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RESEARCH

Boom–bust cycles and asset market participation waves: Momentum, value, risk, and herding

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Abstract
We develop an asset market participation model in which investors base their market entry decisions on the momentum, value and risk of the market. Despite our behavioral framework, the model's fundamental steady state is characterized by standard present-value relations between expected future payouts and the model-implied risk-adjusted return. We derive conditions under which endogenous asset market participation waves and co-evolving boom–bust cycles emerge. Moreover, we show that the asset market may display spontaneous, sharp, and permanent downturns if investors react sensitively to risk, an outcome that goes hand in hand with low asset market participation rates and excess volatility.

Keywords Boom–bust cycles · Asset market participation waves · Momentum · Value and risk · Herding behavior · Feedback loops

JEL Classification D84 · G12 · G41

1 Introduction

There is broad evidence for the occurrence of significant boom–bust cycles across a wide range of asset classes, encompassing stock markets, housing markets and commodity markets. For detailed historical accounts and in-depth discussions of such events, see, amongst others, Glaeser (2013), Brunnermeier and Schnabel (2016) and Greenwood et al. (2019). Although boom–bust cycles can have disastrous effects on the real economy, the economic profession still lacks a profound understanding of what causes them. Motivated by Shiller's (2015) inspiring opening statement, we demonstrate that investors' market entry and exit behavior may lead to endogenous

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asset market participation waves and co-evolving boom–bust cycles. Since the starting point of our investigation – that an asset market experiencing an inflow (outflow) of investors buying (selling) the asset goes up (down) – is in line with basic economic reasoning and is strongly supported by empirical and laboratory evidence, see, e.g., Haruvy and Noussair (2006), Bouchaud et al. (2009), Kirchler et al. (2015) and Razen et al. (2017), a crucial question is: What determines investors' asset market participation? While investors' market entry and exit behavior may depend on a number of influence factors, four of them are particularly relevant for our study.

First, and presumably foremost, Galbraith (1994), Kindleberger and Aliber (2011) and Shiller (2015) point out that investors' asset market participation depends on the asset's momentum. Kindleberger and Aliber (2011) report that a follow-the-leader process may arise during a boom in which investors observe others benefiting from speculative purchases. Hence, more and more investors who had previously been aloof from the market begin to participate in the race for profits, and their market entry pushes asset prices to higher and higher levels. Shiller (2015) adds the finding that asset prices and investors' opinions about their best investment alternative are indeed strongly related. For instance, the fraction of investors who believe that stocks are the best investment alternative rises when stock prices increase. After reviewing the history of financial euphoria, Galbraith (1994) also concludes that a booming market constantly attracts new investors, thereby generating its own momentum. Unfortunately, investors' reaction to market momentum may set a dangerous positive feedback loop in motion: price increases result in an inflow of additional investors who buy the asset, and in doing so push prices even higher.

Second, although asset markets are not well anchored by their fundamental values, it is clear to investors that booms and busts cannot continue forever. Shiller (2015) argues that quantitative anchors – such as dividend-price ratios for stock markets – give investors indications for the appropriate level of the asset market, whether the asset market is overvalued or undervalued, and whether it is a good time to sell or buy. Amongst others, Shiller (2015) provides striking evidence according to which investors' valuation confidence, represented by an index that measures investors' view on whether the stock market is not overvalued, drops during booms and increases during busts. Apparently, investors are aware of the mispricing of a market, particularly if mispricing becomes extreme; and eventually they react to it. Note also that the whole concept of fundamental analysis, such as value investing (Graham and Dodd 1951, Greenwald et al. 2001), is based on this perspective. Fortunately, quantitative anchors may ultimately limit the positive feedback from price changes to further price changes that amplify booms and busts. Put differently: investors' attention to the value of an asset adds a stabilizing negative feedback to the dynamics.

A third factor that affects investors' behavior is risk. As pointed out by Ang et al. (2005), Cohn et al. (2015) and Guiso et al. (2008, 2018), psychological aspects such as fear, lack of trust, and disappointment, resulting from financial turmoil, may prevent investors from participating in risky asset markets. For instance, Guiso et al. (2008) argue that a collapse of a major company, such as the bankruptcy of Enron in 2001 or the notorious Madoff scandal, may change not only investors' views about the distribution of expected payouts, but also their fundamental trust in the entire system that generates those payouts. However, asset markets are also subject to endogenous risk. If the volatility of an asset market increases, for example, investors may exit the asset market, triggering a downward spiral. In fact, Dimson et al. (2002) present data showing that asset market participation is lower in countries with higher asset market volatility. Moreover, laboratory experiments by Guiso et al. (2018) reveal that past risk experiences may prevent investors from participating in risky engagements. The evidence discussed in Cohn et al. (2015) shows a similar direction, causing them to explicitly stress the role that self-reinforcing feedback loops may play in understanding asset market dynamics.

Fourth, Hong et al. (2004), Brown et al. (2008) and Shiller (2015) report that investors' asset market participation depends on their social interactions. Clearly, the more investors' peers participate in an asset market, the more attractive that market seems to those investors. Although the exact dynamic consequences of herding behavior are difficult to anticipate, we can imagine that herding behavior may create inertia. If a market attracts many investors and, consequently, is in a boom state, investors may remain in the market, preventing it from crashing, at least for a while. In a bust state, when only a few people have invested money, the market may remain unpopular for quite a while, delaying a fundamentally justified price recovery. Obviously, investors may temporarily ignore a market's value. To explain this puzzling behavior, Shiller (2015) states that investors may not worry much about apparent contradictions among the views they hold. He argues that there is a willingness to free ride in this case, i.e., to suppose that other investors have already thought through the apparent contradictions and therefore to assume that the others know why such behavior is acceptable.

In this paper, we provide a general asset market participation model that may help us to better comprehend the repeating and potentially harmful phenomenon of boom–bust cycles driven via the inflow and outflow of investors who react to the momentum, value and risk of the market, but who also herd together. Our framework is based on only a few elementary and empirically supported building blocks. Investors can choose between a safe asset and a risky asset. The price of the safe asset, guaranteeing a fixed rate of return, is constant. The price of the risky asset, generating an uncertain payout, increases if additional investors enter the market. Investors' market entry and exit decisions depend on the momentum, value and risk of the risky asset. Risk plays an important role in our model. It contains not only a fundamental component, caused by the uncertainty surrounding future payouts from the asset, but also a speculative component, resulting from price fluctuations induced by investors' market entry and exit behavior. Moreover, investors herd together: the risky asset becomes more popular as investors' asset market participation grows. As it turns out, the dynamics of our model is driven by the iteration of a four-dimensional nonlinear deterministic map, capturing complex interactions between positive and negative feedback loops.

Despite the general characterization of investors' behavioral market entry and exit decisions, our model remains analytically tractable and consistent with standard valuation approaches and risk-return relations. We may summarize our main results as follows:

- *Fundamental steady state*: Our model possesses a unique interior fundamental steady state according to which the risky asset price is equal to the discounted value of expected future payouts. As we will see, investors enter the risky asset market up to the point where the price of the risky asset has reached a level that makes them indifferent between the risky asset and the safe asset. Not all investors are needed to enter the risky asset market to achieve this outcome, implying that asset market participation is limited. Moreover, the discount factor, respectively, the risk-adjusted return, emerges endogenously within our model. In our model specifications, we can express the risk-adjusted return as the sum of the risk-free interest rate and a risk premium, consisting of the product of the market price for risk and the volatility of the risky asset market. It is interesting to see that such classical textbook relations may reappear in a behavioral asset market participation model such as ours.

- *Stability of the fundamental steady state*: Even though investors' market entry and exit decisions and the price adjustment process of the risky asset depend on a host of factors, it is remarkable that the stability of the model's fundamental steady state critically hinges on a straightforward and easy-to-interpret relation that involves the product of four interrelated forces. In fact, we are able to prove analytically that the model's fundamental steady state becomes unstable and gives rise to endogenous oscillatory motion if the fraction of outside investors multiplied by their intensity of switching multiplied by their reaction to the risky asset's momentum multiplied by their price impact on the risky asset exceeds a critical threshold level. To give an example of this chain of arguments: the fundamental steady state loses its stability if investors react too strongly to the risky asset's momentum. However, the fraction of outside investors and their price impact depend on several market characteristics. Amongst others, our model also predicts that a rise in the total number of investors (or the funds available to them) increases the fraction of outside investors, and may thus lead to instability.
- *Functioning of the model*: We can explain the co-evolution of endogenous asset market participation waves and boom–bust cycles as follows. Suppose that the price of the risky asset increases. Some investors then get excited about the risky asset and seek to make a speculative profit by buying it. The resulting demand pressure keeps the momentum of the risky asset alive, at least for a while. At some point, however, the fundamental condition of the risky asset market appears unhealthy, and investors start to sell the risky asset, initiating a downturn. Due to its endogenous price dynamics, the risky asset market appears more risky than is fundamentally warranted. Asset market participation and, consequently, the price of the risky asset may therefore circle below their fundamental steady-state values. Moreover, investors' herding behavior may prolong the duration of boom and bust states. During a boom, for instance, many investors participate in the risky asset market. Since it appears to be popular to be part of the crowd, investors' herding behavior may delay a fundamental price correction.
- *Endogenous risk beliefs*: Investors' reactions to endogenous risk may lead to non-trivial and quite surprising effects involving co-existing regimes and tipping points. For instance, the model gives rise to a low-volatility regime with relatively high risky asset prices and asset market participation rates that co-exists with a high-volatility regime with relatively low risky asset prices and asset market par-

ticipation rates. Small exogenous shocks may then lead to the crossing of tipping points, materializing in spontaneous, sharp, and permanent changes in key market characteristics. Clearly, investors' reactions to risk constitute part of the riskiness of asset markets.

To sum up, our model provides a new and quite general perspective about the connection and consequences of investors' asset market participation behavior and the formation of risky asset prices, including the steady state and stability properties of such an environment, as well as its out-of-equilibrium behavior.

A related stream of literature that also seeks to explain the boom–bust nature of asset markets involves the behavior of heterogeneous interacting investors relying on simple extrapolative and regressive expectations rules to forecast the future direction of a market. For seminal contributions in this direction, see, amongst others, Beja and Goldman (1980), Frankel and Froot (1990), Day and Huang (1990), Kirman (1993), Lux (1995, 1998), Brock and Hommes (1997, 1998) or, more recently, Anufriev and Hommes (2012), Burnside et al. (2016), Glaeser and Nathanson (2017) and Barberis et al. (2015, 2018).[1] While these models also take momentum, value, risk, and herding elements into account, our model jointly considers them in a very natural and general way, ensuring that our main results can be observed for various model specifications. Moreover, asset market participation usually remains constant in these models. In fact, in most of these models, it is assumed that market participants switch between heterogeneous expectation rules subject to their past profitability, the state of the market or depending on investors' actions. In a nutshell, asset price fluctuations are explained as follows. During periods when destabilizing extrapolative expectations dominate the market, asset prices are pushed away from their fundamental values. In contrast, in periods when stabilizing regressive expectations are dominant, asset prices revert back to their fundamental values. A permanent evolutionary competition between these kinds of expectation rules may then keep the cyclical behavior of the market alive. See Dieci and He (2018) for a recent survey.

More closely related to our paper is the work by Schmitt and Westerhoff (2016) and Dieci et al. (2018a), who started taking into account some of the aforementioned observations. In particular, Schmitt and Westerhoff (2016) sketch a quite simple asset market participation model in which an investor's decision whether to enter a risky asset market depends on current market movements and on its mispricing, illustrating that asset market participation waves may trigger and amplify boom–bust cycles. By adding a second risky asset market to such an environment, Dieci et al. (2018a) show that investors' switching between multiple risky asset markets may engender countercyclical dynamics between them. In line with Shiller (2015), investors may take their enthusiasm from one speculative market, say the stock market, to another speculative market, say the housing market, creating a bust in the former market and a boom in the latter one. In contrast to these contributions, we study a considerably

[1] The noise trader approach, summarized by Shleifer and Summers (1990), studies interactions between boundedly rational investors and fully rational investors. Since rational investors face limits to arbitrage, boundedly rational investors can create boom–bust cycles. See, for instance, the influential work by DeLong et al. (1990a, b), Shleifer and Vishny (1997) and Barberis et al. (1998). Limits to arbitrage may also play an important role in our model.

more general model framework. Moreover, we explicitly take into account investors' reactions to exogenous and endogenous risk, resulting in a number of new insights.

Our paper also sheds light on the limited participation puzzle. Standard financial theory predicts that all investors, no matter how risk-averse, should hold some risky assets as long as the risk premium is positive. However, actual asset market participation rates appear surprisingly low. Haliassos and Bertaut (1995) and Campbell (2006) report that less than half of US households possess risky assets, while Guiso and Sodini (2013) present similar evidence for a broad set of European countries. Moreover, asset market participation varies considerably over time. Hong et al. (2004) report that 31.6% of US households owned stock in 1989, increasing to 49.8% in 1998. Asset market participation costs in combination with risk aversion can partially explain the limited participation puzzle (Attanasio et al. 2002, Vissing-Jørgensen 2002), although households' risk aversion has to be quite strong to explain the data. Another explanation may be that households make investment mistakes (Calvet et al. 2007), e.g., due to a lack of financial literacy, or, more generally, owing to cognitive constraints (Grinblatt et al. 2011, van Rooi et al. 2011). However, our model reveals that asset market equilibrium is compatible with a situation in which a large fraction of investors do not participate in the risky asset market. If prices accurately mirror the discounted value of expected future payouts – which is the case at the model's interior steady state – outside investors have no incentive to enter the risky asset market. In a broader sense, this argument is reminiscent of the no-trade theorem by Grossman and Stiglitz (1980). If an additional investor tried to squeeze into the market, it would be overvalued. Moreover, out-of-equilibrium asset market participation rates depend on momentum, value, risk, and herding effects, as most prominently reported by Galbraith (1994), Kindleberger and Aliber (2011) and Shiller (2015), and tend to oscillate below their steady-state values, on average.

Our model also offers insights for understanding the behavior and impact of the discount factor of a risky asset. In an ideal world, risky asset prices should equal their expected discounted payouts. While finance theory has long tried to explain variations in risky asset prices by variations in expected payouts, Campbell and Shiller (1988, 1989) and Cochrane (2011) argue that what really matters is the variability of the discount rate. Within our model, investors' payout expectations are constant. Nevertheless, fluctuations in the risky asset price create fluctuations in the risk premium, which, via a time-varying discount factor, translate into fluctuations of the risky asset's fundamental value. Also consistent with Campbell and Shiller (1988, 1989) and Cochrane (2011), fluctuations in the fundamental value of the risky asset are considerably weaker than fluctuations in the risky asset price, i.e., there is excess volatility.

The remainder of our paper is organized as follows. In Section 2, we develop our model, followed by a discussion of our main analytical results in Section 3. In Section 4, we numerically investigate the dynamic properties of our model. In Section 5, we conclude our work. Appendices A, B and C contain a number of proofs and robustness checks.

2 A general asset market participation model

Let us first outline investors' trading environment and their investment behavior. *Trading environment*: Investors can choose between a safe asset and a risky asset. The safe asset, e.g., a government bond, is perfectly elastically supplied, and guarantees a fixed rate of return r, while the risky asset generates uncertain payouts d_t. As an example, we may consider the dividend process of a stock market index. Moreover, we assume that investors perceive the mean and variance of these payouts with d and v_d, respectively.[2] Since the price of the risky asset adjusts endogenously with respect to investors' market entry and exit behavior, the risky asset market also offers speculative profit opportunities. Accordingly, the risky asset possesses a fundamental and a speculative gain potential, associated with a fundamental and a speculative risk component. *Investment behavior*: Shiller (2015) remarks that investors have cognitive limitations that may render them unable to create optimal portfolios. Although standard economic theory predicts that all investors should hold some risky assets if the risk premium is positive, the fact that actual asset market participation rates are relatively low supports Shiller's (2015) view. Within our model, some investors may enter the risky asset market, while others do not. The number of investors that hold the risky asset, called active investors, is given by n_t. Setting the total number (mass) of investors to N, it follows that the number of inactive investors, also called outside investors, is equal to $N - n_t$.[3]

The starting point of our model is that the price of the risky asset increases in line with the number of active investors, a view that is consistent with elementary economic reasoning and is strongly supported by empirical observations (Haruvy and Noussair 2006, Bouchaud et al. 2009, Kirchler et al. 2015 and Razen et al. 2017). Thus, we model the price of the risky asset as

$$p_t = h(n_t), \tag{1}$$

where function h is such that $h(0) \geq 0$ and $h' > 0$. While this price adjustment function establishes a quite natural relation between investors' asset market participation and the price of the risky asset, it also forms the basis for modeling the boom–bust perspective offered by Galbraith (1994), Kindleberger and Aliber (2011) and Shiller (2015). The only additional restriction we make is that the price of the risky asset does not become negative. Clearly, our price adjustment function is quite general. Amongst others, it may be reconciled with a market maker scenario (Kyle 1985), assuming that each investor buys (sells) a positive amount of the risky asset when he enters (exists) the market.

[2] Note that Campbell and Shiller (1988, 1989) and Cochrane (2011) state that investors' dividend expectations are rather constant. However, our model does not rely on whether investors are right or wrong with respect to their mean and variance beliefs about risky asset payouts nor on the exact nature of the payout-generating process.

[3] An alternative interpretation of our model could be that N represents the total money investors are willing to invest in the risky asset market. The actual money invested in the risky asset market is then given by n_t, and depends on the momentum, value and risk of the risky asset, as well as on the amount of money that investors have already invested in the risky asset market.

Investors repeat their market entry and exit decisions at the beginning of each period, depending on the attractiveness of the safe asset relative to the attractiveness of the risky asset. Since the safe asset guarantees a fixed rate of return, its attractiveness depends on the risk-free interest rate, i.e.,

$$A_t^S = A^S = f(r), \tag{2}$$

where $f' > 0$. Hence, the safe asset becomes more attractive as the risk-free interest rate increases; for a given value of r, attractiveness of the safe asset is constant.

As highlighted by Galbraith (1994), Kindleberger and Aliber (2011) and Shiller (2015), investors become excited about the risky asset if its price increases. Therefore, the attractiveness of the risky asset depends positively on its momentum. However, investors are also aware that asset prices cannot increase forever. According to Graham and Dodd (1951), Greenwald et al. (2001) and Shiller (2015), investors abstain from markets that are (too) overvalued, represented, e.g., by unhealthy payout-price ratios. Undervalued markets, on the other hand, appear attractive to investors because they expect prices to return to fundamentals, at least in the long run. Moreover, Ang et al. (2005), Guiso et al. (2008, 2018) and Cohn et al. (2015) report that investors dislike markets that are subject to a larger risk. Thus, the attractiveness of the risky asset market decreases with payout uncertainty and price variability.

To take these considerations into account, we model the attractiveness of the risky asset by the general function

$$A_t^R = \Phi(\rho_t, \delta_t, \nu_t), \tag{3}$$

where the momentum and value of the risky asset are captured by $\rho_t := \frac{p_t - p_{t-1}}{p_{t-1}}$ and $\delta_t := \frac{d}{p_t}$, respectively, while ν_t represents its risk. Of course, function Φ is such that $\Phi_\rho > 0$, $\Phi_\delta > 0$ and $\Phi_\nu < 0$.[4]

The risk that investors associate with the risky asset consists of a fundamental component, based on their variance beliefs about payouts from the risky asset, and a speculative component, given by their variance beliefs about its price dynamics. Since investors treat the two components separately, we model their risk perception by

$$\nu_t = \nu_d + \nu_{p,t}. \tag{4}$$

Note that investors use standard updating rules to estimate the variance of the risky asset's price dynamics. Following Chiarella et al. (2007), investors keep track of the variance by computing

$$\nu_{p,t} = m\nu_{p,t-1} + m(1-m)(p_t - u_{t-1})^2, \tag{5}$$

[4] While in Section 4 we simulate our model using an attractiveness function which is additive in the momentum, value and risk component, Appendix B suggests that an alternative multiplicative specification produces similar results. A further robustness check is performed in Appendix B by replacing return ρ_t in the attractiveness function by a more general geometric weighted average of past returns.

where their mean estimation satisfies

$$u_t = m u_{t-1} + (1-m) p_t. \tag{6}$$

Investors' memory parameter in the updating process of Eqs. 5 and 6 is restricted to $0 < m < 1$. Since repeated substitutions yield $\nu_{p,t} = \sum_{s=0}^{\infty} \omega_s (p_{t-s} - u_t)^2$ and $u_t = \sum_{s=0}^{\infty} \omega_s p_{t-s}$ with $\omega_s = (1-m)m^s$ and $\sum_{s=0}^{\infty} \omega_s = 1$, investors update their estimation of the variance and mean as time averages with exponentially declining weights. See, e.g., Bretschneider (1986) for related models of joint estimation of mean and variance via exponential smoothing, and the technical report by Finch (2009) for a summary of such techniques.

The higher the relative attractiveness of the risky asset, the more investors opt for that investment alternative. However, investors are also subject to herding behavior. As documented by Hong et al. (2004), Brown et al. (2008) and Shiller (2015), the more investors' peers participate in a market, the more attractive that market seems to those investors. We therefore use exponential replicator dynamics, as put forward by Hofbauer and Sigmund (1988) and Hofbauer and Weibull (1996), to model the number of active investors. As a result, we have

$$n_t = N \frac{n_{t-1} \exp[\lambda A^R_{t-1}]}{n_{t-1} \exp[\lambda A^R_{t-1}] + (N - n_{t-1}) \exp[\lambda A^S_{t-1}]}, \tag{7}$$

where parameter $\lambda > 0$ reflects investors' intensity of switching.[5] Exponential replicator dynamics has a number of economically desirable properties. First, the number of investors opting for the risky asset increases as the risky asset becomes more attractive, ceteris paribus. Second, an increase in investors' intensity of switching implies that more investors enter the more attractive market. Third, a higher (lower) participation in the risky asset market in the previous period tends to lead to a higher (lower) participation in the current period, even if the attractiveness of the risky asset market is relatively low (high).[6] Such a view is reminiscent of herding models invoking word-of-mouth and observational learning, as developed, for instance, by Banerjee (1992), Bikhchandani et al. (1992) and Ellison and Fudenberg (1995).

3 Analytical results

In this section, we characterize the fundamental steady state of our model, discuss its existence and uniqueness, and derive its local asymptotic stability properties. In order to do so, we must first rearrange the dynamic model in a way that makes its iterative structure analytically tractable. Introducing the auxiliary variable $z_t = n_{t-1}$ allows us

[5] Note that, by defining $x_t := \frac{n_t}{N}$, the exponential replicator Eq. 7 can be immediately rewritten in terms of population shares of each strategy, as is common in evolutionary game theory.

[6] In fact, a consequence of the exponential replicator dynamics is that the relative growth of market participation between periods t and $t+1$ increases with the attractiveness differential (see Appendix B for a discussion).

to express the dynamical system of the model as

$$S: \begin{cases} n_{t+1} = N\frac{n_t \exp(\lambda A_t^R)}{n_t \exp(\lambda A_t^R) + (N - n_t)\exp(\lambda A_t^S)} := F(n_t, z_t, \nu_{p,t}) \\ z_{t+1} = n_t \\ u_{t+1} = m u_t + (1-m)h\big(F(n_t, z_t, \nu_{p,t})\big) := G(n_t, z_t, u_t, \nu_{p,t}) \\ \nu_{p,t+1} = m\nu_{p,t} + m(1-m)\Big(h\big(F(n_t, z_t, \nu_{p,t})\big) - u_t\Big)^2 := H(n_t, z_t, u_t, \nu_{p,t}) \end{cases}, \tag{8}$$

where $A_t^S = A^S$, $A_t^R = \Phi(\rho_t, \delta_t, \nu_t)$, $\rho_t = \frac{h(n_t)}{h(z_t)} - 1$, $\delta_t = \frac{d}{h(n_t)}$ and $\nu_t = \nu_d + \nu_{p,t}$. Accordingly, the model's dynamics is now represented by the iteration of a four-dimensional nonlinear deterministic map.

3.1 Existence and uniqueness of the fundamental steady state

Let us denote steady-state quantities with an overbar. At any steady state, we obviously have that $\overline{n} = \overline{z}$, $\overline{u} = h(\overline{n}) = \overline{p}$, $\overline{\rho} = 0$, $\overline{\nu}_p = 0$ and, consequently, $\overline{\nu} = \nu_d$. In the following, we initially focus on the existence and main properties of a hypothetical interior steady state, i.e., such that $0 < \overline{n} < N$. As will be clear, such a steady state does exist, provided that the fixed attractiveness A^S of the safe asset can be mirrored by the attractiveness A_t^R of the riskless asset, i.e., it belongs to the range of A^R. Also, the interior steady state is necessarily unique. As a matter of fact, by imposing the steady-state condition $\overline{n} = n_{t+1} = n_t$ to the first equation in 8, we obtain, for $\overline{n} > 0$:

$$\frac{N \exp(\lambda \overline{A}^R)}{\overline{n}\exp(\lambda \overline{A}^R) + (N - \overline{n})\exp(\lambda A^S)} = 1,$$

i.e.,

$$\exp(\lambda \overline{A}^R) = (1 - \bar{y})\exp(\lambda \overline{A}^R) + \bar{y}\exp(\lambda A^S),$$

where $\bar{y} = (N - \overline{n})/N$. Since both $\bar{y}$ and $1 - \bar{y}$ are strictly positive, this implies

$$\overline{A}^R = A^S, \tag{9}$$

that is, the risky asset and the safe asset are necessarily equally attractive at the interior steady state.[7] Next, by defining

$$\varphi(\delta) := \Phi(0, \delta, \nu_d),$$

[7] Note that there also exists a boundary steady state $\overline{n} = N$. This boundary steady state may even be stable if the steady-state attractiveness of the risky asset is higher than the attractiveness of the safe asset. Although all investors have entered the risky asset market (and no investor has an incentive to exit it), the price of the risky asset remains below its fundamental value. Clearly, limits to arbitrage, as stressed by Shleifer and Vishny (1997), prevent investors' demand pressure from sufficing to push the price of the risky asset towards its fundamental value. See Dieci et al. (2018b) for more insights into such a scenario for a related yet much simpler model framework. The economically uninteresting boundary case $\overline{n} = 0$ is outside the domain of the map defined in Eq. 8.

we can express the no-arbitrage steady-state Eq. 9 by

$$\varphi(\overline{\delta}) = A^S, \tag{10}$$

where $\overline{\delta} = d/h(\overline{n}) = d/\overline{p}$. Due to our qualitative assumptions about the attractiveness function Φ, it is clear that function φ strictly increases in δ. Therefore, for an interior steady state to exist, it is necessary and sufficient that A^S belongs to the range of φ. Moreover, the existence of an interior steady state necessarily implies its uniqueness. More precisely, we can define the payout-price ratio at the interior steady state as the unique solution to Eq. 10, namely

$$\overline{\delta} = \frac{d}{\overline{p}} = \frac{d}{h(\overline{n})} = \varphi^{-1}(A^S) := r_a. \tag{11}$$

Note that r_a can be interpreted as the risk-adjusted return that investors require at the interior steady state. Suppose, for instance, that the attractiveness of the risky asset is due to a high payout-price ratio too far above that of the safe asset. We would then expect an inflow of investors, which elevates the price of the risky asset up to the point where the attractiveness of the risky asset mirrors that of the safe asset. If the payout-price ratio is equal to the risk-adjusted return, investors are indifferent between holding the risky asset and the safe asset, or, put differently, they have no incentive to enter or exit the risky asset market.

The expression defined in Eq. 11 further reveals that $\overline{p} = \frac{d}{r_a}$ and, consequently, that $\overline{n} = h^{-1}(\overline{p}) = h^{-1}(\frac{d}{r_a})$. Note that the fundamental value of the risky asset price, given by the discounted value of expected future payouts, is consistent with standard asset valuation approaches. In the following, the model's unique interior "fundamental" steady state is denoted by FSS. Mathematically, in terms of the dynamical system defined in Eq. 8, the FSS is defined as the following stationary point in the state space:

$$(\overline{n}, \overline{z}, \overline{u}, \overline{v}_p) = \left(h^{-1}\left(\frac{d}{r_a}\right), h^{-1}\left(\frac{d}{r_a}\right), \frac{d}{r_a}, 0\right).$$

Obviously, $\overline{p}$ and $\overline{n}$ increase with d. For instance, if investors expect the risky asset to generate higher payouts, they enter the risky asset market and the resulting demand pressure elevates the price of the risky asset. Moreover, $\overline{p}$ and $\overline{n}$ decrease with r_a.

Given our assumptions about the attractiveness functions of the risky asset and the safe asset, it also follows from Eq. 10 and the implicit function differentiation theorem that the risk-adjusted return increases with the risk-free interest rate and the volatility of the risky asset's payout process. For instance, an increase in the volatility of risky asset payouts makes the risky asset less attractive. As a result, investors leave the risky asset market, an aspect that depresses the price of the risky asset and elevates its risk-adjusted return. Of course, the risk-adjusted return also depends on investors' risk-return attitudes, represented by parameters entering $\overline{A}^R$ and A^S (see Section 4 for an example).

3.2 Local stability conditions

To characterize the stability properties of the FSS, it is helpful to introduce a number of definitions. Let us denote the elasticity of the price adjustment function of the risky asset with respect to asset market participation at the FSS by $\epsilon := \frac{h'(\overline{n})}{h(\overline{n})/\overline{n}}$, the partial derivatives of the general attractiveness function of the risky asset at the FSS by $\beta := \Phi_\rho$, $\gamma := \Phi_\delta$ and $-\theta := \Phi_\nu$, where $\beta, \gamma, \theta > 0$, and the fraction of outside investors at the FSS by $\overline{y} := \frac{N-\overline{n}}{N}$. In Appendix A, we prove the following stability result.

Proposition 1 *The fundamental steady state of the model, characterized by* $r_a = \varphi^{-1}(A^S)$, $\overline{p} = \overline{u} = \frac{d}{r_a}$, $\overline{n} = \overline{z} = h^{-1}(\overline{p})$ *and* $\overline{\nu} = \nu_d$, *is locally asymptotically stable if and only if the double inequality* $\frac{r_a \gamma}{2} - \frac{1}{\lambda \epsilon \overline{y}} < \beta < \frac{1}{\lambda \epsilon \overline{y}}$ *holds, where violation of the first (second) inequality is associated with a Flip (Neimark–Sacker) bifurcation.*

The stability properties of the FSS warrant several comments. Since a violation of the Neimark–Sacker stability condition may set endogenous boom–bust cycles in motion, our focus is on the second inequality.[8] A simple reformulation of this condition yields $\beta \lambda \epsilon \overline{y} < 1$, i.e., the occurrence of endogenous boom–bust cycles depends on the product of four directly related forces. The steady-state fraction of outside investors multiplied by their intensity of switching multiplied by their reaction to the risky asset's momentum multiplied by their price impact on the risky asset at the steady state must remain below unity, otherwise the FSS becomes unstable and cyclical dynamics are set in motion. Let us discuss this chain of arguments in more detail:

i) The effect of β: The FSS becomes unstable if investors react strongly to the momentum of the risky asset, as already anticipated by Galbraith (1994), Kindleberger and Aliber (2011) and Shiller (2015). However, a stronger reaction to the value of the risky asset (i.e., an increasing value for γ) does not re-establish market stability.

ii) The effect of λ: The FSS also becomes unstable if investors' intensity of switching becomes large enough, i.e., if they react strongly to differences in the attractiveness of the safe and the risky asset. Apparently, a higher value of λ reinforces the effect of β, and vice versa. We remark that Brock and Hommes (1997, 1998) also stress the destabilizing effect of investors' intensity of switching.

iii) The effect of $\overline{y}$: Our model may generate cyclical dynamics if the steady-state fraction of outside investors increases, provided that $\beta \lambda \epsilon > 1$. Since the fraction of outside investors is given by $\overline{y} = \frac{N-\overline{n}}{N} = 1 - \frac{\overline{n}}{N}$, we immediately see that the FSS may become unstable if the total number of investors increases. Note that an increase in N merely increases the potential number of investors that may enter the risky asset market. Moreover, the FSS may also lose its stability if $\overline{n}$ decreases. However, for this effect to be clearly determinable, we need to assume that ϵ is

[8] The first inequality can be written as $\gamma < \frac{2\beta}{r_a} + \frac{2}{r_a \lambda \epsilon \overline{y}}$. For reasonable values of the risk-adjusted return, say values below 10%, a violation of this condition requires that investors' reaction to the value of the risky asset is at least 20 times larger than their reaction to the momentum of the risky asset, a constellation that seems to be quite unlikely. Since a Flip bifurcation implies the birth of a period-two cycle, it is furthermore not suitable to explain boom–bust cycles.

constant (the explanation follows below). As already discussed in connection with the properties of the model's FSS, $\overline{n}$ decreases if either d decreases or r_a increases, which, in turn, happens if either r or ν_d increases.

iv) The effect of ϵ: The stability of the FSS becomes compromised if the elasticity ϵ of the price with respect to participation n in the risky asset market (at the FSS) is sufficiently strong. Remember that elasticity is defined by $\epsilon := \overline{n}\frac{h'(\overline{n})}{h(\overline{n})}$ and depends, therefore, on the parameters characterizing function h. Since the steady-state fraction of active investors appears in the nominator and denominator of ϵ, the effect of $\overline{n}$ cannot be determined for our general setup. Thus, in principle, changes in parameters d and r_a may foster or harm the stability of the FSS.

To conclude, it is noteworthy that the FSS of our behavioral asset market participation model is consistent with standard asset valuation approaches and that its stability – despite our general representation of investors' market entry and exit behavior – hinges on a straightforward and easy-to-interpret condition involving the product of four interrelated forces.

4 Numerical results

We are now ready to explore the out-of-equilibrium behavior of the model. In Section 4.1, we first specify our general asset market participation model. In Sections 4.2 to 4.6, we then discuss how changes in investor behavior and fundamental characteristics of the market may affect the model dynamics. We provide some robustness checks with respect to possible alternative model specifications in Appendix B.

4.1 Specification of the model

To be able to simulate the dynamics of our model, we have to specify the price adjustment function of the risky asset and the attractiveness functions of the safe and the risky asset. In this section, we assume that the price of the risky asset is determined according to the isoelastic (power) function

$$p_t = an_t^q, \tag{12}$$

where a and q are positive parameters and $\epsilon = q$ for any $n_t = \overline{n}$. For $q = 1$, the price of the risky asset is proportional to the number of active investors. However, the price adjustment function may also be either strictly concave or strictly convex, depending on whether q is smaller or larger than 1.

For simplicity, the attractiveness of the safe asset is proportional to the risk-free interest rate r, that is

$$A_t^S = A^S = \gamma r, \tag{13}$$

where parameter $\gamma > 0$ captures investors' preference for the fundamental gain potential of the safe asset. Moreover, we formalize the three arguments of the general attractiveness function of the risky asset in an additive manner by

$$A_t^R = \mu \arctan\left(\frac{\beta}{\mu}\rho_t\right) + \gamma\delta_t - \psi\sqrt{\nu_t}. \tag{14}$$

The first term on the right-hand side of Eq. 14 captures the attractiveness of the risky asset, which arises from its momentum. Since $\mu := \frac{2\kappa}{\pi} > 0$ and $\beta\ (= \Phi_\rho) > 0$, a price increase (decrease) makes the risky asset more (less) attractive. However, the momentum component of the asset's attractiveness is S-shaped, reflecting investors' tendency to react more cautiously to more extreme price changes, and is bounded between $\pm\kappa$. The second term on the right-hand side of Eq. 14 reflects the fundamental gain potential of the risky asset. Since $\gamma\ (= \Phi_\delta) > 0$, the risky asset becomes increasingly attractive as its payout-price ratio increases. Note that investors' preference for the fundamental gain potential of the two assets, expressed by parameter γ, appears in Eqs. 13 and 14. The third term on the right-hand side of Eq. 14 represents the perceived risk of the asset. Since $\psi > 0$, investors regard the risky asset as less attractive if its fundamental or speculative risk component increases.

Our specification of the attractiveness functions of the asset yields a number of advantages. In particular, the resulting risk-adjusted return at the steady state is consistent with the one we usually encounter in traditional finance models. To see this, note first that the no-arbitrage condition $\Phi(0, \overline{\delta}, \nu_d) = A^S$ directly yields $\gamma\overline{\delta} - \psi\sqrt{\nu_d} = \gamma r$, implying furthermore that $\overline{\delta} = r + \frac{\psi}{\gamma}\sqrt{\nu_d} = r_a$. A "risk-neutral" setting is recovered in the limiting case $\psi \to 0$. A formal change in our model parameters in which we define $\sqrt{\nu_d} = \sigma\frac{d}{r}$ and $\psi = \frac{\tau\gamma}{d/r}$ reveals that $r_a = r + \tau\sigma$. As in standard asset-pricing models, the risk-adjusted return (at the steady state) comprises the risk-free return and a risk premium, where the risk premium is given by the product of the market price for risk τ and the volatility of the risky asset market, which is equal to the risky asset's fundamental risk σ.[9] With respect to the remaining components of the model's FSS, we have that $\overline{p} = \frac{d}{r+\tau\sigma}$, $\overline{n} = (\frac{d}{a(r+\tau\sigma)})^{\frac{1}{q}}$ and $\overline{\nu} = \nu_d = (\sigma\frac{d}{r})^2$.

Of course, the risky asset market may not always be in equilibrium. Note that we can define a time-varying risk-adjusted return by $r_{a,t} = r + \frac{\tau}{d/r}\sqrt{\nu_t} = r + \tau\sigma\sqrt{\frac{\nu_t}{\nu_d}}$, which additionally reflects the risky asset's speculative (endogenous) risk component, resulting in a time-varying fundamental value $\overline{p}_t = \frac{d}{r_{a,t}}$ and, likewise, in a time-varying fundamental level of the number of active investors, given by $\overline{n}_t = (\frac{\overline{p}_t}{a})^{\frac{1}{q}}$. Since the risk-free interest rate and investors' payout expectations are constant, it is clear that the variability of $\overline{p}_t$ is directly related to $r_{a,t}$, as argued by Campbell and Shiller (1988, 1989) and Cochrane (2011). Clearly, without endogenous risk perceptions, we always have that $r_a = r_{a,t}$, $\overline{p} = \overline{p}_t$ and $\overline{n} = \overline{n}_t$. See Appendix C for more details.

We use the following base parameter setting to illustrate the out-of-equilibrium behavior of the model:

[9] Note that, under our formal change in parameters, the fundamental risk σ is expressed in percentage terms of a benchmark "average" price level $\tilde{p}$, which we take, for simplicity, to be equal to the "risk-neutral" fundamental price, i.e., $\frac{d}{r}$. However, the exact choice of $\tilde{p}$ does not affect the FSS or its stability domain; it only has a mild effect on the out-of-equilibrium behavior of the model.

$$r = 0.01, d = 1.2, \sigma = 0.02, \tau = 0.1, a = 1, q = 1, N = 200,$$
$$\lambda = 1, \beta = 2, \gamma = 20, \kappa = 0.05, m = 0.95.$$

Accordingly, the risk-adjusted return at the steady state is given by $r_a = 0.012$ and the fundamental price of the risky asset is $\overline{p} = 100$. Moreover, we have $\overline{n} = 100$, implying that the steady-state fraction of outside investors amounts to $\overline{y} = 0.5$. Consequently, the dynamics of the model is at the border of instability as $\beta\lambda\overline{y}q = 1$. Since large parts of our simulations rest on $a = 1$ and $q = 1$, we face a convenient parameter constellation, implying that $\overline{p} = \overline{n}$, $\overline{p}_t = \overline{n}_t$ and $p_t = n_t$, i.e., the price of the risky asset is identical to the number of active investors, not only at the FSS, but also out of equilibrium.

4.2 The model's dynamics without endogenous risk beliefs

To be able to appreciate the functioning of our complete model, it is helpful to explore a special case first. Figure 1 depicts the dynamics of the model for our base parameter values, except that we set $\beta = 2.05$, $m = 0$ and $\nu_{p,0} = 0$. Due to $m = 0$ and $\nu_{p,0} = 0$, investors ignore the endogenous (speculative) risk component of the risky asset, i.e., their risk beliefs are given by $\nu_t = \nu_d = \overline{\nu}$. Since investors' reactions to the momentum of the risky asset are slightly above the critical value $\beta^{NS}_{crit} = 2$, our model produces cyclical dynamics. To be precise, the black line in the left panel of Fig. 1 depicts the evolution of the number of active investors, being equal to the price of the risky asset, while the red line shows its fundamental steady-state value, being equal to the fundamental steady-state value of the risky asset price. The right panel of Fig. 1 shows the corresponding dynamics in phase space. Note that the number of active investors and the price of the risky asset oscillate almost symmetrically around their steady-state values. We explain the reason for this outcome and its dependence on investors' risk beliefs in Section 4.3.

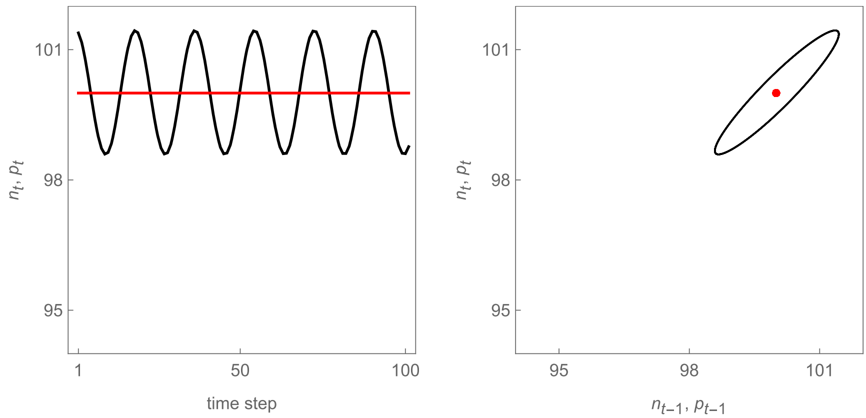

Fig. 1 Asset market participation waves and boom–bust cycles without endogenous risk beliefs. The *left panel* shows the evolution of $n_t = p_t$ (*black*) and $\overline{n} = \overline{p}$ (*red*) for 100 time steps. The *right panel* depicts the corresponding dynamics in phase space. Base parameter setting, except that $\beta = 2.05$, $m = 0$ and $\nu_{p,0} = 0$

Figure 2 portrays the stability and out-of-equilibrium behavior of the model with the help of four bifurcation diagrams. From top left to bottom right, the panels show how the number of active investors (black) and the price of the risky asset (gray) react to an increase in parameters β, λ, $\epsilon = q$ and N, respectively. Recall that p_t may only deviate from n_t if $q \neq 1$. With the exception of the bottom left panel, p_t is thus superimposed by n_t. The remaining parameters correspond to our base parameter setting, except that we keep investors' risk perception constant to the fundamental risk component of the risky asset by assuming $m = 0$ and $\nu_{p,0} = 0$. In line with our analytical results, changes in parameters β, λ and N do not influence the model's FSS. However, $\overline{n}$ decreases with $\epsilon = q$, while $\overline{p}$ is independent of $\epsilon = q$. To understand this, note that $\overline{p} = \overline{n}^q$ and $\overline{n} > 1$. Hence, the price impact of active investors increases with $\epsilon = q$, and fewer investors are needed to push the price of the risky asset towards $\overline{p}$. As

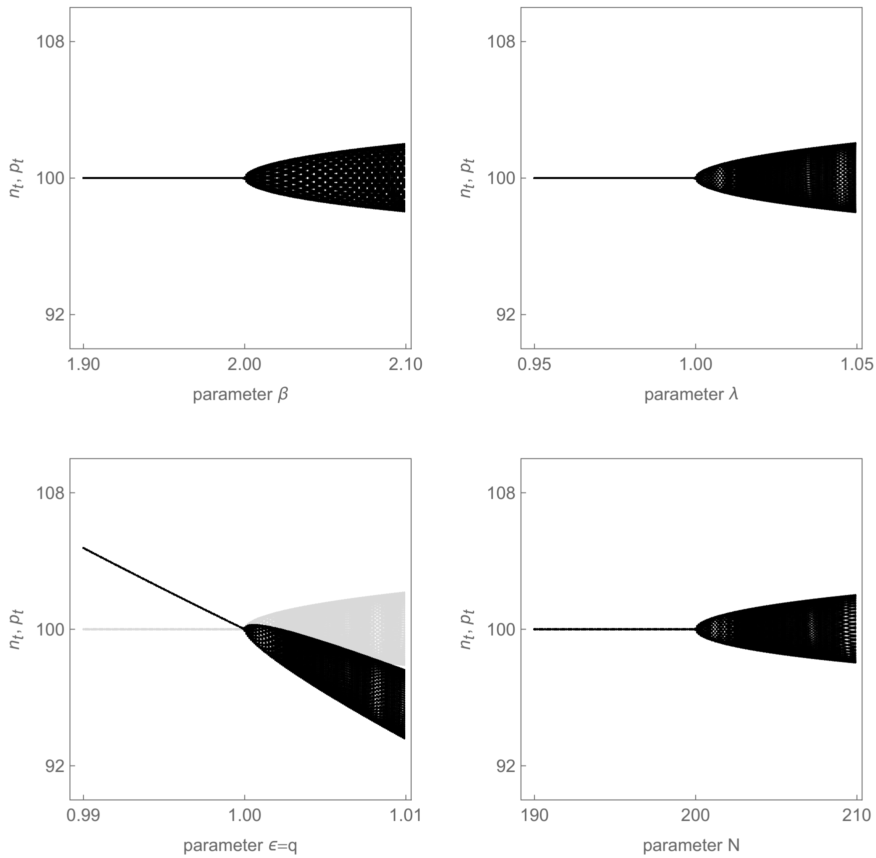

Fig. 2 Illustration of the stability and out-of-equilibrium behavior of the model without endogenous risk beliefs. Bifurcation diagrams of n_t (*black*) and p_t (*gray*) against parameter β (*top left*), parameter λ (*top right*), parameter $\epsilon = q$ (*bottom left*) and parameter N (*bottom right*). The remaining parameters correspond to our base parameter setting, except that we set $m = 0$ and $\nu_{p,0} = 0$. With the exception of the bottom left panel, p_t is superimposed by n_t

furthermore predicted by Proposition 1, we observe that n_t and p_t converge towards the FSS until the parameters under variation cross the critical Neimark–Sacker bifurcation thresholds $\beta^{NS}_{crit} = 2$, $\lambda^{NS}_{crit} = 1$, $\epsilon^{NS}_{crit} = 1$ and $N^{NS}_{crit} = 200$. In each case, we then see the birth of a limit cycle. In particular, n_t and p_t start to oscillate around their steady-state values $\overline{n}$ and $\overline{p}$, and the amplitude of these oscillations grows with the respective bifurcation parameter. Further simulations reveal that the underlying dynamics in all of these out-of-equilibrium scenarios closely resemble the one depicted in Fig. 1, i.e., n_t and p_t circle almost symmetrically around $\overline{n}$ and $\overline{p}$.

4.3 The dynamics of the model with endogenous risk beliefs

Let us now turn to the behavior of our complete model. Figure 3 shows a simulation run that rests on our base parameter setting, except that $\beta = 2.05$. As in Fig. 1, the model's FSS is unstable, and endogenous oscillatory movements occur. However, the dynamics depicted in Fig. 3 are qualitatively different from those depicted in Fig. 1. Since investors now update their variance beliefs, also with respect to the past variability of the risky asset price, the risky asset market appears less attractive to them. As a result, fewer investors enter the risky asset market and, consequently, its price oscillates around a lower average price level than before. In fact, $n_t = p_t$ (black) oscillates around $\overline{n}_t = \overline{p}_t$ (blue) and not around $\overline{n} = \overline{p}$ (red). Put differently, it is the time-varying fundamental value of the risky asset that acts as an anchor for the price of that asset. Note furthermore that fluctuations in $r_{a,t}$ also lead to fluctuations in $\overline{p}_t$, though the volatility of the risky asset price is much higher than the volatility of its time-varying fundamental value. It is important to realize that our model generates all these outcomes, e.g., a limited asset market participation, time-varying discount rates and excess volatility, endogenously. For constant variance beliefs, i.e., $\nu_t = \nu_d$, the

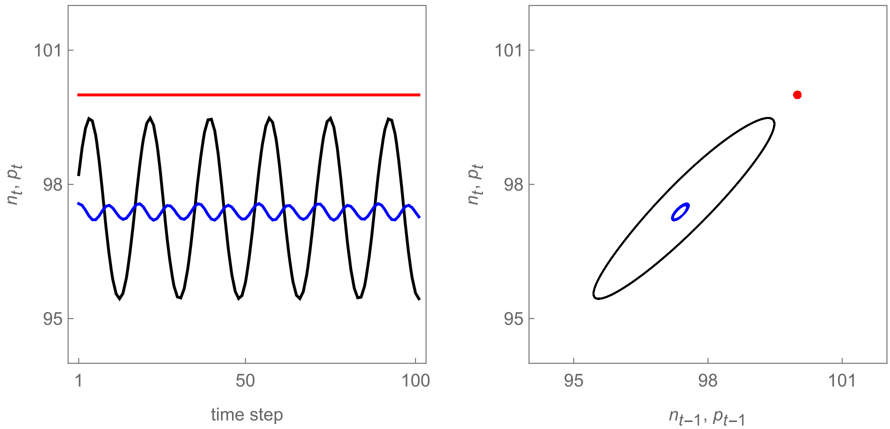

Fig. 3 Asset market participation waves and boom–bust cycles with endogenous risk beliefs. The *left panel* shows the evolution of $n_t = p_t$ (black), $\overline{n}_t = \overline{p}_t$ (blue) and $\overline{n} = \overline{p}$ (red) for 100 time steps. The *right panel* depicts the corresponding dynamics in phase space. Base parameter setting, except that $\beta = 2.05$

number of active investors and the price of the risky asset circle around their constant fundamental steady-state values, as illustrated in Fig. 1.[10]

We may understand the functioning of our complete model as follows. Suppose that the price of the risky asset starts to increase. Investors then get excited about the risky asset, and seek to profit from its momentum. Since more and more investors rush towards the risky asset market, the price increase continues, lending the risky asset market further momentum. At some point, however, the price increase ebbs away and the market begins to reverse its direction. In particular, investors eventually regard the risky asset market as too overvalued, and start to retreat from it. This triggers a self-fulfilling downward movement. Investors leave the risky asset market, which drags it further downwards, and so on, until the payout-price ratio looks so promising again that investors return to the risky asset market, reversing the direction of the market.[11] While investors' reactions to the momentum and value of the risky asset create endogenous oscillations, their reactions to the risk implied by these fluctuations shift the whole dynamics of the risky asset market downwards.

It is remarkable how closely the basic functioning of our model mirrors Galbraith's (1994, p. 3) view on this issue. In particular, he writes that, during an upswing, we observe that "*The price of the object of speculation goes up. Securities, land, objects d'art, and other property, when bought today, are worth more tomorrow. This increase and the prospect attract new buyers; the new buyers assure a further increase. Yet more are attracted; yet more buy; the increase continues. The speculation building on itself provides its own momentum.*" The same holds for Shiller (1990, p. 60) who writes "*when market prices as a whole rise substantially, this creates many success stories of investors, who naively imagine that the same success will come to them if they invest too. Substantial decreases in market prices create many stories of investors' failures, and these discouraging stories drive investors away from the market.*" Shiller (1990) adds that such an explanation may seem unattractive to many economists as investors' behavior appears to be too naive. However, the bull market periods of the 1920s and 1950s and the periods following the crashes of 1929 and 1987 reflect such a behavior. In Section 4.6, we show that investors' market entry and exit behavior may also result in much more irregular boom–bust cycles, letting their behavior appear less naive.

One could argue that these boom–bust cycles cannot continue forever, since investors must eventually learn what is happening and adjust their behavior accordingly. Citing a famous example of a truly smart outside investor who bought high and sold low, Kindleberger and Aliber (2011, p. 47) report that Isaac Newton obtained a large number of shares near the market top of the South Sea Bubble in 1720. After realizing a tremendous loss, Newton dryly remarked that "*I can calculate the motions of the heavenly bodies, but not the madness of people.*" Reviewing 300 years of financial euphoria, Galbraith (1994, p. 11) concludes that investors' behavior and the associated

[10] Note that the two-dimensional phase plots in Figs. 1 and 3 (as well as in the following figures) represent projections of four-dimensional objects onto a two-dimensional plane.

[11] As pointed out by Shiller (2015), major turning points of risky asset markets must naturally occur when almost all investors have entered or exited the market, ending either the demand or the supply pressure on the market. While we can also observe this effect within our model for some more extreme parameter constellations, it is worthwhile to note that our model shows that investors' reactions to value may be sufficient to stop and eventually to reverse the momentum of a market.

boom–bust cycles have occurred "*again and again*" in the past "*in nearly invariant form*", and will also do so in the future. In fact, Galbraith (1994, p. 108) sadly writes that "*there probably is not a great deal that can be done*" by regulators to prevent the repeating emergence of mass euphoria that leads to boom–bust cycles.

The bifurcation diagrams depicted in Fig. 4 illustrate the stability and out-of-equilibrium behavior of our complete model. Note that we can directly compare these with those behaviors reported in Fig. 2. As before, we observe the emergence of cyclical dynamics when the critical Neimark–Sacker bifurcation thresholds $\beta_{crit}^{NS} = 2$, $\lambda_{crit}^{NS} = 1$, $\epsilon_{crit}^{NS} = 1$ and $N_{crit}^{NS} = 200$ are crossed. However, a further increase in these parameters amplifies the amplitude of the cycles, which, in turn, elevates investors' risk perception. As a result, the risky asset market appears less attractive, and the average price level around which the fluctuations occur gradually declines. Of course, the same is true for investors' asset market participation. Both n_t and p_t fluctuate below $\overline{n}$ and $\overline{p}$. In this sense, the phenomenon depicted in Fig. 3 is quite robust, since we observe

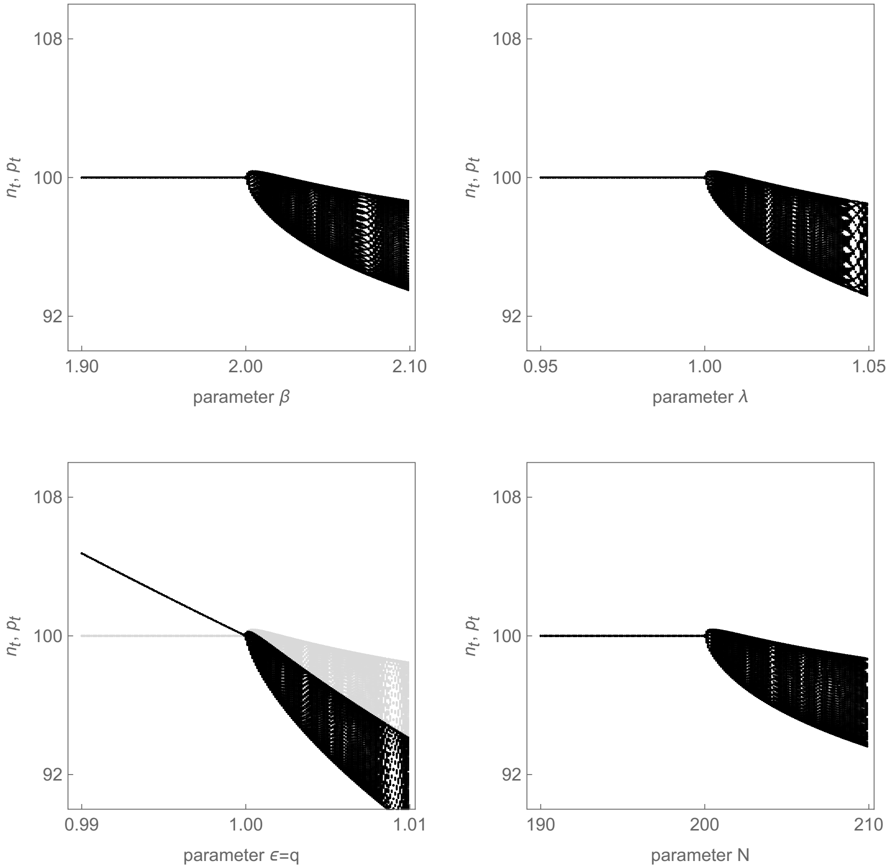

Fig. 4 Illustration of stability and out-of-equilibrium behavior of the model with endogenous risk beliefs. Bifurcation diagrams of n_t (*black*) and p_t (*gray*) against parameter β (*top left*), parameter λ (*top right*), parameter $\epsilon = q$ (*bottom left*) and parameter N (*bottom right*). The remaining parameters correspond to our base parameter setting. With the exception of the bottom left panel, p_t is superimposed by n_t

it for various values of parameters β, λ, $\epsilon = q$ and N beyond the Neimark–Sacker bifurcation boundary.

4.4 Sharp reactions to endogenous risk beliefs

Proposition 1 reveals that the model's FSS becomes unstable for our base parameter setting if investors' reactions to the momentum of the risky asset exceed $\beta_{crit}^{NS} = 2$. One intriguing insight offered by our model is that we may already observe endogenous dynamics for values of β below this threshold, due to the presence of a coexisting attractor. Figure 5, resting on our base parameter setting, except that $\beta = 1.95$ and $\gamma = 5$, provides an example of this phenomenon. Its left panel shows the evolution of $n_t = p_t$ (green) and $\overline{n}_t = \overline{p}_t$ (purple) for 100 time steps, assuming that initial conditions are distant to $\overline{n} = \overline{p}$ (red). Clearly, initial conditions taken close to the FSS yield a convergence of $n_t = p_t$ towards $\overline{n} = \overline{p}$ (not depicted in Fig. 5). While investors' reactions to the value of the risky asset do not affect the local asymptotic stability of the FSS, its impact on the model's out-of-equilibrium behavior may be profound. In particular, decreasing parameter γ from 20 to 5 implies that investors react less aggressively to deviations from the value of the risky asset and, being less anchored to the FSS, the risky asset market is subject to oscillations with a larger amplitude (which can easily be verified by running bifurcation diagrams for p_t against parameter γ). One important consequence of these wilder fluctuations is that the volatility of the risky asset may increase significantly, making the risky asset market less attractive, thereby crowding out investors and depressing its average price level quite dramatically. For instance, the price of the risky asset fluctuates around an average value of around 97 in Fig. 3, and around 90 in Fig. 5. This is also the case for the time-varying fundamental value $\overline{p}_t$. Besides its effects on asset market participation and the price of the risky

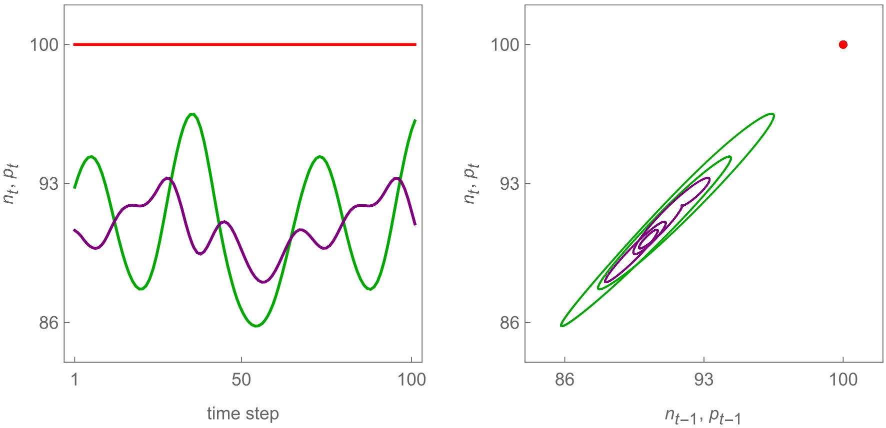

Fig. 5 Co-existence of a locally stable fundamental steady state and a limit cycle. The *left panel* shows the evolution of $n_t = p_t$ (*green*) and $\overline{n}_t = \overline{p}_t$ (*purple*) for 100 time steps; initial conditions are distant to $\overline{n} = \overline{p}$ (*red*). The *right panel* depicts the corresponding dynamics in phase space. Base parameter setting, except that $\beta = 1.95$ and $\gamma = 5$

asset, it is also clear that the co-existing attractor elevates the risk premium, resulting in a higher variability of the fundamental value of the risky asset, although the market continues to display excess volatility.

To get an idea why we observe endogenous dynamics despite the fact that the FSS is locally stable, it is insightful to take a slightly different look at the stability condition $\beta\lambda\epsilon\overline{y} < 1$, by means of a non-rigorous, yet suggestive argument. Clearly, the FSS is locally stable since $\beta = 1.95$, $\lambda = 1$, $\epsilon = 1$ and $\overline{y} = 0.5$. However, the fluctuations of $n_t = p_t$ occur below $\overline{n} = \overline{p} = 100$ and, therefore, the time-varying fraction of outside investors, i.e., $y_t = \frac{N-n_t}{N}$, is larger than 0.5. What does this mean for the current setup? For $\beta = 1.95$, the FSS is locally stable as long as $\overline{y}$ remains below 0.5128. However, the right panel of Fig. 5 reveals that n_t never exceeds 96.5 and, consequently, y_t always remains above 0.5175. Put differently: Once such dynamics is set in motion, through single external shocks, investors' reactions to endogenous risk may result in relatively low asset market participation rates (i.e., in a relatively high number of outside investors), which may prevent a convergence to the FSS. This regime is self-confirming, since it creates an inherently high risk.

Figure 6 shows that we may regard spontaneous, sharp, and permanent shifts of investors' asset market participation and the average price level of the risky asset as a robust property of our model. Again, the four panels of Fig. 6 report bifurcation diagrams of n_t (black and green) and p_t (gray and light green) against parameters β, λ, $\epsilon = q$ and N, respectively. We generated the underlying simulations with our usual parameter setting, except that investors' reactions to the value of the risky asset are given by $\gamma = 5$. The dynamics represented by black and gray (green and light green) originates from initial conditions close to (distant from) the FSS.[12] As long as we take initial conditions close to the FSS, we observe a convergence towards the fixed point of the model – until, of course, the critical bifurcation thresholds $\beta^{NS}_{crit} = 2$, $\lambda^{NS}_{crit} = 1$, $\epsilon^{NS}_{crit} = 1$ and $N^{NS}_{crit} = 200$ are crossed. From then on, we witness a spontaneous, sharp, and permanent downward shift of the dynamics. Clearly, asset market participation and the price of the risky asset start to fluctuate immediately with a high amplitude on a much lower value than before. Interestingly, these dynamics may emerge before the critical bifurcation thresholds, as confirmed by the light green color in the plots. With respect to investors' reactions to the momentum of the risky asset, for instance, we can already detect them for $\beta = 1.94$.

To sum up, there is robust evidence of co-existing attractors according to which a locally stable steady state co-exists above a locally stable high-amplitude limit cycle. Whether the dynamics converges to the one attractor or the other depends on initial conditions. Of course, exogenous shocks may force the dynamics to move from one basin of attraction to another. When these borders, acting as tipping points, are crossed, a fundamentally different dynamic regime emerges, which is either much more stable with higher participation rates and prices or much more volatile with a low participation rate and lower prices.

[12] Note that we can also observe such dynamic phenomena for other parameter constellations. For instance, repeating the simulations depicted in Fig. 6 for our base parameter setting and $\kappa = 0.09$ yields quite similar bifurcation routes. Moreover, we have also detected co-existing attractors for different price adjustment functions and specifications of the attractiveness functions of the safe and the risky asset.

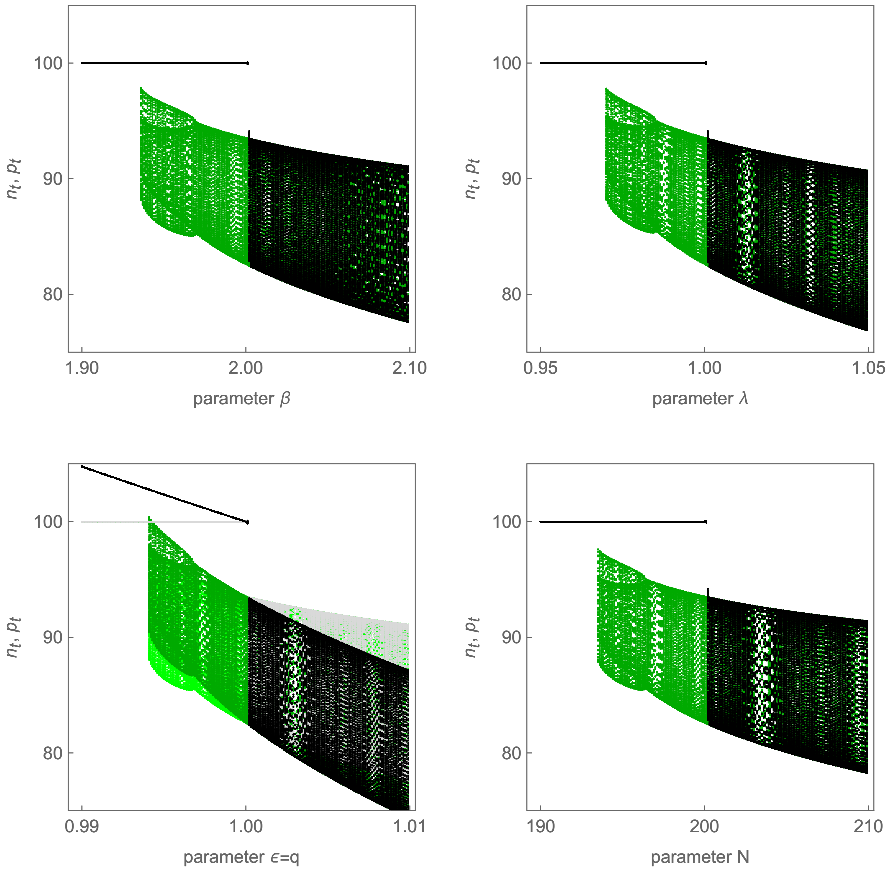

Fig. 6 Illustration of the stability and out-of-equilibrium behavior of the model in the presence of co-existing attractors. Bifurcation diagrams of n_t (*black* and *green*) and p_t (*gray* and *light green*) against parameter β (*top left*), parameter λ (*top right*), parameter $\epsilon = q$ (*bottom left*) and parameter N (*bottom right*), resulting from different initial conditions. The remaining parameters correspond to our base parameter setting, except that $\gamma = 5$. With the exception of the bottom left panel, p_t is superimposed by n_t

4.5 Effects of the discount factor

We now explore how the risk-free interest rate and investors' fundamental risk perception – two crucial parameters that determine the discount factor – may influence the dynamics of our model. The top left panel of Fig. 7 shows a bifurcation diagram of $n_t = p_t$ (black) against parameter r. As can be seen, an increase in the risk-free interest rate, making the risky asset relatively less attractive, reduces investors' asset market participation and, consequently, the price of the risky asset. Moreover, the FSS loses its local asymptotic stability as the interest rate exceeds $r_{crit}^{NS} = 0.01$. At this interest rate level, the steady-state fraction of outside investors becomes larger than $\overline{y}_{crit}^{NS} = 0.5$ and a limit cycle is born. Further increases in the interest rate enlarge the amplitude of these fluctuations and push them gradually downwards. The top right

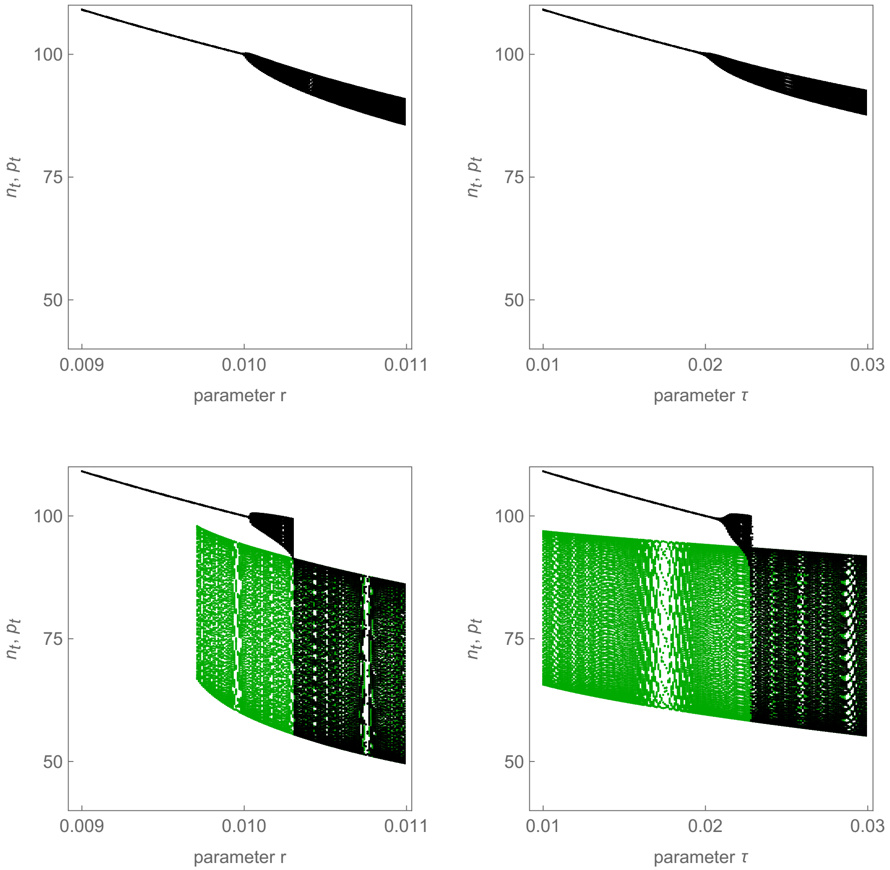

Fig. 7 Effects of the discount rate on the stability and out-of-equilibrium behavior of the model. The *top left* and *top right* panels show bifurcation diagrams of $n_t = p_t$ (*black*) against parameters r and τ, respectively. All other parameters correspond to our base parameter setting. The *bottom panels* show the same, except that $\kappa = 0.4$ and $m = 0.5$. *Green shading* represents a co-existing attractor, resulting from different initial conditions

panel of Fig. 7 shows a quite similar reaction of the model dynamics as parameter τ increases. Since an increase in the market price for risk of the risky asset crowds out investors, the price of the risky asset decreases and the FSS eventually becomes unstable.

The bottom two panels of Fig. 7 repeat this exercise for a slightly different parameter constellation. For $\kappa = 0.4$ and $m = 0.5$, our model may once again generate a spontaneous, sharp, and permanent transition of rather stable dynamics towards rather unstable dynamics. In contrast to Fig. 6, however, we now observe that both a fixed point attractor and a low-volatility attractor (black) may co-exist with a high-amplitude attractor (green), where the latter is located on a much lower level. Such a scenario can also be observed if parameters $\beta, \lambda, \epsilon = q$ or N are used as bifurcation parameters, as well as for other formulations of the price adjustment function and specifications of the attractiveness functions of the safe and the risky asset, as introduced in Appendix B.

Figure 8 provides an example of the latter outcome. The left panel of Fig. 8 presents the evolution of $n_t = p_t$ (black and green), $\overline{n}_t = \overline{p}_t$ (blue and purple) and $\overline{n} = \overline{p}$ (red). The two simulation runs, resting on our base parameter setting, except that $r = 0.0103$, $\kappa = 0.4$ and $m = 0.5$, only differ with respect to their initial conditions. One set of initial conditions leads to low-volatility fluctuations around a relatively high average price level, while the other set of initial conditions leads to high-volatility fluctuations around a relatively low average price level. The former (latter) constellation is associated with relatively high (low) asset market participation rates. Given that the dynamics of the model is due to a four-dimensional nonlinear map, it is difficult to visualize the corresponding basins of attraction of the two limit cycles. Nevertheless, it is clear that sporadic exogenous shocks may enforce intricate attractor switching dynamics. For some time, the dynamics may then be characterized by a rather low level of risk and relatively high asset market participation rates and risky asset prices and then, out of the blue, by considerably higher risk levels, yielding lower asset market participation rates and risky asset prices. As highlighted by Ang et al. (2005), Cohn et al. (2015) and Guiso et al. (2008, 2018), psychological aspects such as fear, lack of trust, and disappointment, resulting from financial market turbulences, may prevent investors from participating in risky asset markets. Interestingly, our model reveals that investors' behavior may create risk, and that their reaction to risk may set a self-reinforcing feedback loop in motion according to which a substantially lower fraction of investors will participate in the risky asset market. The right panel of Fig. 8 depicts the corresponding dynamics in phase space.

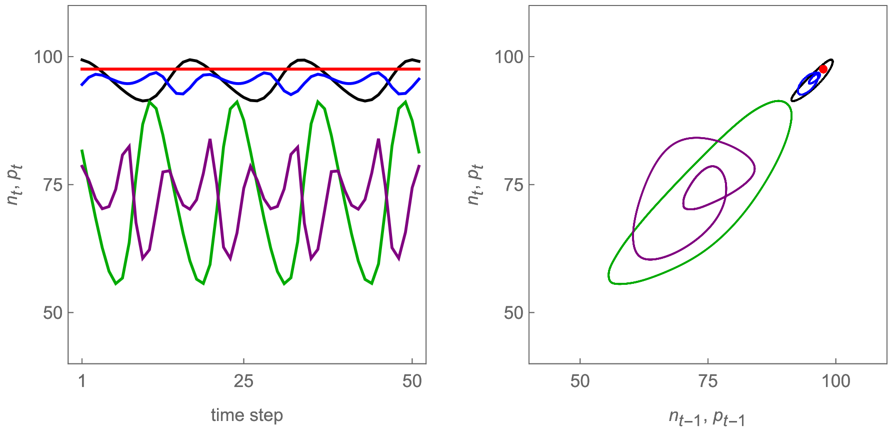

Fig. 8 Co-existence of two qualitatively different limit cycles. The *left panel* shows the evolution of $n_t = p_t$ (*black* and *green*), $\overline{n}_t = \overline{p}_t$ (*blue* and *purple*) and $\overline{n} = \overline{p}$ (*red*) for 50 time steps and two different sets of initial conditions. The *right panel* depicts the corresponding dynamics in phase space

4.6 Further observations

So far, we have seen that our model can produce a number of surprising dynamic phenomena with important economic implications. We conclude the numerical investigation of our model specification by discussing two further fascinating model implications. The top left panel of Fig. 9, resting on our base parameter setting, except that $\lambda = 3$ and $m = 0.5$, shows the evolution of $n_t = p_t$ (black), $\overline{n}_t = \overline{p}_t$ (blue) and $\overline{n} = \overline{p}$ (red) for 40 time steps. Apparently, our model is also able to generate quite complex asset market participation waves and boom–bust dynamics. In particular, the amplitude of the cycles may vary considerably over time. Although it is not our goal to explain the day-to-day vagaries of actual financial markets, these simulations reveal at least that our model is able to produce intricate dynamics, as it is also confirmed by the appearance of the strange attractors in the top right panel of Fig. 9. In fact, complex dynamics is a robust phenomenon of our model – we can observe them for many different parameter constellations. Despite the seemingly more erratic behavior of the market, its general boom–bust nature remains intact. Moreover, investors' behavior appears less naive. Between periods 20 and 35, for instance, the market goes from bottom to top in three consecutive waves. In a broader sense, this also addresses a widespread criticism raised against feedback models, namely that the orderly nature of their boom–bust cycles results in highly predictable price dynamics. Shiller (1990) shows that feedback models do not necessarily create high serial correlation of short-run price changes if investors condition their market outlooks on longer time intervals, e.g., with a distributed lag model. The nonlinearity of our model also yields irregular dynamics, as do model extensions in which investors use different momentum rules (see Appendix B).

Let us finally turn to some possible effects that investors' herding behavior may have in our model. In the bottom left panel of Fig. 9, we see an example in which investors' participation in the risky asset market (black, identical to the price of the risky asset) occasionally increases to very high levels and remains at those levels for quite a while, until it dramatically crashes. The simulation run depicted relies on our base parameter setting, except that $\beta = 1000$, $\kappa = 0.75$ and $m = 0.5$. What is happening here? At the peak of these booms, the risky asset market displays almost no momentum and, given that investors' memory is limited, the endogenous risk component of their variance beliefs quickly ebbs away. Although parameters λ and γ are as in our base parameter setting (see Figs. 1 and 3), investors react only slowly to the value of the risky asset. This is due to investors' herding behavior. Since many investors are active in the risky asset market, they show a tendency to remain active in that market. As pointed out by Hong et al. (2004), Brown et al. (2008) and Shiller (2015), it may simply be popular for investors to be active in the risky asset market, even if it is grossly overvalued.

We can also make this argument more explicit. Exponential replicator dynamics, i.e., Eq. 7 is equivalent to $n_t = N \frac{n_{t-1}}{n_{t-1}+(N-n_{t-1})\exp[\lambda(A^S_{t-1}-A^R_{t-1})]}$. Ignoring the momentum and risk component of the risky asset, which play no significant role in the above dynamics when the market is near its top, and using the current specification yields $n_t = N \frac{n_{t-1}}{n_{t-1}+(N-n_{t-1})\exp[-\lambda\gamma(\frac{d}{p_{t-1}}-r)]}$. Hence, if n_{t-1} approaches N, the fundamental conditions of the market no longer play a role in investor behavior. Shiller (2015)

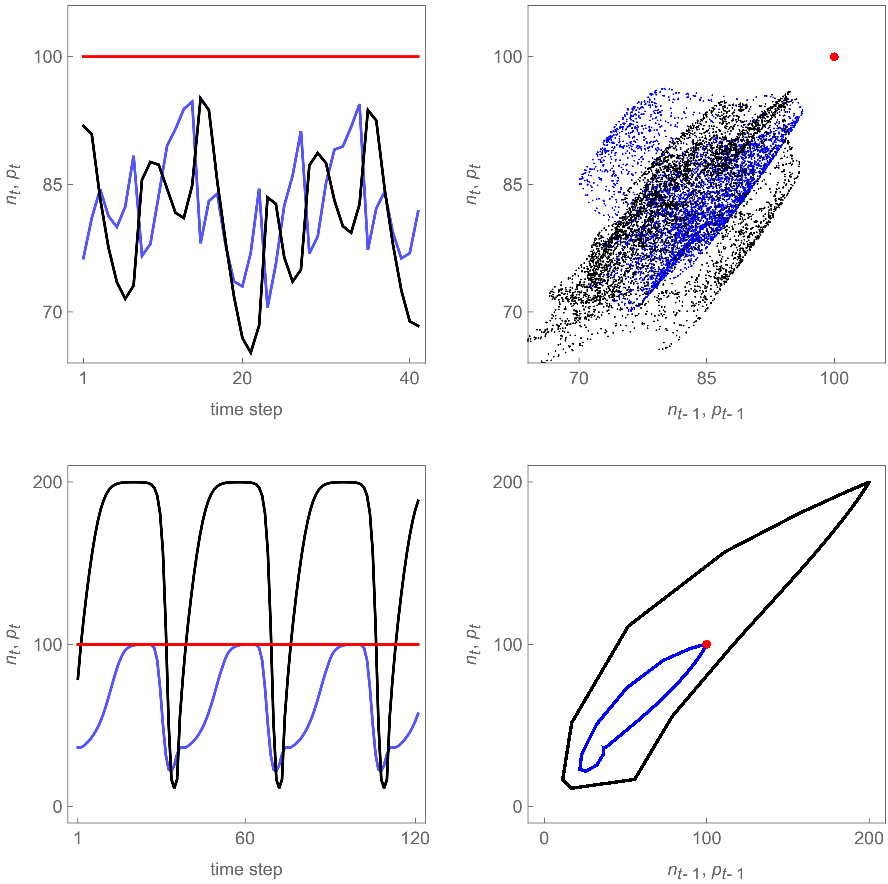

Fig. 9 Complex dynamics and herding behavior. The *top left panel* shows the evolution of $n_t = p_t$ (*black*), $\overline{n}_t = \overline{p}_t$ (*blue*) and $\overline{n} = \overline{p}$ (*red*) for 40 time steps. The *top right panel* depicts the corresponding dynamics in phase space. Base parameter setting, except that $\lambda = 3$ and $m = 0.5$. The *bottom panels* depict the same, except that $\beta = 1000$, $\kappa = 0.75$ and $m = 0.5$

argues that investors, being part of the crowd, may merely assume that other investors know what they are doing. Galbraith (1994, p. 80) recalls a statement by Irving Fisher, expressed in the autumn of 1929, according to which "*stock prices have reached what looks like a permanent high plateau*" and the view by Joseph Stagg Lawrence who commented, as stocks reached their peak, that "*The consensus of judgement of the millions whose valuations function on that admirable market, the Stock Exchange, is that stocks are not at present overvalued ... where is that group of men with the all-embracing wisdom which will entitle them to veto the judgement of this intelligent multitude?*". If that represents the zeitgeist during a boom, as was apparently the case in 1929, it is obviously hard for investors to act against it.

Once the risky asset market starts to collapse, however, investors begin to exit it. The less populated the risky asset market is, the less the persistence-generating herding effect becomes. Alternative parameter settings may yield scenarios in which the risky asset market is stuck in bust states (not depicted). We can therefore conclude that herding may prolong the length of time a risky asset market remains in a boom or bust state by adding inertia to investors' behavior.

5 Conclusions

We develop a quite general and empirically guided asset market participation model to explain the recurrent and potentially harmful phenomenon of boom–bust dynamics observed in many different markets throughout the whole world and at different times. In doing so, we also shed some new light on long-standing and intertwined puzzles of investors' limited asset market participation, time-varying discount factors, excess volatility, and high risk premia. Within our model, we consider a trading environment in which investors can choose between a safe asset and a risky asset. Investors' decisions whether to buy the risky asset depend on the attractiveness of the risky asset relative to the safe asset. While the attractiveness of the safe asset is constant, that of the risky asset changes with respect to its momentum, value and risk, where risk contains an exogenous (fundamental) and an endogenous (speculative) component. However, investors also display some kind of herding behavior, i.e., their decisions depend on other investors' actions. A further natural building block of our model states that the price of the risky asset increases with the number of investors that buy the risky asset. Together, these forces create a complex network with multiple intertwined positive and negative feedback loops that are usually difficult to understand.

Nevertheless, we are able to carry out a rigorous analytical investigation of our setup, which reveals that the dynamical system of the model admits a unique interior fundamental steady state. At the fundamental steady state, the risky asset reflects the present value of its expected future payouts. Remarkably, the risk-adjusted return, arising endogenously within our behavioral model, is equal to the risk-free return plus a risk premium, represented by the product of the market price for risk and the volatility of the risky asset market. We prove that the fundamental steady state becomes unstable if the product of four interrelated forces exceeds a critical threshold. To be precise, if the fraction of outside investors multiplied by their intensity of switching multiplied by their reaction to the momentum of the risky asset multiplied by their impact on the price of the risky asset exceeds unity, endogenous asset market participation waves and co-evolving boom–bust cycles occur.

Roughly speaking, two competing forces are behind such dynamics. First, investors react to the momentum of the risky asset, a behavior that creates a destabilizing positive feedback process. If the price of the risky asset increases, more investors buy the risky asset, prolonging the upward movement of the risky asset market. In contrast, if the price of the risky asset decreases, the risky asset market loses investors and therefore continues its downward movement. Second, investors keep an eye on the value of the risky asset, a behavior that initiates a stabilizing negative feedback process. If the market is overvalued (undervalued), investors leave (enter) the risky asset market, reverting the direction of the market. The interplay between investors' reactions to the momentum and value of the risky asset may already be sufficient for the emergence of endogenous asset market participation waves and co-evolving boom–bust cycles.

However, risk plays an important role in the dynamics, too. Once endogenous dynamics is set in motion, asset market risk builds up. Due to their risk aversion, investors' average market participation declines. As a result, the level around which the aforementioned oscillations occur gradually declines with the volatility of the risky asset market. Moreover, investors' herding behavior tends to prolong the length of time for which the risky asset market may remain in a boom or a bust state. Interestingly, we may also observe a spontaneous, sharp, and permanent downward shift of the average price level in the presence of large risk. In technical terms, this is due to the co-existence of multiple attractors and tipping points. In economic terms, this means that a regime characterized by a rather low volatility and relatively high asset market participation and average prices may co-exist with a regime characterized by a rather high volatility and relatively low asset market participation and average prices. Obviously, single external shocks may push the dynamics from one attractor to another, e.g., from a desirable low-volatility attractor with high asset market participation and high average prices to an undesirable high-volatility attractor with low asset market participation and low average prices. Changes in the market-implied discount rate affect the fundamental value of the risky asset. Nevertheless, the risky asset is excessively volatile and, consequently, goes hand in hand with a high risk premium. Together, these aspects contribute to the instability and riskiness of asset markets and may pose serious threats to the real economy.

Appendix A

To determine the local asymptotic stability of the FSS, we need to build the Jacobian first. To do so, we compute the derivatives of n_{t+1}, z_{t+1}, u_{t+1} and $\nu_{p,t+1}$ with respect to variables n_t, z_t, u_t and $\nu_{p,t}$, respectively. Since we have

$$n_{t+1} = N \frac{n_t \exp(\lambda A_t^R)}{n_t \exp(\lambda A_t^R) + (N - n_t) \exp(\lambda A^S)}, \tag{A1}$$

it is useful to regard the right-hand side of Eq. A1 as a function of n_t and A_t^R, i.e., $n_{t+1} = M(n_t, A_t^R)$, where A_t^R is itself a function of n_t, z_t and $\nu_{p,t}$ since $A_t^R = \Phi(\rho_t, \delta_t, \nu_t) = \Phi\big(\frac{h(n_t)}{h(z_t)} - 1, \frac{d}{h(n_t)}, \nu_d + \nu_{p,t}\big)$. By using the chain rule, we obtain the derivatives of the attractiveness function A_t^R with respect to its arguments n_t, z_t and $\nu_{p,t}$, that is

$$\begin{aligned}
\frac{\partial A_t^R}{\partial n_t} &= \Phi_\rho \frac{h'(n_t)}{h(z_t)} + \Phi_\delta \Big[-d \frac{h'(n_t)}{(h(n_t))^2}\Big], \\
\frac{\partial A_t^R}{\partial z_t} &= \Phi_\rho \Big[-h(n_t) \frac{h'(z_t)}{(h(z_t))^2}\Big], \\
\frac{\partial A_t^R}{\partial \nu_{p,t}} &= \Phi_\nu.
\end{aligned}$$

Recall that the partial derivatives Φ_ρ, Φ_δ and Φ_ν at the FSS are denoted by β, γ and $-\theta$, where $\beta, \gamma, \theta > 0$. Moreover, we define $\alpha := h'(\overline{n}) = h'(\overline{z})$. Since, at the FSS, we also have that $\overline{p} = h(\overline{n}) = h(\overline{z}) = \frac{d}{r_a}$, $\frac{d}{\overline{p}} = \frac{d}{h(\overline{n})} = r_a$ and $\epsilon := \overline{n}\frac{h'(\overline{n})}{h(\overline{n})} = \frac{\overline{n}\alpha}{\overline{p}}$, the above derivatives at the FSS are given by

$$\left.\frac{\partial A_t^R}{\partial n_t}\right|_{FSS} = \beta\frac{\alpha}{\overline{p}} - \gamma\frac{\alpha d}{\overline{p}^2} = \frac{\alpha}{\overline{p}}(\beta - \gamma r_a) = \frac{\epsilon}{\overline{n}}(\beta - \gamma r_a),$$
$$\left.\frac{\partial A_t^R}{\partial z_t}\right|_{FSS} = -\beta\frac{\alpha}{\overline{p}} = -\beta\frac{\epsilon}{\overline{n}},$$
$$\left.\frac{\partial A_t^R}{\partial \nu_{p,t}}\right|_{FSS} = -\theta,$$

where $\overline{n} = \overline{z} = h^{-1}(\frac{d}{r_a})$. Furthermore, the derivatives of $M(n_t, A_t^R)$ with respect to n_t and A_t^R are as follows:

$$\frac{\partial M}{\partial n_t} = N\frac{Y_t \exp(\lambda A_t^R) - n_t \exp(\lambda A_t^R)[\exp(\lambda A_t^R) - \exp(\lambda A^S)]}{Y_t^2},$$
$$\frac{\partial M}{\partial A_t^R} = N\frac{n_t \lambda Y_t \exp(\lambda A_t^R) - n_t^2 \lambda[\exp(\lambda A_t^R)]^2}{Y_t^2},$$

where $Y_t = n_t \exp(\lambda A_t^R) + (N - n_t)\exp(\lambda A^S)$. Since $A^S = \overline{A}^R$ and $\overline{Y} = N\exp(\lambda A^S)$ at the FSS, we get

$$\left.\frac{\partial M}{\partial n_t}\right|_{FSS} = N\frac{\overline{Y}\exp(\lambda A^S)}{\overline{Y}^2} = 1,$$
$$\left.\frac{\partial M}{\partial A_t^R}\right|_{FSS} = N\frac{\overline{n}\lambda N[\exp(\lambda A^S)]^2 - \overline{n}^2\lambda[\exp(\lambda A^S)]^2}{N^2[\exp(\lambda A^S)]^2} = \lambda\overline{n}\left(1 - \frac{\overline{n}}{N}\right).$$

Now, we consider function F, which represents the replicator dynamics defined in Eq. A1. From

$$\frac{\partial F}{\partial n_t} = \frac{\partial M}{\partial n_t} + \frac{\partial M}{\partial A_t^R}\frac{\partial A_t^R}{\partial n_t},$$
$$\frac{\partial F}{\partial z_t} = \frac{\partial M}{\partial A_t^R}\frac{\partial A_t^R}{\partial z_t},$$
$$\frac{\partial F}{\partial \nu_{p,t}} = \frac{\partial M}{\partial A_t^R}\frac{\partial A_t^R}{\partial \nu_{p,t}},$$

we obtain the following derivatives of function F at the FSS:

$$\left.\frac{\partial F}{\partial n_t}\right|_{FSS} = 1 + \lambda\Big(1 - \frac{\overline{n}}{N}\Big)\epsilon(\beta - \gamma r_a), \tag{A2}$$

$$\left.\frac{\partial F}{\partial z_t}\right|_{FSS} = -\lambda\beta\Big(1 - \frac{\overline{n}}{N}\Big)\epsilon, \tag{A3}$$

$$\left.\frac{\partial F}{\partial \nu_{p,t}}\right|_{FSS} = -\lambda\theta\overline{n}\Big(1 - \frac{\overline{n}}{N}\Big). \tag{A4}$$

Next, consider function G, which determines the updating of the price average, i.e.,

$$u_{t+1} = mu_t + (1-m)p_{t+1} = mu_t + (1-m)h\big(F(n_t, z_t, \nu_{p,t})\big) := G(n_t, z_t, u_t, \nu_{p,t}),$$

from which we get

$$\frac{\partial G}{\partial n_t} = (1-m)h'(F(n_t, z_t, \nu_{p,t}))\frac{\partial F}{\partial n_t},$$

$$\frac{\partial G}{\partial z_t} = (1-m)h'(F(n_t, z_t, \nu_{p,t}))\frac{\partial F}{\partial z_t},$$

$$\frac{\partial G}{\partial u_t} = m,$$

$$\frac{\partial G}{\partial \nu_{p,t}} = (1-m)h'(F(n_t, z_t, \nu_{p,t}))\frac{\partial F}{\partial \nu_{p,t}}.$$

Since $F(\overline{n}, \overline{z}, \overline{\nu}_p) = \overline{n}$, $h'(\overline{n}) := \alpha$ and given derivatives A2-A4 of function F, we obtain at the FSS:

$$\left.\frac{\partial G}{\partial n_t}\right|_{FSS} = (1-m)\alpha\Big[1 + \lambda\Big(1 - \frac{\overline{n}}{N}\Big)\epsilon(\beta - \gamma r_a)\Big],$$

$$\left.\frac{\partial G}{\partial z_t}\right|_{FSS} = -(1-m)\alpha\lambda\beta\Big(1 - \frac{\overline{n}}{N}\Big)\epsilon,$$

$$\left.\frac{\partial G}{\partial u_t}\right|_{FSS} = m,$$

$$\left.\frac{\partial G}{\partial \nu_{p,t}}\right|_{FSS} = -(1-m)\alpha\lambda\theta\overline{n}\Big(1 - \frac{\overline{n}}{N}\Big).$$

Finally, we consider function H, which is given by

$$\nu_{p,t+1} = m\nu_{p,t} + m(1-m)(p_{t+1} - u_t)^2 = m\nu_{p,t} + m(1-m)\Big(h\big(F(n_t, z_t, \nu_{p,t})\big) - u_t\Big)^2 := H(n_t, z_t, u_t, \nu_{p,t}),$$

and first compute its derivative with respect to $\nu_{p,t}$, resulting in

$$\frac{\partial H}{\partial \nu_{p,t}} = m + m(1-m)2[h(F(n_t, z_t, \nu_{p,t})) - u_t]h'(F(n_t, z_t, \nu_{p,t}))\frac{\partial F}{\partial \nu_{p,t}}.$$

Since $h(F(\overline{n}, \overline{z}, \overline{\nu}_p)) = h(\overline{n}) = \overline{p} = \overline{u}$, the factor $[h(F(n_t, z_t, \nu_{p,t})) - u_t]$ vanishes at the fundamental steady state, yielding

$$\left.\frac{\partial H}{\partial \nu_{p,t}}\right|_{FSS} = m.$$

The same factor is present in all the remaining derivatives of H, which is why we also have

$$\left.\frac{\partial H}{\partial n_t}\right|_{FSS} = \left.\frac{\partial H}{\partial z_t}\right|_{FSS} = \left.\frac{\partial H}{\partial u_t}\right|_{FSS} = 0.$$

The Jacobian matrix of the dynamical system defined in Eq. 8 at the FSS therefore has the following structure:

$$\mathbf{J} = \begin{pmatrix} \left.\frac{\partial F}{\partial n_t}\right|_{FSS} & \left.\frac{\partial F}{\partial z_t}\right|_{FSS} & 0 & \left.\frac{\partial F}{\partial \nu_{p,t}}\right|_{FSS} \\ 1 & 0 & 0 & 0 \\ \alpha(1-m)\left.\frac{\partial F}{\partial n_t}\right|_{FSS} & \alpha(1-m)\left.\frac{\partial F}{\partial z_t}\right|_{FSS} & m & \alpha(1-m)\left.\frac{\partial F}{\partial \nu_{p,t}}\right|_{FSS} \\ 0 & 0 & 0 & m \end{pmatrix}, \tag{A5}$$

where $\left.\frac{\partial F}{\partial n_t}\right|_{FSS}$, $\left.\frac{\partial F}{\partial z_t}\right|_{FSS}$ and $\left.\frac{\partial F}{\partial \nu_{p,t}}\right|_{FSS}$ are given by Eqs. A2, A3 and A4, respectively.

Since the local asymptotic stability of the FSS depends on the eigenvalues of the Jacobian matrix, i.e., Eq. A5, we next compute its characteristic polynomial, i.e., $\mathcal{P}(\eta) = \det(\mathbf{J}-\eta\mathbf{I})$, and consider the equation

$$\det(\mathbf{J}-\eta\mathbf{I}) = (m-\eta)^2\mathcal{P}_2(\eta) = 0,$$

where the second-degree polynomial $\mathcal{P}_2(\eta)$ is the characteristic polynomial of the two-dimensional submatrix $\mathbf{C}$ in the upper left corner of Eq. A5, namely

$$\mathbf{C} = \begin{pmatrix} 1 + \lambda\left(1 - \frac{\overline{n}}{N}\right)\epsilon(\beta - \gamma r_a) & -\lambda\beta\left(1 - \frac{\overline{n}}{N}\right)\epsilon \\ 1 & 0 \end{pmatrix}.$$

It follows that two out of the four eigenvalues are equal to m, where $0 < m < 1$, implying that the stability properties of $\mathbf{J}$ are fully determined by the remaining two eigenvalues, say η_1 and η_2. They are simultaneously smaller than one in modulus if and only if:

$$\begin{aligned} \mathcal{P}_2(1) &= 1 - Tr(\mathbf{C}) + Det(\mathbf{C}) > 0, \\ \mathcal{P}_2(-1) &= 1 + Tr(\mathbf{C}) + Det(\mathbf{C}) > 0, \\ \mathcal{P}_2(0) &= Det(\mathbf{C}) < 1, \end{aligned} \tag{A6}$$

where $Tr(\mathbf{C}) = 1 + \lambda\left(1 - \frac{\overline{n}}{N}\right)\epsilon(\beta - \gamma r_a)$ and $Det(\mathbf{C}) = \lambda\beta\left(1 - \frac{\overline{n}}{N}\right)\epsilon$. By setting $\overline{x} := \frac{\overline{n}}{N}$ and $\tilde{\lambda} := \lambda\epsilon$, the conditions defined in Eq. A6 can be rearranged, respectively, as follows:

$$\begin{aligned} \tilde{\lambda}(1 - \overline{x})\gamma r_a &> 0, \\ \gamma &< \frac{2}{r_a}\left[\beta + \frac{1}{\tilde{\lambda}(1 - \overline{x})}\right], \\ \beta &< \frac{1}{\tilde{\lambda}(1 - \overline{x})}. \end{aligned} \tag{A7}$$

While the first condition in Eq. A7 is always satisfied, violation of the second (third) condition is associated with a Flip (Neimark–Sacker) bifurcation. Using the original parameters, the second and third condition can be rewritten and summarized by

$$\frac{\gamma r_a}{2} - \frac{1}{\lambda\epsilon\left(1 - \frac{\overline{n}}{N}\right)} < \beta < \frac{1}{\lambda\epsilon\left(1 - \frac{\overline{n}}{N}\right)},$$

where $\overline{n} = h^{-1}(\frac{d}{r_a})$ and $r_a = \varphi^{-1}(A^S)$.

Appendix B

This appendix provides a brief illustration of the robustness of our model's main results to alternative functional specifications of investors' behavioral rules. It suggests that our findings about the steady-state structure of the model, the onset of endogenous fluctuations and the co-existence of different price and volatility regimes are far more general than is indicated by the selected specifications and examples presented in the main body of the paper. In particular, we briefly discuss a different attractiveness specification for the risky asset, a more general rule for return extrapolation and alternative switching rules based on imitative dynamics.

Attractiveness based on a multiplicative risk-adjustment rule

Suppose that, other things being equal, the attractiveness of the risky asset is specified as

$$A_t^R = \Phi(\rho_t, \delta_t, \nu_t) = \frac{1}{1 + \xi\sqrt{\nu_t}}\left[\mu \arctan\left(\frac{b}{\mu}\rho_t\right) + c\delta_t\right],$$

where, again $\mu := \frac{2\kappa}{\pi} > 0,\ b, c > 0$ and $\xi > 0$. With this specification, the tradeoff between the (fundamental and speculative) gain potential and the risk is modeled in a multiplicative fashion. Consistently, we assume $A^S = cr$ for the attractiveness of the safe asset. Therefore, the no-arbitrage condition $\Phi(0, \bar{\delta}, \nu_d) = A^S$ yields $\bar{\delta} = r(1 + \xi\sqrt{\nu_d}) := r_a$. Similar to our illustration in Section 4.1, a formal change in parameters allows us to rewrite the risk-adjusted discount rate in more familiar terms as $r_a = r + \tau\sigma$, where, again, $\nu_d = \left(\sigma\frac{d}{r}\right)^2$ and $\xi = \tau/d$. Likewise, the time-varying risk-adjusted discount rate can again be rewritten as $r_{a,t} = r + \dfrac{\tau r}{d}\sqrt{\nu_t} = r + \tau\sigma\sqrt{\dfrac{\nu_t}{\nu_d}}$. Of course, the results from our general stability analysis still hold. However, note that quantities $\beta := \Phi_\rho$ and $\gamma := \Phi_\delta$ to be used in the steady-state stability conditions (Proposition 1 and Eq. A7) are now given by $\beta := \Phi_\rho = (1+\xi\sqrt{\nu_d})^{-1}b = br/r_a$ and $\gamma := \Phi_\delta = (1 + \xi\sqrt{\nu_d})^{-1}c = cr/r_a$. To illustrate the out-of-equilibrium behavior of the model, we rely again on the parameter setting of Section 4.1, in which the baseline values of parameters $\beta = 2$ and $\gamma = 20$ correspond, respectively, to $b = 2.4$ and $c = 24$. The panels depicted in the top line of Fig. 10, assuming $b = 2.46$, reveal that the main features of the dynamics beyond the Neimark–Sacker bifurcation threshold survive under the new specification. In particular, we observe the emergence of endogenous asset market participation waves and co-evolving boom–bust cycles that are located below their steady-state values.

Return extrapolation through geometric time averages

Assuming again our baseline additive specification of the attractiveness function of the risky asset, we now generalize the rule by which investors perceive and update its momentum, based on a longer history of observations. More precisely, we replace the latest price return $\rho_t := \frac{p_t}{p_{t-1}} - 1$ in the attractiveness function defined in Eq. 14 by the mean return $\bar{\rho}_t$ defined according to the following exponentially weighted (geometric) moving average:

$$1 + \bar{\rho}_t = \left(\frac{p_t}{p_{t-1}}\right)^{1-\eta}\left(\frac{p_{t-1}}{p_{t-2}}\right)^{\eta(1-\eta)}\left(\frac{p_{t-2}}{p_{t-3}}\right)^{\eta^2(1-\eta)} \dots = \sum_{s=0}^{\infty}\left(\frac{p_{t-s}}{p_{t-s-1}}\right)^{\eta^s(1-\eta)},$$

or, in a recursive manner:

$$\bar{\rho}_t = (1 + \rho_t)^{1-\eta}(1 + \bar{\rho}_{t-1})^{\eta} - 1,$$

with $0 \le \eta < 1$, where the particular case studied in the main body of our paper corresponds to $\eta = 0$. The larger parameter η is, the more past data investors use to identify the asset's return potential. Clearly, the steady state of the model – at which the average return $\bar{\rho}$ is equal to 0 – is the same as in our baseline model, although the dimension of the model increases by 1. Simulations reveal that our main qualitative findings survive under this more general setting, provided that $\eta > 0$ is not too large. For instance, the oscillatory dynamics of $n_t = p_t$ presented in the second line of

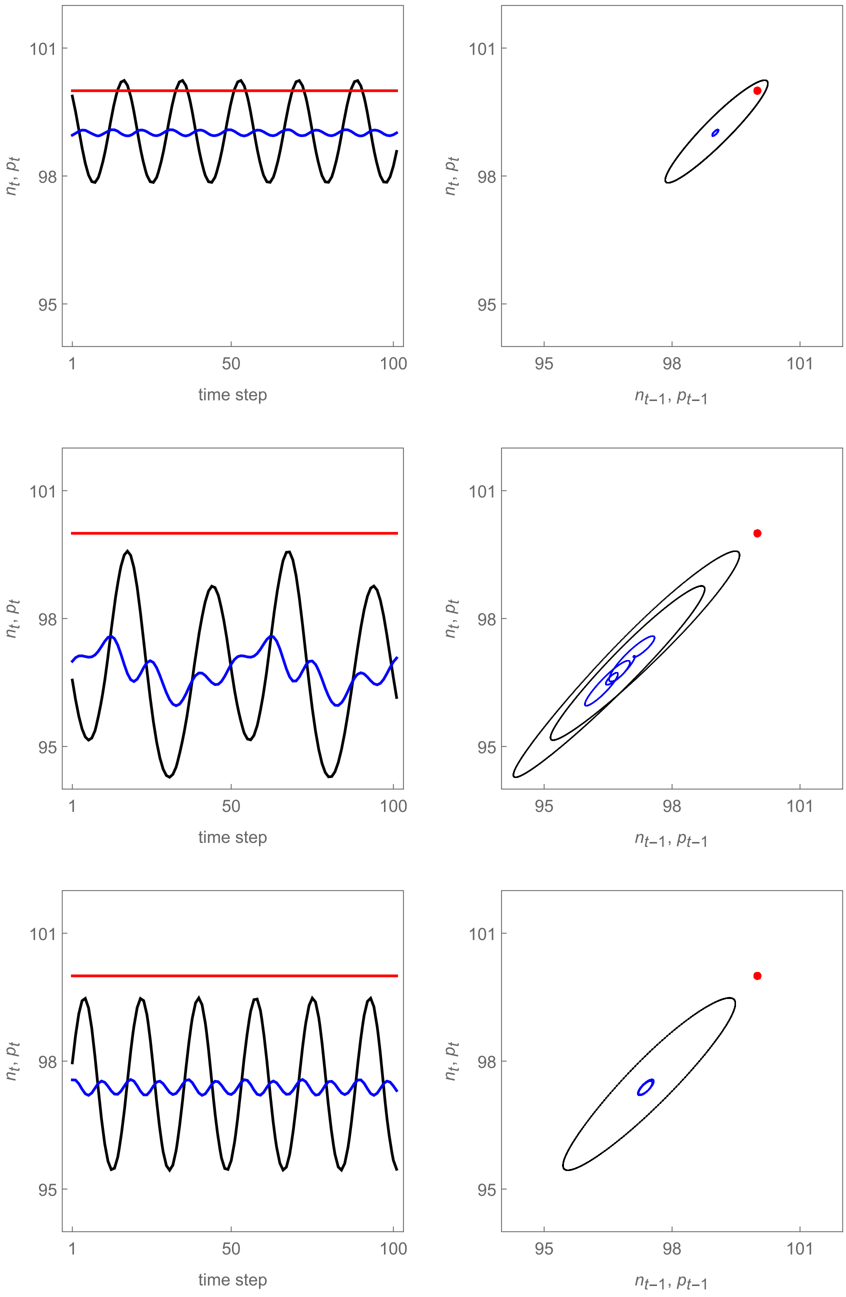

Fig. 10 Asset market participation waves and boom–bust cycles assuming a different attractive specification of the risky asset, a more general rule for return extrapolation and an alternative switching rule based on imitative dynamics, respectively. The *left panels* show the evolution of $n_t = p_t$ (*black*), $\overline{n}_t = \overline{p}_t$ (*blue*) and $\overline{n} = \overline{p}$ (*red*) for 100 time steps. The *right panels* depict the corresponding dynamics in phase space. See Appendix B for the underlying parameter setting

panels in Fig. 10 are based on our standard parameter setting, except that $\beta = 2.25$ and $\eta = 0.5$.

Fundamental steady state under more general imitative dynamics

Although we derived and illustrated our results under a particular evolutionary dynamic rule (the exponential replicator dynamics), the (risk-adjusted) fundamental steady state of our model remains the same under a far more general class of evolutionary dynamics. In fact, the crucial feature of our model behind the equilibrium payout-price ratio defined in Eq. 11 is expressed by Eq. 9, namely, the fact that the risky asset and the safe asset are equally attractive at the interior steady state. A similar property is common to so-called imitative dynamics – which include the exponential replicator rule as a particular case (see Hofbauer and Sigmund 2003 and Sandholm 2010). A class of imitative dynamics, which represents an immediate generalization of the exponential replicator rule as well as of other versions of the replicator dynamics, can be represented, in discrete time, as follows:

$$x_{t+1}^i = \frac{x_t^i w(A_t^i)}{\sum_{k=1}^{n} x_t^k w(A_t^k)}, \quad i = 1, 2, ..., n,$$

where x_t^i is the share of the population that adopts strategy i in period t, A_t^i is the payoff or fitness of strategy i and w (the so-called copying weight) is a strictly positive and increasing function. Even more generally, a characterizing feature of imitative dynamics is that the relative growth rates of the competing strategies exhibit so-called payoff monotonicity, namely, the relative growth rates of strategies are ordered by their payoffs (see Sandholm 2010). For our model with only two possible 'strategies' with payoffs A_t^R and A^S and a fixed total number of investors N, an imitative dynamics in discrete time can therefore be equivalently formalized as the following general evolutionary rule:

$$n_{t+1} = M(n_t, A_t^R) = n_t\, m(n_t, A_t^R),$$

where the one-period growth factor of the population of investors who choose the risky asset, $n_{t+1}/n_t = M(n_t, A_t^R)/n_t := m(n_t, A_t^R)$, is such that:

$$m(n_t, A_t^R) \gtreqless 1 \qquad \Longleftrightarrow \qquad A_t^R \gtreqless A^S.$$

The last condition (capturing payoff monotonicity in our simple case) implies, at the unique interior steady state, $m(\bar{n}, \bar{A}^R) = 1$ and therefore $\bar{A}^R = A^S$, from which steady-state conditions 10 and 11 follow immediately. As an example, an alternative model of imitative dynamics is given by

$$n_{t+1} = n_t + \lambda n_t (A_t^R - \frac{1}{N}(n_t A_t^R + (N - n_t) A^S)),$$

i.e.,

$$n_{t+1} = n_t + \lambda n_t (1 - x_t)(A_t^R - A^S) = n_t(1 + \lambda(1 - x_t)(A_t^R - A^S)),$$

assuming that parameter λ is set such that $n_t \in]0, N[$. The bottom line in panels of Fig. 10 visualizes such dynamics for our standard parameter setting, except that $\beta = 2.05$. It is remarkable that other types of evolutionary rules commonly used in financial market models with behavioral heterogeneity, such as the logit dynamics and its possible extensions with asynchronous updating to capture different degrees of inertia (see Hommes 2013), do not belong to the class of imitative dynamics, and would not result in an interior steady state with the same properties as ours.

Appendix C

In this appendix, we provide a more formal justification for the time-varying fundamental value of the risky asset. Suppose that the attractiveness of the risky asset in period t does not depend on the momentum component, i.e., $\Phi_\rho = 0$ for any δ, ν, which in our framework is equivalent to $\beta = 0$, but only on the current fundamental conditions $\delta_t = d/p_t$ and the current estimate of the risk ν_t. For a given risk estimate ν_t in period t, one may wonder *which* price level $\bar{p}_t$ in period t would satisfy the no-arbitrage condition between the two markets, $A_t^R = A^S$. For the model specified in Section 4, this corresponds to the condition (remember that $\beta = 0$):

$$\gamma \frac{d}{p_t} - \psi \sqrt{\nu_t} = \gamma r, \tag{A8}$$

which results in $\delta_t = \frac{d}{p_t} = r + \frac{\psi}{\gamma}\sqrt{\nu_t} = r + \dfrac{\tau r}{d}\sqrt{\nu_t} = r + \tau\sigma\sqrt{\frac{\nu_t}{\nu_d}} := r_{a,t}$ and $\bar{p}_t = d/r_{a,t}$. Similarly, for the alternative specification of the attractiveness functions discussed in Appendix B, we obtain for $b = 0$

$$\frac{c\frac{d}{p_t}}{1 + \xi\sqrt{\nu_t}} = cr, \tag{A9}$$

implying that $\delta_t = \frac{d}{p_t} = r + r\xi\sqrt{\nu_t} = r + \dfrac{\tau r}{d}\sqrt{\nu_t} = r + \tau\sigma\sqrt{\frac{\nu_t}{\nu_d}} := r_{a,t}$ and $\bar{p}_t = \frac{d}{r_{a,t}}$. The above arguments justify why we may regard $\bar{p}_t$ as the time-varying fundamental value of the risky asset. In fact, $\bar{p}_t$ has a fundamental nature in that (i) it does not depend on the current momentum component and (ii) it is implicitly defined by a hypothetical equilibrium condition in each period, instead of evolving through 'disequilibrium' adjustments of market participation and asset prices, based on attractiveness differentials. Note that if we set $p_t = \bar{p}_t$ in the replicator equation (first equation in 8) and keep ν_t constant, then n_{t+1} would increase (decrease) with respect to n_t *only* due to the momentum component, that is, only if $\rho_t > 0$ ($\rho_t < 0$). This is due to the no-arbitrage conditions Eqs. A8 and A9 above, by which the attractiveness

A^S of the safe asset and the part of the attractiveness A_t^R of the risky asset that depends on δ_t and ν_t neutralize each other in the replicator equation whenever we set $p_t = \bar{p}_t$. If, in addition, we set the momentum component equal to 0 (e.g., $\beta = 0$, or $b = 0$), then n_{t+1} is equal to n_t (and the price would not change either). As a consequence, in each period, the quantity $p_t - \bar{p}_t$ might be interpreted as a price bubble component. As discussed in Section 4 and in Appendix B, $\overline{p}_t$ indeed serves as an anchor for p_t, i.e., the price of the risky asset circles around its time-varying fundamental value. Moreover, the difference (or relation) between the volatility of p_t and the volatility of $\overline{p}_t$ reflects the excess volatility of the risky asset. In line with Campbell and Shiller (1988, 1989) and Cochrane (2011), the driving force of the variability of $\overline{p}_t$ is $r_{a,t}$, as the risk-free interest rate and investors' payout expectations are relatively constant. Finally, note that $\overline{n}_t$ merely indicates the number of active investors needed to bring the price of the risky asset towards its time-varying fundamental value.

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