Kotb, Naira; Brenneisen, Jan-Niklas; Lengnick, Matthias; Proano, Christian; u.a.

Spillover Effects between the Stock Market and the Real Economy in a Mixed-Frequency Agent-Based Macrofinancial Model

Date of secondary publication: 28.05.2026

Version of Record (Published Version), Article

Persistent identifier: urn:nbn:de:bvb:473-irb-115307x

Primary publication

Kotb, Naira; Brenneisen, Jan-Niklas; Lengnick, Matthias; u. a. (2024): Spillover Effects between the Stock Market and the Real Economy in a Mixed-Frequency Agent-Based Macrofinancial Model, in: Jahrbücher für Nationalökonomie und Statistik = Journal of economics and statistics, Berlin: De Gruyter Oldenbourg, vol. 244, no. 4, pp. 331–350, doi: 10.1515/jbnst-2024-0017

DE GRUYTER OLDENBOURG Journal of Economics and Statistics 2024; 244(4): 331–350

Naira Kotb*, Jan-Niklas Brenneisen, Matthias Lengnick,
Christian R. Proaño and Hans-Werner Wohltmann

Spillover Effects Between the Stock Market and the Real Economy in a Mixed-Frequency Agent-Based Macrofinancial Model

https://doi.org/10.1515/jbnst-2024-0017
Received January 13, 2024; accepted September 16, 2024

Abstract: This paper illustrates a behavioral mixed frequency macro-finance model where both real and financial variables are generated on a daily basis. Further, while financial sector data is collected at the same frequency as it is generated (i.e. daily), real data can only be collected on a quarterly basis. Under these circumstances, output and inflation, upon which data is available with a significant delay, become unsuitable as the sole information guide for monetary policy. We suggest that policy makers can deal with this *information problem* by reacting to the variable on which data is collected on high frequency basis: the stock price.

Keywords: new Keynesian model; mixed-frequency macroeconomics; behavioral macroeconomics; optimal monetary policy; macro-finance interaction; heuristic switching;

JEL Classification: E44; E52; G01

1 Introduction

How much additional stability in the real sector justifies a leaning-against-the-wind monetary policy where the policy rate decisively goes beyond the *conventional*

We would like to thank Philipp Hauber for the excellent research assistance.

This article is part of the special issue "Advancing Agent-based Economics" published in the Journal of Economics and Statistics. Access to further articles of this special issue can be obtained at www.degruyter.com/jbnst.

***Corresponding author: Naira Kotb**, Otto-Friedrich-Universität Bamberg, Bamberg, Germany, E-mail: naira.kotb@uni-bamberg.de. https://orcid.org/0000-0002-5441-0801
Jan-Niklas Brenneisen, Matthias Lengnick and Hans-Werner Wohltmann, Christian-Albrechts-Universität zu Kiel, Kiel, Germany
Christian R. Proaño, Otto-Friedrich-Universität Bamberg, Bamberg, Germany

wisdom and reacts to stock price updates? The baggage of appearing to interfere in the stock market is not light to bear. Furthermore, the literature on whether or not, and to what extent, the policy rate should consider leaning, is not settled. In this paper, we focus on the information provided by stock market updates to the policy making process, proposing that a judicious utilization of this information could yield stability benefits that would otherwise be forfeited if this information were overlooked.

The primary aim of this paper is to examine how a monetary policy rule that responds to high-frequency stock price updates, which are correlated with the real sector, affects the macroeconomic and financial stability of an economy. This approach is contrasted with waiting for real data to be collected and published at a slower pace, thereby losing intra-quarter information.

More precisely, we illustrate a behavioral mixed frequency macro-finance model where both real and financial variables are generated on daily basis. Further, while financial sector data is collected at the same frequency as it is generated (i.e. daily), real data can only be collected on a quarterly basis. This situation, in which a variable's evolution through time cannot be observed at all dates it is generated, is known as *temporal aggregation*.

Marcet (1991) explains that temporal aggregation frequently occurs in economics because collecting high-frequency data on certain variables is often prohibitively expensive. However, there is no reason to assume that economic time series are collected at a frequency sufficient to fully capture the movements of the economy. For example, while we typically only have quarterly observations on the gross national product (GNP), it is reasonable to believe that the behaviour of the GNP within a quarter carries relevant information about the structure of the economy, even if this behaviour is unobservable.

The problem of (temporal) aggregation is not only relevant to economic questions and applications, but extends to various other fields, including political science (Shellman 2004), health studies (Lindo 2015; Phelps et al. 2018), and (discrete) choice modelling (Basu and Sullivan 2017; Wong, Brownstone, and Bunch 2019).

The differentiation between the frequency at which agents make their economic decisions and the frequency at which the information relevant to them becomes available is by no means trivial, as the latter may condition the former. For instance, as we will discuss below, if the policy interest rate set by the monetary authorities is specified as a function of the real gross domestic product (GDP) and price inflation, and the realizations of these two variables become observable on a quarterly basis, the policy rate will adopt that quarterly frequency too, even though in principle it could be reset on any given day. Moreover, the greater the discrepancy between the data generating process (DGP) and the data collection process (DCP), the more significant nowcast and forecast biases will become.

In models where agents are boundedly rational and backward-looking, like ours, the discrepancy between the DGP and the DCP is particularly significant. This is due to the dual nature of information and cognitive constraints: on the one hand, these agents must form expectations based on historically available data rather than a model-consistent approach; on the other hand, this data is available to them with a delay due to temporal aggregation. Our paper is closely related to literature where decisions (expectations) are made under cognitive constraints and rely on simple discrete choice models in environments with issues such as data availability, collection delays, or aggregation. Notable examples include the discrete choice approach to environmental and energy decision-making (Grilli and Ferrini 2022) and graph learning (Tomlinson and Benson 2024).

The literature most relevant to our paper includes Kontonikas and Ioannidis (2005), Kontonikas and Montagnoli (2006), Bask (2012), Westerhoff (2012), Naimzada and Pireddu (2013), Lengnick and Wohltmann (2013, 2016), Franke and Sacht (2014), and most recently, Proaño and Kotb (2024). Except for the latter, none of these authors, however, models the discrepancy of the DGP and the DCP explicitly. To address this, we modify the theoretical model by Lengnick and Wohltmann (2016) to explicitly differentiate between the DGP and DCP, and study the implications for policy making. In contrast to Proaño and Kotb (2024), who concentrate on analysing real shocks, our focus lies on financial shocks. We investigate how these shocks propagate to the real economy within a framework where real data are temporally aggregated, rendering real variables alone unsuitable for the purpose of high-frequency policy design.

The remainder of this paper is organized as follows. Section 2 describes our mixed-frequency behavioural macroeconomic model. Section 3 provides the results of the simulation and the associated analysis. Section 4 concludes.

2 The Theoretical Framework

In the following, we set up a behavioural macroeconomic model where all economic activities, both in the financial and the real sectors, take place on a daily basis. Further, we assume that real macroeconomic variables are observable on a quarterly basis given the data collection costs associated with variables such as the GDP and its components in the real world. As we will discuss later, this assumption has important implications since it affects the agents' information sets, and thus their forecasts and economic decisions.

To be as clear as possible in our exposition, we use distinct indices that resemble the different frequencies at which economic decisions are made as well as the

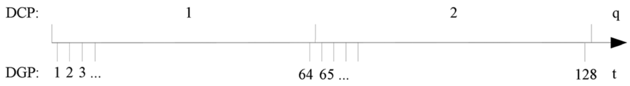

Figure 1: Time scale as indexed by trading days (t) and quarters (q).

frequencies at which data is collected. The index t refers to the daily frequency in our framework. We denote by q the quarterly time index. Further, let T^q denote the number of "trading days", i.e. days with economic activity, in a quarter. Following Lengnick and Wohltmann (2013, 2016), we assume that there is no economic activity (stock market trading, goods production, etc.) on the weekends, so that $T^q = 64 \approx 3 \cdot 30 \cdot \frac{5}{7}$ days per quarter. This implies that a quarter, q, is defined to contain the days $64(q-1)+1, \ldots, 64q$. This is illustrated in Figure 1. The upper part of the figure depicts the evolution of the data collection process, which occurs at a quarterly frequency. The lower part shows the evolution of the data generating process, which occurs at a daily frequency.

2.1 The Real Sector

To facilitate a better understanding of our high-frequency modelling approach and the role of different information sets at different frequencies, we describe first the quarterly law of motions of the Lengnick and Wohltmann (2016) model. Then we derive the corresponding formulations at the daily frequency following Franke and Sacht (2014).

The IS equation derived in Lengnick and Wohltmann (2016) reads

$$x_q = \tilde{E}_q[x_{q+1}] - \frac{1}{\sigma}\left[i_q - \tilde{E}_q[\pi_{q+1}]\right] + c_1\tilde{E}_q[\Delta s_{q+1} - \pi_{q+1}] \tag{1}$$

where x_q represents the output gap, i_q the policy nominal interest rate, s_q the average stock price, σ the risk aversion coefficient, and $\tilde{E}_q[x_{q+1}]$ and $\tilde{E}_q[\pi_{q+1}]$ the agents' aggregate output gap and inflation expectations at quarter q concerning the next quarter $q+1$, respectively. The tilde denotes that expectations are formed in a boundedly rational way in contrast to the rational expectations operator $E_t[\cdot]$.

The new Keynesian Phillips curve (NKPC) reads

$$\pi_q = \beta\tilde{E}_q[\pi_{q+1}] + \gamma x_q - \kappa s_q \tag{2}$$

where $\beta = 1/(1+\bar{r})$, where $\bar{r}$ is the steady state real interest rate in quarterly terms, and $\gamma = \frac{(1-\theta)(1-\beta\theta)(\sigma+\eta)}{\theta}$, where η is the inverse Frisch elasticity of labour supply and θ represents the degree of price stickiness (measured at the quarterly frequency).[1]

As in Franke and Sacht (2014), we denote now with h the time interval of a frequency of interest *relative to the quarterly time interval* (with $0 < h \leq 1$ for time intervals of a higher-than-quarterly frequency). In case of the daily frequency we have $h \equiv 1/T^q$, such that the daily IS-equation is given by

$$x_t \;=\; \tilde{E}_t[x_{t+1}] - \frac{h}{\sigma}\left[i_t - \tilde{E}_t[\pi_{t+1}]\right] + c_1\tilde{E}_t[\Delta s_{t+1} - h\pi_{t+1}] \tag{3}$$

where x_t represents the output gap at day t, i_t the nominal interest rate in quarterly terms, which may be reset at any day t within a quarter, s_t the stock price (observable at day t), and $\tilde{E}_t[\Delta s_{t+1}]$ the average expectation of the stock price change between t and $t+1$.

Note that, following Franke and Sacht (2014), and to maintain consistency in our definitions, the flow variables are uniformly expressed as ‘quarterized’ magnitudes. Accordingly, a nominal interest rate i_t of value a per quarter means that, in an h-economy, the interest is ha over the period $(t, t+h)$. The same applies for the inflation rate: price inflation from period t to period $t+h$ is equal to the product of the quraterized inflation in period $t+h$ (π_{t+h}) and h. Furthermore, the high frequency representation of the model requires the adjustment of preference parameters with a time dimension. Accordingly, the discount factor β becomes $\beta^d = \frac{1}{1+h\bar{r}}$, the Calvo parameter $\theta^d = 1 - h\cdot(1-\theta)$,[2] and $\gamma^d = \frac{(\sigma+\eta)(1-\theta^d)(1-\beta^d\theta^d)}{\theta^d}$. These considerations imply for the dynamics of the quarterized daily inflation rate.

$$h\pi_t = \beta^d h\tilde{E}_t[\pi_{t+1}] + \gamma^d x_t - \kappa s_t. \tag{4}$$

2.2 The Data Collection Process

As previously mentioned, in our model, while financial variables, such as stock prices, are observable daily, data on real sector variables such as the aggregate output gap and aggregate price inflation can only feasibly be gathered on a quarterly basis.

1 The value of γ is obtained by combining the first order condition of the firm problem under Calvo price mechanism, and the households’ first order condition. For more clarification, consult Franke and Sacht (2014).

2 For instance, if $\theta = 0.55$, it means that 45 % of firms reset their prices each quarter. Calculating for daily resets, $\theta^d = 1 - \frac{1}{64}\cdot(1-0.55) \approx 0.993$. Thus, about 0.7 % of firms reset their prices daily, as $1 - 0.993 \approx 0.007$.

In order to model this non-trivial issue in a stylized manner, we assume that the daily values of real variables are unobservable until the current quarter has ended. Once the quarter is complete, all daily values of the real variables from that period become available to the statistical office. The office then collects these values to compute a quarterly aggregate, which is subsequently made available to the public.

$$\text{DCP}:\ x_q := \frac{1}{T^q} \sum_{t=T^q(q-1)+1}^{T^q q} x_t \tag{5}$$

$$\text{DCP}:\ \pi_q := \frac{1}{T^q} \sum_{t=T^q(q-1)+1}^{T^q q} \pi_t. \tag{6}$$

2.3 Expectations of the Real Variables

Differentiating the data availability of the model variables significantly impacts the information sets available to agents and the expectations they form about future variables based on these information sets. This differentiation is particularly crucial in a framework where agents are assumed to be boundedly rational and where heterogeneity may play a significant role. In the following, we use rules of thumb employed by Lengnick and Wohltmann (2016), adjusting them however to the DCP discussed above, i.e.

$$\text{Targeting expectations}: \underset{q}{\overset{tar}{\tilde{E}}}\, y_{q+1} = \bar{y} \qquad y \in \{x, \pi\} \tag{7}$$

$$\text{Static expectations}: \underset{q}{\overset{sta}{\tilde{E}}}\, y_{q+1} = y_{q-1} \tag{8}$$

$$\text{Extrapolating expectations}: \underset{q}{\overset{ext}{\tilde{E}}}\, y_{q+1} = y_{q-1} + \alpha_y \left(y_{q-1} - y_{q-2}\right) \tag{9}$$

with $\alpha_y > 0$. Agents using the first expectational rule-of-thumb (i.e. targeters) simply assume that the variables of interest (x and π) will be at their explicitly announced *targets* of the central bank ($\bar{x}$ and $\bar{\pi}$, respectively). Following, De Grauwe and Macchiarelli (2015) and Lengnick and Wohltmann (2016), we normalise these to zero. Agents using the second heuristic (i.e. static) simply assume that the variables of interest will not change in the next quarter. Finally, agents using the third rule (i.e. extrapolators) add to the most recent quarterly value a *momentum* term. Note that, while the expectations of the targeters remain constant over time, static and extrapolative expectations change only once per quarter when new data is published.

The fractions of agents $\omega_q^{y,j}$ for the three heuristics $j \in \{\text{tar, sta, ext}\}$ are determined by a discrete choice approach with the intensity of choice parameter ϕ.

$$\omega_q^{y,j} = \frac{\exp\left(\phi A_q^{y,j}\right)}{\exp\left(\phi A_q^{y,\text{tar}}\right) + \exp\left(\phi A_q^{y,\text{sta}}\right) + \exp\left(\phi A_q^{y,\text{ext}}\right)} \qquad y \in \{x, \pi\}. \tag{10}$$

The attractivity $A_q^{y,j}$ of a heuristic j is defined as a geometric sum of past squared expectation errors (c.f. De Grauwe (2010, 2011)).

$$A_q^{y,j} = -\left(y_{q-1} - \underset{q-2}{\overset{j}{\tilde{\mathrm{E}}}}[y_{q-1}]\right)^2 - \zeta A_{q-1}^{y,j} \qquad y \in \{x, \pi,\} \tag{11}$$

where the memory coefficient $0 \le \zeta < 1$ determines the speed of agents' forgetting about the past.

Market expectations are then given by a weighted average of the three heuristics.

$$\tilde{E}_q y_{q+1} = \omega_q^{y,tar} \underset{q}{\overset{y,tar}{\tilde{E}}} y_{q+1} + \omega_q^{y,sta} \underset{q}{\overset{y,sta}{\tilde{E}}} y_{q+1} + \omega_q^{y,ext} \underset{q}{\overset{y,ext}{\tilde{E}}} y_{q+1}. \tag{12}$$

Note that the settings explained above capture the notion that, while the real variables themselves are generated daily, the quarterly nature of the collection and publication process leads to expectations, weights and attractivity values that follow a quarterly frequency (i.e. change only once per quarter). Hence, the subscript q in equations (7)–(12).

Figure 2 illustrates the processes of data generation and collection for the real variables. The solid black line represents the true generating process of the variable, which is only observed at specific time points (e.g. the last day of each quarter when data is collected). Various agents employ different rules of thumb to interpret this observed data and form expectations about the future.

2.4 The Financial Sector

As in Westerhoff (2008) and Lengnick and Wohltmann (2016), our theoretical framework depicts the financial sector with two types of traders: chartists and fundamentalists. These traders formulate their expectations regarding future stock prices based on the following rules.

$$\underset{t}{\overset{C}{\tilde{E}}}[s_{t+1}] = s_{t-1} + k^c [s_{t-1} - s_{t-2}] \tag{13}$$

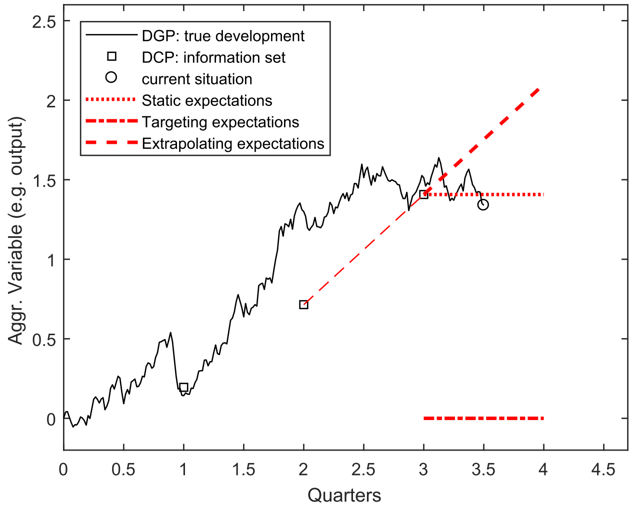

Figure 2: Illustration of boundedly rational expectations in a model with daily DGP but quarterly DCP.

$$\tilde{E}_t^F [s_{t+1}] = s_{t-1} + k^f \left[s_{t-1}^f - s_{t-1} \right] \tag{14}$$

where k^c and k^f are both positive and s_t^f represents the agents' perception about the "fundamental" stock price. Since the true fundamental stock price cannot be known with certainty, agents have to form beliefs about it.[3] Following earlier work,[4] we model the agents' perception to be positively correlated with real economic conditions

$$s_t^f = g\, x_{q-1} \qquad \text{with} \quad g \geq 0. \tag{15}$$

The excess stock demand functions are given by

$$D_t^i = \ell \left(\tilde{E}_t^i [s_{t+1}] - s_t \right) \qquad i \in \{C, F\}. \tag{16}$$

We also allow for a third excess demand function $D_t^{NT} = 0$ which prescribes a "no-trading" position in that period.

Traders choose between these three different rules according to their respective attractivity which is determined as a function of past profits.

3 See also De Grauwe and Kaltwasser (2012) and Bernanke and Gertler (2000).

4 Consult Lengnick and Wohltmann (2013), Section 2.3 for more details.

$$A_t^i = (\exp(s_{t-1}) - \exp(s_{t-2}))D_{t-2}^i + dA_{t-1}^i \qquad i \in \{C, F\} \tag{17}$$

$$A_t^{NT} = 0 \tag{18}$$

with the corresponding relative weights being determined by

$$\omega_t^i = \frac{\exp\left(e A_t^i\right)}{\exp\left(e A_t^C\right) + \exp\left(e A_t^F\right) + \exp\left(e A_t^{NT}\right)} \qquad i \in \{C, F, NT\} \tag{19}$$

where e represents the intensity of choice among the different strategies.

Market expectations of the stock price are a weighted average of chartists' and fundamentalists' expectations.

$$\tilde{E}_t\, s_{t+1} = \omega_t^C \tilde{E}_t^C\, s_{t+1} + \omega_t^F \tilde{E}_t^F\, s_{t+1}. \tag{20}$$

Following Lengnick and Wohltmann (2016), the non-speculative households' demand for stocks is given by[5]

$$d_t = x_{q-1} - c_2\,(s_{t-1} - p_{q-1}) - c_3\, i_t. \tag{21}$$

where p_q is the goods price in a quarter q. Finally, the actual stock price is determined by the following price impact function (see Westerhoff 2008)

$$s_t = s_{t-1} + a\left(\omega_t^C D_t^C + \omega_t^F D_t^F + k\,(d_t - d_{t-1})\right) + \epsilon_t^s \tag{22}$$

that relates stock price changes positively to excess demand, where ϵ_t^s is an AR(1) process in the form $\epsilon_t^s = \rho^s \epsilon_{t-1}^s + v_t$, where v_t is a W.N shock with mean zero and standard error σ^s.

2.5 The Monetary Policy

As previously discussed, the monetary authorities can only observe quarterly values of the aggregates x_q and π_q, but not their daily values. Accordingly, they react with a delay and only imperfectly to real sector developments, namely through their "quarterly lenses".

The monetary authorities can try to circumvent this problem by reacting to financial market data which is available at a more frequent basis than macroeconomic data. If the policy rate is set to react linearly to stock price deviations from its fundamental value, the interest rule would be

5 Note that equation (21) could easily be adjusted to also include the net demand of other agents, for example, direct interventions by the central bank.

$$i_t = \delta_\pi\left(\pi_{q-1} - \pi^*\right) + \delta_x x_{q-1} + \delta_s\left(s_t - s_t^f\right). \tag{23}$$

3 Model Simulation and Analysis

In this section, we numerically analyze the monetary policy implications of the mixed frequency setup. Our examination is specifically directed towards stock price shocks, narrowing the focus on the spillover effects from the financial sector to the real sector within the context of the aforementioned mixed frequency model. To facilitate this analysis, we commence with a brief overview of the calibration process.

3.1 Calibration

Our calibration is, besides a few exceptions, fairly in line with Lengnick and Wohltmann (2016). The calibration of the real sector is summarized in Table 1. We set the inverse Frisch elasticity of labor supply, η, to unity. In line with the estimation results of Smets and Wouters (2007), we choose an intertemporal rate of substitution, σ, of 1.4. A steady state real interest rate, $\bar{r}$, of 0.01 implies a quarterly discount factor, β, of 0.99. Following Coibion and Gorodnichenko (2011), we set the quarterly degree of price stickiness, θ, to 0.55. We adopt the same calibration as Lengnick and Wohltmann (2016) for the learning and noise parameters with an intensity of choice of 10, a memory coefficient of 0.5 and extrapolation parameters of 0.2.

Table 1: Calibration of the real sector.

Variable	Value
Structure	
Risk aversion coefficient	$\sigma = 1.4$
Inverse Frisch elasticity of labor supply	$\eta = 1$
Steady state real interest rate	$\bar{r} = 0.01$
Calvo pricing rigidity (quarterly)	$\theta = 0.55$
Bounded Rationality, Learning	
Extrapolation parameter in heuristics	$\alpha_x = \alpha_\pi = 0.2$
Memory coefficient	$\zeta = 0.5$
Intensity of choice	$\phi = 10$

Table 2: Calibration of the financial sector.

Variable	Value
Structure	
Excess demand impact on stock price	$a = 1$
Expected stock price growth impact on demand	$\ell = 1$
Bounded Rationality, Learning	
Memory coefficient	$d = 0.95$
Rationality of choice	$e = 30$
Chartists' degree of extrapolation	$k^c = 0.04$
Fundamentalists' perceived stock price reversal speed	$k^f = 0.04$
Noise	
Stock price shock standard deviation	$\sigma^s = 0.01$

We summarize the financial sector calibration in Table 2. Here, we only deviate from Lengnick and Wohltmann (2016) by choosing a slightly lower memory coefficient, d, of 0.95 and a more moderate rationality of choice parameter, e, of 30.[6]

Finally, we state the macro-finance interaction parameters in Table 3. These parameters link the two sectors and, therefore, lie at the heart of our setup. We keep the calibration of the household stock demand equation ($c_2 = 0.5$ and $c_3 = 1$) and the interaction parameters in the financial sector, $g = 0.5$ and $k = 0.2$ of Lengnick and Wohltmann (2016). However, we adjust the parameters measuring the impact of stock price in PC and IS equation, κ and c_1, to $1e-3$ and 0.9, respectively.

3.2 Instability and Responding to the Stock Price

Figures 3 and 4 illustrate the variance of output, inflation, and stock prices at different values of the policy parameters δ_x and δ_s, as well as δ_π and δ_s, respectively. These results are derived from a Monte Carlo experiment with 500 runs. Upon closer examination of the figures, it becomes evident that the value of δ_x as well as that of δ_π have minimal relevance to the stability of real or financial variables. This outcome is not surprising considering that the delayed and aggregated availability of data on

6 The mixed-frequency considerations in our model generate an even higher degree of complexity than in the original model. For a large set of parameter combinations we observe explosive behaviour. By taming the financial sector fluctuations in this way, we obtain a more stable system. A stability analysis of our model is not the purpose of this paper and, thus, left for further research.

Table 3: Calibration of the macro-finance interaction.

Variable	Value
Real sector	
Stock price cost effect in Phillips curve	$\kappa = 0.001$
Stock price effect in IS equation	$c_1 = 0.9$
Household stock demand	
Sensitivity to real stock price	$c_2 = 0.5$
Sensitivity to nominal interest rate	$c_3 = 1$
Financial sector	
Bias in perceived fundamental stock price to output	$g = 0.5$
Impact of household stock demand on excess demand	$k = 0.2$
Monetary Policy	
Respond to output	$\delta_x = 0.5$
Respond to inflation	$\delta_\pi = 1.5$
Respond to stock price	$\delta_s = 0.2$

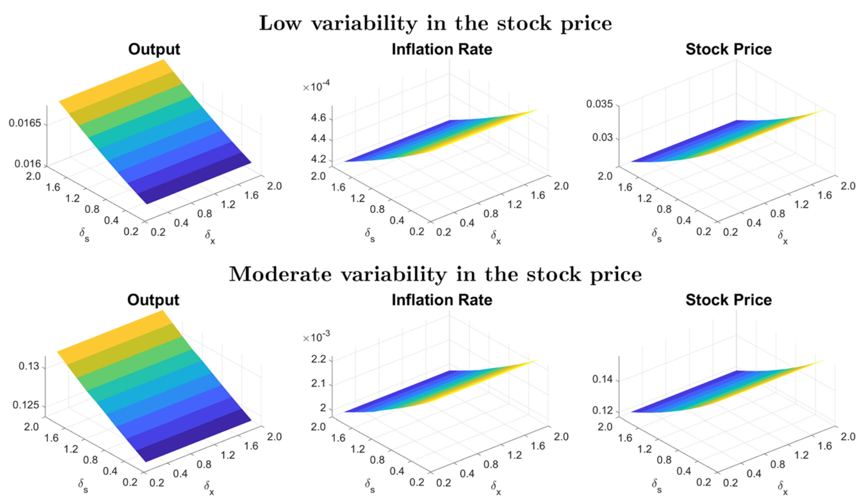

Figure 3: A Monte Carlo experiment of 500 runs, each for 400 quarters (25,600 days). Illustrated are the variances of output, inflation and the stock price at different values for the policy parameters δ_x and δ_s. First row: $\sigma_s = 0.01$. Second row: $\sigma_s = 0.02$.

output and inflation renders them unsuitable as the sole information guide for high-frequency policy-making purposes.

On the contrary, the value of δ_s holds significant importance for the stability of both the real and financial sectors. Regardless of the magnitude of the stock price

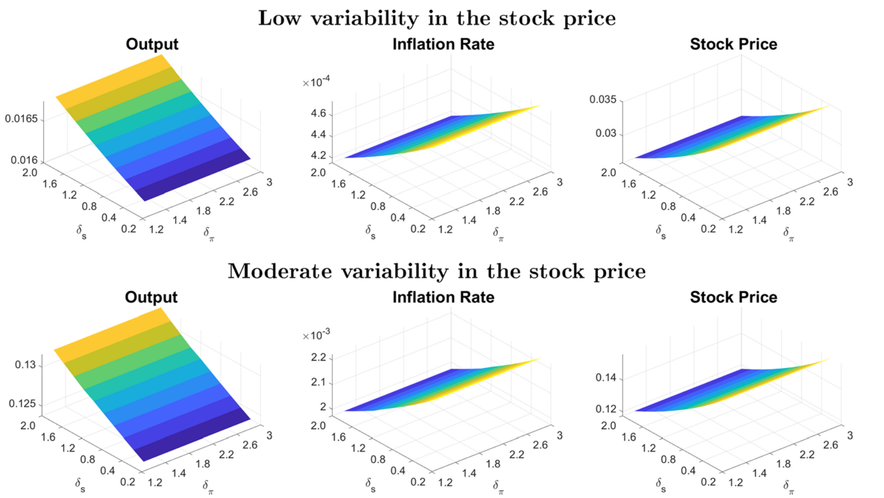

Figure 4: A Monte Carlo experiment of 500 runs, each for 400 quarters (25,600 days). Illustrated are the variances of output, inflation and the stock price at different values for the policy parameters δ_π and δ_s. First row: $\sigma_s = 0.01$. Second row: $\sigma_s = 0.02$.

shocks, a heightened response to stock price updates contributes to greater stability (lower variance) in the stock price and inflation rate. However, drawing conclusions for output is more nuanced. For high values of the stock price shock standard error, increasing the strength of policy response to the stock price may stabilize output.[7] Yet, at moderate values of the standard errors, the instability introduced by the aggressive movements of the policy rate may outweigh the stability benefits derived from stabilizing the stock price and inflation.[8]

These results can be explained by closely examining the output gap and inflation equations, specifically equations (3) and (4). As we can see in equation (4), the inflation rate is directly affected by the stock price value s_t. A policy rate that reacts strongly and contemporaneously to stock price updates, minimizes the switching behaviour in the stock market and stabilizes the stock price following a financial shock. This leads to a more stable inflation rate, as per equation (4).

In equation (3), it is evident that the output gap is directly influenced by the expected change in the stock price, $\Delta E_t s_{t+1}$, rather than the contemporaneous stock

7 The impact of leaning on output will be further explained in additional figures.

8 In fact, as will be demonstrated later, increasing the value of δ_s to as high as 20 or 30 continues to provide stability benefits for the stock price and inflation rate, but can lead to instability in output.

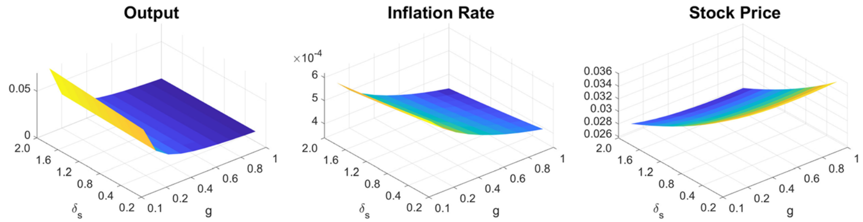

Figure 5: A Monte Carlo experiment of 500 runs, each for 400 quarters (25,600 days). Illustrated are the variances of output, inflation and the stock price at different values for the policy parameter, δ_s, and the parameter g.

price value. By combining equations (13) through (15) with equation (20), we can compute $\Delta E_t s_{t+1}$ as follows.

$$\Delta E_t s_{t+1} = \left(\omega_t^C + k^c \omega_t^C - k^f \omega_t^F + \omega_t^F\right) s_{t-1} - k^c \omega_t^C s_{t-2} - s_t + k^f \omega_t^F g x_{q-1}. \tag{24}$$

This expression is a smoother function of the stock price compared to its level, s_t. Conversely, x_t is directly influenced by the policy rate, i_t. Therefore, the stability gains from targeting the stock price level are only justified when stock market variation is extremely high.

Figure 5 illustrates the variances of output, inflation and stock price at different combinations of the policy parameter δ_s and the parameter g, which determines the value perceived by the fundamentalists as the fundamental value for the stock price (see equation (15) and Table 3). The model variables exhibit less variation when fundamentalists' perception of stock price value adheres more closely to real output, and when monetary policymakers react to stock price updates.

To further illustrate the stability benefits and costs of the policy parameter δ_s, Figure 6 shows the simulation dynamics of output, inflation, the stock price, the stock price shock, and the interest rate as they are generated on a daily basis, and the first two as they are collected on a quarterly basis. The weights of extrapolators in inflation and output gap expectations, as well as chartists in stock price expectations, are also shown, providing evidence of the intensity of switching in the model. Large values of the policy parameter δ_s are used to clearly observe its effects on the model's dynamics.

The figure shows that a high reaction to stock price deviations leads to a highly stable stock price and a minimized switching behaviour. This stability in switching behaviour is also evident in inflation expectations, resulting in a more stable inflation rate.

By carefully comparing the red dotted line, which represents a high positive reaction to the stock price, with the blue solid line, which represents a no-leaning

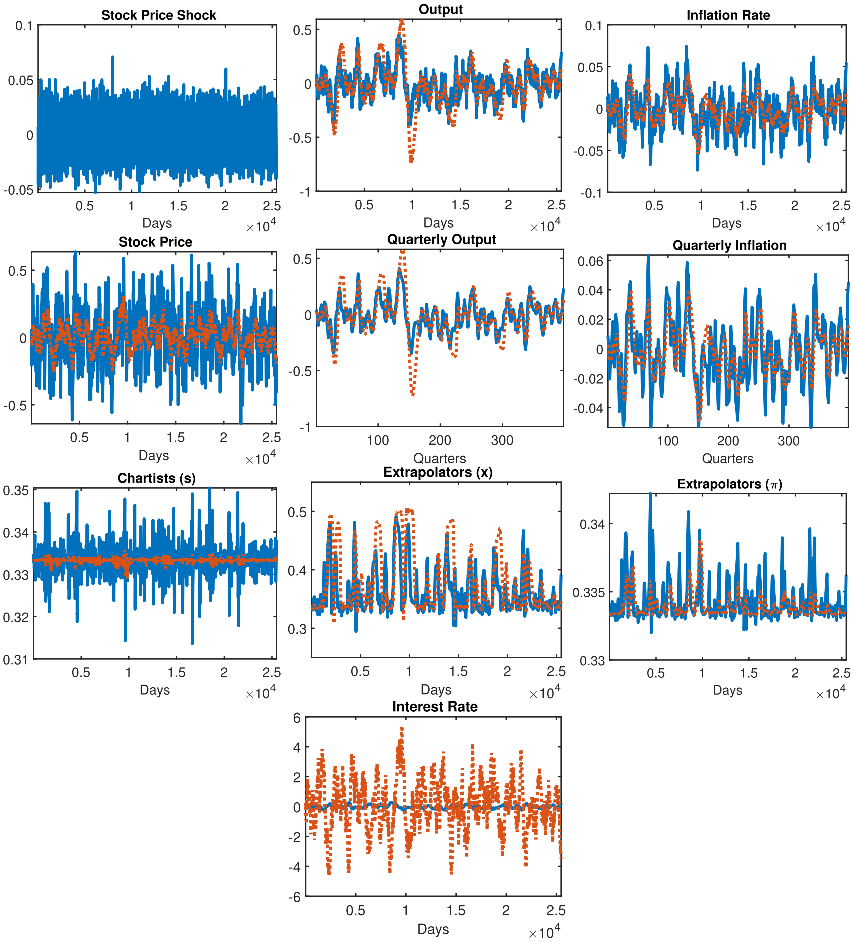

Figure 6: A simulation for 400 quarters (25,600 days). Blue solid line: $\delta_s = 0$. Red dotted line: $\delta_s = 18$. $\sigma_s = 0.01$.

policy, we can see that the leaning policy minimizes the sharp variations caused by intense switching behaviour. This leads to smoother business cycles of booms and busts for the stock price and inflation. However, this comes at the cost of a slightly more unstable output gap and an extremely variable interest rate.

Figure 7 repeats the exercise with a higher value for the stock price shock standard deviation. The results are even more pronounced. A leaning monetary policy minimizes the switching behaviour in the stock market and inflation expectations, leading to smoother cycles in both inflation and the stock price. The output

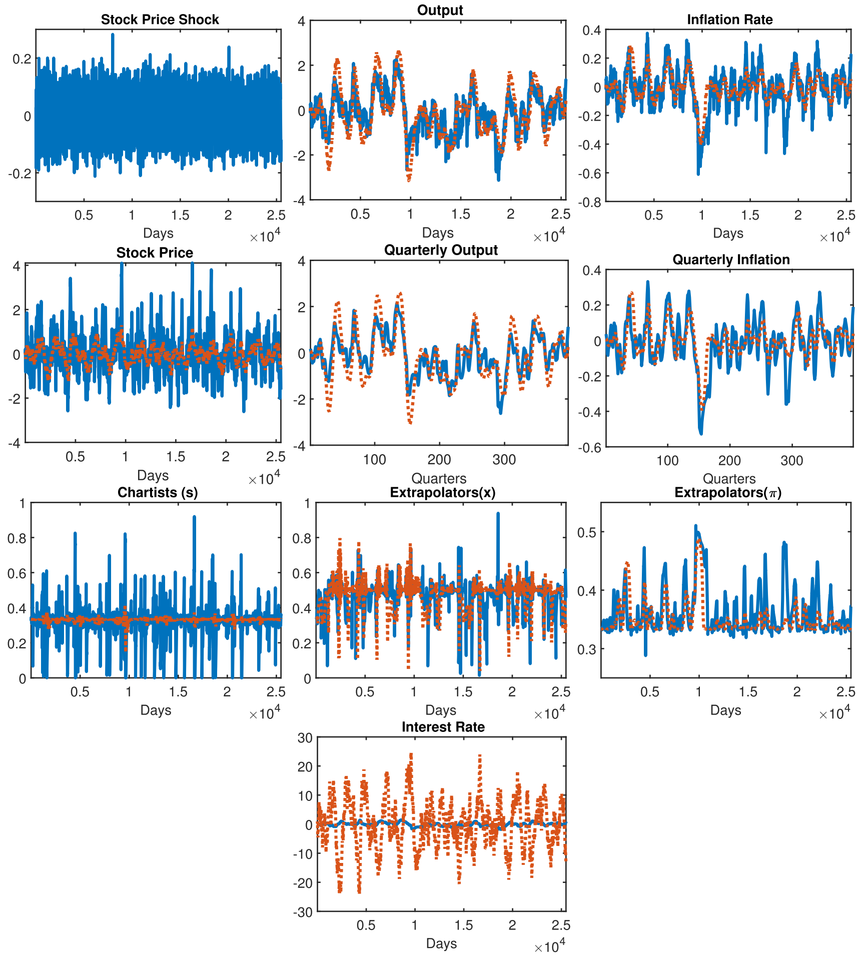

Figure 7: A simulation for 400 quarters (25,600 days). Blue solid line: $\delta_s = 0$. Red dotted line: $\delta_s = 20$. $\sigma_s = 0.04$.

cycles are also slightly more smoothed. However, the interest rate *absorbs* all of this *removed* instability.

Comparable findings can be derived by examining the impulse responses of the model variables following a one standard deviation stock price shock.[9] Figure 8 shows the results of a positive shock. Figure 9 shows the results of a negative shock. In the first case (second case), the inflation rate reacts slightly negative (positive) as per

9 Appendix A describes how the impulse response functions are computed.

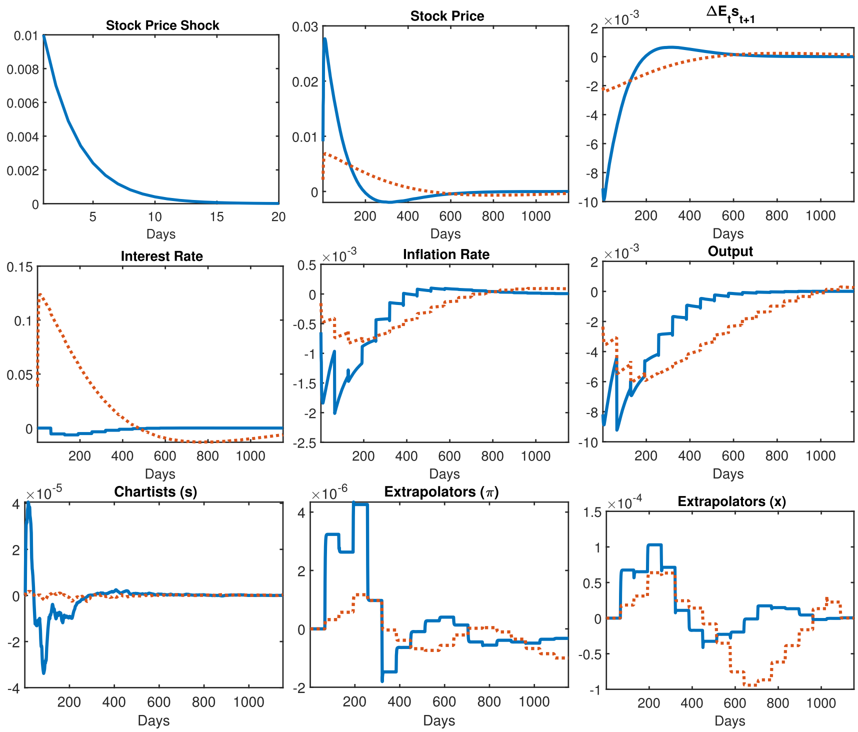

Figure 8: Impulse responses to a positive one standard deviation stock price shock under δ_s = 0 (blue solid line) and δ_s = 18 (red dotted line).

equation (4). When the policy rate is leaning, the stock price is instantaneously stabilized, and the switching behaviour is minimized. The balance between fundamental and chartist expectations results in expected stock prices that are only marginally negative (positive). Consequently, output also responds slightly negatively (positively) as described in equation (3).

It is noteworthy to mention that the stepped form pattern of the inflation rate and output (blue solid lines) arises from the fact that the most significant changes to these variables occur at the end of each quarter when data is collected and expectations are updated. When the policy rate leans (red dotted lines), this stepped form pattern becomes smoother due to the daily frequency of interest rate impacts on the movement of these variables.

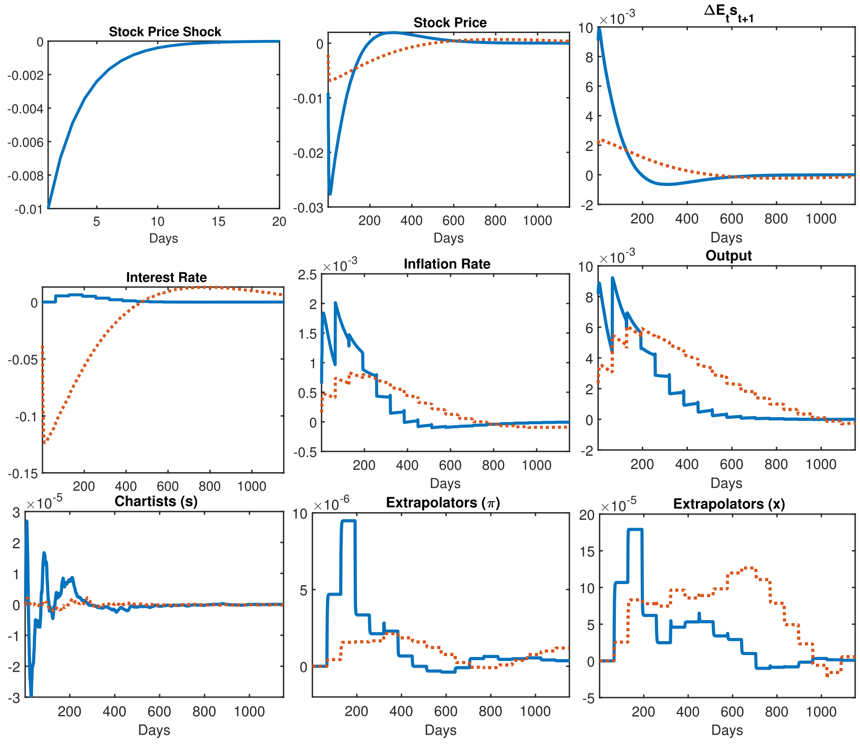

Figure 9: Impulse responses to a negative one standard deviation stock price shock under $\delta_s = 0$ (blue solid line) and $\delta_s = 18$ (red dotted line).

4 Concluding Remarks

"From the point of view of macroeconomic stability, and particularly of equilibrium determinacy, the conventional wisdom appears to be that an explicit response to stock prices is a bad idea" (Airaudo, Nisticó, and Zanna 2015, p. 1274). Airaudo, Nisticó, and Zanna (2015) showed that a policy response to the stock price could be indeed stabilizing, when the real sector and the financial sector are structurally related. We show that such a policy response can be even more justified, when real data is temporally aggregated. Under these circumstances, output and inflation, upon which data is available with a significant delay, become unsuitable as the sole information guide for monetary policy. The policy makers can extract the missing information by considering leaning against the stock price.

We demonstrate that a positive response to stock prices can enhance stability in both the real and financial sectors when a structural relationship exists between them. However, this benefit comes with the potential drawback of an unstable and highly variable policy rate. Additionally, under the assumption of bounded rationality, agents switching between different heuristics leads to nonlinear reactions to shocks and more pronounced dynamic spikes, especially as the switching intensifies. Monetary policy can mitigate this effect by reacting contemporaneously to stock price updates, leading to more stable stock prices and a minimized switching behaviour, which in turn results in smoother real and financial cycles. These benefits must be weighed against the costs of variation in the policy rate.

Our model is a stylized attempt to capture the information gains of leaning against the wind in an environment of temporal aggregation and boundedly rational behaviour. Further research can provide an econometric basis for this analysis, analyse real shocks, and study models under different real-financial spillover channels than those examined in our model.

Appendix A: Impulse Response Analysis

To calculate impulse response functions, we follow the steps of the experiment discussed in Lengnick and Wohltmann (2013). These steps are described as follows:

1. Generate model dynamics for one particular random seed.
2. Generate the dynamics again with the same random seed, but with ϵ^s_{129} increased (decreased) by 0.01. In other words, at time $t = 129$, which is the first day of the third quarter, the value of the stock price shock is higher (lower) than the same shock at the same time in the previous step with an amount 0.01.
3. Calculate the difference between the trajectories of steps 1 and 2 which gives the isolated impact of the additional shock.
4. Repeat steps 1–3 for 500 times.

References

Airaudo, M., S. Nisticó, and L.-f. Zanna. 2015. “Learning, Monetary Policy, and Asset Prices.” *Journal of Money, Credit and Banking* 47 (7): 1273–307.

Bask, M. 2012. “Asset Price Misalignments and Monetary Policy.” *International Journal of Finance & Economics* 540. https://doi.org/10.1002/ijfe.450.

Basu, A., and S. D. Sullivan. 2017. “Toward a Hedonic Value Framework in Health Care.” *Value in Health* 20 (2): 261–5.

Bernanke, B., and M. Gertler. 2000. *Monetary Policy and Asset Price Volatility*. Working Paper 7559. National Bureau of Economic Research. https://doi.org/10.3386/w7559

Coibion, O., and Y. Gorodnichenko. 2011. “Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation.” *American Economic Review* 101 (1): 341–70.

De Grauwe, P. 2010. "The Scientific Foundation of Dynamic Stochastic General Equilibrium (DSGE) Models." *Public Choice* 144 (3–4): 413–43.

De Grauwe, P. 2011. "Animal Spirits and Monetary Policy." *Economic Theory* 47 (2–3): 423–57.

De Grauwe, P., and P. Kaltwasser. 2012. "Animal Spirits in the Foreign Exchange Market." *Journal of Economic Dynamics and Control* 36: 1176–92.

De Grauwe, P., and C. Macchiarelli. 2015. "Animal Spirits and Credit Cycles." *Journal of Economic Dynamics and Control* 59 (C): 95–117.

Franke, R., and S. Sacht. 2014. "Some Observations on the High-Frequency Versions of a Standard New-Keynesian Model." *Bulletin of Economic Research* 66 (1): 72–94.

Grilli, G., and S. Ferrini. 2022. "Discrete Choice Modeling in Environmental and Energy Decision-Making: An Introduction to the Special Issue." *Journal of Environmental Planning and Management* 65 (7): 1203–9.

Kontonikas, A., and C. Ioannidis. 2005. "Should Monetary Policy Respond to Asset Price Misalignments?" *Economic Modelling* 22: 1105–21.

Kontonikas, A., and A. Montagnoli. 2006. "Optimal Monetary Policy and Asset Price Misalignments." *Scottish Journal of Political Economy* 53: 636–54.

Lengnick, M., and H.-W. Wohltmann. 2013. "Agent-based Financial Markets and New Keynesian Macroeconomics: A Synthesis." *Journal of Economic Interaction and Coordination* 8: 1–32.

Lengnick, M., and H.-W. Wohltmann. 2016. "Optimal Monetary Policy in a New Keynesian Model with Animal Spirits and Financial Markets." *Journal of Economic Dynamics and Control* 64: 148–65.

Lindo, J. 2015. "Aggregation and the Estimated Effects of Economic Conditions on Health." *Journal of Health Economics* 40 (C): 83–96.

Marcet, A. 1991. "Temporal Aggregation of Economic Time Series." In *Rational Expectations Econometrics*, edited by L. P. Hansen, and T. J. Sargent, 237–81. San Francisco: Westview Press.

Naimzada, A. K., and M. Pireddu. 2013. "Dynamic Behavior of Real and Stock Markets with Varying Degree of Interaction." *DEMS Working Paper* 245: 17.

Phelps, C., D. Lakdawalla, A. Basu, M. Drummond, A. Towse, and P. Danzon. 2018. "Approaches to Aggregation and Decision Making – A Health Economics Approach: An ISPOR Special Task Force Report [5]." *Value in Health* 21: 146–54.

Proaño, C., and N. Kotb. 2024. *Monetary Policy, Stock Prices and Temporal Aggregation in a New Keynesian Model with Behavioural Expectations*. University of Bamberg. unpublished manuscript.

Shellman, S. M. 2004. "Time Series Intervals and Statistical Inference: The Effects of Temporal Aggregation on Event Data Analysis." *Political Analysis* 12 (1): 97–104.

Smets, F., and R. Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *The American Economic Review* 97 (3): 586–606.

Tomlinson, K., and A. R. Benson. 2024. "Graph-Based Methods for Discrete Choice." *Network Science* 12 (1): 21–40.

Westerhoff, F. 2008. "The Use of Agent-Based Financial Market Models to Test the Effectiveness of Regulatory Policies." *Jahrbücher für Nationalökonomie und Statistik* 228: 195–227.

Westerhoff, F. 2012. "Interactions between the Real Economy and the Stock Market: A Simple Agent-Based Approach." *Discrete Dynamics in Nature and Society* 2012: 1–21.

Wong, T., D. Brownstone, and D. S. Bunch. 2019. "Aggregation Biases in Discrete Choice Models." *Journal of Choice Modelling* 31: 210–21.