Markovich, Natalia M.; Krieger, Udo R.

Statistical Analysis and Modeling of Skype VoIP Flows

Date of secondary publication: 15.04.2026

Accepted Manuscript (Postprint), Article

Persistent identifier: urn:nbn:de:bvb:473-irb-114738x

Primary publication

Markovich, Natalia M.; Krieger, Udo R. (2010): Statistical Analysis and Modeling of Skype VoIP Flows, in: Computer communications, Amsterdam: Elsevier, Vol. 33, Supplement 1, Special Issue: Heterogeneous Networks: Traffic Engineering and Performance Evaluation, pp. S11–S21, doi: 10.1016/j.comcom.2010.04.029.

Statistical analysis and modeling of Skype VoIP flows

Natalia M. Markovich [a], Udo R. Krieger [b,*]

[a] *Institute of Control Sciences, Russian Academy of Sciences, Profsoyuznaya 65, 117997 Moscow, Russia*
[b] *Otto-Friedrich University, D-96052 Bamberg, Germany*

A B S T R A C T

The traffic characterization of adaptive real-time applications provides a challenging engineering task of the current Internet. Here the arrival and packet length processes arising from a unidirectional VoIP flow of Skype packets are investigated by statistical means. The mean traffic load offered by the packet flow to a virtual link of limited capacity in a finite observation period is calculated and the mean and upper bounds for the time without loss are derived. A possible characterization of the packet arrival process by a properly chosen non-linear GARCH (1,1) model is presented.

Keywords:
Skype VoIP flows
Statistical traffic characterization
Traffic modeling
GARCH (1,1)

1. Introduction

Considering teleservices in telecommunication networks, voice has dominated the transport in classical public-switched telephone networks (PSTNs) and the integrated services digital network (ISDN) in the last decades. However, the rise of the Internet during the last fifteen years has initiated a dramatic shift towards data communication using the TCP/IP protocol suite which dominates the transport in current transmission networks. The latter development has also reflected on the traditional voice communication and created a strong demand for advanced voice-over-IP (VoIP) services. In this regard, Skype [29] has become a powerful service platform for the generation and transport of VoIP based on the concept of a peer-to-peer overlay network.

In recent years, this transport of voice traffic by the Internet has also initiated a shift towards more efficient source coding of voice samples by variable bitrate (VBR) schemes. They mainly operate with narrowband and wideband sampling rates from 8 to 32 kHz. A prominent representative of this category is given by the Internet Speech Audio Codec (iSAC) of the manufacturer Global IP Solutions (GIPS, see [27,28]). It is a channel-adaptive wideband VBR codec with an inband signaling of its variable frame length that applies a sampling rate of 16 kHz and can carry voice frames with speech samples from 30 to 60 ms. This proprietary codec is also used by Skype clients. In contrast to that, traditional voice services arising from PSTN and ISDN have applied narrowband and wideband sampling and source coding by algorithms with constant bitrate based on standards like G.711 in the A-law and μ-law variants, the linear predictive coding, long term prediction, regular pulse excitation (LPC-LTP-RPC) full rate coding scheme of a Global System for Mobile Communications (GSM) used in wide area mobile networks or the popular Internet encoding schemes G.723 and G.729, respectively. Therefore, the resulting voice flows can be easily characterized by the pure packet inter-arrival times (IATs) within a flow since the frame size is constant or determined by a discrete distribution with a few atoms if silence detection and comfort noise generation techniques are used.

From an economic and technical perspective, it is an interesting issue whether VoIP service should by implemented by virtual-circuit transmission on top of an IP/UDP/RTP stack including signaling by the session initiation protocol (SIP) with an overhead of at least 54 bytes in a LAN and a fixed length of voice packets in the range of 114–214 bytes or whether it is better to apply the principles of overlay networking established by the peer-to-peer transport paradigm of Skype. From the teletraffic and network design perspectives of an Internet service provider (ISP) or an operator of a wired or mobile network infrastructure it is therefore a natural and important requirement to characterize the traffic features of P2P voice traffic generated by communicating Skype clients.

Considering quality of service (QoS), quality of experience (QoE) and teletraffic engineering (TE) aspects of a voice service in general, the basic features of the processing chain of sampling, source coding and packetization of a compound of voice samples play the most important role. The applied sampling rate and segment length determine the packetization process and the resulting IATs and lengths of IP packets or Ethernet frames that are issued on a wired or wireless physical link by a generated voice stream of communicating peers. If the codec can adaptively respond to varying network conditions like the iSAC of GIPS which is used by Skype clients, VoIP traffic

* Corresponding author.
E-mail addresses: markovic@ipu.rssi.ru (N.M. Markovich), udo.krieger@ieee.org

characterization becomes a difficult task. The reason is that both the IATs between Skype packets and their corresponding packet lengths (PL) are dependent random entities in this case.

Following [14,16] we present in this comprehensive study a new statistical methodology to accomplish the traffic characterization of network dependent, VBR encoded peer-to-peer voice traffic which is arising from communicating Skype clients. The data set that is used in our paper to illustrate the typical features of unidirectional packet flows has been gathered from a transmission path with several wired and wireless links of prototypical scenarios between two Skype clients within a LAN testbed at Otto-Friedrich University Bamberg (cf. [14,16]). As the iSAC codec of a Skype client clearly determines the traffic behavior of unidirectional VoIP flows, we believe that a more extensive experimentation and the collection of a huge number of flows will not reveal any deeper inside which can be exploited in the model development of P2P VoIP flows. Here the proposed common mathematical methodology is just illustrated by our flow data. At the same time the applied statistical methods can be simply reproduced with any other Skype data including aggregated traffic.

This proposed methodology differentiates our study from existing investigations (see e.g. [1,2,4,7,9,22]), which intend either to analyze the architecture, security issues, transport protocols or overlay structure of a Skype peer-to-peer network or try to identify Skype traffic within a gathered set of Internet flows at a router interface. In [15] simple methods for the detection of heavy-tailed features and dependence in traffic data have been considered.

In [14] we have already shown that IATs $\{X_1,\ldots,X_n\}$ and PLs $\{Y_1,\ldots,Y_n\}$ arising from a unidirectional Skype flow are moderate long-range dependent (LRD) and self-similar processes. The IATs and PLs are heavy- and light-tailed distributed random variables (r.v.), respectively. Due to this heaviness of the tails and the long-range dependence it is necessary to investigate the statistical characteristics of unidirectional packet flows arising from variable bitrate encoded traffic very carefully since the classical teletraffic results are no longer valid.

Further, we have investigated the user satisfaction of Skype traffic in [16]. For this purpose, we have considered the cluster behavior of the IATs and PLs. It is generated by their individual dependence structure and specified by the overshooting above a threshold determined by an empirical quantile of a flow (see Fig. 1). We have also estimated the loss and delay characteristics of a flow which influence the user satisfaction. More specifically, we have evaluated the mean delivery time variation of packets per cluster, the overall and the mean byte losses for a virtual link with fixed capacity and the high quantiles of the extremes of IATs and PLs. The latter characteristics reveal the extremal values which arise with high probabilities close to 100%. This consideration of extremal conditions is more important than the ordinary conditions of a flow since, for example, frequently incoming large packets may influence the user satisfaction more than arriving packets of ordinary lengths.

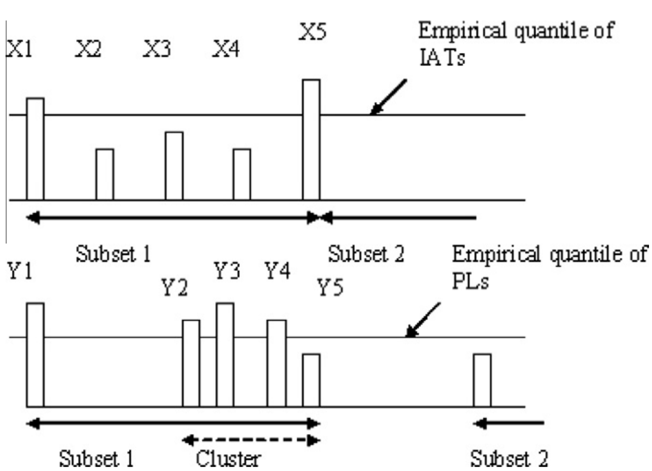

Fig. 1. The IATs $\{X_i\}$ are partitioned into subsets by exceedances over their empirical quantile. This determines the partition of PLs $\{Y_i\}$. If the partition of PLs were provided by their own quantile the subsets could be different.

In this study we transform the arrival process of the byte volumes arising from Skype VoIP packets of independent subsets into a renewal process and evaluate the mean traffic load in a finite time interval $[0,t]$. Then we determine the overall traffic load in this period by estimating the mean number of such volumes arriving up to time t by the renewal function. Following [16] the packet stream is approximated by a continuous byte flow and a bufferless fluid model is applied which describes a physical link with given capacity C. Then exceedances of the PLs Y_i over some threshold related to $X_i \cdot C$ cause a loss since the corresponding large packets cannot be delivered completely by the channel in time. We shall investigate the periods between the epochs of these exceedances, i.e. periods without losses, and find their high quantiles. The later indicate the upper bounds of lossless periods which may be exceeded just with a very small probability close to zero. Besides, we motivate that the packet arrival process can be modelled by a simple non-linear GARCH (1,1) process.

From a mathematical perspective, our analysis is significantly based on a partition of the flow data into subsets of different sizes which are statistically independent. Then one can deal with representatives of these subsets, e.g. maxima or cumulative sums of IATs or PLs over these subsets, as independent statistical entities. The key question is how to partition the data into such subsets. Here it is our main idea to identify the locations in a bivariate time series $\{X_i, Y_i\}$ arising from the exceedances of one r.v. over its empirical quantile, e.g. X_i, as the boundaries in the measured subsets of the other r.v. Y_i (see Fig. 1). In a way similar to [16] we use the exceedances of the IATs over some of their quantiles to partition the PLs in Subsections 3.1, and 3.2. This procedure reflects an intuitive understanding that packets or subsets of packets arriving after long time intervals one after another (which correspond to large IATs) should be independent. In Subsection 3.3 the PLs are partitioned into subsets by exceedances over the own quantile. In [17] the random frame sizes of a video stream have been partitioned in a similar manner into independent subsets by exceedances of the frame size over the own quantile.

In this respect, it is the main objective of our study to lay the foundation for a rigorous mathematical derivation of measurement-driven teletraffic models regarding single flows of P2P VoIP traffic. The rapid development of real-time voice services in the Internet supports such a non-parametric approach.

The paper is organized as follows. In Section 2 the used data sets of the Skype VoIP flows are described. In Subsection 3.1 an advanced statistical analysis and partitioning of the gathered data set into independent blocks are provided. In Subsection 3.2 we develop a renewal model to estimate the overall volume of packets offered to a link of limited capacity in a finite observation interval $[0,t]$. The mean and most likely upper bounds of the time without loss are estimated in Subsection 3.3. Simple steps of the analysis are proposed in Subsection 3.4. In Section 4 we fit the dependence in the IATs between Skype packets by a simple non-linear GARCH (1,1) process. Finally, the conclusions are presented.

2. Collection of Skype flow data

A comprehensive measurement of all aspects of the voice transport by Skype requires the installation of Skype clients in a controlled clean-room environment on a global scale such as PlanetLab and the complete instrumentation of all network elements along the involved paths between the communicating peers. Due to its complexity such a monitoring scenario is completely out of scope.

Following many existing approaches we have therefore extracted our basic data sets from a much simpler experimental setting which consists of two communicating Skype clients and a single packet monitor at the receiving peer (cf. [4,14,16]). In the testbed used at Otto-Friedrich University Bamberg we have established a transmission path between a sending mobile client (host 2 at 192.168.182.22) and a receiving mobile client (host 1 at 192.168.1.4) along several wired and two wireless links (see Fig. 2, cf. [14,16]). This setting represents a typical VoIP over WLAN scenario. It is currently used by a majority of Skype subscribers since both the sending and the receiving mobile host first traverse private IEEE802.11 WLAN segments 1 and 5, respectively, with DSL attachments to the public Internet and then an Internet path 2, 3, 4 which includes a tier-1 carrier exchange point between two ISP networks.

The VoIP packet flows which have been monitored by Ethereal at the receiving host (at 192.168.1.4) have been generated by short representative voice sessions of 135 s arising from a mixture of monologues, dialogs and music clips with German and English male and female speakers. The data set used here to illustrate the features of a unidirectional Skype flow has been isolated from the UDP packet stream. The latter is transporting the VBR encoded

Table 1
Description of the data arising from a Skype flow.

r.v.	Sample size	Min.	Max.	Mean	St. Dev.
IATs (s)	4605	$1.9 \cdot 10^{-5}$	$2.01 \cdot 10^{-1}$	$3.1 \cdot 10^{-2}$	$8.635 \cdot 10^{-3}$
PLs (bytes)	4605	45	284	160.27	25.808

voice samples of an adaptive iSAC encoder used by the Skype client that has responded to the varying network conditions during the experiment. The descriptive statistics of the flow data are stated in Table 1 (see also [14,16]).

3. Characterization of the offered traffic load

3.1. An advanced statistical study of Skype traffic

Let $X_1, X_2, \ldots, X_n$ denote the sequence of n IATs between the packets of a unidirectional Skype flow and $Y_1, Y_2, \ldots, Y_n$ be the associated transported PLs.

To evaluate the byte traffic load within a fixed time interval (see Subsection 3.2), we need the cumulative IATs, i.e. the arrival times of the cumulative PLs within independent subsets of the underlying measured data of a flow. By this statistical decomposition of the data intrinsic renewal points within a flow should be identified.

In [16] such a partition of the IATs between Skype packets $X_1, \ldots, X_n$ into disjoint independent subsets (data blocks) has al-

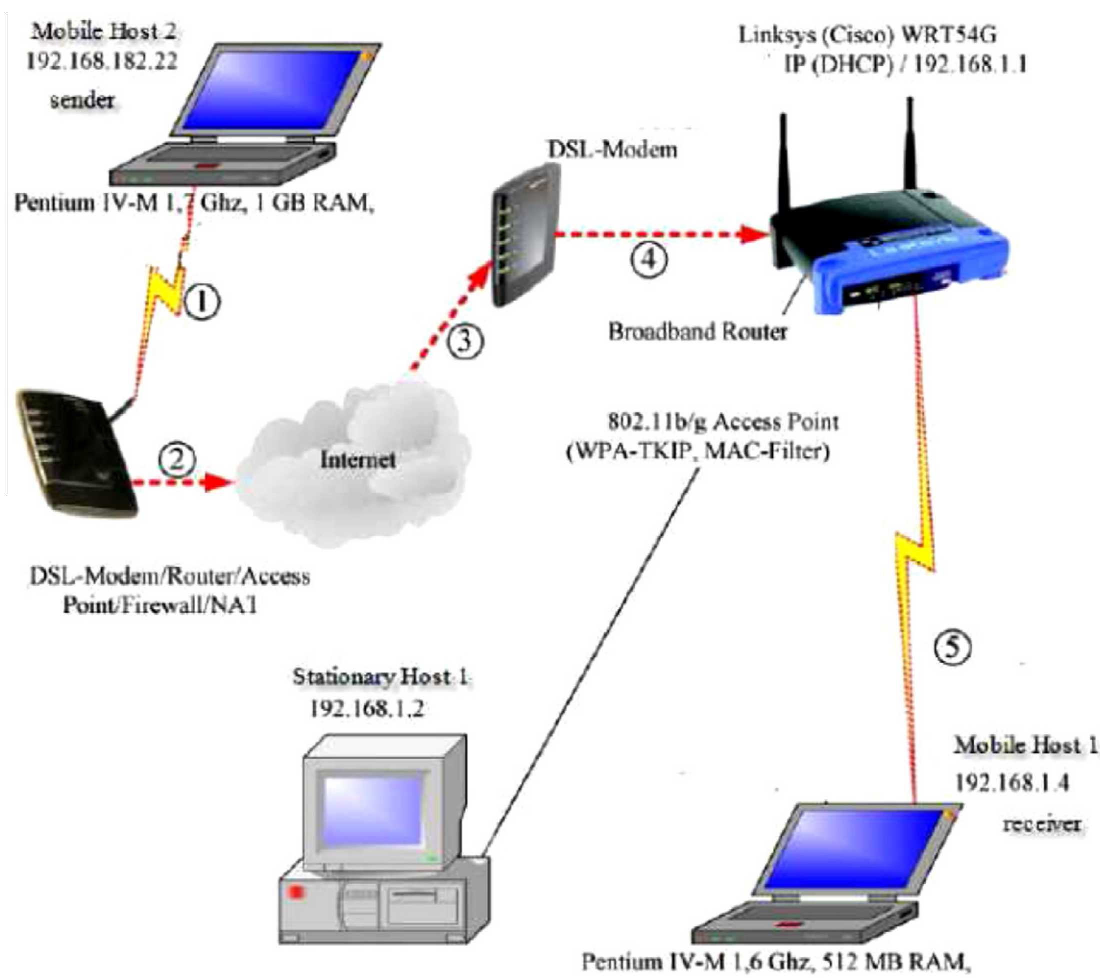

Fig. 2. LAN testbed for voice communicaton by Skype clients.

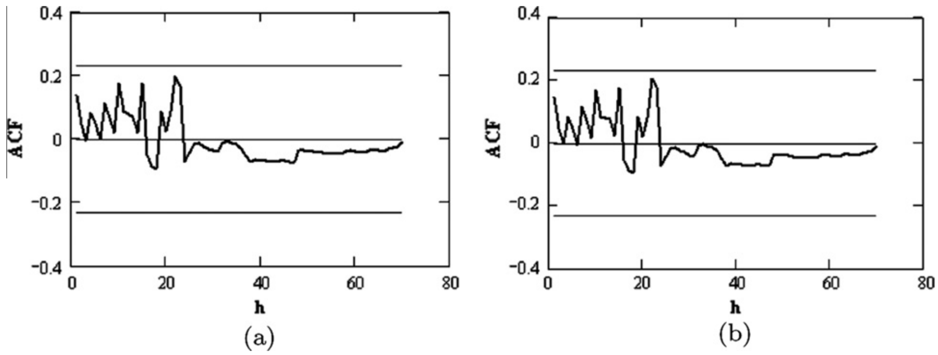

Fig. 3. ACF of the cumulative packet lengths within subsets (a) and the lengths of subsets (b) (see also [16]) obtained by a 98.4% quantile of the IATs between Skype packets with a Gaussian 95% confidence interval $\pm 1.96/\sqrt{n}$. The values of the sample ACF are close to 0 for large lags and lies inside the confidence interval. This feature allows us to suppose the independence of $\{V_j\}$ and $\{L_j\}$.

ready been considered. For this purpose, the locations of exceedances of the IATs over some empirical quantile of the measured IATs have been proposed as boundaries between these subsets. Roughly speaking, we select sufficiently long IATs as boundaries between subsets.

It has been found that a 98.4% empirical quantile provides $N_s = 72$ subsets with independent interval lengths

$$L_j = \sum_{i=k_j}^{k_j+N_j-1} X_i, \qquad j = 1, \ldots, N_s, \tag{1}$$

in the time domain. N_j is the random number of packets in the jth subset depending on the quantile of X_i, $N_0 = 0$, and $k_j = \sum_{m=0}^{j-1} N_m + 1$ is the number of the first packet in the jth subset.

Here, we provide a new statistical analysis of the lengths L_j and volumes $V_j = \sum_{k_j=1}^{k_j+N_j-1} Y_i$ of these subsets. First, it is necessary to check the independence, stationarity and finiteness of the variance of these random variables (r.v.). The independence and stationarity of $\{L_j\}$ obtained by a 98.4% empirical quantile of the IATs has been shown in [14]. To prove the independence, the sample autocorrelation function (ACF)

$$\hat{\rho}(h) = \sum_{t=1}^{n-h} (X_t - \overline{X}_n)(X_{t+h} - \overline{X}_n) \Big/ \sum_{t=1}^{n} (X_t - \overline{X}_n)^2$$

and Runde's Portmanteau test [21] have been applied. One has to apply the latter test instead of the better known Ljung-Box test since the variance of $\{L_j\}$ is shown to be infinite. The same features are evidently valid for $\{V_j\}$. Moreover, the dependence structure of $\{L_j\}$ and $\{V_j\}$ is the same. One can conclude this feature from the coincidence of their sample ACFs (see Fig. 3). Although the ACF at lag h is not defined when the variance is infinite, one can use the classical formula $\hat{\rho}(h)$ in this case, too (cf. [20, p. 349]). The further analysis aims to investigate the distributions and finiteness of the variances of both $\{L_j\}$ and $\{V_j\}$. Those are required to evaluate the mean and variance of the overall traffic load at time t (see Subsection 3.2).

To check whether the variances of $\{L_j\}$ and $\{V_j\}$ are finite, we use the plots of the "running moment estimates" and of the "running tail index estimates" following the line of [26] as a very rough tool. For this purpose, a parameter or a moment estimate is plotted against an increasing number of observations used in the estimation. If the moment of the distribution is finite, the sequential plot converges to the true value of this moment and the corresponding confidence interval becomes narrow. Indeed, this method works better if the sample size is large. In our case this size is very moderate, i.e. there are only 72 observations (subsets). This is the payment to deal with independent data. The estimation by dependent data and especially by LRD data (the IATs in our illustrative data set have shown an LRD property, cf. [14]) may lead to huge misleading variances of the estimation. This difficulty particularly concerns the estimation of the probability density function, see [13, Section 2.3.1], or even the application of the empirical distribution function (DF) as a model of the DF.[1] A large sample size is absolutely not helpful in this case. Despite of the moderate sample size the further investigation provides some valuable results.

The mean excess function $e(u) = \mathbb{E}(X - u | X > u)$ is a simple tool to check the heaviness of the tail. For various thresholds u and the underlying sequence $X_1, \ldots, X_n$ its sample estimate is determined by the formula

$$e_n(u) = \sum_{i=1}^{n} (X_i - u)\mathbf{1}(X_i > u) \Big/ \sum_{i=1}^{n} \mathbf{1}(X_i > u), \tag{2}$$

where $\mathbf{1}(A)$ denotes the indicator function of an event A. The increase or decrease of this function means that the distribution is heavy- or light-tailed, respectively. A constant value $1/\lambda$ corresponds to an exponential distribution with intensity λ. Particularly, the linear increase indicates that the distribution obeys a law of Pareto type. At the area of the largest thresholds $e_n(u)$ is not reliable due to a lack of observations which can exceed such u's.

Fig. 4 shows that $e_n(u)$ of the lengths of subsets (in seconds) is close to a linear shape with constant intervals and even intervals of decrease. Therefore, the distribution might follow a mixture of a Pareto type law with exponential and light-tailed increments with the domination of a heavy-tailed Pareto increment. The volume of packets within subsets (in bytes) tends to be Pareto-like heavy-tail distributed.

Apart from these plots we have analyzed the tail index of the distribution that drives the heaviness of tail. If the distribution of the underlying random sequence is regularly varying, its distribution function (DF) is determined by

$$1 - F(x) = x^{-\alpha}\ell(x), \tag{3}$$

where $\ell(x)$ is a slowly varying function, i.e. $\lim_{x \to \infty} \ell(tx)/\ell(x) = 1$, $\forall t > 0$.[2] Positive constants provide the simplest examples of $\ell(x)$. For regularly varying distributions the tail index α indicates the number of finite moments of the distribution. The βth moment is finite if $\beta < \alpha$ holds and infinite otherwise.

[1] According to the Glivenko–Cantelli theorem the empirical DF converges uniformly to the corresponding DF for sufficiently large sample size if the r.v.s are independent.

[2] The class of regularly varying distributions covers a wide class of heavy-tailed distributions due to many possible examples of $\ell(x)$. It includes the Pareto, Fréchet, and Zipf laws.

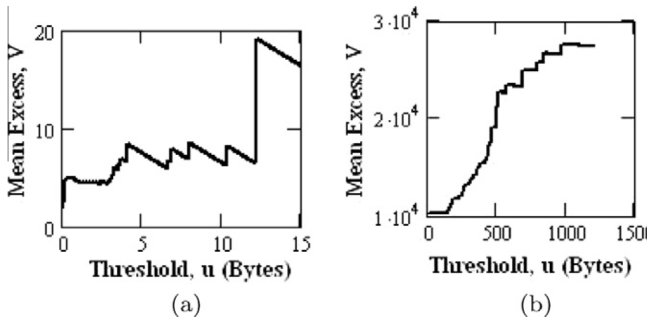

Fig. 4. The mean excess function of the lengths of subsets (a) and cumulative packet lengths within subsets (b) obtained by the 98.4% quantile of the inter-arrival times between Skype packets.

Table 2
Estimates of the extreme value index γ with the 95% confidence intervals of the Skype flow characteristics.

r.v.	Hill's estimate	Ratio estimate	Moment estimate
Lengths of subsets (s)	0.641 [0.516,0.845]	0.681 [0.55,0.894]	0.676 [0.201,1.152]
Volume of subsets (bytes)	0.647 [0.521,0.855]	0.651 [0.527,0.854]	0.592 [0.186,1.03]

To estimate α, the well-known Hill's, ratio and moment estimators have been used. The Hill's estimator is determined by

$$\hat{\gamma}^H(n,k_0) = \frac{1}{k_0}\sum_{i=1}^{k_0} \ln X_{(n-i+1)} - \ln X_{(n-k_0)}, \quad (4)$$

where $X_{(1)} \leqslant X_{(2)} \leqslant \cdots \leqslant X_{(n)}$ are the order statistics of the sample $X^n = \{X_1, X_2, \ldots, X_n\}$ and k_0 is the number of the largest order statistics (i.e. a smoothing parameter). The description and theoretical properties of other estimators can be found in [13–16]. The corresponding estimates of the extreme value indices (EVIs) $\gamma = 1/\alpha$ of $\{L_j\}$ and $\{V_j\}$ are summarized in Table 2.[3]

One may conclude from Table 2 and Figs. 5(a) and (c) that the variances of both the lengths and volumes of subsets are infinite since their distributions are regularly varying and $1 < \alpha < 2$ holds.[4] Figs. 5(b) and (d) indicate that the standard deviation of both the cumulative volumes and lengths within selected subsets increases as the number of observations involved in the estimation process increases.

One can roughly check whether a distribution is regularly varying. For this purpose, one can plot $\log(1-F(x))$ against $\log(x)$ in the ordered sample points. Note that $\log(1-F(x)) \sim -\alpha\log(x) + c$ holds for regularly varying distributions. In Fig. 6 the empirical DF

$$F_n(t) = (1/n)\sum_{i=1}^{n} \mathbf{1}(t \geqslant X_i), \quad (5)$$

calculated by $\{L_j\}$ and $\{V_j\}$ is used in the log–log plot instead of the unknown $F(x)$. The value $\alpha = 1.56$ corresponds to our finding of $\gamma = 0.64$ and reflects the tail (see Figs. 5(a), and (c)). Hence, one may conclude that the distributions of $\{L_j\}$ and $\{V_j\}$ could be mixtures of two regularly varying distributions. For $\{L_j\}$ the increments with lighter tails are suppressed.

[3] The formulas of the confidence intervals of the tail index estimates are given in [16]. The estimates of γ for $\{L_j\}$ are taken from Table 2 in [16].
[4] This corresponds to stable distributions.

3.2. Estimating the offered traffic load

The characterization of a measured traffic process including both the arrival times and the associated PLs constitutes an important task of traffic control. It requires the determination of the overall byte traffic in a finite time interval $[0,t]$ and can be applied in on-line decision processes like capacity allocation and network control actions such as connection admission control (CAC).

We consider the partition of a flow into subsets of independent lengths $\{L_j\}$ that has been developed in the previous section. Then the overall volume of packets offered to a virtual link of finite capacity in a finite observation period $[0,t]$ is determined by

$$V^*(t) = \sum_{j=1}^{N_t} V_j = \sum_{j=1}^{N_t}\sum_{i=1}^{N_j} Y_i.$$

N_t denotes the number of subsets before time t, $N_t = \max\{n\colon t_n < t\}$, and $t_n = \sum_{i=1}^{n} L_i$ is the cumulative time interval corresponding to the n subsets. The interval length L_i of the ith subset in the time domain is determined by (1). For simplicity and without loss of generality, we re-enumerate the packets within subsets from 1 to N_j, where N_j denotes the random number of packets in the jth subset.

Further, we assume that the volume V_j of the jth subset is concentrated at the end of the time interval L_j and independent of the latter. One can consider the appearances of the volumes $\{V_j\}$ as events and $\{L_j\}$, $j = 1,2,\ldots,N_s$ as corresponding IATs between these events. Then the arrival process of volumes $\{V_j\}$ can be considered as a renewal process[5] since the independence and the common distribution of the $\{L_j\}$ have been shown in Subsection 3.1.

Since the number N_t of subsets at time t and the volumes V_j of subsets arriving in the future are independent and V_j are assumed to be independent of L_j, the expectation of the overall volume at time t can be determined by Wald's equation

$$\mathbb{E}(V^*(t)) = \mathbb{E}\left(\sum_{j=1}^{N_t} V_j\right) = \mathbb{E}(N_t)\mathbb{E}(V_j) \quad (6)$$

and the variance by

$$Var(V^*(t)) = Var\left(\sum_{j=1}^{N_t} V_j\right) = Var(V_j)\mathbb{E}(N_t) + \left(\mathbb{E}(V_j)\right)^2 Var(N_t), \quad (7)$$

if the second moment of the volume exists, i.e. $\mathbb{E}V_j^2 < \infty$ (cf. [13,23]).

For our illustrative data $Var(V_j)$ is infinite and hence, $Var(V^*(t))$ is infinite, too. The infiniteness of the variance of the cumulative packet volumes of subsets is not evident taking into account that the distribution of the PLs is light-tailed with a finite variance (cf. [14]). It might be caused by the infiniteness of the third moment of the IATs between packets. Really, similarly to (7) the variance of the volume V_j can be determined by

$$Var(V_j(t)) = Var\left(\sum_{i=1}^{N_j} Y_i\right) = Var(Y_i)\mathbb{E}(N_j) + (\mathbb{E}(Y_i))^2 Var(N_j),$$

if $\{Y_j\}$ and $\{X_j\}$ were sets of i.i.d r.v.s., independent of each other and N_j a stopping time. Then the variance $Var(N_j)$ may be approximated by

$$Var(N_j) \approx \frac{\sigma^2}{\mu^3}L_j + \left\{\frac{2\sigma^2}{\mu^2} + \frac{3}{4} + \frac{5\sigma^4}{4\mu^4} - \frac{2\mu_3}{3\mu^3}\right\}, \quad (8)$$

for large time interval length L_j, where μ, σ^2 and μ_3 are the first three moments of the distribution of the IATs between packets. Eq. (8) follows from the corresponding result for $Var(N_t)$ obtained in [8]. It was shown in [14,16] that the tail index of the IATs $\{X_i\}$

[5] A sequence of r.v.s is called a renewal process if the IATs of the events are independent and have a common distribution, cf. [5].

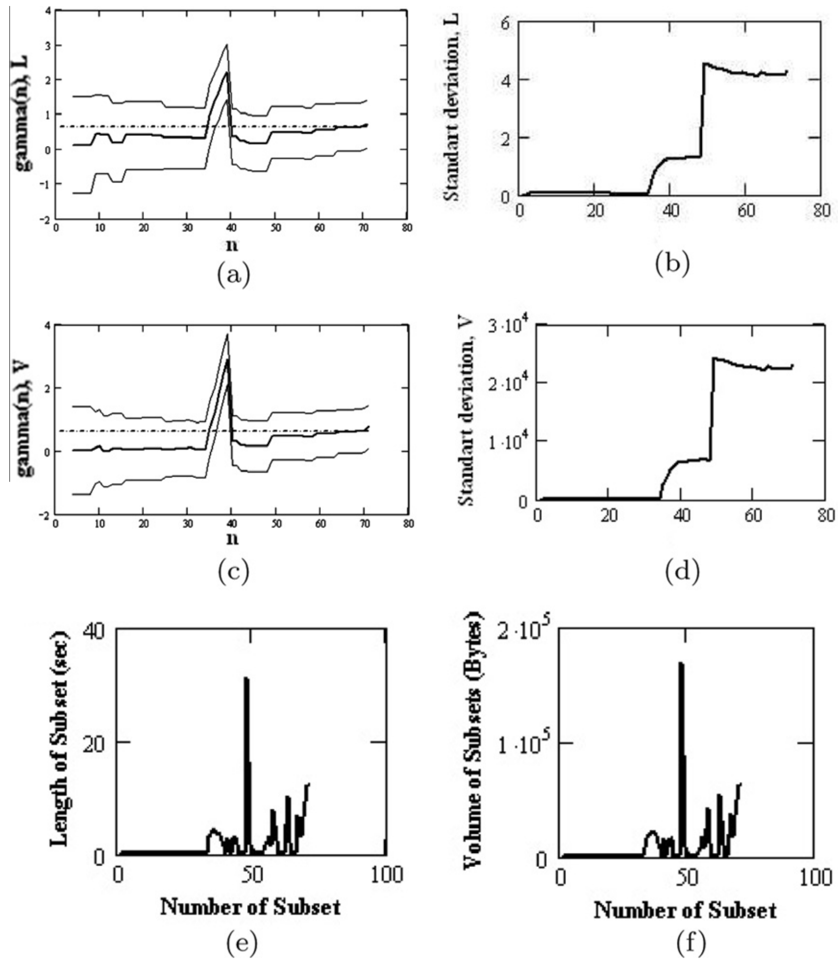

Fig. 5. The Hill plot with an asymptotical 95% Gaussian confidence interval and the standard deviation of the lengths of subsets (a) and (b); of the volume of subsets (c) and (d), respectively. $\gamma = 0.64$ (dashed line) on (a) and (c) corresponds to the intervals of stability of both Hill plots and hence, it is accepted as an appropriate value of γ. The standard deviation does not converge to a finite value as n increases. The bursts on the Hill plots are caused by bursts among the lengths of subsets (*e*) and of the volume of subsets (*f*).

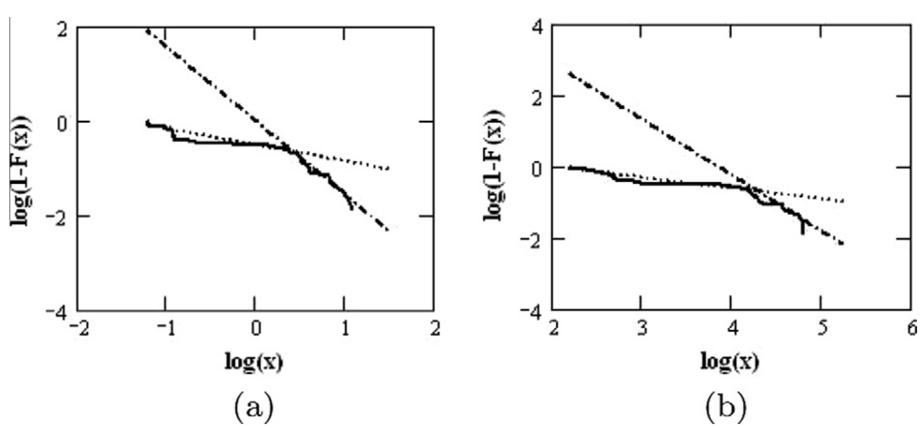

Fig. 6. The Log–log plots (solid lines) of the lengths of subsets (a) and the cumulative packet lengths within subsets (b) $-0.35\log(L_{(i)}) - 0.5$ and $-1.56\log(L_{(i)})$ (a) and $-0.3\log(V_{(i)}) + 0.6$ and $-1.56\log(V_{(i)}) + 6$ (b) (dotted and dashed lines, respectively) are plotted against $\log(L_{(i)})$ and $\log(V_{(i)})$. $\{L_{(i)}\}$ and $\{V_{(i)}\}$, $i = \overline{1, N_s}$, are the order statistics of the samples $L_1, \ldots, L_{N_s}$ and $V_1, \ldots, V_{N_s}$.

of our illustrative data set satisfies the condition $2 < \alpha < 3$. Since the distribution of the IATs has a Pareto type law, this implies the infiniteness of μ_3 and, hence, the unboundedness of $Var(N_j)$ and $Var(V_j(t))$.

Considering (6) and (7), according to its definition

$$H(t) = \mathbb{E}(N_t) = \sum_{n=1}^{\infty} \mathbb{P}\{t_n < t\}, \qquad (9)$$

denotes the renewal function (cf. [13]). It exhibits simple analytic forms just for a few distributions, e.g. $H(t) = \lambda t$ for an exponential distribution with intensity λ. When the distribution of $\{L_i\}$ is unknown or one can expect a complicated mixture of classical distributions as a model like in Subsection 3.1, it is better to use nonparametric estimators of $H(t)$. We have applied a so-called histogram-type estimator proposed in [13]. It offers a good alternative to a similar estimator proposed by Frees [6], since it provides similar accuracy but with a sufficiently fewer number of calculations. Another advantage is given by the opportunity to see how the renewal functions of different distributions which can only be approximated by numerical methods look like. The histogram-type estimator is based on the simple idea to replace the DF $\mathbb{P}\{t_n < t\}$ of t_n in (9) by the empirical DF (i.e. an unbiased estimate) and the infinite sum by a finite one. It is determined by

$$\widetilde{H}(t,k,N) = \sum_{n=1}^{k} \frac{1}{N_n} \sum_{i=1}^{N_n} \mathbf{1}(t \geqslant t_n^i), \tag{10}$$

where $t_n^i = \sum_{q=1+n(i-1)}^{n \cdot i} \tau_q,\ i = 1,\dots,N_n,\ N_n = [\frac{N}{n}],\ n = 1,\dots,k,$ are observations of the r.v. t_n using suitable IATs $\{\tau_j, j = 1,\dots,N\}$. In our consideration we replace $\{\tau_j\}$ and N by $\{L_j\}$ and N_s to estimate the mean number of subsets (or volumes) at time t.

To select the parameter k for a fixed t we use the formula

$$k^* = \arg\min\{k : \widetilde{H}(t,k,N) = \widetilde{H}(t,k+1,N),\ k = \overline{1, N-1}\}.$$

More details about these methods can be found in [13, Chapter 8].

$\widetilde{H}(t,k,N)$ can only be applied for sufficiently small time intervals $[0,t]$. Due to the limited number of inter-arrivals $\{L_i\}$ of subsets that are at our disposal the histogram-type estimate becomes constant after a sufficiently large time t, i.e. $\widetilde{H}(t,k,N) = k$ holds for $t \in [t_{\max}(k), \infty)$, where

$$t_{\max}(k) = \max_{1 \leqslant n \leqslant k} \max_{1 \leqslant i \leqslant N_n} t_n^i \leqslant \sum_{i=1}^{N} \tau_i$$

and $k \leqslant N$ is some fixed number. Therefore, for large t it is better to use an asymptotic linear approximation to fit $H(t)$. In Figs. 7(a) and (b) the estimate (10) of the renewal function of the volumes $\{V_j\}$ (that is the mean number of such volumes at time t) is shown at small and large time intervals. Since the mean excess function of $\{L_j\}$ indicates that the distribution of $\{L_j\}$ may include exponential increments (see Fig. 4(a)) we compare $\widetilde{H}(t,k,N)$ with the renewal function of the exponential distribution $H(t) = \lambda t$ in Fig. 7(a). The intensity λ is approximated by $1/\hat{\mu} = 0.532$, where $\hat{\mu}$ is the mean of $\{L_j\}$. The comparison shows that $\{L_j\}$ is not exponentially distributed and the sequence $\{V_j\}$ cannot be considered as a Poisson process.

Regarding the linear approximation for the large t one cannot use the well-known formula

$$H(t) = \frac{t}{\mu} + \frac{\sigma^2}{2\mu^2} - \frac{1}{2} + \circ(1),$$

since the variance σ^2 of $\{L_j\}$ is not finite as it was shown in Subsection 3.1. Therefore, we use the approximation

$$H(t) = \frac{t}{\mu} + \frac{t^2(1 - F(t))}{\mu^2(\alpha - 1)(2 - \alpha)} + o(1), \tag{11}$$

for $t \to \infty$ (cf. [24]), where $F(t)$ is the DF of L_j.[6] (11) is valid for regularly varying distributions (3) and $1 < \alpha < 2$. It has been shown in Subsection 3.1 that the distribution of $\{L_j\}$ is regularly varying and that the tail index α of $\{L_j\}$ falls into the interval $[1.468, 1.56]$. Hence, we can take $\alpha = 1.5$ in (11) and the estimate $t^{-\alpha}$ instead of $1 - F(t)$ and apply the approximation

[6] The notation $o(1)$ means that the approximation is valid up to an arbitrary constant.

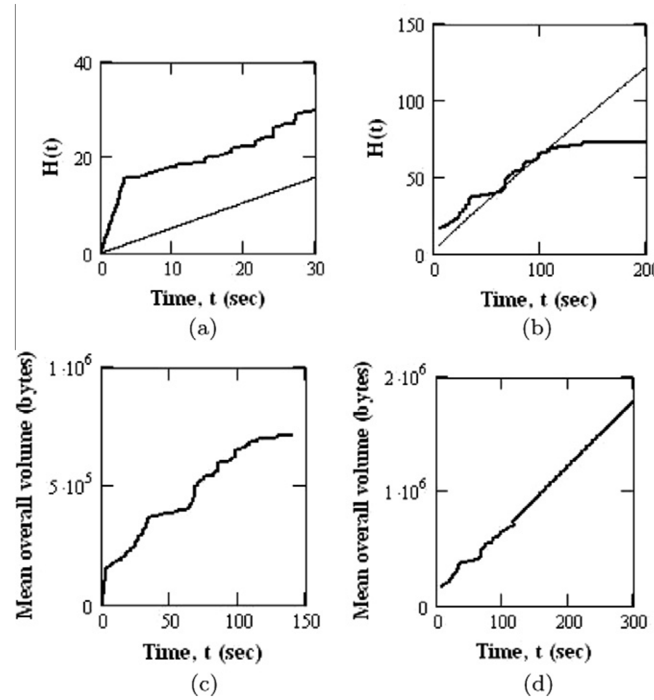

Fig. 7. The estimate (10) of the renewal function of cumulative packet lengths $\{V_j\}$ within subsets (solid line) for the time interval $[0.01, 30]$ s (a) and $[5, 200]$ s (b). The exponential model in (a) and the linear approximation (12) in (b) are used as alternatives (thin solid lines). For $t > 138.914$ $\widetilde{H}(t,k,N) = k^* = 72$ should be replaced by a linear model. The estimate of the mean overall volume $\mathbb{E}(V^*(t))$ in time intervals $[0, t_{\max}] = [0, 138.914]$ (c) and $[5, 300]$ (d).

$$\widehat{H}(t) = \frac{t}{\hat{\mu}} + \frac{t^{2-\alpha}}{\hat{\mu}^2(\alpha - 1)(2 - \alpha)}, \tag{12}$$

for $t > t_{\max}(k)$ (see Fig. 7(b)). $\hat{\mu}$ is the average of $\{L_j\}$.

One can estimate the mean overall volume $\mathbb{E}(V^*(t))$ at time t by the formula

$$\overline{V^*}(t) = H^*(t)\overline{V}. \tag{13}$$

Here $\overline{V}$ is the sample average of $\{V_j\}$ that can be used to estimate $\mathbb{E}(V_i)$ in (6),

$$H^*(t) = \begin{cases} \widetilde{H}(t,k,N), & t \leqslant t_{\max}, \\ \frac{t}{\hat{\mu}} + \frac{\widehat{\sigma}^2}{2\hat{\mu}^2} - \frac{1}{2}, & t > t_{\max},\ \sigma^2 < \infty, \\ \widehat{H}(t), & t > t_{\max},\ \sigma^2 = \infty, \end{cases} \tag{14}$$

and $\widehat{\sigma}$ is the standard deviation of $\{L_j\}$.

The sample average of $\overline{V} = 1.018 \cdot 10^4$ bytes is used in (13) to estimate $\mathbb{E}(V_i)$. Figs. 7(c), and (d) show the mean offered load of the packets corresponding to a pre-defined time t. In Fig. 7(d) the estimate of $H(t)$ coincides with $\widetilde{H}(t,k,N)$ at $[5, 138.914]$ and with (12) where $\alpha = 1.5$ at $(138.914, 300]$.

3.3. Estimating the periods without loss

Now we consider the packet load offered in a finite interval $[0,t]$ to a virtual link of finite capacity, e.g. a wireless channel. Let us use a bufferless fluid model and assume that the packet stream is approximated by a continuous flow. Then it is an important issue to evaluate the time without loss.

The loss is generated by the exceedances of the PLs over some threshold u which corresponds to the channel capacity for a unit time. Obviously, large and frequent exceedances may generate a big loss. Furthermore, the inter-exceedance times (IETs) imply the periods without loss.

The question arises which threshold u generates a good pattern of inter-exceedances and what is the minimal capacity of the logical channel corresponding to this threshold that provides the longest period without loss.

For this purpose, we will investigate the quantiles of the IET to understand the frequency of the exceedances and evaluate the lossless time for a fixed threshold u.

Let $\{Y_j^*\},\ j = 1, 2, \dots, N^*$, denote those packets whose lengths exceed u. We consider the IETs $T(u)$, i.e. the time intervals between any two consecutive events $\{Y_i > u\}$ and $\{Y_{i+j} > u\}$ (see the second picture in Fig. 1).

The quantile $x = x_p$ of $T(u)$ related to the level $1-p$, $p \in (0,1)$, indicates the duration of a phase without loss that cannot be exceeded with probability $1-p$, i.e. $\mathbb{P}\{T(u) \leqslant x_p\} = 1-p$. We shall consider high quantiles that are close to 100%. These are approximately equal to the most likely upper bounds of the duration without loss. Usually, high quantiles are located beyond the range of the sample and, hence, cannot be evaluated by means of the empirical DF or other estimators based on the order statistics of the gathered observations.

We select the quantile of the PL $\{Y_j\}$ as threshold u such that the resulting IETs are independent events. Then $\{Y_j^*\}$ are the corresponding change-packets of this partition. In Subsections 3.1 and 3.2 we have considered the quantiles of the IATs as u and the exceedances of IATs over this u in a similar manner as change inter-arrivals between independent subsets of Skype packets. This decomposition is necessary since many statistical tools are only consistent for independent data, e.g. the empirical DF and high quantile estimators which we shall apply further.

First we select the 98.5% empirical quantile of the PLs which is equal to $u = 200.026$ bytes. The descriptive statistic of the corresponding IETs and the related scatter plot of the IETs versus the change-packet lengths $\{Y_j^*\}$ are shown in Table 3 and Fig. 8(a). The data hugs the orthogonal axis. Hence, change-packets with the lengths about 199.526 bytes correspond to several IETs and the IETs about 0.069 s correspond to several change-packet lengths. It implies the mutual independence of the IETs and change-packet lengths. The mean IET indicates the mean time without loss.

Now we show the independence of the obtained IETs. For this purpose, we first check the finiteness of their variance by means of different tools. The mean excess function is partly constant and partly linear (see Fig. 8(b)). It shows that the distribution of the IETs contains exponential and Pareto increments. The estimate of the mean excess function is not reliable at the interval of the largest thresholds $u > 4.5$, where the function decreases due to the few observations there. Anyway, the heavy-tailed Pareto component dominates all light-tailed increments at the tail of the distribution and the mixed distribution is heavy-tailed regularly varying. The Hill's and ratio estimates of the tail index both indicate that the tail index is given by $\alpha = 2.222$. Since for a regularly varying distribution the tail index determines the number of finite moments, then the fact $\alpha > 2$ implies that the variance of the IETs is finite. Hence, we can apply the Ljung-Box test [12] to check the null hypothesis regarding independence. According to this test the statistic

$$Q = n(n+2)\sum_{j=1}^{h} \hat{\rho}^2(j)/(n-j)$$

has approximately a chi-square distribution with h degrees of freedom in case of a process with a finite variance. The i.i.d. hypothesis should be rejected at level η if $Q > \chi^2_\eta(h)$, where $\chi^2_\eta(h)$ is the ηth quantile of the chi-square distribution with h degrees of freedom, i.e. $Pr\{\chi^2 > \chi^2_\eta(h)\} = \eta,\ 0 < \eta < 1$. One can conclude from Table 4 that the Ljung-Box test accepts the null hypothesis regarding independence at the 5% level. Taking a threshold less than 200.026 bytes leads to dependent subsets and shorter time intervals without loss. Larger thresholds lead to independent but rare subsets. Then we have not enough IETs to perform any statistical analysis. We can have no loss for some sufficiently large capacity. Hence, the selected value of $u \approx 200$ bytes corresponds to the minimal reasonable capacity. The resulting capacity is calculated by $\{u/X_i\}$ and its mean value is equal to 11.77 kbps. It is known that the iSAC codec adjusts the transmission rate from 10 to 32 kbps (cf. [27,28]).

Now we can calculate the high quantiles of the inter-exceedance times by Weissman's estimator [25]

$$x_p^w = X_{(n-k_0)}\left(\frac{k_0+1}{(n+1)p}\right)^{\hat{\gamma}}, \quad k_0 = 1, \dots, n-1, \tag{15}$$

and a similar estimator

$$x_p^c = X_{(n-k_0)}\left(-0.5 + \sqrt{0.25 + \frac{p \cdot n \cdot c(\hat{\gamma})}{k_0}}\right)^{-\hat{\gamma}}, \tag{16}$$

with

$$c(\gamma) \sim 1 + X_{(n-k_0)}^{-1/\gamma} + X_{(n-k_0)}^{-2/\gamma}, \tag{17}$$

given in [13, Section 6.2]. Both estimators are obtained from a Pareto tail model and assume that at least a weak independence of the underlying sequence $X_1, \dots, X_n$ holds. $X_{(1)} \leqslant X_{(2)} \leqslant \dots \leqslant X_{(n)}$ denotes the order statistics of the latter sample and $\hat{\gamma}$ is some estimate of the EVI γ of the underlying r.v. X. k_0 is the same parameter as in Hill's estimate (4) and selected by a bootstrap method with 200 bootstrap resamples (see [13, Section 6.4.2]). The quantile estimate x_p^c is better than x_p^w with regard to the mean squared errors (cf. [13]). The high quantile estimates together with tolerant 95% bootstrap

Table 3
Description of the inter-exceedance times arising from the 98.5% empirical quantile of the packet length.

u (bytes)	Inter-exceedance times (s)				
	Sample size	Min.	Max.	Mean	St. Dev.
200.026	69	0.049	12.48	1.348	2.081

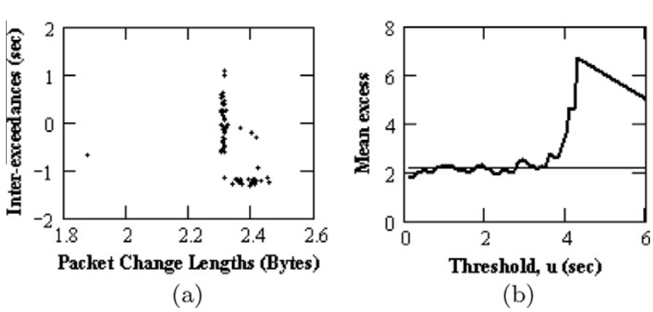

Fig. 8. Scatter plot of IETs versus change-packet length in a logarithmic scale (a) and the mean excess function of IETs (b).

Table 4
Ljung-Box test results.

$\hat{\alpha}$	Lags	Q	$\chi^2_{0.05}(h)$
Selection of IETs by the 98.5% quantile of PLs			
	10	9.198	18.3
2.222	20	13.589	31.4
	30	15.546	43.8

Table 5
High quantiles of the inter-exceedance times in seconds arising from the 98.5% empirical quantile of the packet length with 95% bootstrap confidence intervals.

Estimator	High quantile estimate			
	97.1%	98.6%	99%	99.9%
x^w	15.561 (−17.867,42.337)	27.384 (−52.951,93.741)	39.153 (−89.561,136.501)	452.147 $(-7.165\cdot10^4, 7.468\cdot10^4)$
x^c	5.948 (−2.307,9.609)	6.391 (−8.404,20.2)	8.838 (−4.342,16.544)	93.461 (−583.519,672.399)
Empirical quantile				100%
	4.291	9.48		12.48

confidence intervals built by 100 bootstrap resamples are shown in Table 5. The method to calculate the latter intervals is given in [13, Section 6.4.2]. The empirical quantiles are too rough and do not show all desirable high quantiles. Nevertheless, x_p^c is closer to the empirical quantiles of lower percentages than x_p^w.

3.4. Data analysis of adaptive VBR encoded real-time flows

To summarize our findings we propose the following simple analysis of VBR encoded flow data:

1. Find the empirical 98% quantile of the IATs by the inversion of the empirical DF (5). This high quantile may correspond to independent or approximately independent subsets of packets. It is not necessary to test their independence in this case.
2. Calculate the time lengths $\{L_j\}$ of IATs of subsets by formula (1). Check the finiteness of the variance of $\{L_j\}$. For this purpose, estimate the mean excess function $e(u)$ by formula (2) and look at its plot. The distribution of L_j is heavy-tailed and its variance may not be finite if the plot of $e_n(u)$ contains intervals with increasing segments. Then one may estimate the tail index γ by the Hill's, ratio and moment estimators[7] and compare a slope of the plot $\log(1-\widehat{F}(x))$ versus $\log(x)$ with $-1/\hat{\gamma}$. The variance is finite if $1/\hat{\gamma} > 2$ and infinite otherwise in case that the distribution of L_j is expected to be heavy-tailed regularly varying. Plot the 'running moment estimates' if the distribution does not look like a regularly varying one.
3. Select a time t. Calculate the mean overall volume at time t by formulas (13) and (14).
4. Find the most likely upper bounds for the periods without loss. For this purpose, find the empirical 98% quantile of the PLs as a threshold u and calculate the corresponding IETs $T(u)$. Estimate the 99% and 99.9% quantiles of $T(u)$ by formulas (15)–(17) for $p = 0.1$ and $p = 0.01$, respectively.

4. Modeling the packet arrival process

The analysis of the IATs $\{X_t, t = 1,\dots,n\}$, $n = 4605$, between Skype packets performed in [14] has revealed that the IAT process constitutes a moderate LRD heavy-tailed stream. Measures of dispersion and shape of the used flow data are summarized in Table 6. They illustrate the asymmetric heavy-tailed behavior of the marginal distribution of the IATs X_i deviating from a normal distribution. Therefore, short-term correlated linear models like an autoregressive process (AR (p)) or autoregressive moving average process (ARMA (p,q)) (cf. [3]) cannot be used to cover the main features of a Skype packet flow. It has been observed that the IAT process does not exhibit a clear weak stationarity property due to the variation of its variance over blocks of the IATs (cf. [14]).

[7] For details of the tail index estimation see [13].

4.1. Characterization by non-linear processes

To explain the dependence in the data and the observed time-varying conditional variance, it is recommended to use a GARCH (p,q) process as a more suitable fine-grained model of the packet inter-arrival process (cf. [18,19]). It has a multiplicative form

$$X_t = \mu + \sigma_t Z_t, \quad t \in \mathbb{Z}, \tag{18}$$

where $\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p}\alpha_j\sigma_{t-j}^2 + \sum_{i=1}^{q}\beta_i X_{t-i}^2$ holds. Here $\mu = \mathbb{E}(X_t)$, $\{Z_t\}$ is an iid sequence, and $\{\sigma_t\}$ is a stochastic process with non-negative values. It is important that σ_t and Z_t are mutually independent for fixed t.

The interpretation of this model is as follows. The direction of the IAT changes is determined by the sign of Z_t, independently from the magnitude of this change which is modelled by the volatility σ_t. Due to the independence of σ_t and Z_t it is clear that $Var(X_t) = \sigma_t^2$ assuming, without loss of generality, that Z_t has zero mean and variance 1. This implies that the variance of X_t varies conditionally on the past values $X_{t-1}, \sigma_{t-1}, X_{t-2}, \sigma_{t-2}, \dots$

Some modifications of GARCH like FARIMA (p,d,q)-GARCH (r,s) may be considered as alternative (cf. [3,10,11]). However, the problem is to estimate the numerous parameters of these models. We can only consider simple models with a few parameters since it requires the more data the more parameters we wish to estimate. Moreover, the estimation of the parameters is a rather complicated optimization problem. In this respect a GARCH (1,1) model (18) with

$$\sigma_t^2 = \alpha_0 + \alpha_1\sigma_{t-1}^2 + \beta_1 X_{t-1}^2 \tag{19}$$

and with only three parameters helps to simplify the estimation and fits the real-life data rather good.

The parameters α_0, α_1 and β_1 are often estimated by the Gaussian quasi-maximum likelihood estimator. For this purpose it is assumed that the noise Z_t is Gaussian. Despite the IAT process is heavy-tailed the application of non-Gaussian (e.g. a heavy-tailed) noise can lead to non-consistent maximum likelihood estimates (cf. [18]). To estimate the parameters one maximizes the log-likelihood function

$$L_n(\alpha_0, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q) = -\frac{1}{2n}\sum_{t=1}^{n}\left(2\log(\sigma_t) + \sigma_t^{-2}X_t^2\right). \tag{20}$$

4.2. Identification of the GARCH (1,1) components

We have applied the GARCH (1,1) to the IAT data. For simplicity we consider $\widehat{Y}_t = X_t - \hat{\mu}$ for the mean $\hat{\mu} = 3.1 \cdot 10^{-2}$ and model it by $\sigma_t Z_t$. To make iteration runs by means of (19) we take the initial value $\hat{\sigma}_0 = 1$ and $\widehat{Y}_0 = 0$. It is known that the influence of the initial values disappears for large t. Then the maximization of the log-likelihood function (20) provides the parameters $\alpha_0 = 0.000001$, $\alpha_1 = 0.0877$ and $\beta_1 = 0.299$. We have calculated the residuals $\widehat{Z}_t = \widehat{Y}_t/\hat{\sigma}_t$ with these values of α_0, α_1 and β_1 and found that the distribution of the

Table 6
Statistics of the IATs arising from a Skype flow.

r.v.	1. Quartil (25%)	3. Quartil (75%)	Median	Skewness	Kurtosis
IAT (s)	$2.63{\cdot}10^{-2}$	$3.35{\cdot}10^{-2}$	$3.031\cdot10^{-2}$	5.183	79.75

residuals is fitted by a Student's distribution with $d \in \{1.9, 2\}$ degrees of freedom (see Fig. 9). Note that the tail of the Student's distribution is Pareto-like and therefore it is regularly varying.

One can estimate the tail index k of the σ_t by the equation

$$\frac{1}{n}\sum_{i=1}^{n}\left(\alpha_1\widehat{Z}_i^2+\beta_1\right)^{k/2}=1, \tag{21}$$

numerically. The latter follows from Kesten's theoretical result related to a GARCH (1,1) process and the property of regularly varying distributions (see Breiman's result in [18]). Assuming that Z_t is Student distributed with $d \in \{1.765, 1.9, 2\}$, we have generated one million realizations of Z_t and solved (21). Then we get $\hat{k} \in \{1.443, 1.735, 1.727\}$, respectively, as tail index estimates of σ_t. We have selected $\hat{k} = 1.735$ which corresponds to the better QQ-plot for $d = 1.9$ in Fig. 9(a). It follows from the extreme value theory that

$$\mathbb{P}\{\sigma_1 > x\} \sim c\cdot x^{-1.735},$$
$$\mathbb{P}\{Y_1 > x\} = \mathbb{P}\{\sigma_1 Z_1 > x\} \sim \mathbb{E}|Z_1|^{1.735}\mathbb{P}\{\sigma_1 > x\}$$

holds for positive c as $x \to \infty$ since Z_t and σ_t are independent.

We have checked the quality of the obtained GARCH (1,1) model using Breiman's result [13,p. 5.], similarly to [20, pp. 238–239]. According to this result, if a r.v. Z is regularly varying with the tail index $\alpha_Z > 0$ and a r.v. σ is non-negative and independent of Z with the tail index $\alpha_\sigma > 0$, then their product has a tail index equal to $\min(\alpha_Z, \alpha_\sigma)$. As Kesten's and Breiman's theorems are asymptotical results and can only be considered as recommendations for data sets of moderate size, we have further evaluated the quality of the modeling and fitting procedure. The Student model does not fit the outliers perfectly (see Fig. 9). For this reason, we have estimated the tail indices of the normalized IATs $\widehat{Y}_t$ and the residuals $\widehat{Z}_t$ directly by the Hill's, moment and ratio estimates and then determined the tail index of the σ_t by (21). The estimates of the tail indices are summarized in Table 7 and illustrated in Fig. 10. One can recognize the small difference regarding the estimated value $\hat{k} = 1.735$ arising from the Student distributed Z_t's for degree $d = 1.9$ and the estimated value 1.765 determined by the residuals $\widehat{Z}_t$ of the original IATs. Since the value $\min(1.9, 1.765) = 1.765$ determined by Breiman's results is rather close to the Ratio estimate 1.852 of the normalized IATs, we can conclude that the proposed GARCH (1,1) process may fit the IATs. This minimal value 1.765 of the tail index also enters the 95% confidence intervals of the Hill's, ratio and moment estimates.

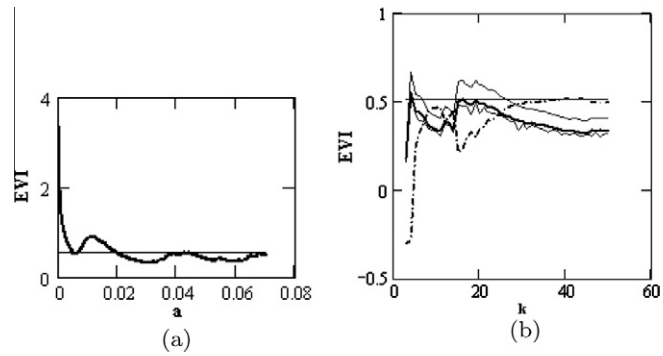

Fig. 10. Ratio estimate (a); Hill's estimate (solid line) with 95% Gaussian confidence interval (thin solid line) and moment estimate (dashed line) (b) of IATs.

In summary, a GARCH (1,1) process with properly selected parameters can be used as a versatile, but complex fine-grained model of the packet inter-arrival times.

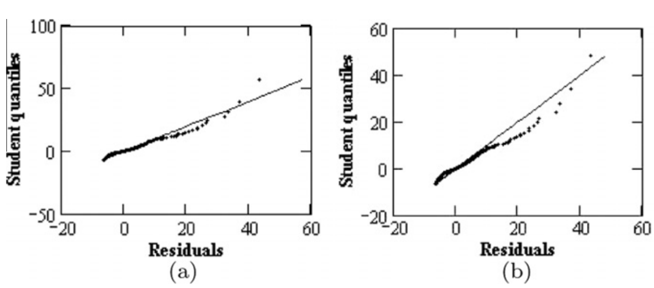

Fig. 9. QQ-plots of the GARCH (1,1) residuals against the quantiles of a Student's distribution with $d = 1.9$ (a) and $d = 2$ (b) degrees of freedom.

Table 7
Estimation of the tail index α of the normalized IATs $\widehat{Y}_t$ with 95% confidence intervals, the residuals $\widehat{Z}_t$ and the volatility σ_t of the GARCH (1,1) process.

$\widehat{Y}_t$			$\widehat{Z}_t$	σ_t
Hill	Moment	Ratio		
1.949 [1.511,2.387]	1.949 [1.215,4.926]	1.852 [1.416,2.288]	1.9	1.765

5. Conclusions

We have presented a new statistical characterization of unidirectional packet flows generated by VoIP traffic of Skype clients. The mean byte traffic which is offered to a virtual link of finite capacity in a finite observation interval $[0, t]$ has been evaluated. For this purpose, we have separated the IATs into independent subsets (i.e. blocks of flow data). Then we have considered the appearance of the sub-volumes within these subsets as renewal process assuming that the latter are concentrated at the end of the subsets. This allows us to estimate the mean number of sub-volumes within time t as the renewal function and to evaluate the mean byte traffic in different desirable time intervals. Moreover, we can predict the mean traffic load at the time exceeding the observation interval by a linear model.

We have also evaluated the period without loss, namely, the mean time without loss (1.348 s) and the high quantiles of this time (about 1.5 min for the 99.9% quantile). The high quantiles provide approximate upper bounds of the possible transmission time of the considered Skype packets without any loss. We found that the minimal capacity which is required to provide such upper bounds is 11.77 kbps.

We have modelled the dependence in the IAT process between Skype packets by a non-linear process. We have realized that a simple GARCH (1,1) process with properly determined values of the parameters can provide a suitable model for our illustrative flow data. Since it is found that the IATs of a Skype flow are an LRD process with a time-varying variance (which seems to be a common property of IATs arising from different adaptive codecs in different networks), one can recommend such a GARCH (1,1) process with heavy-tailed noise as a simple, but versatile model of packet inter-arrival times and adapt its coefficients to any available measurements.

In conclusion, we have shown by our statistical study how complicated traffic models arising from network dependent variable bitrate traffic can be exploited and simplified by an intrinsic renewal structure. The latter allows us to cope with various analysis and design tasks such as capacity allocation and the evaluation of losses. We are convinced that the proposed statistical methodology can be used to characterize successfully not only unidirectional Skype flows considered here but also other network dependent multi-media streams with variable bitrate such as slice-encoded MPEG4 video traffic (see [17]). Following this approach, both single streams and composite traffic with a few aggregated streams of a triple play environment flowing, for instance, along DSL or cable lines to a user in his private home can be investigated.

Acknowledgment

The authors acknowledge the partial support by the research actions IST-FP6 NoE EuroFGI, COST IC0703 and BMBF MDA08/15.

References

[1] S.A. Baset, H. Schulzrinne, An analysis of the Skype peer-to-peer Internet telephony protocol, in: Proceedings of the 25th IEEE International Conference on Computer Communications INFOCOM 2006, Barcelona, April, 2006.
[2] D. Bonfiglio et al., Revealing Skype traffic: when randomness plays with you, in: Proceedings of ACM SIGCOMM'07, Kyoto, Japan, August 27–31, 2007.
[3] P.J. Brockwell, R.A. Davis, Introduction to Time Series and Forecasting, second ed., Springer Texts in Statistics, New York, 2002.
[4] K.-T. Chen, C.-Y. Huang, P. Huang, C.-L. Lei, Quantifying Skype user satisfaction, in: Proceedings of ACM SIGCOMM'06, Pisa, Italy, September 11–15, 2006.
[5] W. Feller, An Introduction to Probability Theory and Its Applications, second ed., vol. II, Wiley, New York, 1971.
[6] E.W. Frees, Warranty analysis and renewal function estimation, Naval Research Logistics Quarterly 33 (1986) 361–372.
[7] S.L. Garfinkel, VoIP and Skype security, Skype Security Overview, Rev 1.6, 1/26/05, 2005.
[8] D.P. Heyman, M.J. Sobel, Stochastic Models in Operations Research, vol. I, McGraw-Hill, New York, 1982.
[9] T. Hoßfeld, A. Binzenhöfer, Analysis of Skype VoIP traffic in UMTS: end-to-end QoS and QoE measurements, Computer Networks 52 (3) (2008) 650–666.
[10] J.R.M. Hosking, Fractional differencing, Biometrika 68 (1) (1981) 165–176.
[11] G. Li, W.K. Li, Least absolute deviation estimation for fractionally integrated autoregressive moving average time series models with conditional heteroscedasticity, Biometrika 95 (2) (2008) 399–414.
[12] G.M. Ljung, G.E.P. Box, On a measure of lack of fit in time series models, Biometrika 65 (1978) 297–303.
[13] N.M. Markovich, Nonparametric Estimation of Univariate Heavy-Tailed Data, J. Wiley & Sons, Chichester, 2007.
[14] N.M. Markovich, U.R. Krieger, Statistical analysis of VoIP flows generated by Skype users, in: Proceedings of IEEE International Workshop on Traffic Management and Traffic Engineering for the Future Internet, FITraMEn, Porto, Portugal, December 11–12, 2008.
[15] N.M. Markovich, U.R. Krieger, Statistical traffic data arising from the Internet, in: D.D. Kouvatsos (Ed.), Traffic and Performance Engineering for Heterogeneous Networks, Series in Information Science and Technology, vol. 1, River Publishers, Aalborg, Denmark, 2009, pp. 41–60 (Chapter 2).
[16] N.M. Markovich, U.R. Krieger, Statistical characterization of QoS aspects arising from the transport of Skype VoIP flows, in: Proceedings of the First International Conference on Evolving Internet 2009, Cannes, France, August 23–29, 2009, pp. 9–14.
[17] N.M. Markovich, A. Undheim, P. Emstad, Classification of slice-based VBR video traffic and estimation of link loss by exceedance, Computer Networks 53 (7) (2009) 1137–1153.
[18] T. Mikosch, Modeling dependence and tails of financial time series, Laboratory of Actuarial Mathematics, University of Copenhagen, Working paper no. 181, 2002.
[19] T. Mikosch, C. Stărică, Limit theory for the sample autocorrelations and extremes of a GARCH(1,1) process, Annals of Statistics 28 (5) (2000) 1427–1451.
[20] S.I. Resnick, Heavy-Tail Phenomena: Probabilistic and Statistical Modeling, Springer, New York, 2006.
[21] R. Runde, The asymptotic null distribution of the Box-Pierce Q-statistic for random variables with infinite variance, Journal of Econometrics 78 (1997) 205–216.
[22] K. Suh, D.R. Figueiredo, J. Kurose, D. Towsley, Characterizing and detecting relayed traffic: a case study using Skype, in: Proceedings of INFOCOM 2006, Barcelona, Spain, 2006.
[23] K.S. Trivedi, Probability & Statistics with Reliability, Queuing, and Computer Science Applications, Prentice Hall of India, 1997.
[24] J.L. Teugels, Renewal theorems when the first or the second moment is infinite, Annals of Statistics 39 (1968) 1210–1219.
[25] I. Weissman, Estimation of parameters and large quantiles based on the k largest observations, Journal of American Statistical Association 73 (1978) 812–815.
[26] W. Willinger, D. Alderson, L. Li, A pragmatic approach to dealing with high-variability in network measurements, in: Proceedings of Internet Measurement Conference (IMC) 2004, Taormina, Sicily, Italy, October 25–27, 2004, pp. 88–100.
[27] Global IP Solutions, San Francisco, USA, GIPS iSAC, <http://www.gipscorp.com/files/english/datasheets/iSAC.pdf>.
[28] T. le Grand, P. Jones, RTP payload format for the iSAC codec, IETF internet draft, July 12, 2008, <http://tools.ietf.org/id/draft-legrand-rtp-isac-00.txt>.
[29] Skype Technologies S.A., Luxembourg, <http://www.skype.com>.