10.20378/irb-59552

Mathematical Proof Assistants for Teaching Logic: The LogiKEy Methodology

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We understand logic education as an interdisciplinary challenge, which we want to tackle with the support of modern computer technologies. Thus, we have been researching and experimenting with an uniform approach towards teaching logic at undergraduate level for mixed groups of computer science, mathematics and philosophy students, with the support of automated reasoning tools.

Moreover, motivated by applications in artificial intelligence, knowledge representation and natural language semantics, we have realized that classical logic alone is not sufficient. As a consequence, we have focused from the very beginning in teaching modal and non-classical logics. During the last years we have started to rely in the logico-pluralistic LogiKEy methodology [2], which explicitly harnesses interactive and automated theorem proving technology as integrated in modern mathematical proof assistants. Our methodology is in fact independent of a concrete choice in this regard, as long as the system's logic is based upon higher-order logic HOL (i.e. logics extending Church's simple type theory [4]). So far we have mostly focused on utilizing the Isabelle/HOL proof assistant [8], by harnessing its ecosystem of integrated theorem provers (via Sledgehammer [5]) and model generators (e.g. Nitpick [6]).

The LogiKEy framework supports plurality at different layers; cf. Fig. 1. Classical higher-order logic (HOL) is fixed as a *universal meta-logic* [1] at the base layer (L0), on top of which a plurality of (combinations of) object logics can become encoded (layer L1). Employing these logical notions we can now articulate a variety of logic-based domain-specific languages (DSLs [7]), theories and ontologies at the next layer (L2), thus enabling the modelling and automated assessment of different application scenarios (layer L3). These linked layers, as featured in the LogiKEy approach, facilitate fruitful interdisciplinary collaboration between specialists (and students) in different AI-related domains and domain experts in the design and development of knowledge-based systems.

The LogiKEy methodology builds upon previous work on the *shallow semantical embedding* [1] of *quantified* non-classical logics in HOL (starting with modal logics in [3]) and has been employed in teaching since summer 2016 with the awarded *Computational Metaphysics* course at the Freie Universität Berlin.[1]. Ever since, the approach has been applied in several other courses where it has enabled concise and elegant encoding of prominent logic puzzles (e.g. in epistemic logic), foundations of mathematics and arguments in philosophy that require the use of non-trivial combinations of quantified non-classical logics.

We list below some of the teaching venues in which the LogiKEy methodology has been applied.

- **(Invited lecture course) Higher-Order Modal Logics: Automation and Applications**. Benzmüller, C. & Woltzenlogel-Paleo, B. The 11th Reasoning Web Summer School, Berlin, Germany, 2015.
- **(Invited lecture course) Higher-Order Modal Logics**. Benzmüller, C. & Woltzenlogel-Paleo, B. Logic Summer School, ANU Canberra, Australia, 2015.
- **(Invited lecture course) Computational Metaphysics.** Benzmüller, C. Thematic trimester 'Current Issues in the Philosophy of Practice of Mathematics & Informatics', Centre International de Mathématiques et d'Informatique de Toulouse (CIMI), France. June, 2016.

[1] [9]; see also `https://www.fu-berlin.de/campusleben/lernen-und-lehren/2016/160428-lehrpreis/index.html`

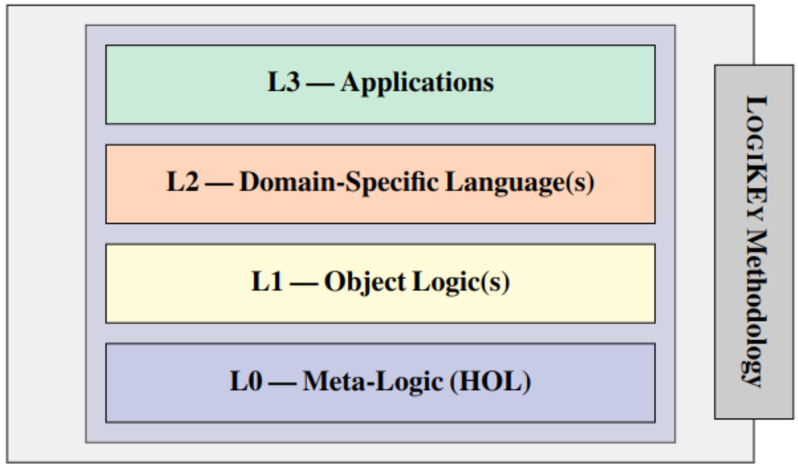

Fig. 1: LogiKEy methodology.

- **Interactive and Automated Theorem Proving for Non-Classical Logics.** Benzmüller, C., Steen, A., & Wisnieswki, M. Tutorial at GCAI-2016, 2016.
- **(Invited tutorial) A Universal Logic Theorem Proving Approach.** Benzmüller, C. Berkeley-Stanford Circle in Logic and Philosophy, San Francisco, USA, 2016.
- **(Invited Tutorial) Tutorial on Universal Logic Theorem Proving in HOL.** Benzmüller, C. ILIAS group, University of Luxembourg, 2017.
- **(Invited Tutorial) Universal Logical Reasoning via Semantical Embeddings in HOL.** Benzmüller, C. 2nd CLE Colloquium for Philosophy and History of Formal Sciences (CLE4Science), University of Campinas, Brasil, 2017.
- **(Invited Tutorial) Automated Reasoning in Higher-order and Non-classical Logics.**. Benzmüller, C. Pontifícia Universidade Católica do Rio Grande do Sul (PUCRS), Porto Alegre, Brasil, 2017.
- **From computational metaphysics towards computational pseudo-ethics.** Benzmüller, C. Bath IMI Thematic Semester (U Bath, UK): Algorithms, Accountability and Ethics in Machine Learning, 2018.
- **Universal Logical Reasoning.** Benzmüller, C. Zhejiang University, Hangzhou, China, 2020. Invited Lecture Course (Part of course on Logic in AI).[2]
- **AISE-UL: Universelle Logik & Universelles Schließen.** Benzmüller, C., & Fuenmayor, D. University of Bamberg, 2022.[3]
- **Blockseminar AISE: Automated Theorem Proving and the TPTP.** Benzmüller, C., & Fuenmayor, D. University of Bamberg and University of Luxembourg and University of Greifswald, 2022.[4]

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[2] Course website: `https://www.xixilogic.org/events/2020/08/course-logic-in-ai/`

[3] Winter 2022/23, 2+2 SWS, 6 ECTS (3 ECTS as seminar in Philosophy), Website `https://vc.uni-bamberg.de/enrol/index.php?id=57735`

[4] Seminar, Summer 2022, 2 SWS, 3 ECTS, Website `https://vc.uni-bamberg.de/enrol/index.php?id=54718`

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