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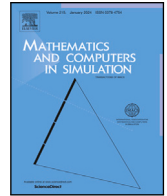


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Original articles

## Contagious popular stories, stock market participation, and boom–bust cycles<sup>☆</sup>

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### ABSTRACT

We study a model in which investors' stock market participation hinges on contagious popular stories. Two opposing narratives exist that either advocate investing in the stock market or abstaining from it. Investors' adherence to these narratives depends on the current behavior of the stock market and the social interactions among investors. For instance, stories that advocate investing in the stock market appear more plausible to investors during boom periods and when such behavior is common among peers. We identify different constellations in which waves of market entry and exit, driven by contagious popular stories, create boom–bust stock market dynamics.

### 1. Introduction

As emphasized by Shiller [1,2,3,4], the mass of people is not well informed when making investment decisions. Most people do not follow the news closely and rarely get the facts right. Yet, their investment decisions shape stock market behavior. This raises the question of how people make their investment decisions. Shiller argues that contagious popular stories guide people's loose thinking and ultimately dictate their investment decisions. Often, two opposing narratives dominate the public debate: one advocates investing in the stock market and the other warns against it. Shiller posits that the wave-like adherence to such opposing contagious stories is a significant driver of the boom–bust behavior of stock markets, a channel that needs to be better understood.<sup>1</sup>

We develop a simple model that captures Shiller's basic reasoning. Our model can be summarized as follows. Instead of holding an optimal portfolio, investors' follow one of two popular stories. One narrative favors investing in the stock market, while the other one does not. The diffusion of these narratives depends on the behavior of the stock market, as expressed by its current price level and trend, and on social interactions among investors. The narrative that promotes investing in the stock market receives more attention when the stock market is booming and when many investors are already following it. Finally, the stock price depends on market participation: an inflow (outflow) of investors increases (decreases) the stock price.

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<sup>1</sup> Galbraith [5] and Kindleberger and Aliber [6] express similar views.

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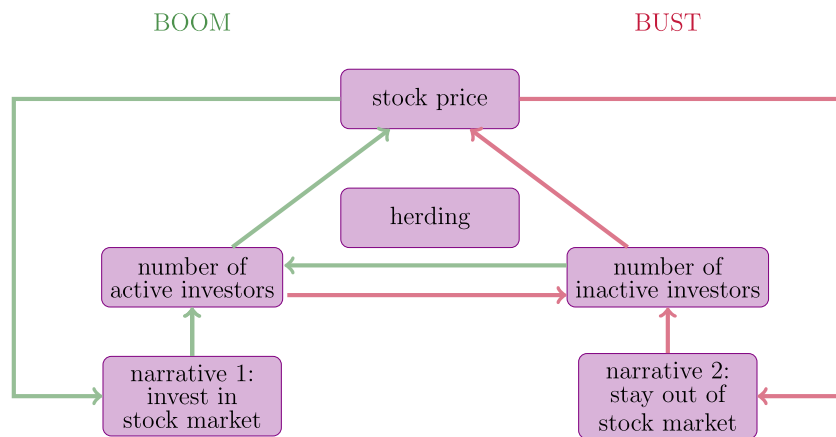


Fig. 1. Feedback dynamics. Green arrows depict the dynamics of a booming stock market. Rising stock prices make the “invest in the stock market” narrative more popular. Investors’ stock market participation is further driven by herding behavior. Red arrows illustrate the dynamics of a stock market bust. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Our setup includes an important feedback loop, visualized in Fig. 1. The diffusion of contagious popular stories causes waves of market entry and exit, which trigger boom–bust stock market dynamics, and these dynamics, in turn, impact the spread of contagious popular stories.

We use a mix of analytical and numerical tools to explore this feedback loop. The dynamics of our model are driven by the iteration of a two-dimensional nonlinear map. The stock price, which equals the market share of active investors, is bounded in the unit interval. As long as investors’ herding behavior is not too strong, the model has a unique middle steady state where half of the investors are active in the stock market. This steady state may lose its local stability if investors react more strongly to the current stock market trend via a Neimark–Sacker bifurcation or if they herd more strongly via a Pitchfork bifurcation. Let us briefly discuss these two scenarios.

In the Neimark–Sacker bifurcation scenario, the middle steady state becomes unstable, leading to endogenous stock price oscillations around it. This is a first example where repeated waves of market entry and exit, driven by the emergence of opposing contagious popular stories, create endogenous boom–bust dynamics as envisioned by Shiller. Interestingly, the simulations reveal that the bubble patterns exhibit a slow build-up and rapid burst, as is frequently observed in real stock markets.

In the Pitchfork bifurcation scenario, the middle steady state becomes unstable and two stable symmetric low and high steady states emerge. In these steady states, either more or less than half of the investors are active in the stock market, resulting in a steady-state stock price above or below the middle steady state. These additional steady states may lose stability through two consecutive Neimark–Sacker bifurcations. Since the higher steady state loses stability before the lower one as investors’ reaction to current price trends or herding behavior increases, we may observe endogenous cyclical stock market dynamics first around the higher steady-state price and then around the lower steady-state price. Our stock market model thus gives rise to coexisting attractors, an aspect that may lead to intricate attractor-switching dynamics in the presence of noise.

We proceed as follows. In Section 2, we discuss related work. In Section 3, we develop our model. In Sections 4 and 5, we use a mix of analytical and numerical tools to discuss the functioning of our model. In Section 6, we conclude our paper.

## 2. Related work

Stock market participation varies considerably over time. For instance, [7] report that 31.6 percent of U.S. households owned stocks in 1989, rising to 49.8 percent in 1998. Our paper seeks to contribute to the literature that explores the relationship between investors’ stock market participation and the emergence of stock market bubbles. Two important factors that drive investor participation in the stock market are the following.

First, [7,8], and [2] argue that investor participation in the stock market depends on social interactions. Clearly, the more investors participate in the stock market, the more attractive it appears to them. In general, this type of herding behavior may create inertia. In a boom state, when holding stocks is popular, investors remain in the stock market, preventing it from crashing. In a bust state, when holding stocks is unpopular, investors will exit the stock market, preventing a price recovery. We will also encounter such inertia effects in our model — they may give rise to multiple steady states.

Second, [5,6], and [2] argue that investor participation in the stock market depends on stock price momentum. Reviewing historical stock market bubbles, [5] observes that a booming market constantly attracts new investors, thereby generating its own momentum. Kindleberger and Aliber [6] report that during a boom, a follow-the-leader process can occur in which investors observe others benefiting from speculative purchases. Consequently, more and more investors who had previously stayed away from the market begin to join the race for profits, and their market entry behavior pushes stock prices to ever higher levels. Shiller [2] adds that stock prices and investors’ opinions about their best investment alternatives are strongly related. For instance, the proportion

of investors who believe that stocks are the best investment alternative increases when the stock market rises. Note that investors’ reactions to stock market momentum may set a positive feedback loop in motion: stock price increases result in an influx of additional investors who buy stocks, pushing stock prices even higher. This effect also plays an important role in our model, as it can destabilize the dynamics of the stock market.

There are a few related models that study how the market entry and exit behavior of investors may affect stock market dynamics.<sup>2</sup> Alfi et al. [12,13,14] investigate models in which the number of investors active in the stock market self-organizes in a way that generates realistic stock price dynamics. Iori [15,16,17] proposes models in which investors’ stock market participation depends positively on their peers’ actions and negatively on trade frictions, such as trading costs. Westerhoff and Dieci [18] develop a model in which stock market trading becomes more widespread when it is profitable, a factor that results in bubbles and excess volatility. Moreover, they show that the imposition of transaction taxes may crowd out destabilizing behavior and thereby tame stock market dynamics.

More closely related to our work is the paper by [19]. They develop a model in which investors’ market entry decisions depend on current market movements and the fundamental state of the stock market. In other words, investors tend to enter (or exit) the stock market during periods of rising (or falling) stock prices and exit (or enter) the stock market during periods of overvaluation (or undervaluation), a behavior that may lead to boom–bust stock market dynamics. Blaurock et al. [20] propose a model in which speculators’ market entry decisions are influenced by herding behavior and risk, with speculators’ order placement dependent on price trends, market misalignments, and fundamental news. They show that herding-induced waves of market entry can trigger bubbles and volatility outbursts, but that higher stock market risk eventually reduces speculator participation in the stock market, leading to a reduction in mispricing and volatility. Dieci et al. [21,22] develop a model in which investors participate in stock, bond, or housing markets, influenced by herding effects, price trends, and market mispricing. They demonstrate that the endogenous dynamics of the stock and housing markets are countercyclical to each other when investors react strongly to price trends. This cross-feedback reflects investors’ tendency to transfer their enthusiasm from one speculative market to another. Dieci et al. [23] study a model in which investors base their market entry decisions on momentum, value, risk, and social interactions, showing that waves of stock market participation may trigger coevolving boom–bust cycles, with sensitive risk responses leading to spontaneous, sharp, and persistent downturns and excess volatility.

The latter contributions have in common that the market share of active stock market investors follows the exponential replicator dynamics proposed by [24,25]. In contrast, our paper employs the transition probability approach by [26] to model investor participation in the stock market.<sup>3</sup> Inspired by [1–4], we assume that these transition probabilities depend on social interactions and on the behavior of the stock market transmitted via opposing contagious popular stories. As a result, we obtain an analytical tractable model that may produce endogenous boom–bust dynamics and coexisting attractors. A third difference to the aforementioned contributions is that investors’ behavior does not depend on the stock market’s fundamental value. Indeed, there is no fundamental value in our model. This model feature also mirrors one of Shiller’s [1] famous arguments: people lack objective models for valuing risky assets.

### 3. A simple market entry model

Stock prices are determined as in [1]. Accordingly, there are  $n$  investors in total. Rather than maintaining a well-balanced portfolio, investors think in terms of what might be the best investment opportunity for them. Consequently, they either invest in the stock market or they abstain from it. The number of active and inactive investors is given by  $n_t^A$  and  $n_t^I$ , respectively. By normalizing the mass of investors to  $n = 1$ , we can also consider  $n_t^A$  and  $n_t^I$  as the market shares of active and inactive investors, respectively, with  $0 \leq n_t^A \leq 1$  and  $n_t^A + n_t^I = 1$ . Now, let  $d_t$  be the value of shares demanded by a single active investor. Clearly,  $d_t$  denotes the shares demanded by a single investor at the current price multiplied by the current price. Market equilibrium requires that the total demand of shares is equal to the total supply of shares, i.e.,

$$\frac{d_t}{p_t} n_t^A = s, \tag{1}$$

where  $p_t$  stands for the stock price and  $s$  for the constant number of outside shares. It follows that

$$p_t = \frac{d_t}{s} n_t^A. \tag{2}$$

Assuming that investors’ demand elasticity is unitary, we can regard  $d_t$  as an exogenously given constant, say  $d_t = d$ . For ease of exposition, we set  $d/s = 1$  and obtain

$$p_t = n_t^A. \tag{3}$$

The stock price matches the market share of active investors and is bounded between  $0 \leq p_t \leq 1$ . Quite naturally, Shiller’s pricing rule [1] implies that an influx of investors increases the stock price, and vice versa.

<sup>2</sup> Gardini et al. [9] study a stock market in which speculators exhibit “new economic era” thinking. Similarly, [10,11] study stock market models in which competing narratives influence speculators’ perception of the stock market’s fundamental value.

<sup>3</sup> Lux [27,28,29] uses the transition probability approach to capture speculators’ herding-dependent switching between technical and fundamental trading rules. See [30–32], and [33] for additional herding models.

We use the transition probability approach proposed by [26] to model the market shares of active investors. Applied to our setup, the key idea is that the market shares of active and inactive investors in the next period depend on their current market shares and the probabilities of switching from one group to the other. In the following,  $prob_t^{IA}$  describes the probability that an inactive investor will become active, while  $prob_t^{AI}$  describes the probability that an active investor will become inactive. For a continuum of investors, the market share of active investors is given by

$$n_t^A = n_{t-1}^A + n_{t-1}^I prob_t^{IA} - n_{t-1}^A prob_t^{AI}. \tag{4}$$

Accordingly, the market share of active investors increases by  $n_{t-1}^I prob_t^{IA}$  and decreases by  $n_{t-1}^A prob_t^{AI}$ . The market share of inactive investors is equal to

$$n_t^I = 1 - n_t^A. \tag{5}$$

See [27,34,35] for a further discussion of the transition probability approach.

The two transition probabilities depend on a so-called switching index  $a_t$ , which reflects the relative influence of the “invest in the stock market” narrative versus its counterpart, the “abstain from the stock market” narrative. An increase in  $a_t$  increases  $prob_t^{IA}$  and decreases  $prob_t^{AI}$ . To be more precise, the probability that an inactive investor will become active and that an active investor will become inactive is given by

$$prob_t^{IA} = \nu Exp[a_t] \tag{6}$$

and

$$prob_t^{AI} = \nu Exp[-a_t], \tag{7}$$

respectively. The flexibility parameter  $0 < \nu < 1$  captures how quickly investors react to the switching index. For  $a_t = 0$ , for instance, the transition probabilities are  $prob_t^{IA} = prob_t^{AI} = \nu$ .<sup>4</sup> While (6) and (7) ensure positive transition probabilities, they remain below one if the switching index is bounded and parameter  $\nu$  is sufficiently small.

Inspired by [2–4], we assume that investor participation in the stock market is driven by contagious popular stories. When the stock market is booming, popular stories that advocate investing in stock markets receive more attention. Such narratives become contagious through social interactions among investors. Here, we specify the switching index as follows:

$$a_t = \alpha_x (n_{t-1}^A - n_{t-1}^I) + 2p_{t-1} \mu Arctan \left[ \frac{\delta}{\mu} (p_{t-1} - p_{t-2}) \right], \tag{8}$$

with  $\mu = \frac{2}{\pi} \kappa$ . The first term on the right-hand side of (8) captures the impact of investors’ social interactions. The belief that investing in the stock market is the right thing to do increases with the market share of active investors. Parameter  $\alpha_x > 0$  measures the strength of investors’ herding behavior. The larger the parameter  $\alpha_x$ , the stronger the investors’ herding behavior. The second term on the right-hand side of (8), consisting of two multiplicative components, reflects how the relevance of the two popular stories depends on the behavior of the stock market. First, the belief that investing in the stock market is the right thing to do increases as the stock price rises. Parameter  $\delta > 0$  controls the strength of the investors’ reaction to the stock price trend. The larger the parameter  $\delta$ , the stronger the investors react to the stock price trend. The S-shaped component  $Arctan \left[ \frac{\delta}{\mu} z_{t-1} \right]$  is bounded between  $-\kappa$  and  $\kappa$ , where we have defined  $z_{t-1} := p_{t-1} - p_{t-2}$ . Its upper and lower limits are reached when  $z_{t-1} \rightarrow \infty$  and  $z_{t-1} \rightarrow -\infty$ , respectively. Clearly, an increase in the parameter  $\kappa$  enhances investors’ overall sensitivity to popular stories. Second, this component is weighted by  $2p_{t-1}$ , revealing that this effect increases in line with the stock price. In fact, people get more excited about investing in the stock market when it reaches new highs.<sup>5</sup>

#### 4. Analytical results

Defining the difference in market shares as  $x_t = n_t^A - n_t^I$ , noting that  $n_t^A = \frac{1+x_t}{2}$  and  $n_t^I = \frac{1-x_t}{2}$ , and introducing the auxiliary variable  $y_t = x_{t-1}$ , it follows that the dynamics of our model are driven by the two-dimensional nonlinear map

$$M := \begin{cases} x_t = x_{t-1} + (1 - x_{t-1})\nu Exp[a_t] - (1 + x_{t-1})\nu Exp[-a_t] \\ y_t = x_{t-1} \end{cases}, \tag{9}$$

with  $a_t = \alpha_x x_{t-1} + (1 + x_{t-1})\mu Arctan \left[ \frac{\delta}{2\mu} (x_{t-1} - y_{t-1}) \right]$ .

The following proposition, proven in Appendix, summarizes our main results:

<sup>4</sup> Note that for  $a_t = a = 0$ , the market share of active investors is due to  $n_t^A = (1 - 2\nu)n_{t-1}^A + \nu$ . The steady state of this map, given by  $\bar{n}^A = \frac{1}{2}$ , is globally stable. This is an important property of the transition probability approach: when the switching index is equal to zero, dynamics are set in motion, resulting in a convergence to  $\bar{n}^A = \frac{1}{2}$ .

<sup>5</sup> For analytical convenience, we multiply the price signal in (8) by 2. Moreover, note that the second term on the right-hand side of (8) vanishes at any steady state. As a result, the switching index is zero, positive, or negative, depending on the difference in the steady-state market shares of active and inactive investors.

**Proposition 1.** *The dynamics of our model are driven by the two-dimensional nonlinear map  $M$ , where different cases could be distinguished.*

- (i) *For  $0 < \alpha_x < 1$ , there is a middle steady state  $\bar{x}_M = 0$ , i.e.,  $\bar{p}_M = \bar{n}_M^A = \bar{n}_M^I = 0.5$ . For  $\alpha_x > 1$ , there are three steady states, namely a low steady state  $\bar{x}_L$ , a middle steady state  $\bar{x}_M$ , and a high steady state  $\bar{x}_H$ , such that  $-1 < \bar{x}_L < \bar{x}_M < \bar{x}_H < 1$ , i.e.,  $0 < \bar{p}_L = \bar{n}_L^A < \bar{p}_M = \bar{n}_M^A = 0.5 < \bar{p}_H = \bar{n}_H^A < 1$ . The low and high steady states  $\bar{x}_{L,H}$  are symmetrically located around the middle steady state  $\bar{x}_M$  and obtained by solving  $Tanh[\alpha_x \bar{x}] = \bar{x}$ . For  $\alpha_x \rightarrow \infty$ , we have that  $\bar{x}_L \rightarrow -1$  and  $\bar{x}_H \rightarrow 1$ .*
- (ii) *The middle steady state  $\bar{x}_M$  is locally stable if and only if inequalities  $\nu\delta < 1$  and  $\alpha_x < 1$  jointly hold. A separate violation of the first inequality is associated with the emergence of a Neimark–Sacker bifurcation. A separate violation of the second inequality is associated with the emergence of a Pitchfork bifurcation. The low and high steady states  $\bar{x}_{L,H}$  are locally stable if and only if  $\nu\delta < (1 - \bar{x}_{L,H})^{-\frac{1}{2}}(1 + \bar{x}_{L,H})^{-\frac{3}{2}}$ . A violation of this stability condition is associated with the emergence of a Neimark–Sacker bifurcation. Notably, for  $\bar{x}_L \rightarrow -1$  and  $\bar{x}_H \rightarrow 1$ , the stability condition is satisfied for any value of  $\delta\nu$ . In addition, the high steady state  $\bar{x}_H$  undergoes a Neimark–Sacker bifurcation for lower values of  $\delta\nu$  than the low steady state  $\bar{x}_L$ . Finally, the low steady state  $\bar{x}_L$  is always stable if  $\delta\nu < 1$  and the high steady state  $\bar{x}_H$  is always stable if  $\delta\nu < \frac{4}{3\sqrt{3}}$ .*

Fig. 2 visualizes the results detailed in Proposition 1. In the top-left panel of Fig. 2, the red curve represents  $Tanh[\alpha_x x]$  for  $\alpha_x = 0.5$ , while the gray curve represents the bisector  $x$ . The top-right panel of Fig. 2 shows the same for  $\alpha_x = \frac{5 \log 9}{8} \approx 1.373$ . When  $\alpha_x < 1$ , the two curves intersect only at the middle steady state  $\bar{x}_M = 0$ , denoted by a cyan dot. For  $\alpha_x > 1$ , however, the two curves intersect three times: at the low steady state  $\bar{x}_L < 0$  (dark purple dot), at the middle steady state  $\bar{x}_M = 0$  (cyan dot), and at the high steady state  $\bar{x}_H > 0$  (light purple dot). We can thus conclude that the possible emergence of multiple steady states depends only on  $\alpha_x$ , reflecting the strength of investors’ herding behavior.

The bottom-left panel of Fig. 2 illustrates the existence and the local stability domain of the steady states with respect to  $\alpha_x$ . For  $0 < \alpha_x < 1$ , only the middle steady state, represented by the cyan line, exists and is locally stable, if in addition,  $\nu\delta < 1$  holds. At  $\alpha_x = 1$ , a Pitchfork bifurcation occurs, causing the middle steady state to lose its local stability. The high steady state  $\bar{x}_H$  and the low steady state  $\bar{x}_L$  emerge symmetrically around the middle steady state  $\bar{x}_M$ , represented by the light purple and dark purple curves, respectively. The dashed cyan line indicates the unstable middle steady state.

The low and high steady states  $\bar{x}_{L,H}$  are locally stable as long as the condition  $\nu\delta < (1 - \bar{x}_{L,H})^{-\frac{1}{2}}(1 + \bar{x}_{L,H})^{-\frac{3}{2}}$  holds. This condition is represented in the bottom-right panel of Fig. 2. It achieves its minimum at  $\bar{x}_H = 0.5$  ( $\bar{n}_H^A = 0.75$ ), indicating that for  $\delta\nu < \frac{4}{3\sqrt{3}} \approx 0.77$ , both steady states  $\bar{x}_L$  and  $\bar{x}_H$  are always stable if they exist. This also indicates that the symmetrical steady states lose their local stability at distinct critical values of  $\delta\nu$ . Considering  $\bar{x}_M = 0$  ( $\bar{n}_M^A = 0.5$ ), the inequality becomes  $\nu\delta < 1$ . Therefore, if this inequality holds,  $\bar{x}_L$  is always stable. Finally, if  $\bar{x}_L$  and  $\bar{x}_H$  approach  $-1$  and  $1$ , respectively, the condition is satisfied for every value of  $\delta\nu$ .

### 5. Numerical results

In this section, we numerically illustrate the analytical properties of the model and deepen our analysis. We keep parameters  $\nu$  and  $\kappa$  equal to  $\nu = 0.05$  and  $\kappa = 1$  and vary parameters  $\alpha_x$  and  $\delta$ . In Section 5.1, we first discuss the bifurcation behavior of our model. After this technical investigation, we explain the economic functioning of our model in Section 5.2.

#### 5.1. Bifurcation structures

Fig. 3 presents the main bifurcation routes of our model. The top-left panel of Fig. 3 shows a bifurcation diagram plotting the market share of active investors  $n_t^A$  against parameter  $\delta$  for  $\alpha_x = 0.95$ . For  $\delta < 20$ , the middle steady state  $\bar{n}_M^A$  is locally stable, represented by the cyan line. At  $\delta = 20$ , the middle steady state loses its local stability via a Neimark–Sacker bifurcation, giving rise to periodic or quasi-periodic dynamics, depicted in magenta.

The top-right panel of Fig. 3 shows a bifurcation diagram plotting the market share of active investors  $n_t^A$  against parameter  $\alpha_x$  for  $\delta = \frac{125}{8} = 15.625$  (implying that  $\nu\delta \approx 0.78$ ). For  $\alpha_x < 1$ , the middle steady state  $\bar{n}_M^A = 0$  is locally stable, represented by the cyan line. At  $\alpha_x = 1$ , the middle steady state loses its local stability and two symmetric steady states  $\bar{n}_H^A$  and  $\bar{n}_L^A$  emerge. Depending on the initial value conditions, the number of active investors is higher or lower than  $\bar{n}_M^A = 0.5$ , represented by a light purple and a dark purple line, respectively. For  $\alpha_x \approx 1.056$ ,  $\bar{n}_H^A \approx 0.7$  loses its local stability via a Neimark–Sacker bifurcation, while  $\bar{n}_L^A \approx 0.3$  remains locally stable.<sup>6</sup> Between  $\alpha_x \approx 1.056$  and  $\alpha_x = \frac{5 \text{Log}[4]}{6} \approx 1.155$ , the low steady state  $\bar{n}_L^A$  coexists with a periodic or quasi-periodic attractor, represented in magenta. For  $\alpha_x > \frac{5 \text{Log}[4]}{6} \approx 1.155$ , the two steady states  $\bar{n}_H^A > 0.8$  and  $\bar{n}_L^A < 0.2$  are locally stable.

The bottom-left panel of Fig. 3 shows a bifurcation diagram plotting the market share of active investors  $n_t^A$  against parameter  $\delta$  for  $\alpha_x = \frac{5 \text{Log}[4]}{6} \approx 1.155$  and for a combination of initial values of  $(n_{t-1}^A, n_t^A) = (0.5, 0.51)$ . For  $\delta < \frac{125}{8} = 15.625$ , the high steady state,  $\bar{n}_H^A = 0.8$ , is locally stable. At  $\delta = \frac{125}{8} = 15.625$ , the high steady state loses its local stability via a Neimark–Sacker bifurcation and periodic or quasi-periodic dynamics emerge. At  $\delta \approx 17.375$ , the generic trajectory starts to converge toward the low steady state  $\bar{n}_L^A = 0.2$ . At  $\delta = 62.5$ , the low steady state loses its local stability via a Neimark–Sacker bifurcation and periodic or quasi-periodic dynamics emerge.

<sup>6</sup> It follows that  $\bar{x}_H^A \approx 0.4$  and  $\bar{x}_L^A \approx -0.4$ .

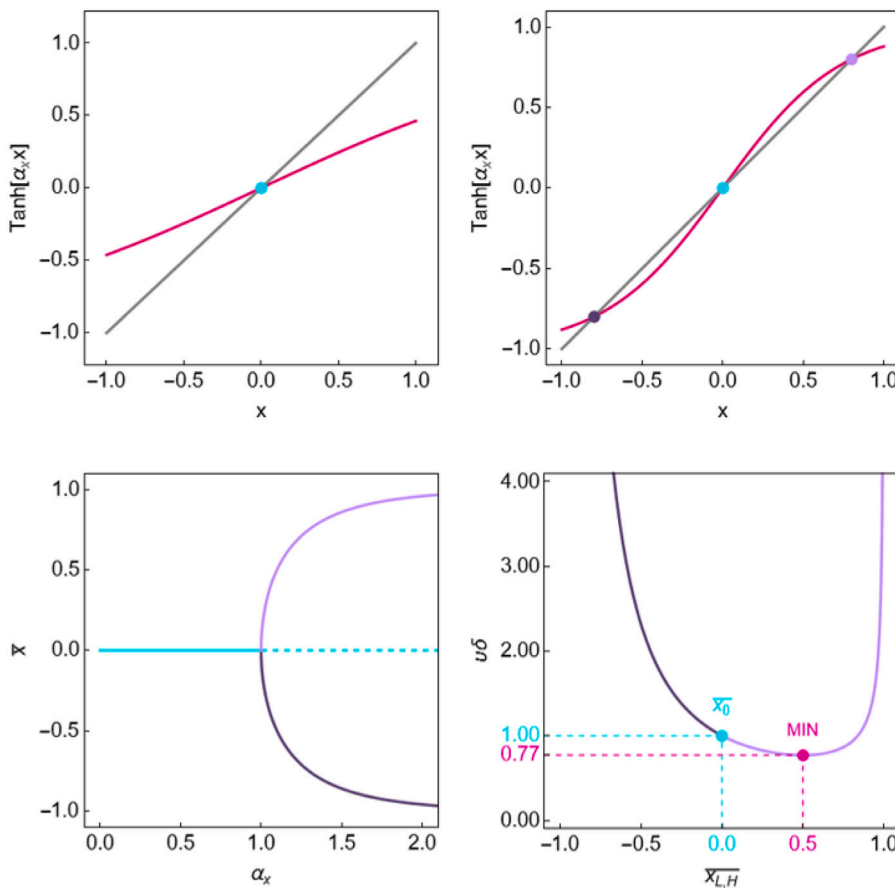


Fig. 2. The top-left panel shows  $Tanh[\alpha_x x]$  in red and  $x$  in gray for  $\alpha_x = 0.5$ . The top-right panel shows the same for  $\alpha_x = \frac{5Log[9]}{8} \approx 1.373$ . The bottom-left panel shows the evolution of the steady states with respect to  $\alpha_x$ . The bottom-right panel shows the stability condition for  $\bar{x}_{L,H}$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The bottom-right panel of Fig. 3 shows a bifurcation diagram plotting the market share of active investors  $n_i^A$  against parameter  $\delta$  for  $\alpha_x = \frac{5Log[4]}{6} \approx 1.155$  and for a combination of initial values  $(n_{i-1}^A, n_i^A) = (0.5, 0.49)$ . For  $\delta < 62.5$ , the low steady state is locally stable and the combination of initial values lies in its basin of attraction. At  $\delta = 62.5$ , the low steady state  $\bar{n}_L^A$  loses its local stability via a Neimark–Sacker bifurcation and endogenous dynamics are set in motion.

As we have seen, our model may produce coexisting attractors. Let us explore this issue in more detail. Fig. 4 depicts the basins of attraction of the low steady state and the coexisting quasi-periodic attractors. Specifically, the left panel of Fig. 4, based on  $\alpha_x = \frac{5Log[4]}{6} \approx 1.155$  and  $\delta = 17$  shows the basins of attraction for different combinations of initial values  $(n_{i-1}^A, n_i^A)$ . The lighter shade of the dark purple area visualizes initial value combinations converging toward the low steady state  $(\bar{n}_L^A, \bar{n}_L^A) = (0.2, 0.2)$ , marked in dark purple. The light pink area visualizes initial value combinations that generate quasi-periodic dynamics, shown in magenta. The right panel of Fig. 4 shows the same for  $\delta = 17.37$ . The low steady state is still given by  $(\bar{n}_L^A, \bar{n}_L^A) = (0.2, 0.2)$ . However, by increasing parameter  $\delta$ , the amplitude of the quasi-periodic attractor has increased, while its basin of attraction has decreased. At  $\delta \approx 17.373$ , a homoclinic bifurcation occurs, i.e., the quasi-periodic attractor collides with the unstable middle steady state  $\bar{n}_M^A$  and its basin of attraction disappears.<sup>7</sup>

Fig. 5 summarizes our model’s main bifurcation properties. The left panel of Fig. 5 shows a two-dimensional bifurcation diagram in which parameter  $\delta$ , which ranges from 0 to 80, is plotted against parameter  $\alpha_x$ , which ranges from 0 to 2. We use the initial values  $(n_{i-1}^A, n_i^A) = (\bar{n}_M^A, \bar{n}_M^A + 0.001)$  if  $\alpha_x < 1$  and  $(n_{i-1}^A, n_i^A) = (\bar{n}_H^A, \bar{n}_H^A + 0.001)$  otherwise. The right panel shows an enlargement for  $15 \leq \delta \leq 21$  and  $0.9 \leq \alpha_x \leq 1.1$ . The cyan area reflects parameter combinations where the dynamics converge toward the middle steady state. The light purple and dark purple areas mark parameter combinations where the dynamics converge toward the high and low steady state, respectively. All other colored areas, in particular the magenta area, visualize parameter combinations that lead to periodic or quasi-periodic dynamics. The following results can be observed. For  $\alpha_x < 1$  and  $\delta < 20$ , the dynamics converge

<sup>7</sup> We thank Laura Gardini for explaining this concept to us.

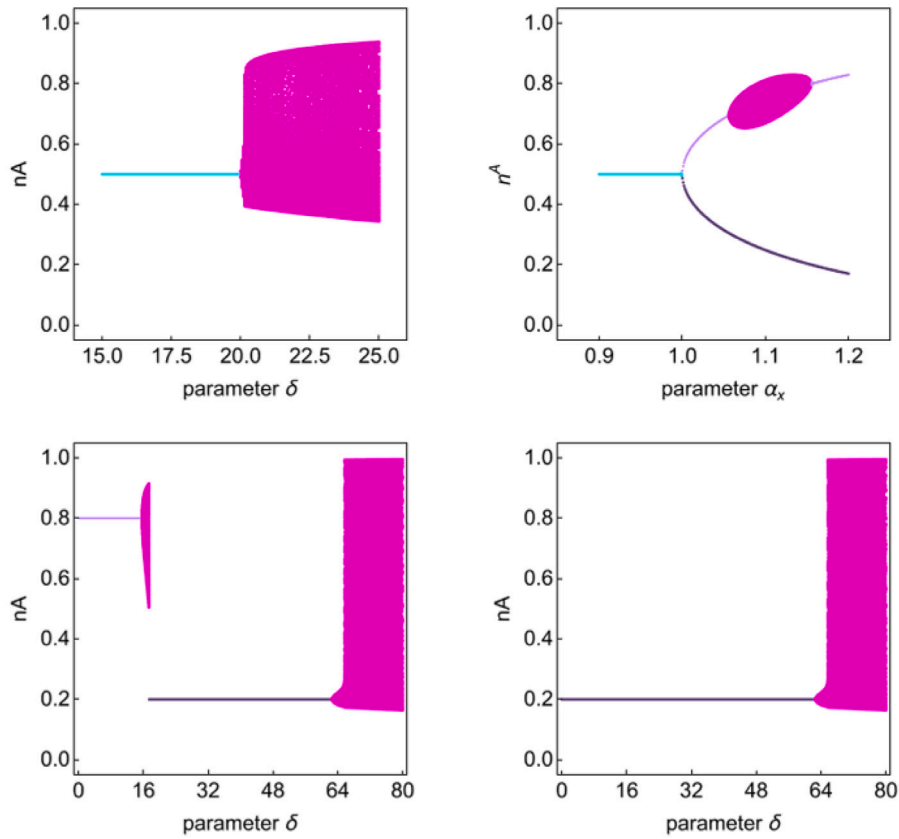


Fig. 3. The top-left panel shows a bifurcation diagram plotting  $n_t^A$  against  $\delta$  for  $\alpha_x = 0.95$ . The top-right panel shows a bifurcation diagram plotting  $n_t^A$  against  $\alpha_x$  for  $\delta = \frac{125}{8}$ . The bottom-left panel shows a bifurcation diagram plotting  $n_t^A$  against  $\delta$  for  $\alpha_x = \frac{5 \text{Log}[4]}{6}$  and for initial values  $(n_{t-1}^A, n_t^A) = (0.5, 0.51)$ . The bottom-right panel shows the same for initial values  $(n_{t-1}^A, n_t^A) = (0.5, 0.49)$ .

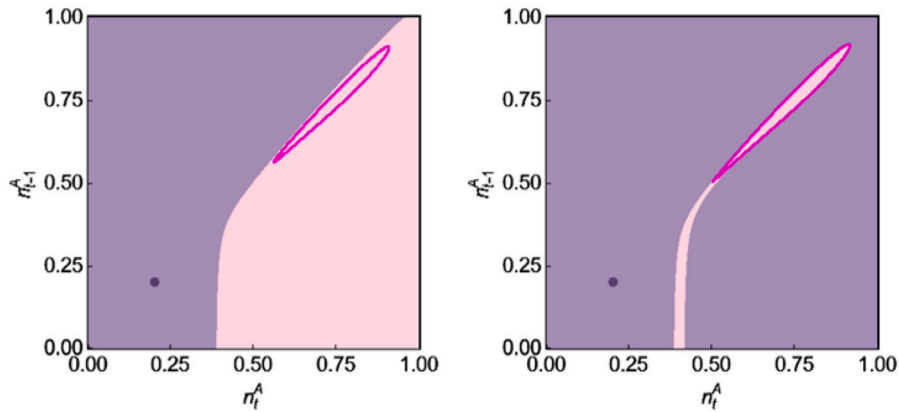


Fig. 4. The left panel shows the basin of attractions for  $\alpha_x = \frac{5 \text{Log}[4]}{6}$  and  $\delta = 17$  for initial values  $(n_{t-1}^A, n_t^A)$ . The right panel shows the same except that  $\delta = 17.37$ .

to the middle steady state. For  $\alpha_x > 1$  and low values of parameter  $\delta$ , the dynamics converge toward the high steady state. Keeping parameter  $\alpha_x$  constant and increasing parameter  $\delta$  destabilizes the high steady state and triggers periodic or quasi-periodic dynamics. Further increase of parameter  $\delta$  causes the dynamics to converge toward the low steady state. Finally, keeping parameter  $\delta$  high and decreasing parameter  $\alpha_x$  destabilizes the low steady state and causes periodic or quasi-periodic dynamics.

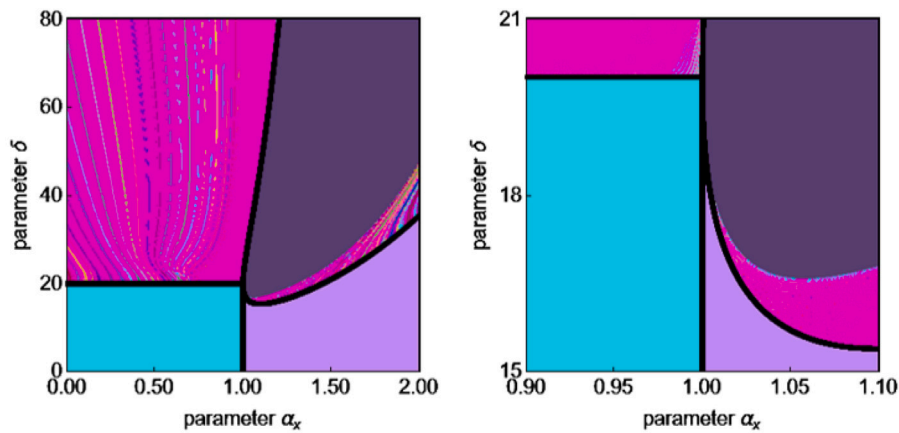


Fig. 5. The left panel shows a two-dimensional bifurcation diagram in which parameter  $\alpha_x$  ranges from 0 to 2 and parameter  $\delta$  ranges from 0 to 80. The right panel shows an enlargement.

### 5.2. Economic functioning of the model

In this section, we explain the economic functioning of our model. In Fig. 6, we discuss the emergence of boom–bust stock market dynamics. In Fig. 7, we repeat this exercise — yet, we allow for some exogenous shocks. Recall that the stock price is identical to the market share of active investors. Fig. 6 portrays different boom–bust dynamics in the time domain. The top-left panel of Fig. 6 shows the evolution of the market share of active investors  $n_t^A$  for  $\alpha_x = 0.95$  and  $\delta = 20$ . The cyan dashed line represents the middle steady state  $\bar{n}_M^A = 0$ . The market share of active investors fluctuates around the middle steady state, but at a higher level, i.e., approximately between 0.38 and 0.90. These dynamics occur after the Neimark–Sacker bifurcation of the middle steady state  $\bar{n}_M^A$ , as depicted in the corresponding top-left panel of Fig. 3.

In economic terms, we can understand how our model functions as follows. Suppose there is an initial inflow of investors, which raises the stock price. Then, the combined effects of (i) a larger number of active investors, (ii) a higher stock price, and (iii) a positive stock price trend lead more investors to believe in the “invest in the stock market” narrative. As a result, more investors enter the stock market, pushing stock prices even higher. After a while, the stock market begins to slow down. With so many investors already participating in the stock market, fewer new investors are available to join, and the pace of the stock market’s rise diminishes. Consequently, the appeal of the “invest in the stock market” narrative fades, causing some investors to exit the stock market. Once this regime change occurs, the alternative “stay out of the stock market” narrative gains popularity, triggering a stock market crash. The crash continues until investor exodus reaches its lowest point. At this point, the stock market begins to recover. Initially, the recovery is slow because both investor participation in the stock market and stock prices remain low. Over time, as participation and prices increase, the momentum of the stock market picks up again—until it finally slows down and the crash scenario described above repeats itself.

The top-right panel of Fig. 6 shows the evolution of the market share of active investors  $n_t^A$  for  $\alpha_x = \frac{5 \text{Log}[4]}{6} \approx 1.155$  and  $\delta = 17$ , assuming a combination of initial values of  $(n_{t-1}^A, n_t^A) = (0.5, 0.51)$ . The light purple dashed line represents the high steady state  $\bar{n}_H^A = 0.8$ . The market share of active investors fluctuates around the high steady state, but at a lower level, i.e., approximately between 0.56 and 0.90. See the corresponding bifurcation diagrams in the top-right and bottom-right panels of Fig. 3. The economic functioning of our model is the same as before, except that investors’ increased herding behavior results in higher average stock prices.

The bottom-left panel of Fig. 6 shows the evolution of the market share of active investors for  $\alpha_x = \frac{5 \text{Log}[4]}{6} \approx 1.155$  and  $\delta = 65$ , i.e., “near” the secondary Neimark–Sacker bifurcation point. The dark purple line represents the low steady state  $\bar{n}_L^A = 0.2$ . The dynamics fluctuate around the low steady state, but at a slightly higher level, i.e., approximately between 0.18 and 0.24. See the corresponding bifurcation diagram in the bottom-right panel of Fig. 3. Why is the amplitude of the stock market’s boom–bust dynamics lower than before? The reason is that the fluctuations take place around a lower price level, which is why investors react less sensitively to the “invest in the stock market” narrative.

The bottom-right panel of Fig. 6 shows the evolution of the market share of active investors for  $\alpha_x = \frac{5 \text{Log}[4]}{6} \approx 1.155$  and  $\delta = 80$ , i.e., further way from the secondary Neimark–Sacker bifurcation point. The amplitude of the dynamics is large, fluctuating approximately between 0.16 and 1. See the bifurcation diagram in the corresponding bottom-right panel of Fig. 3. Note that investors’ herding behavior is the same as in the last two panels. The more significant boom–bust dynamics are triggered by investors’ reaction to the stock market behavior, expressed by parameter  $\delta$ .

Finally, we explore how exogenous shocks affect the dynamics of our model by adding the noise term  $\epsilon_t \sim \mathcal{N}(0, 0.01)$  to the market share of active investors, assuming that  $n_t^A$  remains in the unit interval. Fig. 7 shows some examples. The top-left panel of Fig. 7 shows the evolution of the market share of active investors  $n_t^A$  for  $\alpha_x = 0.95$  and  $\delta = 20$ . As in the top-left panel of Fig. 6, the

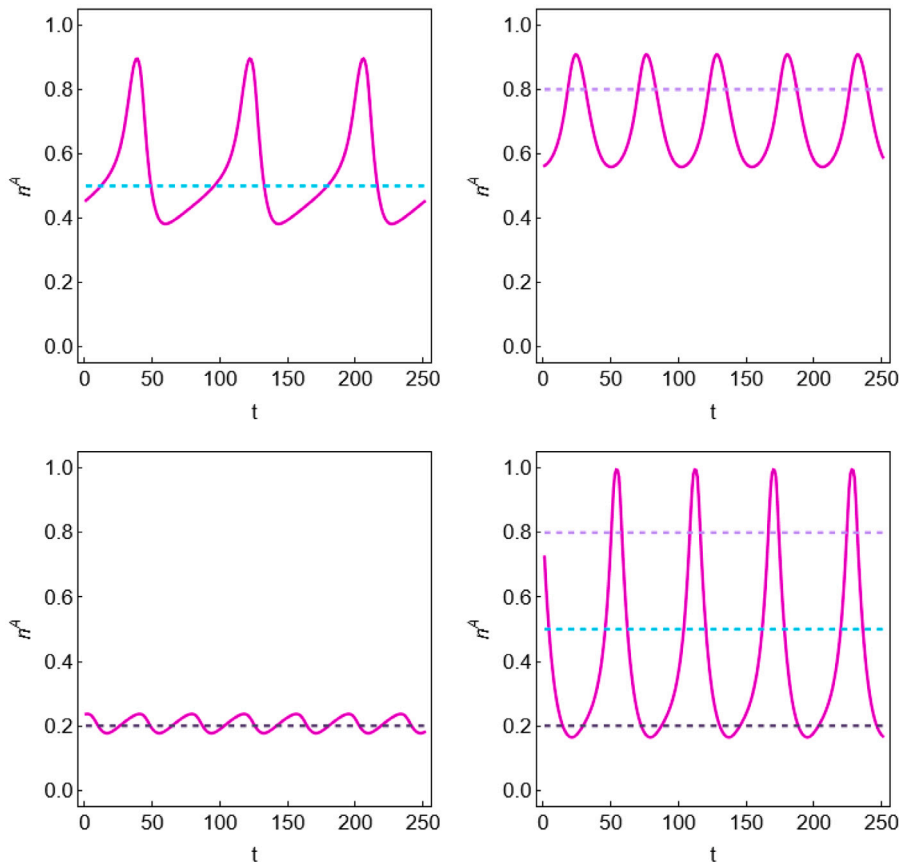


Fig. 6. The evolution of the market share of active investors in the time domain. Top left:  $\alpha_x = 0.95$  and  $\delta = 20$ . Top right:  $\alpha_x = \frac{5\text{Log}[4]}{6}$  and  $\delta = 17$ . Bottom left:  $\alpha_x = \frac{5\text{Log}[4]}{6}$  and  $\delta = 65$ . Bottom right:  $\alpha_x = \frac{5\text{Log}[4]}{6}$  and  $\delta = 80$ .

market share of active investors fluctuates around the middle steady state, indicating that in the presence of exogenous shocks the general boom–bust behavior of the stock market remains intact.

The top-right panel of Fig. 7 shows the evolution of the market share of active investors  $n_t^A$  for  $\alpha_x = \frac{5\text{Log}[4]}{6} \approx 1.155$  and  $\delta = 17$ . Note that the dynamics fluctuate around the low steady state for a long time before a bubble occurs. After the market share of active investors crashes, the dynamics again fluctuate around the low steady state. In fact, for this parameter combination, a locally stable low steady state coexists with a quasi-periodic attractor. Their basins of attraction are depicted in the left panel of Fig. 4. Note that in the case where the dynamics are attracted to the low steady state, a sufficiently strong shock is needed to switch to the basin of attraction of the quasi-periodic attractor. Such shocks occur only rarely. This periodic attractor, however, lies at the border of its basin of attraction. Hence, a small shock can push the dynamics back into to the basin of attraction of the low steady state. Such shocks happen more frequently.

The bottom-left panel of Fig. 7 shows the evolution of the market share of active investors for  $\alpha_x = \frac{5\text{Log}[4]}{6} \approx 1.155$  and  $\delta = 65$ . In contrast to the bottom-left panel of Fig. 6, where the dynamics fluctuate around the low steady state with low amplitude, random disturbances can cause the market share of active investors to rapidly approach a value close to 1 before swiftly returning to the low steady state.

The bottom-right panel of Fig. 7 shows the evolution of the market share of active investors for  $\alpha_x = \frac{5\text{Log}[4]}{6} \approx 1.155$  and  $\delta = 80$ . As can be seen, the general boom–bust dynamics remain intact, but it can happen that the stock market dynamics remain in the bust state for some time, as around period 250.

### 6. Conclusions

Shiller [1,2,3,4] argues that contagious popular stories are a major driver of the boom–bust behavior of stock markets. In this paper, we aim to provide a model that supports his line of reasoning. For simplicity, we limit our attention to two opposing narratives: one that advocates investing in the stock market and one that cautions against it. Investors, who are boundedly rational and uninformed about the intricacies of stock markets, follow one of these narratives. The “invest in the stock market” narrative becomes more popular with them when the stock market is booming and many of their peers invest in it. Conversely, the “avoid the

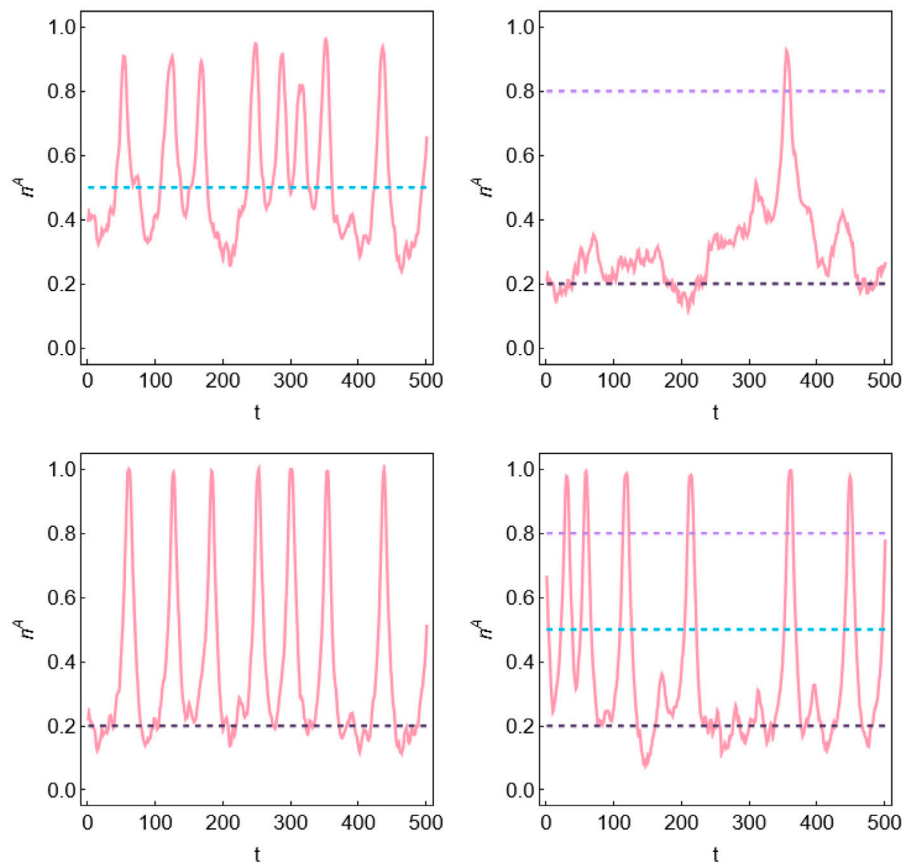


Fig. 7. The stochastic evolution of the market share of active investors in the time domain. Random shocks  $\varepsilon_t \sim \mathcal{N}(0, 0.01)$  have been added to the market share of active investors. The other parameters are as in Fig. 6.

stock market” narrative gains traction when the stock market is underperforming and when stock market investment is relatively uncommon. We close our model by assuming that an influx (outflux) of investors increases (decreases) the stock price.

Our model entails an important feedback loop. Suppose the stock price rises due to an influx of investors. An increase in the stock price, associated with a higher number of active investors, leads more investors to believe in the “invest in the stock market” narrative. As a result, more investors enter the stock market, pushing its price even higher. At some point, however, this momentum automatically wanes. Note that as more investors participate in the stock market, fewer can join them. Clearly, the stock market boom must eventually lose momentum. Then, two generic scenarios may unfold.

- First, when herding behavior among investors is strong enough, the stock market may become locked in a boom state, leading to a permanently overvalued stock market. A similar scenario could occur in a bust state, suggesting that multiple steady states are possible.
- Second, when investors’ attention to stock market performance is high (and their herding behavior is not too pronounced), the “invest in the stock market” narrative loses its appeal as the stock market momentum slows. When this happens, some investors choose to exit the stock market, causing a downward price movement. This reaction recurs in a bust state, producing cyclical stock market dynamics. Interestingly, such cycles can be centered around low, medium, and high price levels, which on average are associated with low, medium, and high levels of stock market participation, respectively.

Overall, our model supports Shiller’s view that contagious popular stories can trigger significant endogenous stock market dynamics. Remarkably, our simple model can generate a variety of boom–bust patterns. For instance, stock prices may circle around low, medium and high price levels. In the presence of exogenous shocks, these scenarios interact as the stock price enters and exits different basins of attraction of coexisting attractors.

While we present a qualitative analysis of Shiller’s view, future research could aim to align our model more closely with empirical data. For instance, the Method of Simulated Moments, as introduced by [36], could be used to estimate our model. A key advantage of this method is that it does not require data on investors’ actual stock market participation; instead, the model can be estimated using stock price data. Combining our market entry model with a standard chartist–fundamentalist model could also be a valuable

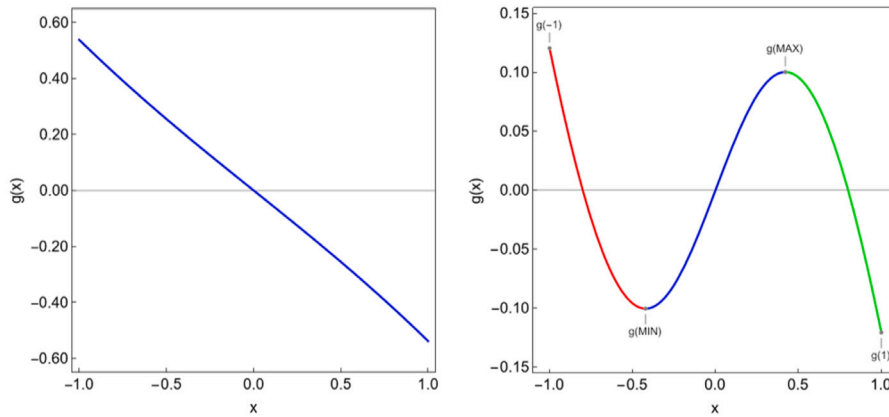


Fig. 8. The panels show  $g(x)$  for  $\alpha_x < 1$  (left) and  $g(x)$  for  $\alpha_x > 1$  (right).

endeavor. We conjecture that investors’ market entry and exit behavior drive larger long-term fluctuations that are decoupled from the stock market’s fundamental value, while the speculative trading behavior of chartists and fundamentalists introduces additional short-term fluctuations. These short-term fluctuations may occur around the long-term movements but ultimately anchor the stock market to its fundamental value. To the best of our knowledge, no existing analytically tractable models integrate these two perspectives.

**CRedit authorship contribution statement**

**Sarah Mignot:** Writing – review & editing, Visualization, Formal analysis, Conceptualization. **Frank Westerhoff:** Writing – review & editing, Visualization, Formal analysis, Conceptualization.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Appendix**

*Existence of low, middle, and high steady states*

To compute the model’s steady states, we set  $x_{t-1} = y_{t-1} = \bar{x} = \bar{y}$  and obtain from map  $M$  that

$$\bar{x} = \frac{Exp[\alpha_x \bar{x}] - Exp[-\alpha_x \bar{x}]}{Exp[\alpha_x \bar{x}] + Exp[-\alpha_x \bar{x}]} = Tanh[\alpha_x \bar{x}]. \tag{10}$$

To evaluate the number of steady states, we define  $g(x) := Tanh[\alpha_x x] - x$  for  $-1 < x < 1$ . We then get  $g(-1) = Tanh[-\alpha_x] + 1 > 0$ ,  $g(0) = 0$ , and  $g(1) = Tanh[\alpha_x] - 1 < 0$ . Moreover, the first partial derivative of  $g(x)$  is  $g'(x) = \alpha_x(1 - Tanh^2[\alpha_x x]) - 1$ . We can then distinguish two cases:

- Case  $\alpha_x < 1$ : Since  $0 < Tanh^2[\alpha_x x] < 1$ ,  $g'(x) < 0$  for every  $x \in (-1, 1)$ . It follows from the Intermediate Value Theorem that for  $\alpha_x < 1$ ,  $\bar{x}_M = 0$  is the unique solution.
- Case  $\alpha_x > 1$ :  $g'(x)$  is negative when  $-1 < x < -\frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right]$ , positive when  $-\frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right] < x < \frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right]$ , and negative again for  $\frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right] < x < 1$ . Consequently,  $g(x)$  has a local minimum at  $x^{MIN} = -\frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right]$  and a local maximum at  $x^{MAX} = \frac{1}{\alpha_x} Arctanh\left[\frac{1-\alpha_x}{\alpha_x}\right]$ , and  $g(x^{MIN}) < 0 < g(x^{MAX})$ . It follows from the Intermediate Value Theorem that three steady states exist, with  $\bar{x}_L < \bar{x}_M < \bar{x}_H$ .

In summary, we have shown that for  $\alpha_x < 1$ , only a middle steady state  $\bar{x}_M = 0$  exists, leading to  $\bar{n}_M^A = 1 - \bar{n}_M^A = 0.5$ . For  $\alpha_x > 1$ , three steady states exist: The middle steady state  $\bar{x}_M = 0$  and the low and high steady states  $\bar{x}_L, \bar{x}_H$ , which are symmetrically positioned around the middle steady state  $\bar{x}_M = 0$ , implying  $0 < \bar{n}_L^A < 0.5 < \bar{n}_H^A < 1$ . Notably,  $\alpha_x = \frac{Arctanh[\bar{x}_L, \bar{x}_H]}{\bar{x}_L, \bar{x}_H}$ . Fig. 8 visualizes the proof.

*Local stability domain of the middle steady state*

To study the local stability properties of the model’s middle steady state, we have to evaluate the Jacobian matrix of (9) at  $\overline{x_M} = 0$ . From

$$J(\overline{x_M}) = \begin{bmatrix} 1 + (2(\alpha_x - 1) + \delta)\nu & -\delta\nu \\ 1 & 0 \end{bmatrix}, \tag{11}$$

we obtain the characteristic polynomial

$$P(\lambda) = \lambda^2 + a_1\lambda + a_2, \tag{12}$$

with  $a_1 = -(1 + (2(\alpha_x - 1) + \delta)\nu)$  and  $a_2 = \delta\nu$ . Necessary and sufficient conditions ensuring that the two eigenvalues of (12) are less than one in modulus require that (i)  $1 + a_1 + a_2 > 0$ , (ii)  $1 - a_1 + a_2 > 0$  and (iii)  $1 - a_2 > 0$ . See [37]. Condition (ii) is always satisfied. Conditions (i) and (iii) yield

$$\alpha_x < 1 \tag{13}$$

and

$$\nu\delta < 1, \tag{14}$$

respectively. A separate violation of (13) is associated with the emergence of a Pitchfork bifurcation; a separate violation of (14) is associated with a Neimark–Sacker bifurcation.

*Local stability domain of the low and high steady states*

To study the local stability properties of the model’s low and high steady states, we need to study the characteristic polynomial of the Jacobian matrix of (9) for  $\alpha_x = \frac{\text{Arctanh}[\overline{x_{L,H}}]}{\overline{x_{L,H}}}$ . This characteristic polynomial is given by

$$P(\lambda) = \lambda^2 + a_1\lambda + a_2, \tag{15}$$

with

$$a_1 = -1 + \frac{\nu(2 + \delta(\overline{x_{L,H}} - 1)(1 + \overline{x_{L,H}})^2)}{(1 - \overline{x_{L,H}})^{\frac{1}{2}}} - \frac{2\nu(1 - \overline{x_{L,H}})^{\frac{1}{2}} \text{Arctanh}[-\overline{x_{L,H}}]}{\overline{x_{L,H}}}$$

and

$$a_2 = \delta\nu(1 - \overline{x_{L,H}})^{\frac{1}{2}}(1 + \overline{x_{L,H}})^{\frac{3}{2}}.$$

Necessary and sufficient conditions ensuring that the two eigenvalues of (15) are less than one in modulus requires that (i)  $1 + a_1 + a_2 > 0$ , (ii)  $1 - a_1 + a_2 > 0$  and (iii)  $1 - a_2 > 0$ . If  $\overline{x_{L,H}}$  exist, conditions (i) and (ii) are always satisfied. Condition (iii) requires that

$$\nu\delta < \frac{1}{(1 - \overline{x_{L,H}})^{\frac{1}{2}}(1 + \overline{x_{L,H}})^{\frac{3}{2}}}. \tag{16}$$

For  $\overline{x_L} \rightarrow -1$  and  $\overline{x_H} \rightarrow 1$ , the right-hand side of the inequality tends to infinity, indicating that condition (iii) is satisfied for any value of  $\nu\delta$  in this case. Furthermore, the derivative of the expression with respect to  $\overline{x_{L,H}}$  is given by

$$\frac{\partial}{\partial \overline{x_{L,H}}} \left( (1 - \overline{x_{L,H}})^{\frac{1}{2}}(1 + \overline{x_{L,H}})^{\frac{3}{2}} \right)^{-1} = \frac{-1 + 2\overline{x_{L,H}}}{(1 - \overline{x_{L,H}})^{\frac{3}{2}}(1 + \overline{x_{L,H}})^{\frac{5}{2}}}. \tag{17}$$

Hence, the right-hand side of the inequality reaches its minimum when  $\overline{x_H} = 0.5$ , i.e.,  $\overline{x_H^A} = 0.75$ . This observation has two important implications:

- If  $\delta\nu < \frac{4}{3\sqrt{3}}$ , then  $\overline{x_{L,H}}$  are always stable.
- The critical bifurcation value  $\delta\nu$  must be smaller for  $\overline{x_H}$  than for  $\overline{x_L}$ .

Considering  $\overline{x_M} = 0$ , we find that  $\nu\delta < 1$ , implying that for  $\nu\delta < 1$ ,  $\overline{x_L}$  is always stable.

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