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Date of secondary publication: 27.04.2026

Accepted Manuscript (Postprint), Conferenceobject

Persistent identifier: urn:nbn:de:bvb:473-irb-114826x

Primary publication

Krieger, Udo R.; Markovich, Natalia M. (2020): Reliability Analysis of a Multipath Transport System in Fog Computing, in: Piotr Gaj, Wojciech Gumiński, and Andrzej Kwiecien (Ed.), Computer Networks: 27th International Conference, CN 2020, Gdańsk, Poland, June 23–24, 2020, Proceedings, Cham: Springer International Publishing, pp. 101–116, doi: 10.1007/978-3-030-50719-0_9.

Reliability Analysis of a Multipath Transport System in Fog Computing

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Abstract. We consider a fog computing approach with function virtualization in an IoT scenario that uses an SDN/NFV protocol stack and multipath communication between its clients and servers at the transport and session layers. We analyze the reliability of the associated redundant transport system comprising two logical channels that are susceptible to random failures. We model the error-prone system with a single repair unit and independent phase-type distributed repair times by a Marshall-Olkin failure model. The failure processes of both channels are described by general Markov-modulated Poisson processes (MMPPs) that are associated with the corresponding failure times and that are driven by the transitions of a common random environment. First we identify the generator matrix of the associated continuous-time Markov chain that is determined by the interarrival times of the Markov-modulated failure processes and the independent phase-type distributed repair times and the Kronecker-product structures of their associated parameter matrices. Then we show that the steady-state distribution of the restoration model can be effectively calculated by a semiconvergent iterative aggregation-disaggregation method for block matrices. Finally, we compute the associated reliability function and hazard rate of the multipath transport system.

Keywords: Fog computing · Marshall-Olkin failure model · Reliability function · Markov-modulated arrival process · Phase-type distributed repair times

1 Introduction

In recent years the cloud computing approach has been refined by mobile edge and fog computing to cover the technical challenges of new applications arising from the rapidly evolving Internet-of-Things (IoT) (cf. [1,2,8]). These architectures try to integrate new services based on advanced multimedia and machine-to-machine communication into the associated computing, storage, and internetworking infrastructures. They are based on modern software-defined networks

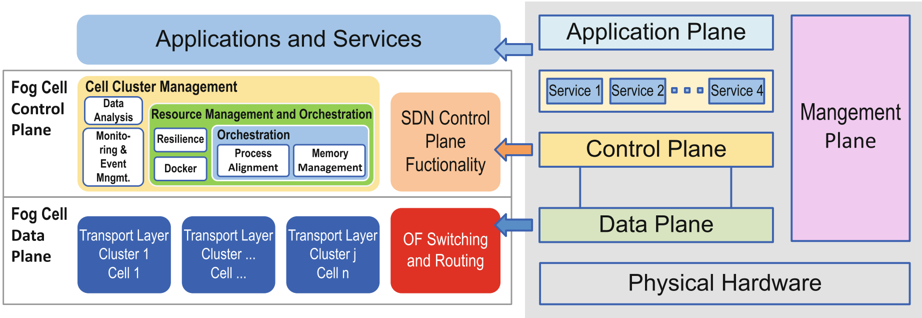

Fig. 1. Protocol stack of fog computing in a virtualized software environment.

(SDN), network function virtualization (NFV), and microservice concepts (cf. [14,24]). The fog computing architecture can be derived from the classical clear separation of functionalities into an application and services plane, a control plane, a data plane, and a management plane (see Fig. 1, cf. [7,9]).

In this context of client-server processing it has been realized that a multipath communication which is established at the transport and session layers of the SDN/NFV protocol stack can substantially improve the capacity and reliability of the required fast interprocess communication. Adapted new transport layer protocols such as multipath TCP or multipath QUIC can be applied to establish redundant transport paths between clients and servers. The required multi-homing is realized by a use of multiple interfaces and will also be supported in the upcoming mobile settings of the local area and wide area 5G standards (see Fig. 2, cf. [3,5,20]).

In this paper we investigate a basic multipath transport system comprising two logical transport channels which provide redundant high-speed interprocess communication paths between a client as sender and a server as receiver in an SDN/NFV/5G-RAN environment. Due to the presumed existence of an exclusive, virtualized restoration function in the management plane, we describe the impact of a functional outage of each transport channel subject to random errors by a generalized Marshall-Olkin failure model (cf. [13,21–23]). We assume here that the entire redundant system is managed by a scalable, virtualized management system applying container virtualization techniques like Docker or Kubernetes (cf. [11,12]). It can instantiate a single repair function as virtual network function and provides a restoration of the original transport status after a generally distributed, nonnegative restoration period. We approximate the latter entity by random variables with phase-type (PH) distributions (cf. [25]).

Our main goal is to derive a Markovian reliability model of this redundant transport system in a random environment and to compute its reliability function and hazard rate. For this purpose we apply an effective computational solution method to a finite continuous-time Markov chain. In this way we enhance our related previous study [19] to a new setup of the underlying reliability model in a highly relevant technical context of fog computing. The new model exhibits a much more sophisticated algebraic structure of its generator matrix due to the involved three correlated Markov-modulated Poisson arrival processes of the failure patterns and the engagement of a single virtualized repairman function within the management layer of the fog computing architecture (see Fig. 3).

The paper is organized as follows. In Sect. 2 we describe the multipath transport system with two error-prone channels and a generalized Marshall-Olkin failure model. In Sect. 3 we derive the associated finite Markov chain with its generator matrix and calculate its related steady-state vector by an effective semiconvergent iteration scheme. In Sect. 4 we compute the reliability function and hazard rate of the multipath transport system. Finally, some conclusions and an outlook on further performance studies are presented.

2 Characterizing the Reliability of a Multipath Transport System by a Generalized Marshall-Olkin Model

We consider the hierarchical logical structure of a fog computing system that is deployed as edge computing infrastructure within the continuum between the cloud and the edge devices of an IoT environment (see Fig. 2, cf. [2,8]).

2.1 Description of a Transport System Related to Fog Computing

We assume that the fog computing environment is constructed by means of lightweight virtualization technology based on Linux containers and the orchestration framework Docker (cf. [9,12]). Dedicated stationary or mobile fog gateways provide the first logical entrance gates of its computing, storage and networking infrastructure. They realize aggregation points of the collected data streams generated by IoT sensor systems of corresponding smart edge devices in an associated fog cell (cf. Fig. 2, [9]). These generated data flows may be preprocessed and then forwarded to fog nodes in a higher logical layer or to processing and storage nodes in a distributed cloud infrastructure (cf. Fig. 2, see [7,26]). The latter nodes may also interact with a blockchain to support an immutable event history and the secure transfer of anonymous data elements in an underlying IoT infrastructure (cf. [7,26]). The minimal set of redundant transport paths between a fog node and more powerful nodes in the fog hierarchy or edge and cloud computing nodes, respectively, is modelled by two logical channels operating in hot stand-by mode. These entities are subject to random failures that may strike either one or both channels simultaneously. We suppose that each erroneous channel is immediately handled by independent repair activities which are triggered by a virtual surveillance function. The latter is realized in the management layer of the fog computing architecture (see Fig. 3).

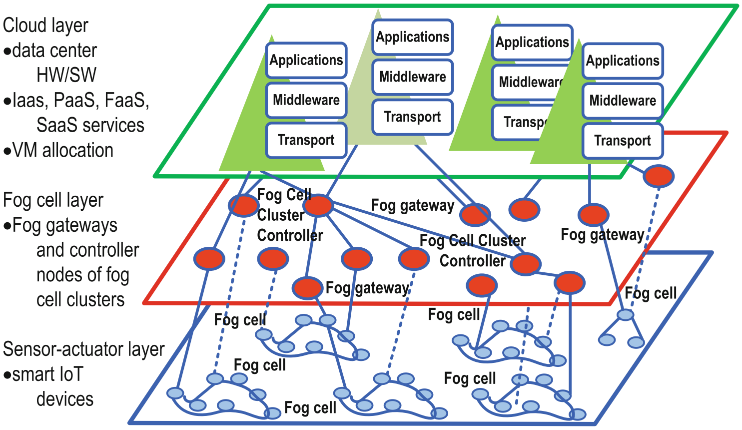

Fig. 2. A hierarchical fog computing architecture supporting IoT data processing in the fog cells (see also [7,26]).

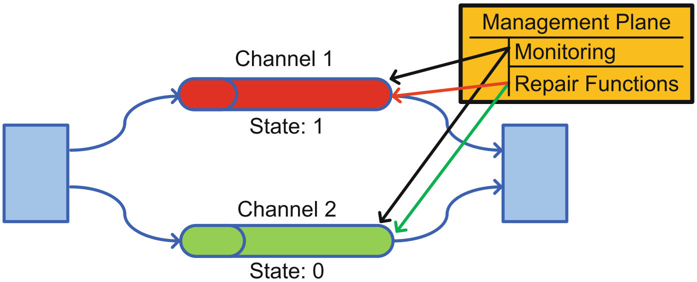

Fig. 3. A logical channel model of the redundant transport system with an operational channel 2 and an erroneous channel 1.

2.2 A Generalized Marshall-Olkin Failure Model of the Multipath Transport System

The considered multipath transport system comprises these two coupled logical channels. We suppose that they exhibit an identical logical structure. Therefore, we conclude that the error events are governed by a failure model of Marshall-Olkin type (see Figs. 2, 3, also [13,21–23]).

The transfer function of each channel is hit by different kinds of errors that are triggered by a common internal or external environment. We can assume that a system failure occurs if the throughput along a channel drops below a certain predefined threshold or a total outage of the transport functions is observed. Then the latter error patterns are modelled by a Marshall-Olkin failure model with three correlated Markov-modulated Poisson processes (MMPPs) (cf. [10]) which are driving these failures on the individual channels 1 and 2, respectively, or strike both of them simultaneously (cf. [13,21–23]). This MMPP class of Markovian arrival processes is an important subset the well-known general Markovian arrival processes (MAPs) (cf. [6]). We suppose that the latter MMPP processes can be described by the state of a common Markov-modulating environment $\{Y(t), t \geq 0\}$ in continuous time with a finite state space $\Sigma_Y = \{1, \dots, K\}, K \in \mathbb{N}$ and an irreducible $K-$state generator matrix $Q \in \mathbb{R}^{K \times K}$. Its associated unique steady-state probability vector is denoted by $p \in \mathbb{R}^K$. It is determined by the solution of the linear system $p^T \cdot Q = 0, p^T \cdot e = 1$ with the vector of all ones $e \in \mathbb{R}^K$.

In the following, we apply the order relation $0 \ll x$ for vectors $x \in \mathbb{R}^N$. It shows that all components $x_i > 0$ of a vector $x \in \mathbb{R}^N$ are positive. In contrast, the order relation $0 < x$ indicates that $x \in \mathbb{R}^N$ is a nonnegative, non-zero vector, i.e. $0 \leq x_i$ for all $i \in \{1, \dots N\}$ and $0 < x_i$ holds for at least one i (cf. [4]).

Considering a given state $Y(t) = j \in \Sigma_Y$ of the modulating environment, we assume that the interarrival times of any isolated failures imposed on channel 1 and 2 appear as independent exponentially distributed events with mean values $1/\lambda_{1j}$ and $1/\lambda_{2j}$, respectively, whereas a common failure is governed by the mean values $1/\lambda_{3j}$.

Let $0 \ll \lambda_1 = (\lambda_{11}, \dots, \lambda_{1K})^T \in \mathbb{R}^K$, $0 \ll \lambda_2 = (\lambda_{21}, \dots, \lambda_{2K})^T \in \mathbb{R}^K$, and $0 \ll \lambda_3 = (\lambda_{31}, \dots, \lambda_{3K})^T \in \mathbb{R}^K$ be the positive column vectors of these associated arrival rates and $\Lambda_1 = \mathrm{Diag}(\lambda_1) > 0, \Lambda_2 = \mathrm{Diag}(\lambda_2) > 0$, $\Lambda_3 = \mathrm{Diag}(\lambda_3) > 0$ denote the corresponding diagonal-positive diagonal matrices of these arrival rate vectors of the failures in the random environment Y. Let $\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3$ be the arrival rate matrix of the superimposed MMPP arrival process of all correlated errors. Then the mean arrival rates of these three basic point processes are given by $\widehat{\lambda}_i = p^t \cdot \Lambda_i \cdot e = p^t \cdot \lambda_i, i \in \{1, 2, 3\}$, and $\widehat{\lambda} = p^t \cdot \Lambda \cdot e = \widehat{\lambda}_1 + \widehat{\lambda}_2 + \widehat{\lambda}_3$ holds (cf. [10]). We set $\overline{\Lambda}_1 = \Lambda_1 + \Lambda_3$ and $\overline{\Lambda}_2 = \Lambda_2 + \Lambda_3$ as arrival rate matrices of two corresponding MMPP processes that arise from a superposition of the streams 1 and 3 as well as 2 and 3, respectively.

Furthermore, we suppose that the initiated repair processes after an isolated error of channel 1 or 2, respectively, or the single maintenance process of both channels after a simultaneous outage are described by independent, phase-type distributed repair times R_1, R_2, R_3, respectively. Their stochastic characteristics are governed by general phase-type distributions

$$F_1(x) = \mathbb{P}\{R_1 \leq x\} = 1 - \beta^T \cdot \exp(T \cdot x) \cdot e, \quad (1)$$

$$F_2(x) = \mathbb{P}\{R_2 \leq x\} = 1 - \alpha^T \cdot \exp(S \cdot x) \cdot e, \quad (2)$$

$$F_3(x) = \mathbb{P}\{R_3 \leq x\} = 1 - \gamma^T \cdot \exp(U \cdot x) \cdot e \quad (3)$$

with the corresponding probability densities on the support set $[0, \infty) \subset \mathbb{R}$

$$f_1(x) = d\mathbb{P}\{R_1 \leq x\}/dt = \beta^T \cdot \exp(T \cdot x) \cdot T^0, \quad (4)$$

$$f_2(x) = d\mathbb{P}\{R_2 \leq x\}/dt = \alpha^T \cdot \exp(S \cdot x) \cdot S^0, \quad (5)$$

$$f_3(x) = d\mathbb{P}\{R_3 \leq x\}/dt = \gamma^T \cdot \exp(U \cdot x) \cdot U^0. \quad (6)$$

Here e denotes the vector of all ones of corresponding dimension. It means that three finite state phase-type representation matrices

$$(T,\beta), T \in \mathbb{R}^{n_1 \times n_1}, 0 < \beta \in \mathbb{R}^{n_1}, T^0 = -T \cdot e > 0, \tag{7}$$
$$(S,\alpha), S \in \mathbb{R}^{n_2 \times n_2}, 0 < \alpha \in \mathbb{R}^{n_2}, S^0 = -S \cdot e > 0, \tag{8}$$
$$(U,\gamma), U \in \mathbb{R}^{n_3 \times n_3}, 0 < \gamma \in \mathbb{R}^{n_3}, U^0 = -U \cdot e > 0 \tag{9}$$

with n_1, n_2, and n_3 states are used. Then the associated mean repair times are given by

$$\mathbb{E}(R_1) = 1/\mu_1 = -\beta^T \cdot T^{-1} \cdot e, \quad \mathbb{E}(R_2) = 1/\mu_2 = -\alpha^T \cdot S^{-1} \cdot e, \tag{10}$$
$$\mathbb{E}(R_3) = 1/\mu_3 = -\gamma^T \cdot U^{-1} \cdot e \tag{11}$$

and their variances are determined by

$$\mathbb{V}ar(R_1) = 2\beta^T \cdot T^{-2} \cdot e - (\beta^T \cdot T^{-1} \cdot e)^2, \tag{12}$$
$$\mathbb{V}ar(R_2) = 2\alpha^T \cdot S^{-2} \cdot e - (\alpha^T \cdot S^{-1} \cdot e)^2, \tag{13}$$
$$\mathbb{V}ar(R_3) = 2\gamma^T \cdot U^{-2} \cdot e - (\gamma^T \cdot U^{-1} \cdot e)^2. \tag{14}$$

Then the overall state of the multipath transport system can be described for $t \geq 0$ by a vector process

$$Z(t) = (X(t), M(t), Y(t)) = ((X_1(t), X_2(t)), (M_1(t), M_2(t), M_3(t)), Y(t)) \tag{15}$$

on the finite state space $\Sigma \subset \{0,1\}^2 \times \{0,1,\ldots,n_1\} \times \{0,1,\ldots,n_2\} \times \{0,1,\ldots,n_3\} \times \{1,\ldots,K\}$. The binary tuple $X(t) = (X_1(t), X_2(t)) = (i_1, i_2) \in \Sigma_X = \{0,1\}^2$ indicates by $X_1(t) = i_1 = 1$ or $X_2(t) = i_2 = 1$ that at time t a failure has occurred in channel 1 or 2, respectively, and the related channel is repaired by a virtual maintenance function of the transport system. A state $i_1 = 0$ or $i_2 = 0$ indicates a proper operation of the respective transport channel. $X(t) = (0,0)$ corresponds to the initial operational state and the failure state is determined by $X(t) = (1,1)$ where no further operation is possible until the maintenance process has been successfully executed on both channels. The common maintenance component

$$M(t) = (M_1(t), M_2(t), M_3(t)) = m = (m_1, m_2, m_3) \in \Sigma_M, \tag{16}$$

$\Sigma_M \subseteq \{0,1,\ldots,n_1\} \times \{0,1,\ldots,n_2\} \times \{0,1,\ldots,n_3\}$, records the phases $m = (m_1, m_2, m_3) \geq 0$ of the running repair processes for a state $i_1 = 1$ or $i_2 = 1$. Here a state $m_k = 0, k \in \{1,2,3\}$ indicates an idle repair function for a given related level component $i_l = 0, l \in \{1,2\}$.

We arrange the state variable $Z(t)$ and its overall state space Σ in such a way that the level process $X(t) = (X_1(t), X_2(t)) \in \Sigma_X$ is the leading indicator variable of the continuous-time Markov chain with the subspace $\Sigma_X = \{0,1,2,3\}$. Its four states are arranged according to a lexicographical ordering, i.e., it is

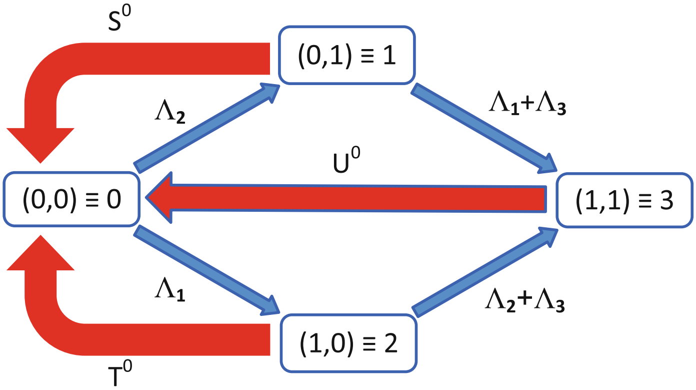

Fig. 4. Model of the states and transitions with their associated rate vectors and matrices of the related failure arrival epochs and maintenance completion events.

given by a binary encoding $0 \equiv (0,0), 1 \equiv (0,1), 2 \equiv (1,0), 3 \equiv (1,1)$. The phase variable $(M(t), Y(t)) \in \Sigma_{(M,Y)}$ with the phase state space

$$\begin{aligned} \Sigma_{(M,Y)} = \quad & \{(0,0,0)\} \times \{1,\dots,K\} \\ & \cup \{0\} \times \{1,\dots,n_2\} \times \{0\} \times \{1,\dots,K\} \\ & \cup \{1,\dots,n_1\} \times \{0\} \times \{0\} \times \{1,\dots,K\} \\ & \cup \{0\} \times \{0\} \times \{1,\dots,n_3\} \times \{1,\dots,K\} \end{aligned} \tag{17}$$

indicates the residual set of the microstates.

The initial state $Z(t) = z = (x, m, y)$ with $x = (0,0) \in \Sigma_X$ consists of the $j_0 = n_0 \cdot K = K, n_0 = 1$, microstates $(x, m, y) \in \{((0,0),(0,0,0))\} \times \{1,\dots,K\}$, whereas the final error state with $x = (1,1) \in \Sigma_X$ comprises the $j_3 = n_3 \cdot K$ microstates $\{(1,1)\} \times \{(0,0)\} \times \{1,\dots,n_3\} \times \{1,\dots,K\}$. The two failure states with $x \in \{(0,1),(1,0)\} \subset \Sigma_X$ with one channel under repair consist of $j_1 = n_2 \cdot K$ and $j_2 = n_1 \cdot K$ microstates $\{(0,1)\} \times \{0\} \times \{1,\dots,n_2\} \times \{0\} \times \{1,\dots,K\}$ and $\{(1,0)\} \times \{1,\dots,n_1\} \times \{0\} \times \{0\} \times \{1,\dots,K\}$, respectively. A model of the state space with the associated transition vectors and matrices of the corresponding failure arrival epochs and maintenance completion events is illustrated in Fig. 4.

2.3 Analysis of a Simplified Marshall-Olkin Failure Model of the Redundant Transport System

We now consider the simplified Marshall-Olkin failure model with three independent Poisson input streams as failure triggers whose arrival rates are given by $\widehat{\lambda}_i = p^t \cdot \Lambda_i \cdot e = p^t \cdot \lambda_i, i \in \{1,2,3\}$. Furthermore, we may select the previously specified three independent phase-type distributed repair processes R_1, R_2, R_3 with the finite means $1/\mu_i = \mathbb{E}(R_i), i \in \{1,2,3\}$. Applying the steady-state results of Rykov et al. [23, Theorem 2], one can determine the steady-state probabilities $\Pi^{(S)} = (\Pi_0^{(S)}, \Pi_1^{(S)}, \Pi_2^{(S)}, \Pi_3^{(S)})^T \in \mathbb{R}^4$ of this simplified Marshall-Olkin failure model on the corresponding levels $i \in \Sigma_X = \{0,1,2,3\}$ in the following form:

$$\Pi_1^{(S)} = \Pi_0^{(S)} \cdot \frac{\widehat{\lambda}_1}{\widehat{\lambda}_2 + \widehat{\lambda}_3} \cdot \left[1 - \beta^T \cdot \left((\widehat{\lambda}_2 + \widehat{\lambda}_3)I - T\right)^{-1} \cdot T^0\right] \tag{18}$$

$$\Pi_2^{(S)} = \Pi_0^{(S)} \cdot \frac{\widehat{\lambda}_2}{\widehat{\lambda}_1 + \widehat{\lambda}_3} \cdot \left[1 - \alpha^T \cdot \left((\widehat{\lambda}_1 + \widehat{\lambda}_3)I - S\right)^{-1} \cdot S^0\right] \tag{19}$$

$$\begin{aligned} \Pi_3^{(S)} = \frac{\Pi_0^{(S)}}{\mu_3} \cdot \Bigg(\widehat{\lambda}_1 \cdot \left[1 - \beta^T \cdot \left((\widehat{\lambda}_2 + \widehat{\lambda}_3)I - T\right)^{-1} \cdot T^0\right] \\ + \widehat{\lambda}_2 \cdot \left[1 - \alpha^T \cdot \left((\widehat{\lambda}_1 + \widehat{\lambda}_3)I - S\right)^{-1} \cdot S^0\right] + \widehat{\lambda}_3\Bigg) \end{aligned} \tag{20}$$

$$\begin{aligned} \Pi_0^{(S)} = \Bigg(1 + \frac{\widehat{\lambda}_1}{\widehat{\lambda}_2 + \widehat{\lambda}_3} \cdot \left[1 - \beta^T \cdot \left((\widehat{\lambda}_2 + \widehat{\lambda}_3)I - T\right)^{-1} \cdot T^0\right] \cdot \left[1 + \frac{\widehat{\lambda}_2 + \widehat{\lambda}_3}{\mu_3}\right] \\ + \frac{\widehat{\lambda}_2}{\widehat{\lambda}_1 + \widehat{\lambda}_3} \cdot \left[1 - \alpha^T \cdot \left((\widehat{\lambda}_1 + \widehat{\lambda}_3)I - S\right)^{-1} \cdot S^0\right] \cdot \left[1 + \frac{\widehat{\lambda}_1 + \widehat{\lambda}_3}{\mu_3}\right] + \frac{\widehat{\lambda}_3}{\mu_3}\Bigg)^{-1} \end{aligned} \tag{21}$$

This vector $\Pi^{(S)} \in \mathbb{R}^4$ can be used to approximate the initial steady-state solution $\alpha(x^{(0)})$ of the first aggregation system (47) that is triggering the disaggregation step (48) and the following iteration step (49) of the IAD approach in Subsect. 3.3.

3 Analyzing the Markov Model of the Transport System

In the following we investigate the finite continuous-time Markov chain (CTMC) $\{Z(t), t \geq 0\}$ that is used to analyze the reliability behavior of the described multipath transport system subject to the sketched generalized Marshall-Olkin failure model of its basic redundant, erroneous transport channels.

3.1 Generator Matrix of the Finite Markov Chain

In the following we consider the three different state sets $\{IOS, FS, IES\} \subset \mathcal{P}(\Sigma_X)$ comprising the initial operational state (IOS) $X(t) = x = (0,0) \in \Sigma_X$, the complete failure state (FS) $X(t) = x = (1,1) \in \Sigma_X$, and the cluster of isolated error states (IES) $X(t) = x \in \{(0,1),(1,0)\} \subset \Sigma_X$. Then the resulting generator matrix A of this finite CTMC $Z(t)$ has a block structure on the corresponding microstates $(x, m, y) \in \Sigma$ which is related to a redundant system with the Marshall-Olkin failure behavior (cf. [13,21–23]):

$$A = \begin{pmatrix} A_{00} & A_{01} & A_{02} & A_{03} \\ A_{10} & A_{11} & 0 & A_{13} \\ A_{20} & 0 & A_{22} & A_{23} \\ A_{30} & 0 & 0 & A_{33} \end{pmatrix} \in \mathbb{R}^{N \times N}, \tag{22}$$

The transition behavior of the failure interarrivals in the random environment Y is driven by the irreducible generator matrix Q. The three PH-type driven repair processes are governed by (T, β), (S, α), (U, γ) that run independently of each other. Subsequently, we define the corresponding blocks A_{ij} of the generator matrix A in terms of the Kronecker product and Kronecker sum, i.e. $F \otimes E = (F_{ij} \cdot E)_{ij}$, and $F \oplus E = F \otimes I_l + I_m \otimes E$ for block matrices $F \in \mathbb{R}^{m \times m}, E \in \mathbb{R}^{l \times l}$ and identity matrices I_l, I_m as well as vectors of all ones e_l, e_m of appropriate dimensions $l > 0, m > 0$:

$$A_{00} = 1 \otimes (Q - \Lambda) = Q - \Lambda \tag{23}$$
$$A_{01} = 1 \otimes \alpha^T \otimes 1 \otimes \Lambda_2 = \alpha^T \otimes \Lambda_2 \tag{24}$$
$$A_{02} = \beta^T \otimes 1 \otimes 1 \otimes \Lambda_1 = \beta^T \otimes \Lambda_1 \tag{25}$$
$$A_{03} = 1 \otimes 1 \otimes \gamma^T \otimes \Lambda_3 = \gamma^T \otimes \Lambda_3 \tag{26}$$
$$A_{10} = 1 \otimes S^0 \otimes 1 \otimes I_K = S^0 \otimes I_K \tag{27}$$
$$\begin{aligned} A_{11} &= 1 \otimes S \otimes 1 \otimes I_K + 1 \otimes I_{n_2} \otimes 1 \otimes (Q - \Lambda_1 - \Lambda_3) \\ &= S \oplus (Q - \Lambda_1 - \Lambda_3) \end{aligned} \tag{28}$$
$$A_{13} = 1 \otimes e_{n_2} \otimes \gamma^T \otimes (\Lambda_1 + \Lambda_3) = e_{n_2} \otimes \gamma^T \otimes (\Lambda_1 + \Lambda_3) \tag{29}$$
$$A_{20} = T^0 \otimes 1 \otimes 1 \otimes I_K = T^0 \otimes I_K \tag{30}$$
$$\begin{aligned} A_{22} &= T \otimes 1 \otimes 1 \otimes I_K + I_{n_1} \otimes 1 \otimes 1 \otimes (Q - \Lambda_2 - \Lambda_3) \\ &= T \oplus (Q - \Lambda_2 - \Lambda_3) \end{aligned} \tag{31}$$
$$A_{23} = e_{n_1} \otimes 1 \otimes \gamma^T \otimes (\Lambda_2 + \Lambda_3) = e_{n_1} \otimes \gamma^T \otimes (\Lambda_2 + \Lambda_3) \tag{32}$$
$$A_{30} = 1 \otimes 1 \otimes U^0 \otimes I_K = U^0 \otimes I_K \tag{33}$$
$$A_{33} = 1 \otimes 1 \otimes U \otimes I_K + 1 \otimes 1 \otimes I_{n_3} \otimes Q = U \oplus Q \tag{34}$$
$$A_{31} = A_{32} = A_{12} = A_{21} = 0 \tag{35}$$

Then the part of the generator matrix A on the operational states $OS = \{0, 1, 2\} \equiv \{(0, 0), (0, 1), (1, 0)\} \subset \Sigma_X$ excluding the failure state $FS = \{3\} \equiv \{(1, 1)\} \subset \Sigma_X$ is defined by the block matrix

$$A_O = \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & 0 \\ A_{20} & 0 & A_{22} \end{pmatrix} \in \mathbb{R}^{M \times M} \tag{36}$$

with $M = K \cdot (n_0 + n_1 + n_2) = K \cdot (1 + n_1 + n_2)$ states.

3.2 Calculation of the Steady-State Vector

In the following we suppose that an irreducible generator matrix Q of the Markovian environment and three irreducible phase-type generators $T + T^0\beta^T, S + S^0\alpha^T, U + U^0\gamma^T$ are given. Then we denote by $\Pi^T = (\Pi_0^T, \Pi_1^T, \Pi_2^T, \Pi_3^T) \gg 0$ the resulting partitioned, unique steady-state row vector of the irreducible Markov chain $Z(t)$.

We can calculate Π by efficient numerical solution methods for finite ergodic Markov chains such as direct or iterative solution techniques of the balance equations $\Pi^T \cdot A = 0, \quad \Pi^T \cdot e = 1$, for instance, by applying aggregation-disaggregation methods such as an additive or multiplicative Schwarz decomposition method or any other iteration scheme derived from an M-splitting (cf. [4, 15–18, 25]).

Let $\widetilde{A} = -A^T$ denote the irreducible M-matrix associated with the generator matrix A and $A = L + U - \Delta$ be the Jacobi block-matrix decomposition into the diagonal block matrix $\Delta = -\mathrm{Diag}(A_{00}, A_{11}, A_{22}, A_{33})$, and lower- and upper-diagonal block matrices

$$L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ A_{10} & 0 & 0 & 0 \\ A_{20} & 0 & 0 & 0 \\ A_{30} & 0 & 0 & 0 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & A_{01} & A_{02} & A_{03} \\ 0 & 0 & 0 & A_{13} \\ 0 & 0 & 0 & A_{23} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{37}$$

respectively. Then we define the associated M-splitting $\widetilde{A} = -A^T = M - N$ with the corresponding transposed matrices of the block-matrix decomposition

$$M = \Delta^T = -\begin{pmatrix} {A_{00}}^T & 0 & 0 & 0 \\ 0 & {A_{11}}^T & 0 & 0 \\ 0 & 0 & {A_{22}}^T & 0 \\ 0 & 0 & 0 & {A_{33}}^T \end{pmatrix} \tag{38}$$

$$N = L^T + U^T = \begin{pmatrix} 0 & {A_{10}}^T & {A_{20}}^T & {A_{30}}^T \\ {A_{01}}^T & 0 & 0 & 0 \\ {A_{02}}^T & 0 & 0 & 0 \\ {A_{03}}^T & {A_{13}}^T & {A_{23}}^T & 0 \end{pmatrix}. \tag{39}$$

We get the iteration matrix $J = M^{-1} \cdot N = [\Delta^T]^{-1} \cdot [L^T + U^T]$ and the associated nonnegative matrix

$$\widetilde{T} = I_N - \widetilde{A} \cdot M^{-1} = N \cdot M^{-1} \tag{40}$$

$$= \begin{pmatrix} 0 & {A_{10}}^T \cdot [-A_{11}]^{-T} & {A_{20}}^T \cdot [-A_{22}]^{-T} & {A_{30}}^T \cdot [-A_{33}]^{-T} \\ {A_{01}}^T \cdot [-A_{00}]^{-T} & 0 & 0 & 0 \\ {A_{02}}^T \cdot [-A_{00}]^{-T} & 0 & 0 & 0 \\ {A_{03}}^T \cdot [-A_{00}]^{-T} & {A_{13}}^T \cdot [-A_{11}]^{-T} & {A_{23}}^T \cdot [-A_{22}]^{-T} & 0 \end{pmatrix}$$

$$\widetilde{T} = \begin{pmatrix} 0 & (S^0)^T \otimes I_K & (T^0)^T \otimes I_K & (U^0)^T \otimes I_K \\ \alpha \otimes \Lambda_2 & 0 & 0 & 0 \\ \beta \otimes \Lambda_1 & 0 & 0 & 0 \\ \gamma \otimes \Lambda_3 & [e_{n_2}]^T \otimes \gamma \otimes (\Lambda_1 + \Lambda_3) & [e_{n_1}]^T \otimes \gamma \otimes (\Lambda_2 + \Lambda_3) & 0 \end{pmatrix} \tag{41}$$

$$\cdot \begin{pmatrix} [Q^T - \Lambda]^{-1} & 0 & 0 & 0 \\ 0 & [S^T \oplus (Q^T - \Lambda_1 - \Lambda_3)]^{-1} & 0 & 0 \\ 0 & 0 & [T^T \oplus (Q^T - \Lambda_2 - \Lambda_3)]^{-1} & 0 \\ 0 & 0 & 0 & [U^T \oplus Q^T]^{-1} \end{pmatrix}$$

with the property $M^{-1} \cdot \widetilde{T} \cdot M = J$. This stochastic matrix $\widetilde{T}$ extends the structure of the Marshall-Olkin reliability model to a block matrix. Its reduction to single elements by means of the aggregation-disaggregation approach will yield a stochastic matrix $B \equiv B(x) = R \cdot T \cdot P(x) \in \mathbb{R}^{4\times 4}$ subject to an aggregation matrix R and a prolongation matrix $P(x)$ for any probability vector $0 < x \in \mathbb{R}^N$, $e^T \cdot x = 1$ (see (43)). The latter reflects the connectivity graph of the original Marshall-Olkin reliability model.

Then the column-stochastic block structured matrix

$$T = I_N - \omega \widetilde{A} \cdot M^{-1} = (1-\omega) I_N + \omega \widetilde{T} \tag{42}$$

is a semiconvergent, nonnegative matrix for any scaling $\omega \in (0,1)$ (cf. [4,15,25]). Thus, the algebraically similar iteration matrix $J_\omega = I_N - \omega M^{-1} \cdot \widetilde{A} = (1 - \omega) I_N + \omega \widetilde{J}$ which has the same spectrum as T is also semiconvergent for any $\omega \in (0,1)$.

Based on the block-matrix decomposition of A in (22) we determine a partition $\Gamma = \{J_0, J_1, J_2, J_3\}$ into $m = 4$ subsets of the state space $\Sigma = \{1, \ldots, N\}, N = n_0 \cdot K + n_2 \cdot K + n_1 \cdot K + n_3 \cdot K, n_0 = 1$, with the four disjoint subsets $J_0 = \{1, \ldots, K\}, J_1 = \{K+1, \ldots, (1+n_2) \cdot K\}, J_2 = \{(1+n_2) \cdot K + 1, \ldots, (1+n_1+n_2) \cdot K\}$, $J_3 = \{(1+n_1+n_2) \cdot K + 1, \ldots, (1+n_1+n_2+n_3) \cdot K\}$.

3.3 Application of a Semiconvergent IAD-Algorithm for M-Matrices

In the following we apply an iterative aggregation-disaggregation (IAD) algorithm that is semiconvergent to the unique steady-state vector $\Pi^T = (\Pi_i^T)_i \gg 0$ with its positive components Π_i^T on the partition set J_i for each state $i \in \{0,1,2,3\}$ (cf. [17,18]). The IAD-algorithm includes three basic matrices. First, we generate an aggregation matrix R and a prolongation matrix $P(x)$,

$$R = \begin{pmatrix} e_{J_0}^T & 0 & 0 & 0 \\ 0 & e_{J_1}^T & 0 & 0 \\ 0 & 0 & e_{J_2}^T & 0 \\ 0 & 0 & 0 & e_{J_3}^T \end{pmatrix} \in \mathbb{R}^{4\times N}, \quad P(x) = \begin{pmatrix} y_0 & 0 & 0 & 0 \\ 0 & y_1 & 0 & 0 \\ 0 & 0 & y_2 & 0 \\ 0 & 0 & 0 & y_3 \end{pmatrix} \in \mathbb{R}^{N\times 4}, \tag{43}$$

for $0 < x = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^N$, $e^T \cdot x = 1$, in terms of

$$[\alpha(x)]_j = e_{J_j}^T \cdot x_j \qquad [y(x)]_j = x_j / [\alpha(x)]_j \tag{44}$$

provided that $x_j > 0$ holds for its component on set $J_j, j \in \{0,1,2,3\}$, and we use the uniform distribution in case of $x_j = 0$ for a given j. Here $e_{J_0} \in \mathbb{R}^{n_0 K}, e_{J_1} \in \mathbb{R}^{n_2 K}, e_{J_2} \in \mathbb{R}^{n_1 K}, e_{J_3} \in \mathbb{R}^{n_3 K}$, $e_4 \in \mathbb{R}^4, e \in \mathbb{R}^N$ denote the vectors of all ones.

Applying the iteration matrix $T = T(\omega)$ in (42) and such a nonnegative vector $x \in \mathbb{R}^N$ we get the corresponding aggregated matrix $B(x) \in \mathbb{R}^{4\times 4}$ in terms of

$$B(x) = R \cdot T \cdot P(x). \tag{45}$$

We further use $r(x) = ||(I_N - T) \cdot x||_1$ with the identity matrix I_N for the L_1-norm $||x||_1 = \sum_1^N |x_i|$ in $\mathbb{R}^N$.

Then the IAD-algorithm reads as follows:

1. We choose four real numbers $\omega, \epsilon, c_1, c_2 \in (0, 1)$ and set $k = 0$.
   First, we select the steady-state vector of the simplified Marshall-Olkin model as initial vector $\alpha(x^{(0)}) = \Pi^{(S)} \in \mathbb{R}^4$, cf. (18)–(21). Then we construct the initial probability vector $x^{(0)} = (x_i^{(0)})_i \gg 0, e^T \cdot x^{(0)} = 1$, by expanding $\alpha(x^{(0)})$ uniformly on each subset $J_i, i \in \{0, 1, 2, 3\}$ in terms of

$$x_i^{(0)} = \frac{\alpha(x^{(0)})_i}{n_i} \cdot e_{J_i} \tag{46}$$

   Then we go to step 3.
2. We solve

$$B(x^{(k)}) \cdot \alpha(x^{(k)}) = \alpha(x^{(k)}) \tag{47}$$

   subject to $e_4^T \cdot \alpha(x^{(k)}) = 1, \quad \alpha(x^{(k)}) > 0.$
3. We compute

$$\widetilde{x} = P(x^{(k)}) \cdot \alpha(x^{(k)}). \tag{48}$$

4. We compute

$$x^{(k+1)} = T \cdot \widetilde{x}. \tag{49}$$

5. If

$$r(\widetilde{x}) \leq c_1 \cdot r(x^{(k)})$$

   then go to step 6
   else compute

$$x^{(k+1)} = T^h \cdot \widetilde{x} \tag{50}$$

   for $h > 1$ such that $r(x^{(k+1)}) \leq c_2 \cdot r(x^{(k)})$
   endif
6. If

$$||x^{(k+1)} - x^{(k)}||_1 / ||x^{(k)}||_1 < \epsilon \tag{51}$$

   then go to step 7
   else

$$k = k + 1,$$

   and go to step 2
   endif
7. At the end we perform a normalization after a successful convergence test:

$$\Pi = \frac{M^{-1} \cdot x^{(k+1)}}{e^T \cdot M^{-1} \cdot x^{(k+1)}} \tag{52}$$

The existing convergence theory related to numerical solution methods for finite Markov chains has revealed that the semiconvergence of this specific IAD-algorithm to the probability vector Π can be proven (cf. [17,18,25]). The specific selection of its initial vector $x^{(0)}$ in (46) by the simplified Marshall-Olkin model shall guarantee this required local convergence behavior of our approach.

4 Computing the Reliability Function and Hazard Rate of the Redundant Transport System

The reliability of the error-prone multipath transport system is characterized by the dwell time $D_T \geq 0$ in the set of the operational states $\widehat{O} = \{z = (x, h, y) \in \Sigma \mid x \in OS \subset \Sigma_X\}$ of the overall state space Σ subject to the start in one of those states $z \in \widehat{O}$ in the steady-state regime with the steady-state row vector $\Pi_O^T = (\Pi_0^T, \Pi_1^T, \Pi_2^T) \gg 0$ and its positive components $\Pi_i^T \gg 0$ associated with each non-failure state $i \in OS = \{0, 1, 2\} \subset \Sigma_X$.

Then we can calculate the reliability function $F_R(t) = \mathbb{P}\{D_T > t\}$ as time-dependent probability of the Markov chain $Z(t)$ to reside in a state $z \in \widehat{O}$ up to time $t > 0$ given that a capturing in the absorbing states $\widehat{F} = \{z = (x, h, y) \in \Sigma \mid x \in FS \subset \Sigma_X\}$ does not occur before that epoch (cf. (36), see also [13,19]):

$$\begin{aligned} F_R(t) &= \mathbb{P}\{Z(0) \in \widehat{O}\} \cdot \mathbb{P}\{D_T > t \mid Z(0) \in \widehat{O}\} \\ &= \mathbb{P}\{Z(0) \in \widehat{O}\} \cdot \mathbb{P}\{Z(t) \notin \widehat{F} \mid Z(0) \in \widehat{O}\} = \Pi_O^T \cdot \exp(A_O t) \cdot e \end{aligned} \tag{53}$$

The computation of the matrix exponential $\exp(A_O t)$ can be effectively performed by means of a uniformization approach (cf. [25]).

Let $D = \text{Diag}(D_{ii}) > 0$ denote the diagonal matrix determined by the positive diagonal elements $D_{ii} = -(A_O)_{ii} > 0, i \in \{1, \ldots, M\}$, of the M-matrix $-A_O$ in (36). We define a constant $\gamma = \max_{1 \leq i \leq M}(D_{ii}) > 0$ and use the sub-stochastic submatrix of the generator matrix $P_O = (P_{ij}), 1 \leq i \leq M, 1 \leq j \leq M$, that is determined by the transition probabilities $P_O = I_M + A_O/\gamma$ at the embedded time epochs of transition events in the Markov chain $Z(t)$. Then it holds $A_O = \gamma \cdot (P_O - I_M)$ with the identity matrix $I_M \in \mathbb{R}^{M \times M}$. It induces a simple representation as matrix exponential

$$\begin{aligned} R_O(t) &= \exp(A_O \cdot t) = \exp(\gamma t \cdot (P_O - I_M)) \\ &= \left[\sum_{n=0}^{\infty} \frac{(\gamma t)^n}{n!} \exp(-\gamma t) \cdot ((P_O)^n)_{ij} \right]_{1 \leq i,j \leq M} . \end{aligned} \tag{54}$$

The latter form allows a fast computation of the reliability function $F_R(t)$ in (53) in terms of the matrix-vector product

$$F_R(t) = \Pi_O^T \cdot R_O(t) \cdot e = \sum_{n=0}^{\infty} \frac{(\gamma t)^n}{n!} \exp(-\gamma t) \cdot {\Pi_O}^T \cdot (P_O)^n \cdot e \tag{55}$$

by means of a Poisson distribution with parameter γt and the consecutive powers of the sub-stochastic matrix P_O (cf. [19,25]).

If we assume to start in steady state with the probability distribution Π_O, then the hazard rate $h(t) : [0, \infty) \rightarrow \mathbb{R}$ of the reliability model can be simply calculated by the Markov chain with the absorbing failure states $z \in \widehat{F}$ of $FS = \{(1,1)\}$ as a simple phase-type model:

$$h(t) = \frac{d[1 - F_R(t)]}{dt} \cdot [F_R(t)]^{-1} = \frac{\Pi_O^T \cdot \exp(A_O t) \cdot (-A_O) \cdot e}{\Pi_O^T \cdot \exp(A_O t) \cdot e} \quad t \geq 0. \quad (56)$$

Due to $A \cdot e = 0$ and $\gamma^T \cdot e = 1$ we get

$$\begin{aligned} \Delta = -A_O \cdot e &= \begin{pmatrix} A_{03} \\ A_{13} \\ A_{23} \end{pmatrix} \cdot e = \begin{pmatrix} \gamma^T \otimes \Lambda_3 \\ e_{n_2} \otimes \gamma^T \otimes (\Lambda_1 + \Lambda_3) \\ e_{n_1} \otimes \gamma^T \otimes (\Lambda_2 + \Lambda_3) \end{pmatrix} \cdot e \\ &= \begin{pmatrix} \gamma^T \otimes \lambda_3 \\ e_{n_2} \otimes \gamma^T \otimes (\lambda_1 + \lambda_3) \\ e_{n_1} \otimes \gamma^T \otimes (\lambda_2 + \lambda_3) \end{pmatrix} \cdot e = \begin{pmatrix} \lambda_3 \\ e_{n_2} \otimes (\lambda_1 + \lambda_3) \\ e_{n_1} \otimes (\lambda_2 + \lambda_3) \end{pmatrix} \end{aligned} \quad (57)$$

with vectors of all ones e of appropriate dimensions. Inserting the uniformization representation (55), we conclude that

$$\begin{aligned} h = \lim_{t \to \infty} h(t) &= \frac{\Pi_O^T \cdot P_O \cdot (-A_O) \cdot e}{\Pi_O^T \cdot P_O \cdot e} \\ &= \frac{\Pi_O^T \cdot P_O \cdot \Delta}{\Pi_O^T \cdot [e - \Delta/\gamma]} = \frac{\Pi_O^T \cdot P_O \cdot \Delta}{1 - \Pi_O^T \cdot \Delta/\gamma} \end{aligned} \quad (58)$$

holds for the asymptotic regime $t \rightarrow \infty$ and we approach a corresponding exponential distribution with mean $1/h$ in this asymptotic regime. This outcome expands the results of Kozyrev, Rykov et al. [13,22] to the developed generalized Marshall-Olkin failure model.

5 Conclusions

We have considered the application of a fog computing paradigm with function virtualization to an IoT scenario that is supported by an SDN/NFV protocol stack and a multipath communication between its clients and servers (cf. [8,14,20,24]).

It has been our major goal to model this error-prone multipath transport system with a single repairman and independent phase-type distributed repair times by a generalized Marshall-Olkin failure model. For this purpose the failure processes of the incorporated two logical transport channels between a client-server pair have been described by three Markov-modulated Poisson failure processes that are driven by the transitions of a common random environment. The restoration processes were modelled by general phase-type distributed repair times.

First we have identified the generator matrix of the derived finite, continuous-time Markov chain of the reliability model in terms of associated Kronecker products of the parameter matrices which are related to the Markov-modulated interarrival times of failures and the phase-type distributed repair times. Then we have revealed that the steady-state distribution of this restoration model of Marshall-Olkin type can be effectively computed by means of an iterative aggregation-disaggregation method that has been derived from a Jacobi splitting of an associated block structured M-matrix. The latter scheme has used a closed-form representation of the steady-state vector of a simpler Marshall-Olkin failure model derived by Rykov, Kozyrev et al. [23]. Finally, we have used this outcome to compute the reliability function and the hazard rate of the multipath transport system by means of an appropriately defined finite, absorbing Markov chain and we have revealed its form in the asymptotic regime of time.

Our future work will focus on the sensitivity analysis of the reliability function and hazard rate with regard to the properties of the Markov-modulated arrival processes. Moreover, we shall consider the application of the Marshall-Olkin failure model to other services in SDN/NFV networks with an integrated 5G RAN that can support fog and mobile edge computing (cf. [1,7,14]).

Acknowledgment. N.M. Markovich was partly supported by the Russian Foundation for Basic Research (grant 19-01-00090).

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