the interarrival times of failures of channel 1 and 2 occur as independent exponentially distributed events with mean values $1/\lambda_{1j}$ and $1/\lambda_{2j}$, respectively. Let

$$0 \ll \lambda_1 = (\lambda_{11}, \dots, \lambda_{1K})^T \in \mathbb{R}^K$$

and

$$0 \ll \lambda_2 = (\lambda_{21}, \dots, \lambda_{2K})^T \in \mathbb{R}^K$$

denote the corresponding positive column vectors of the associated arrival rates and $\Lambda_1 = \mathrm{Diag}(\lambda_1) > 0, \Lambda_2 = \mathrm{Diag}(\lambda_2) > 0$ be the associated diagonal-positive diagonal matrices of these arrival rate vectors of the failures in the random Markovian environment Y. Let $\Lambda = \Lambda_1 + \Lambda_2$.

We suppose that each erroneous channel is immediately handled by independent repair activities. The associated independently distributed repair times R_1, R_2 of channel 1 and 2, respectively, are governed by general phase-type distributions

$$F_1(t) = \mathbb{P}\{R_1 \leq t\} = 1 - \beta^T \cdot \exp(T \cdot t) \cdot e, \tag{1}$$

and

$$F_2(t) = \mathbb{P}\{R_2 \leq t\} = 1 - \alpha^T \cdot \exp(S \cdot t) \cdot e \tag{2}$$

with the corresponding probability densities

$$f_1(t) = d\mathbb{P}\{R_1 \leq t\}/dt = \beta^T \cdot \exp(T \cdot t) \cdot T^0,$$
$$f_2(t) = d\mathbb{P}\{R_2 \leq t\}/dt = \alpha^T \cdot \exp(S \cdot t) \cdot S^0$$

for channels 1 and 2. Here e denotes the vector of all ones of corresponding dimension. It means that the two finite state phase-type representation matrices

$$(T, \beta), T \in \mathbb{R}^{n_1 \times n_1}, 0 < \beta \in \mathbb{R}^{n_1}, T^0 = -T \cdot e > 0$$

and

$$(S, \alpha), S \in \mathbb{R}^{n_2 \times n_2}, 0 < \alpha \in \mathbb{R}^{n_2}, S^0 = -S \cdot e > 0$$

with n_1 and n_2 states are used for channel 1 and 2, respectively. Then the associated mean repair times are given by

$$\mathbb{E}(R_1) = -\beta^T \cdot T^{-1} \cdot e, \tag{3}$$
$$\mathbb{E}(R_2) = -\alpha^T \cdot S^{-1} \cdot e \tag{4}$$

and their variances are determined by

$$\mathbb{V}(R_1) = \beta^T \cdot T^{-2} \cdot e - (\beta^T \cdot T^{-1} \cdot e)^2, \tag{5}$$
$$\mathbb{V}(R_2) = \alpha^T \cdot S^{-2} \cdot e - (\alpha^T \cdot S^{-1} \cdot e)^2. \tag{6}$$

3 Steady-State Analysis of the Reliable Transport System

3.1 A Markov Model of the Redundant Transport System

The redundant technical system comprising these two coupled logical (or physical) channels of identical logical structure with failure events whose parameters are driven by a common Markov-modulated environment Y and independent, phase-type distributed repair times R_1, R_2, respectively, can be described by a state-vector process

$$Z(t) = (X(t), H(t), Y(t)) = ((X_1(t), X_2(t)), (H_1(t), H_2(t)), Y(t)), \ t \geq 0 \quad (7)$$

on the finite state space Σ which can be embedded into the set $\{0,1\}^2 \times \{0,1,\ldots,n_1\} \times \{0,1,\ldots,n_2\} \times \{1,\ldots,K\}$. The tuple

$$X(t) = (X_1(t), X_2(t))$$

indicates by $X_1(t) = i_1 = 1$ or $X_2(t) = i_2 = 1$ that at time t a failure has occurred in channel 1 or 2, respectively, and the related channel of the system is under repair. A state $i_1 = 0$ or $i_2 = 0$ indicates a proper operation of the corresponding channel. The initial operational state is given by $X(t) = (0,0)$ and the error state by $X(t) = (1,1)$ where no further operation is possible until the repair of at least one channel has been executed successfully. The component

$$H(t) = (H_1(t), H_2(t))) = (j_1, j_2) \in \{0,1,\ldots,n_1\} \times \{0,1,\ldots,n_2\}$$

records the phases $h = (j_1, j_2) \geq 0$ of the running repair processes for a state $i_1 = 1$ or $i_2 = 1$ where a state $j_l = 0$ indicates an idle repair function for given $i_l = 0, l \in \{1,2\}$.

The state variable $Z(t)$ and its state space are arranged such that $X(t) = (X_1(t), X_2(t))$ is the leading system indicator variable of the continuous-time Markov chain (CTMC) with a subspace $\Sigma_X = \{0,1,2,3\}$ of four states that is arranged according to a lexicographical ordering, i.e.

$$0 \equiv (0,0), 1 \equiv (0,1), 2 \equiv (1,0), 3 \equiv (1,1).$$

$(H(t), Y(t)) \in \Sigma_{(H,Y)}$ with

$$\begin{aligned}\Sigma_{(H,Y)} = \quad & \{(0,0)\} \times \{1,\ldots,K\} \\ & \cup \{0\} \times \{1,\ldots,n_2\} \times \{1,\ldots,K\} \\ & \cup \{1,\ldots,n_1\} \times \{0\} \times \{1,\ldots,K\} \\ & \cup \{1,\ldots,n_1\} \times \{1,\ldots,n_2\} \times \{1,\ldots,K\}\end{aligned}$$

indicates the residual phase space.

The initial state

$$Z(t) = z = (x, h, y) \quad \text{with} \quad x = (0,0) \in \Sigma_X$$

consists of the $j_1 = K$ microstates $(x, h, y) \in \{((0,0),(0,0))\} \times \{1,\ldots,K\}$, whereas the final error state with $x = (1,1) \in \Sigma_X$ comprises the $j_4 = n_1 \cdot n_2 \cdot K$ microstates $\{(1,1)\} \times \{1,\ldots,n_1\} \times \{1,\ldots,n_2\} \times \{1,\ldots,K\}$. The two failure states with $x \in \{(0,1),(1,0)\} \subset \Sigma_X$ with one channel under repair consists of $j_1 = n_2 \cdot K$ and $j_2 = n_1 \cdot K$ microstates $\{(0,1)\} \times \{0\} \times \{1,\ldots,n_2\} \times \{1,\ldots,K\}$ and $\{(1,0)\} \times \{1,\ldots,n_1\} \times \{0\} \times \{1,\ldots,K\}$, respectively.

3.2 Generator Matrix of the Markov Model

The resulting generator matrix A of this finite CTMC $Z(t)$ has a block structure of the following form:

$$A = \begin{pmatrix} A_{00} & A_{01} & A_{02} & 0 \\ A_{10} & A_{11} & 0 & A_{13} \\ A_{20} & 0 & A_{22} & A_{23} \\ 0 & A_{31} & A_{32} & A_{33} \end{pmatrix} \in \mathbb{R}^{N \times N}, \tag{8}$$

Analyzing the transition behavior of the failure interarrivals in the Markovian environment Y driven by the irreducible generator matrix Q and the PH-type repairs governed by (T,β), (S,α) that run independently of each other, we define the corresponding blocks A_{ij} of the generator matrix A in terms of the Kronecker product and the Kronecker sum, i.e. $B \otimes C = (B_{ij} \cdot C)_{ij}$,

$$B \oplus C = B \otimes I_l + I_m \otimes C$$

for block matrices $B \in \mathbb{R}^{m \times m}, C \in \mathbb{R}^{l \times l}$ and identity matrices I_l, I_m of appropriate dimensions $l > 0, m > 0$:

$$\begin{aligned}
A_{00} &= 1 \otimes (Q - \Lambda) = Q - \Lambda \\
A_{01} &= 1 \otimes \alpha^T \otimes \Lambda_2 = \alpha^T \otimes \Lambda_2 \\
A_{02} &= \beta^T \otimes 1 \otimes \Lambda_1 = \beta^T \otimes \Lambda_1 \\
A_{10} &= 1 \otimes S^0 \otimes I_K = S^0 \otimes I_K \\
A_{11} &= 1 \otimes S \otimes I_K + 1 \otimes I_{n_2} \otimes (Q - \Lambda_1) = S \oplus (Q - \Lambda_1) \\
A_{13} &= \beta^T \otimes I_{n_2} \otimes \Lambda_1 \\
A_{20} &= T^0 \otimes 1 \otimes I_K = T^0 \otimes I_K \\
A_{22} &= T \otimes 1 \otimes I_K + I_{n_1} \otimes 1 \otimes (Q - \Lambda_2) = T \oplus (Q - \Lambda_2) \\
A_{23} &= I_{n_1} \otimes \alpha^T \otimes \Lambda_2 \\
A_{31} &= T^0 \otimes I_{n_2} \otimes I_K \\
A_{32} &= I_{n_1} \otimes S^0 \otimes I_K \\
A_{33} &= T \otimes I_{n_2} \otimes I_K + I_{n_1} \otimes S \otimes I_K + I_{n_1} \otimes I_{n_2} \otimes Q = (T \oplus S) \oplus Q \\
A_{03} &= A_{30} = A_{12} = A_{21} = 0
\end{aligned}$$

We define the part of the generator matrix A on the operational states

$$O = \{0,1,2\} \equiv \{(0,0),(0,1),(1,0)\} \subset \Sigma_X$$

excluding the failure state

$$F = \{3\} \equiv \{(1,1)\} \subset \Sigma_X$$

by the block matrix

$$A_O = \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & 0 \\ A_{20} & 0 & A_{22} \end{pmatrix} \in \mathbb{R}^{M \times M} \tag{9}$$

with $M = K \cdot (1 + n_1 + n_2)$.

3.3 Calculation of the Steady-State Vector

Let $\Pi^T = (\Pi_0^T, \Pi_1^T, \Pi_2^T, \Pi_3^T) \gg 0$ denote the partitioned, unique steady-state row vector of the irreducible Markov chain $Z(t)$ supposing an irreducible Markovian environment matrix Q and two irreducible phase-type generators $T + T^0\beta^T, S + S^0\alpha^T$.

We may calculate Π by efficient numerical solution methods for ergodic, finite Markov chains such as direct or iterative solution techniques of the balance equations

$$\Pi^T \cdot A = 0, \quad \Pi^T \cdot e = 1, \tag{10}$$

for instance, aggregation-disaggregation methods such as an additive or multiplicative Schwarz decomposition method or another iterative scheme derived from an M-splitting (cf. [4,15–18,24]).

Let $\widetilde{A} = -A^T$ denote the irreducible M-matrix associated with the generator matrix A and

$$A = L + U - \Delta$$

be the block matrix decomposition into the diagonal block matrix

$$\Delta = -\mathrm{Diag}(A_{00}, A_{11}, A_{22}, A_{33}), \tag{11}$$

and lower- and upper-diagonal block matrices

$$L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ A_{10} & 0 & 0 & 0 \\ A_{20} & 0 & 0 & 0 \\ 0 & A_{31} & A_{32} & 0 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & A_{01} & A_{02} & 0 \\ 0 & 0 & 0 & A_{13} \\ 0 & 0 & 0 & A_{23} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

respectively. Then we define the associated M-splitting

$$\widetilde{A} = -A^T = M - N$$

with the corresponding transposed matrices of the block matrix decomposition $M = \Delta^T - U^T, N = L^T$ and get the iteration matrix

$$J = M^{-1} \cdot N = [\Delta^T - U^T]^{-1} \cdot L^T$$

and the nonnegative matrix

$$\widetilde{T} = I - \widetilde{A} \cdot M^{-1} = N \cdot M^{-1} \tag{13}$$

with the property $M^{-1} \cdot \widetilde{T} \cdot M = J$. Then the column-stochastic matrix

$$T = I - \omega \widetilde{A} \cdot M^{-1} = (1-\omega)I + \omega \widetilde{T} \tag{14}$$

is a semiconvergent, nonnegative matrix for any scaling $\omega \in (0,1)$ (cf. [4,15,24]).

Based on the block matrix decomposition of A in (8) we determine a partition $\Gamma = \{J_1, J_2, J_3, J_4\}$ into $m = 4$ subsets of the state space $\Sigma = \{1, \ldots, N\}$, $N = K + n_2 \cdot K + n_1 \cdot K + (n_1 \cdot n_2) \cdot K$, with the four disjoint subsets $J_1 = \{1, \ldots, K\}$, $J_2 = \{K+1, \ldots, (1+n_2) \cdot K\}$, $J_3 = \{(1+n_2) \cdot K + 1, \ldots, (1+n_1+n_2) \cdot K\}$, $J_4 = \{(1+n_1+n_2) \cdot K + 1, \ldots, (1+n_1+n_2+n_1 \cdot n_2) \cdot K\}$.

Then the following iterative aggregation-disaggregation (IAD) algorithm is semiconvergent to the unique, normalized steady-state solution vector

$$\Pi^T = (\Pi_0^T, \Pi_1^T, \Pi_2^T, \Pi_3^T) \gg 0$$

with its components $\Pi_i^T \gg 0$ on the partition set J_{i+1} for each state $i \in \{0, 1, 2, 3\}$ (cf. [17,18]):

IAD-Algorithm for M-Matrices with Block Structure

1. We choose an initial probability vector

$$x^{(0)} \gg 0, e^T \cdot x^{(0)} = 1,$$

e.g., the uniform distribution, and real numbers $\epsilon, c_1, c_2 \in (0,1)$. We construct an aggregation matrix R and a prolongation matrix $P(x)$ for $0 < x \in \mathbb{R}^N$, $e^T \cdot x = 1$, by

$$R = \begin{pmatrix} e_{J_1}^T & 0 & 0 & 0 \\ 0 & e_{J_2}^T & 0 & 0 \\ 0 & 0 & e_{J_3}^T & 0 \\ 0 & 0 & 0 & e_{J_4}^T \end{pmatrix} \in \mathbb{R}^{4 \times N},$$

$$P(x) = \begin{pmatrix} y_1 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 \\ 0 & 0 & y_3 & 0 \\ 0 & 0 & 0 & y_4 \end{pmatrix} \in \mathbb{R}^{N \times 4},$$

and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^N$, in terms of

$$[\alpha(x)]_j = e^T \cdot x_j$$

and

$$[y(x)]_j = x_j / [\alpha(x)]_j$$

provided that $x_j > 0$ holds and the uniform distribution in case of $x_j = 0$ for given $j \in \{1, \ldots, 4\}$. Using the iteration matrix T in (14) and $x \in \mathbb{R}^N$ we get the associated aggregated matrix $B(x) \in \mathbb{R}^{4 \times 4}$ by

$$B(x) = R \cdot T \cdot P(x).$$

We set $k = 0$ and

$$r(x) = ||(I - T) \cdot x||_1$$

for the L_1-norm $||x||_1 = \sum_1^N |x_i|$ in $\mathbb{R}^N$.

2. We solve

$$B(x^{(k)}) \cdot \alpha(x^{(k)}) = \alpha(x^{(k)})$$

subject to

$$e^T \cdot \alpha(x^{(k)}) = 1, \alpha(x^{(k)}) > 0$$

and compute

$$\widetilde{x} = P(x^{(k)}) \cdot \alpha(x^{(k)}).$$

3. We compute

$$x^{(k+1)} = T \cdot \widetilde{x}.$$

4. If

$$r(\widetilde{x}) \leq c_1 \cdot r(x^{(k)})$$

then go to step 5
else compute

$$x^{(k+1)} = T^h \cdot \widetilde{x}$$

for $h > 1$ such that

$$r(x^{(k+1)}) \leq c_2 \cdot r(x^{(k)})$$

endif

5. If

$$||x^{(k+1)} - x^{(k)}||_1 / ||x^{(k)}||_1 < \epsilon$$

then step 6
else

$$k = k + 1,$$

and go to step 2
endif

6. At the end we perform a normalization after a successful convergence test:

$$\Pi = \frac{M^{-1} \cdot x^{(k+1)}}{e^T \cdot M^{-1} \cdot x^{(k+1)}}$$

Regarding the semiconvergence of this specific IAD-algorithm to the probability vector Π, we refer to the existing convergence theory related to numerical solution methods for finite Markov chains (cf. [17,18,24]).

4 Computing the Reliability Function of the Redundant Transport System

The reliability of the transport system is characterized by the sojourn time $S_T \geq 0$ in the set of the operational states $\widehat{O} = \{z = (x, h, y) \in \Sigma \mid x \in O \subset \Sigma_X\}$ of the overall state space Σ subject to the start in one of those states $z \in \widehat{O}$ in the steady-state regime with the steady-state row vector

$$\Pi_O^T = (\Pi_0^T, \Pi_1^T, \Pi_2^T) \gg 0$$

and its positive components $\Pi_i^T \gg 0$ associated with each non-failure state $i \in O = \{0, 1, 2\} \subset \Sigma_X$.

Then we can calculate the reliability function $F_R(t)$ as time-dependent probability of the Markov chain $Z(t)$ to reside in a state $z \in \widehat{O}$ up to time $t > 0$ given that the capturing in the absorbing states $\widehat{F} = \{z = (x, h, y) \in \Sigma \mid x \in F \subset \Sigma_X\}$ does not occur (see (9)):

$$\begin{aligned} F_R(t) &= \mathbb{P}\{S_T > t \mid Z(0) \in \widehat{O}\} \cdot \mathbb{P}\{Z(0) \in \widehat{O}\} \\ &= \mathbb{P}\{Z(t) \notin \widehat{F} \mid Z(0) \in \widehat{O}\} \cdot \mathbb{P}\{Z(0) \in \widehat{O}\} \qquad (15) \\ &= \Pi_O^T \cdot \exp(A_O t) \cdot e \qquad (16) \end{aligned}$$

The computation of the exponential matrix $\exp(A_O t)$ can be effectively performed by means of a uniformization approach (cf. [24]).

Let $D = \mathrm{Diag}(D_{ii}) > 0$ denote the diagonal matrix determined by the positive diagonal elements $D_{ii} = -(A_O)_{ii} > 0, i \in \{1, \ldots, M\}$, of the M-matrix $-A_O$ in (9). We set

$$\gamma = \max_{1 \leq i \leq M} (D_{ii}) > 0$$

and define the sub-stochastic matrix $P_O = (P_{ij}), 1 \leq i \leq M, 1 \leq j \leq M$, by the transition probabilities

$$P_{ij} = \begin{cases} A_{ij}/\gamma \geq 0, & i \neq j \\ 1 + A_{ii}/\gamma \geq 0, & i = j \end{cases} \quad \text{or} \quad P_O = I_M + A_O/\gamma$$

at the embedded time epochs of events in the Markov chain $Z(t)$. Then it holds

$$A_O = \gamma \cdot (P_O - I_M)$$

with the identity matrix $I_M \in \mathbb{R}^{M \times M}$. It implies the representation

$$\begin{aligned} R_O(t) &= \exp(A_o \cdot t) = \exp(\gamma t \cdot (P_O - I_M)) \\ &= \left[\sum_{n=0}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t} \cdot ((P_O)^n)_{ij} \right]_{1 \leq i,j \leq M} \qquad (17) \end{aligned}$$

that enables a fast computation of the reliability function $F_R(t)$ in (16) as matrix-vector product

$$\begin{aligned} F_R(t) &= \Pi_O^T \cdot R_O(t) \cdot e \\ &= \sum_{n=0}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t} \cdot {\Pi_O}^T \cdot (P_O)^n \cdot e \qquad (18) \end{aligned}$$

by means of a Poisson distribution with parameter γt and the n-th powers of the sub-stochastic matrix P_O.

5 Conclusions

In our study we have considered a multipath communication that is realized between clients and servers of new cloud computing concepts like fog and mobile edge computing at the transport and session layers of an underlying SDN/NFV network (cf. [7,14,20,23]). The latter network integrates diverse radio technologies into its attached RAN environments and supports the efficient interprocess communication of new services between the clients and servers.

We have modelled the reliability of the associated redundant transport system comprising two logical channels, that are susceptible to failures and restored with general phase-type distributed repair times, and computed its corresponding reliability function. The failure processes of both channels are described by Markov-modulated failure times that are driven by the transitions of a common Markovian environment.

First we have modelled this error-prone system with repair by a finite state, continuous-time Markov chain. We have identified its generator matrix in terms of associated Kronecker products of the underlying parameter matrices which are related to the Markov-modulated interarrival times of failures and the phase-type distributed repair times. We have shown that the steady-state distribution of the restoration model can be effectively computed by an iterative aggregation-disaggregation method and then computed the associated reliability function of the transport system by means of an appropriately defined finite, absorbing Markov chain.

Our further studies will elaborate on the extension of the reliability model to more general random environments. We will further consider its application to SDN/NFV networks with an integrated 5G RAN that can support fog and mobile edge computing in advanced IoT and machine-to-machine communication scenarios discussed in the literature (cf. [1,6,14,19]).

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