

A MARKOVIAN APPROACH FOR MODELLING AND ANALYSIS OF ADVANCED TELECOMMUNICATION NETWORKS

Udo R. KRIEGER

Research Institute of the DBP Telekom, P.O. Box 100003, D-6100 Darmstadt, F.R.G.

Michael SCZITTNICK

Informatik IV, Universität Dortmund, P.O. Box 500500, D-4600 Dortmund 50, F.R.G.

We describe a Markovian approach for modelling and analyzing modern telecommunication networks. The presented techniques are provided by a convenient software tool called MACOM. Applying MACOM to the analysis of adaptive routing schemes for telephone networks and models of B-ISDN, we illustrate the benefits of the proposed approach.

1. Introduction

In the last decade, telecommunication networks have evolved towards ISDN comprising SPC exchanges and efficient signalling procedures based on a separate, powerful signalling network. The digital telephone network with CCS CCITT No. 7 was a starting-point of this development. Considering the routing of calls or packets in circuit- or packet-switched networks, the use of computer-controlled exchanges offers new possibilities. Therefore, considerable attention has been devoted to the investigation of new adaptive routing schemes and improved congestion-control mechanisms which can be employed in digital networks.

Usually, the evaluation and selection of new strategies is based on stochastic models derived from teletraffic theory. Normally, a system is described by a queueing network. As the proposed new strategies such as state-dependent routing schemes for circuit-switched networks violate the restrictions of classical queueing networks of BCMP or Kelly type, simulation seems to be in most cases the only method of analysis. To overcome the difficulties of simulation, e.g., the well-known problem of simulating rare loss events with probabilities in the range of 10^{-9} arising from the investigation of ATM networks, a software package called MACOM (Markovian analysis of communication systems) has been developed (cf. [17], [12], [14]). MACOM provides an interactive environment for modelling and analysis of modern communication networks with adaptive routing and congestion control based on numerical solution methods for continuous-time Markov chains (CTMC) with finite state spaces. In this paper we only want to outline the basic concepts of the Markovian approach implemented by MACOM. A detailed description is given in [17], [14], [11], [12]. Here it is our main objective to demon-

strate the versatility of the tool. Therefore, MACOM is applied to study advanced circuit-switched networks which employ adaptive routing strategies such as mutual overflow routing (cf. [8], [13], [1] - see Figure 1). Furthermore, we show that MACOM can also be used to investigate teletraffic systems arising from modelling small parts of a B-ISDN. It is one of the benefits of MACOM that an analyst can restrict his attention to his genuine task of modelling a performance problem at the high logical level of queueing networks, instead of dealing with the sophisticated relations of mathematical objects like the transitions between states in Markov chains. The tool relieves the user from the burden of translating his queueing model into the underlying Markov chain. Moreover, it offers the automatic evaluation of specified performance measures associated with the generated model. By this means a versatile user-friendly tool for modelling and analysis of modern communication systems is provided.

The paper is organized as follows: In Section 2 we outline the basic concepts implemented by MACOM. Section 3 illustrates the investigation of an adaptive routing strategy for circuit-switched networks and the study of a teletraffic model describing a part of a B-ISDN by means of MACOM. Finally, the conclusion summarizes the findings of the paper.

2. Concepts of MACOM

The tool MACOM combines state-of-the art techniques in Markov modelling and software design. It provides an environment for the management of modelling data, the description and evaluation of models and the representation of results. The specification of models is supported by a graphical user interface and enhanced by convenient window techniques (cf. Figure

2). Besides the construction of models by combining predefined graphical elements, the tool offers functions for the specification of evaluation schemes and series of experiments.

The model world of MACOM is built upon concepts known from product-form queueing networks. To cope with phenomena typically arising in communication networks, these concepts have been extended by new features, e.g., capacity restrictions, losses, state-dependent routing, sources generating special Semi-Markovian arrival streams or batch arrival streams, and service times with phase-type distributions. The features of the components are chosen such that the Markovian nature of the resulting models is guaranteed. The following elements are available in MACOM:

- *Sources* generate customers (load entities) of certain classes (types) identified by the name of a class. The main attribute is given by the stochastic structure of a generated load stream. MACOM offers Poisson streams, renewal streams with different interarrival-time distributions (Phase-type, Coxian, Hyperexponential, Erlang) and Semi-Markovian streams (MMPP, MAP). A source can generate single load entities or batches of fixed size.
- *Load-controlled sources* generate customers only if they are visited by other entities. They can be used to model acknowledgements, splitting operations or changes of classes.
- *Stations* correspond to service facilities with multiple homogeneous servers and finite waiting room (delay-loss systems) or without waiting places (loss systems). Moreover, the capacity of a customer type at a station may be limited to a fixed number. If the acceptance of incoming customers violates this restriction, they will be rejected. They depart from the network via loss exits or overflow to other components. Several service disciplines are available, e.g., priority, polling, random, infinite server, processor sharing. The main attribute of a class of customers is given by the service-time distribution (Exponential, Coxian, Hyperexponential, Erlang).
- *Sinks and loss exits* describe departures of customers from the network. Loss exits are used to remove customers which are blocked and rejected due to capacity restrictions of network components.
- *Links* represent unconditional, directed transitions of customers between the components.
- *Routing elements* called conexas describe switches with one entry and two distinct exits. They are used to model probabilistic or state-dependent routing. In the latter case, a Boolean expression determines the exit chosen by a customer. This

expression may depend on the actual state of the network. It is specified in terms of the occupation processes of queues and the status of several conditions used to take a routing decision. The access to values of the actual state is provided by predefined functions, e.g., $pop(st, cl)$ for the population of class cl at station st .

- *Synchronization elements* called syncs are used to model complex multi-queue scheduling strategies or special semaphor mechanisms. They are based on two concepts. On the one hand, a customer may wait in the finite waiting room of a sync until a condition is fulfilled before leaving the sync. Such conditions are specified by Boolean expressions, too. They contain standard operators and predefined functions providing access to the actual state of the network. On the other hand, a customer may change a global integer variable by leaving a sync. It is part of the state of a model and identified by its name. It may only vary within a specified range and it is available in expressions, too.

Combining these elements by means of a graphical editor supported by a mouse, the structure of a model is defined. Adding textual information to the graphical components in related popup menus, the attributes of the model elements are determined.

The model world allows the description of complex systems, e.g., communication networks employing congestion-control mechanisms or adaptive routing. Nevertheless, the Markovian nature of a model still offers the possibility of calculating the steady-state vector of the underlying finite CTMC by numerical methods. It is a major benefit of MACOM that the generator matrix of the Markov chain arising from a graphical model is automatically generated. Moreover, both stationary and transient characteristics of the underlying CTMC may be computed. Although the size of the state space is limited by the available memory capacity of the used computer, MACOM can normally cope with large models with up to 100000 states. The numerical solver of the tool employs advanced direct and iterative methods for the solution of finite Markov chains (cf. [11], [12]). Iterative procedures may be accelerated applying relaxation techniques or inserting some aggregation-disaggregation steps (cf. [10], [16]). The relevant performance measures of a model are specified in a separate evaluation description. This process is supported by the graphical editor, too. The calculation of default measures, e.g., the population of stations or utilizations, may be determined by popup menus. Additionally, a user may define certain random variables and relevant probabilities in terms of expressions of state variables, e.g., end-to-end blocking probabilities or the aggregated mean population in a part of the model. Furthermore, means of counting measures like throughputs or loss rates can be specified

by selecting a set of links. All transitions along these links are taken into account to evaluate the resulting rates. Other measures derived from the original network state or counting measures, e.g., call-congestion rates as ratios of loss and arrival rates, mean delay times by Little's formula or cost functions, may be calculated, too.

To study the influence of model parameters on the performance measures, series of experiments may be performed. For this purpose, it is only necessary to select a model and an evaluation description and to set the formal parameters of the model including control information for the solution process. Exploiting intermediate results like the structure of the generator matrix, MACOM considerably reduces the computational efforts of experiment series. This provides a user-friendly efficient tool for modelling and analysis of communication systems.

3. Telecommunication network analysis by means of MACOM

3.1. Investigation of an adaptive routing scheme for telephone networks

In the last decade, considerable efforts have been made to study new adaptive routing schemes, e.g., DAR (cf. [2]) or variants of least loaded path routing (cf. [4]), which may be employed in circuit-switched digital networks like the modern telephone network. An adaptive variant of alternative routing is provided by a randomized version of a cyclic routing scheme (cf. [5]) which is called mutual overflow routing (MOR) in the case of two alternative paths (cf. [8], [9], [13], [1]). MOR is a sender-initiated load balancing strategy with local information horizon. It can be used to distribute the offered load uniformly among two equivalent routes depending on their congestion which is specified in terms of the corresponding end-to-end call-congestion rates. A simple variant of MOR has been derived from the use of learning stochastic automata as routing controllers (cf. [18]). Let us consider a part of a network, depicted in Figure 1, comprising four digital exchanges EX_0, EX_1, EX_2, EX_3 . Regarding a set of two equivalent alternative paths, i.e. $\{(0, 1, 3), (0, 2, 3)\}$, the offered traffic of A Erlangs is split up by a factor p . Each portion is offered to different routes of first choice, i.e., $A_{01} = pA$ Erlangs to route 1 and $A_{02} = (1-p)A$ to route 2. In the case of blocking, each portion may use the other path. The question arises whether there exists a suitable splitting formula. It should take into account only that information about the call congestion of links and routes which is locally available at the controlling exchange 0 of the considered origin-destination pair $(0, 3)$. Here, we assume that a link-by-link signalling procedure is used and that global monitoring and control is not available.

Therefore, schemes employing loss minimization procedures based on global information about the network status, e.g., maximum revenue strategies, are not considered. Let us denote the end-to-end call congestion on route $i \in \{1, 2\}$ by E_i and the call congestion on link i by B_i , $i \in \{1, 2, 3, 4\}$. Regarding the behaviour of MOR in the long run, Warko [18] has derived the following formula for the splitting factor p :

$$(1) \quad \begin{aligned} p &= \text{Prob}\{\text{call is offered to route 1}\} \\ &= \frac{E_2 - B_2 E_1}{E_1(1 - B_2) + E_2(1 - B_1)} \end{aligned}$$

$$(2) \quad \begin{aligned} 1 - p &= \text{Prob}\{\text{call is offered to route 2}\} \\ &= \frac{E_1 - B_1 E_2}{E_1(1 - B_2) + E_2(1 - B_1)} \end{aligned}$$

Hence, p is proportional to $W_1 = E_2 - B_2 E_1 = E_1(1 - B_2) + 1 - E_1 - (1 - E_2)$ and $1 - p$ is related to $W_2 = E_1 - B_1 E_2 = E_2(1 - B_1) + 1 - E_2 - (1 - E_1)$ since

$$(3) \quad p = \text{Prob}\{\text{route 1}\} = \frac{W_1}{W_1 + W_2} = \frac{C}{W_2},$$

$$(4) \quad 1 - p = \text{Prob}\{\text{route 2}\} = \frac{W_2}{W_1 + W_2} = \frac{C}{W_1}$$

holds, given $C = 1/(1/W_1 + 1/W_2) = W_1 W_2 / (W_1 + W_2)$. In analogy to DAR, the proposed MOR scheme with traffic splitting by formulas (1), (2) has a stochastic interpretation (cf. [7]). Assume that the call-blocking probabilities on different links and, hence, along different paths are independent. Then W_1 obviously specifies the probability that a call is offered to route 2 finding a free line on link 2, but it is blocked on the second link 4, or that it is blocked on 2, thus overflowing to route 1, and it is successfully carried along this route. The corresponding event is denoted by Ω_2 . In both cases, it is better to choose route 1 immediately. This strategy is implemented by setting the selection probability p of route 1 proportional to W_1 .

This interpretation proposes the following implementation of MOR. Let $t = 0$ be the instant of an arrival which has changed the preference list to $(2, 1)$, i.e., a new call is first offered to route 2 and overflows to route 1 in the case of blocking. Then we change the preference list only if blocking event Ω_2 occurs. We model the offered traffic by a Poisson stream of load A . Furthermore, we assume that the call-congestion rates are fixed and do not depend on the number of calls carried by a route. Then the mean number of calls $Z_2(t)$ up to the change of the preference list $(2, 1)$ to $(1, 2)$ at time $t > 0$, given $Z_2(0) = 0$, coincides with the mean of a geometric distribution with parameter $q = 1 - W_1$, i.e., it is $1/W_1$. Now it is evident that the selection probabilities are given by (3), (4). Experiments (cf. [18]) reveal that this approximation reflects rather accurately the behaviour of the MOR strategy. By this means the preference list of the routing scheme is updated according to the rare blocking events. This

avoids the inefficient call-by-call update cycle associated with automata routing schemes.

Normally, direct traffic carried by the links of alternative paths is protected by trunk reservation. If we model all the traffic by Poisson streams, the individual call-congestion rates of the streams on a link can be calculated by the well-known trunk reservation model (cf. [3]). Let R be the trunk reservation parameter and assume that a protected stream of load A_1 and an unprotected stream of load A_2 are offered to N lines. The loss probability of the protected traffic is denoted by $L_1(N, R, A_1, A_2)$, that of the unprotected one by $L_2(N, R, A_1, A_2)$.

Supposing the independence of call-blocking probabilities on the links, the end-to-end call-congestion rates E_i can be approximated by $E_1 = 1 - (1 - B_1)(1 - B_3)$ and $E_2 = 1 - (1 - B_2)(1 - B_4)$. Then the splitting factor $p = p(\vec{B})$ may be approximated by the fixed point $\vec{B} = (B_1, B_2, B_3, B_4)$ determined by the following modified Erlang fixed-point approach (cf. [6], [9]),

$$\begin{aligned} B_1 &= L_2(N_1, R, A_1, A[p(\vec{B}) + B_2(1 - p(\vec{B}))](1 - B_3)) \\ B_2 &= L_2(N_2, R, A_2, A[1 - p(\vec{B}) + B_1p(\vec{B})](1 - B_4)) \\ B_3 &= L_2(N_3, R, A_3, A[p(\vec{B}) + B_2(1 - p(\vec{B}))](1 - B_1)) \\ B_4 &= L_2(N_4, R, A_4, A[1 - p(\vec{B}) + B_1p(\vec{B})](1 - B_2)) \end{aligned}$$

provided that $p = p(\vec{B})$ is given by (1).

To evaluate the quality of this approximation (MOR-FXP), the performance of MOR, defined in terms of the resulting overall throughput (THP) in the network, was calculated by means of MACOM. Furthermore, the performance of the scheme was compared with a static bifurcation variant of fixed routing (FXR) (cf. [15]) and a simplified, static version of least loaded path routing (LLP) (cf. [4]). The performance of these strategies has been evaluated by means of MACOM, too. Some results are provided by Table 3. Compared to the optimal splitting factor of MOR, the calculated fixed point $p(\vec{B})$ yields a satisfactory throughput. But it is not in all cases close to the optimum. Regarding the LLP strategy, the splitting factor p is a fictitious parameter resulting from the actual distribution of the offered load among both routes.

3.2. Investigation of a teletraffic model for a part of a B-ISDN

As existing communication networks evolve rapidly towards ISDN, modelling and analysis of such systems has become an important issue of teletraffic theory. Based on STM bearer services in the network, advanced versions of ISDN may carry different kinds of services with distinct bandwidth requirements, e.g., connection-oriented, continuous bit stream services like voice or video and connection-less, bursty services like data traffic or mail.

MACOM can be used to model a link of such a network

employing hybrid switching technology and multi-slot connections for services with high bandwidth requirements. Normally, a combined delay-loss system is derived as basic teletraffic model (cf. [15, §12-3]). Calls of circuit-switched (CS) traffic are lost if not enough free channels are available in the trunk group (loss-system) whereas load units of packet-switched (PS) traffic are allowed to wait for transmission in a finite buffer (delay-loss-system). The fair access to the channels is guaranteed by trunk reservation or priority schemes.

Let us consider two consecutive links with N_1 and N_2 channels carrying a CS-traffic T_c with intensity λ_c and mean holding time $1/\mu_c$. Additionally, both links carry local PS-traffic T_{p1}, T_{p2} with intensities $\lambda_{p1}, \lambda_{p2}$ and equal mean holding time $1/\mu_p$. Each call or packet seizes one channel. The buffer sizes are denoted by B_1, B_2 . The service times are supposed to be exponentially distributed whereas the interarrival times of both service classes follow distinct, 2-phase Coxian distributions. The corresponding MACOM model is depicted in Figure 3.

In this model we use a trunk reservation scheme as access control. A fixed number of channels is reserved in link $i \in \{1, 2\}$ for each service class, namely, R_{ci} for CS- and R_{pi} for PS-traffic. The rest of the channels is shared by both classes.

The burstiness of the offered streams may be specified by the coefficients of variation (CV) of the corresponding interarrival-time distributions. Experiment series performed by MACOM reveal their influence on the call- and time-congestion rates experienced on both links. Some results are shown in Tables 1, 2, where the parameters are set to $\lambda_c = 0.01$, $\mu_c^{-1} = 100$, $\lambda_{p1} = \lambda_{p2} = 5$, $\mu_p^{-1} = 0.1$, $N_1 = 10$, $N_2 = 9$, $B_1 = 4$, $B_2 = 4$, $R_{c1} = 4$, $R_{c2} = 4$, $R_{p1} = 2$, $R_{p2} = 1$. Obviously, the congestion rates are very sensitive to the burstiness of the arrival streams.

CV of T_c	time cong. T_c	call cong. T_c	time cong. T_{p2}
0.8	1.43E-5	8.51E-6	1.81E-2
1.0	5.11E-5	5.11E-5	2.05E-2
1.2	1.00E-4	1.26E-4	2.29E-2
2.0	2.29E-4	3.92E-4	2.57E-2

Table 1: Congestion rates varying the CV of T_c

CV of T_{p1} and T_{p2}	time cong. T_c	time cong. T_{p2}	call cong. T_{p2}
0.8	3.85E-5	1.57E-2	1.54E-2
1.0	5.11E-5	2.05E-2	2.05E-2
1.2	6.84E-5	2.66E-2	2.70E-2
2.0	9.84E-5	3.98E-2	4.34E-2

Table 2: Congestion rates varying the CV of T_{p1} and T_{p2} simultaneously

4. Conclusion

The performed investigations illustrate that the tool MACOM considerably reduces the efforts necessary for modelling and analyzing selected parts of modern telecommunication networks. Its application relieves a system designer from the burden of investigating a communication system at the low level of abstraction associated with a detailed, sophisticated Markovian model. Furthermore, it offers him the chance to draw his attention to his genuine task of problem analysis and modelling by means of queueing networks.

References

- [1] L.D. Fossett and M. Liotine. The traffic engineering benefits of flexible routing in international networks. In *Proceedings Network Planning in the 1990's, Palma de Mallorca, Spain*, pp. 325 – 331, North-Holland, Amsterdam, 1989.
- [2] R. J. Gibbens, F. P. Kelly, and P. B. Key. Dynamic alternative routing - modelling and behaviour. In *Proceedings ITC 12, Turino, Italy*, pp. 3.4A.3.1 – 3.4A.3.7, 1988.
- [3] A. Girard. Blocking probability of noninteger groups with trunk reservation. *IEEE Trans. on Communications*, 33(2), 113–120, 1985.
- [4] A. Girard and M.A. Bell. Blocking evaluation for networks with residual capacity adaptive routing. *IEEE Trans. on Communications*, 37(12), 1372–1380, 1989.
- [5] M.M. Jung and J. de Boer. Cyclic overflow of calls offered to subgroups of a full-availability group of lines. In *Proceedings ITC 8, Melbourne, Australia*, pp. 426-1–426-6, #3, 1976.
- [6] F. P. Kelly. Routing in circuit-switched networks: optimization, shadow prices and decentralization. *Advances of Applied Probability*, 20, 112–144, 1988.
- [7] P. B. Key. Implied cost methodology and software tools for a fully connected network with DAR and trunk reservation. *British Telecom Technol. Journal*, 6(3), 52–65, 1988.
- [8] U. Krieger. Analysis of a loss system with mutual overflow. In *International Teletraffic Seminar on Teletraffic and Network, Beijing, September 12–16, 1988, Proceedings*, pp. 331–340, 1988.
- [9] U. Krieger. Analysis of a loss system with mutual overflow and external traffic. In *Stochastische Modelle und Methoden in der Informationstechnik, ITG-Fachbericht 107*, pp. 145–153, VDE-Verlag, Berlin, 1989.
- [10] U. Krieger. Analysis of a loss system with mutual overflow in a Markovian environment. In *Proceedings of the First International Conference on the Numerical Solution of Markov Chains, Raleigh, North Carolina, January 8–10, 1990*, pp. 328–348, Marcel Dekker, New York, 1990.
- [11] U. Krieger. *Computational Methods For Markovian Queueing Models*. Technical Report, 4302 TB 10E, Forschungsinstitut der DBP, Darmstadt, 1989.
- [12] U. Krieger, B. Müller-Clostermann, and M. Sczittnick. Modeling and analysis of communication systems based on computational methods for Markov chains. *IEEE Journal on Selected Areas in Communications*, Vol. 8, December 1990.
- [13] F. Le Gall and J. Bernussou. An analytical formulation for grade of service determination in telephone networks. *IEEE Trans. on Communications*, 31(3), 420–424, 1983.
- [14] B. Müller-Clostermann, M. Sczittnick, and U. Krieger. *Modelling and analysis of modern telecommunication networks by Markovian techniques: Foundations, algorithms and examples*. Technical Report, Nr. 341, Informatik IV, Universität Dortmund, 1990.
- [15] M. Schwartz. *Telecommunication Networks: Protocols, Modeling and Analysis*. Addison-Wesley, New York, 1987.
- [16] P.J. Schweitzer. A survey of aggregation-disaggregation in large Markov chains. In *Proceedings of the First International Conference on the Numerical Solution of Markov Chains, Raleigh, North Carolina, January 8–10, 1990*, pp. 53–80, Marcel Dekker, New York, 1990.
- [17] M. Sczittnick and B. Müller-Clostermann. MACOM - A tool for the Markovian analysis of communication systems. In *Proceedings of the Fourth International Conference on Data Communication Systems and their Performance, Barcelona, Spain, June 20 – 22, 1990*.
- [18] J. Warko. *Adaptive Verkehrslenkung in hierarchischen leitungsvermittelten Netzen*. Technical Report 4302 TB 6, Forschungsinstitut der DBP, Darmstadt, 1986.

Network parameters										Performance of the routing schemes							
offered load					structure					MOR-FXP		MOR		FXR		LLP	
A	A ₁	A ₂	A ₃	A ₄	N ₁	N ₂	N ₃	N ₄	R	p	THP	p	THP	p	THP	p	THP
17.	10.	0.	3.	30.	12	10	13	12	2	0.97	24.96	0.99	24.97	0.99	24.92	0.87	24.97
17.	3.	0.	30.	10.	12	10	13	12	2	0.43	25.67	0.01	25.86	0.01	25.86	0.39	25.93
17.	5.	0.	35.	3.	12	10	13	12	2	0.31	26.59	0.01	27.00	0.01	27.00	0.12	27.02
17.	5.	0.	5.	5.	12	10	13	12	2	0.62	23.10	0.65	23.11	0.41	22.71	0.57	23.56

Table 3: Comparison of different routing strategies

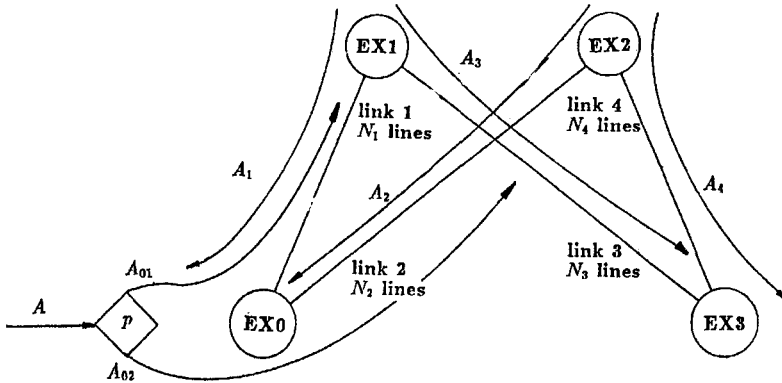


Figure 1: Part of a circuit-switched network with mutual overflow routing

Legend

- Station
- Source
- Sink
- Routing element
- Synchronization element
- Load-controlled source
- Loss exit
- Link

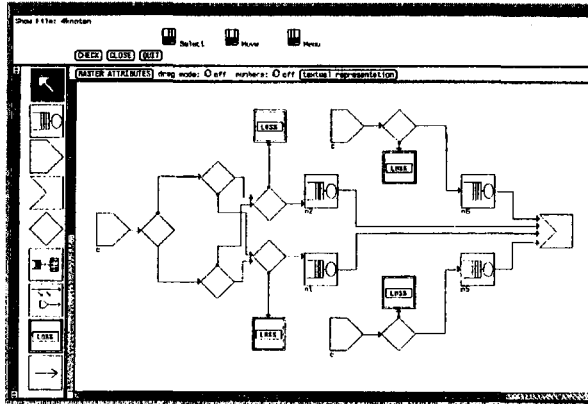


Figure 2: MACOM model of a circuit-switched network with mutual overflow routing

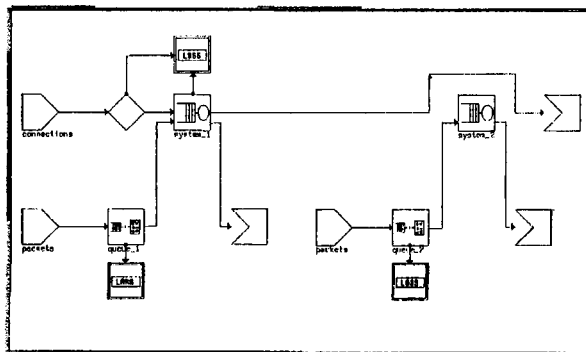


Figure 3: MACOM model of two consecutive links in B-ISDN