2 | New Perspectives on Logistics and Supply Chain Management

On B2C E-commerce Logistics, Economies of Scale, Marketplace Platforms, and the Difficult Competitive Position of Smaller Retailers

Christian Straubert

UNIVERSITY OF BAMBERG
CHAIR OF OPERATIONS MANAGEMENT & LOGISTICS

2 New Perspectives on Logistics and Supply Chain Management

UNIVERSITY OF BAMBERG
CHAIR OF OPERATIONS MANAGEMENT & LOGISTICS

New Perspectives on Logistics and Supply Chain Management

Volume 2

Edited by
Prof. Dr. Eric Sucky

University of Bamberg
Chair of Operations Management & Logistics
Feldkirchenstr. 21, 96052 Bamberg

The successful management of internal and cross-company value-added processes, value chains, and entire networks depends in particular on the purposeful application of existing and advanced methods and concepts of production and logistics management and operations research, the use of innovative information and communication technologies, and theoretical and practical findings in process management. The series aims to publish new research results in the areas of production and logistics management, supply chain management, and e-commerce. Publications that contribute to scientific progress in these areas are selected.

On B2C E-commerce Logistics, Economies of Scale, Marketplace Platforms, and the Difficult Competitive Position of Smaller Retailers

Christian Straubert

UNIVERSITY OF BAMBERG
CHAIR OF OPERATIONS MANAGEMENT & LOGISTICS

The thesis at hand has been submitted to the Faculty of Social Sciences, Economics, and Business Administration of the University of Bamberg as a dissertation.

Reviewers:

Prof. Dr. Eric Sucky,	University of Bamberg
Prof. Dr. Alexander Pflaum,	University of Bamberg
Prof. Dr. Björn Ivens,	University of Bamberg
Prof. Dr. Alexander Dobhan,	Technical University of Applied Sciences Würzburg-Schweinfurt

Date of Defense: March 6, 2025

Christian Straubert 0000-0003-0342-1524

Otto-Friedrich-Universität Bamberg, 2025
URN: urn:nbn:de:bvb:473-irb-1077166
DOI: https://doi.org/10.20378/irb-107716
eISSN: 3052-2021

Preface

Electronic commerce (e-commerce) has experienced significant growth, and the COVID-19 pandemic has accelerated this trend. Of particular interest in the context of Mr. Straubert's dissertation are B2C e-marketplaces, especially hybrid marketplaces. B2C e-marketplaces are essentially online marketplaces where supply and demand come together, similar to a physical market. In terms of design and organization, a distinction can be made between online shops and online marketplaces. While in online shops the operator is the sole seller of products and/or services (one-to-many), in online marketplaces the (platform) operator acts as an independent third party, merely acting as an intermediary for the products and services of a large number of sellers (many-to-many). In an online marketplace, products and services from different sellers are offered under one brand or domain. Prominent examples are Amazon Marketplace, eBay, Otto Market and Zalando Marketplace (the so-called Zalando Partner Program). However, the operator of an online marketplace can also operate its own online shop (e.g. Amazon, Otto or Zalando). A further distinction can therefore be made between pure B2C e-marketplaces (i.e. the provider of a pure B2C e-marketplace operates the platform but does not offer any goods for sale itself) and so-called hybrid B2C e-marketplaces (i.e. the provider of a hybrid B2C e-marketplace operates the platform and offers goods for sale itself).

From a logistical point of view, a pure B2C marketplace is what is known as drop shipping. When a customer orders goods from a seller (partner) via the B2C marketplace, the seller is informed and delivers the order from its own warehouse. From the point of view of the B2C marketplace, drop shipping means that the goods are delivered directly from the seller to the customer. For the B2C marketplace operator, the appeal of this business model lies in the fact that, unlike an online shop, growth is possible without major investment. In recent years, however, B2C marketplace operators have begun to offer logistics services in addition to drop shipping. Companies such as Amazon, Otto and Zalando, which operate both an online shop and an online marketplace, can use their existing logistics infrastructure to offer fulfillment services to sellers or partners. In such cases, all activities from order acceptance, warehousing, picking, packing and shipping of goods to activities related to returns management could be handled by the operator of the online marketplace. Otto uses the Hermes Fulfilment Group (a wholly owned subsidiary of the Otto Group) for this purpose. Amazon operates this service under the name Fulfillment by Amazon (FBA). Amazon provides sellers with storage capacity and handles shipping and other logistics services.

Finally, retailers can also act as pure logistics service providers. In these cases, customers may order directly from the online shop of a manufacturer, but the retailer handles the logistics processes, for example, the delivery from the retailer's warehouse (where the manufacturer's goods are stored) to handling any returns. It is clear that logistics is of critical importance in B2C e-commerce due to the rapid growth, the increasing flow of goods in forward and reverse logistics, and the increasingly smaller shipment sizes. Mr. Straubert aptly notes: "It is likely that B2C e-commerce logistics will play an important role in this context. Logistics is one of the most important, if not the most important, part of an e-tailer's value chain. Procurement, inbound logistics, warehousing, picking, packing and delivery often account for the majority of an e-tailer's costs, and the fulfillment process is also one of the few that is visible to the customer". Based on these considerations, Christian Straubert's dissertation addresses, in particular, but not exclusively, practically relevant and theoretically significant problems in the context of B2C e-commerce logistics.

The first step is to examine fundamental questions in the context under consideration:

- What are the current trends in B2C e-commerce logistics?
- What logistics strategies can be identified based on the current trends in B2C e-commerce logistics?
- How does differentiation through logistics services influence the overall competition in the B2C e-commerce market?

Building on this, customer loyalty programs (e.g. Amazon Prime) come into focus. First, Mr. Straubert analyzes the benefits of subscribing to Amazon Prime. However, only if a third-party seller uses the "Fulfillment by Amazon" program are their offers Prime-eligible and therefore more attractive to Prime subscribers. Therefore, Mr. Straubert addresses the following additional questions:

- How much do shoppers like to buy from different seller types?
- What determines how much customers are willing to buy from different seller types?

Another important mechanism in B2C e-marketplaces is that third-party sellers pay a commission to the marketplace operator when they sell something on the marketplace. Focusing on these commission payments and inventory management in the case of hybrid B2C e-marketplaces (i.e., the provider of a hybrid B2C e-marketplace operates the platform and offers goods for sale itself), Mr. Straubert analyzes the coopetition between a marketplace operator and its third-party sellers. However, online merchants do not necessarily have to offer their goods on a B2C e-commerce marketplace. What about online retailers that use logistics outsourcing but do not operate within a marketplace platform? For cases where an online retailer outsources all logistics processes except inventory management, Mr. Straubert formulates and analyzes a continuous approximation location-inventory model with exact inventory costs and nonlinear lead-time penalties.

In summary, Mr. Straubert impressively manages to draw a thread through the various aspects of his dissertation. Based on the relevant trends identified for logistics in B2C e-commerce, he categorizes alternative logistics strategies used by B2C online retailers. From the analysis of these logistics strategies, it can be concluded that only companies with a logistics industry leader strategy have the opportunity to gain significant market share in the long run. This is due to the importance of logistics services to customers and the strong economies of scale that can be achieved in B2C e-commerce logistics. Consequently, the significant platform of the Amazon marketplace is analyzed from the perspective of customers, third-party sellers, and competition between marketplace operators and third-party sellers. Among many other findings, a causal chain emerges that is relevant and significant for many areas of business administration: (1) Fast shipping that is perceived as free of charge is very important to customers in B2C e-commerce. However, this integral part of the Prime subscription basically only applies to products offered by Amazon itself or by third-party sellers using Amazon's Fulfillment by Amazon (FBA) program. (2) As a result, the free, fast shipping made possible by FBA and the Prime subscription has become an important customer expectation, implicitly pressuring third-party sellers to use FBA. (3) When a third-party seller uses FBA on Amazon's marketplace, and both the marketplace operator and the third-party seller offer the same or substitutable products, direct competition exists. This must be taken into account when making decisions about the quantities of products offered.

This dissertation impressively reflects Mr. Christian Straubert's outstanding technical and methodological expertise. Overall, Mr. Straubert's work makes a significant and lasting contribution to scientific progress not only in the context of e-commerce and e-commerce logistics, but also for the platform economy and the management of business ecosystems as a whole. It is also highly relevant to business practice.

Univ.-Prof. Dr. Eric Sucky

Abstract

This PhD thesis is about logistics and its importance in B2C e-commerce retailing of physical goods (e-tailing). Logistics encompasses a wide range of activities in e-tailing, including the purchase of goods, their storage, as well as the packing and shipping of orders. In many cases, these logistical processes are at the core of an e-tailer's value chain. Consequently, inexpensive, fast, and reliable logistics are also frequently emphasized by e-tailers as part of their marketing.

A prominent example of this is the *Amazon Prime* subscription (membership-based free fast shipping) and the *Fulfillment by Amazon* service, which third-party sellers on the *Amazon marketplace* can use to tap into *Amazon's* logistics capabilities (i.e., logistics outsourcing). At the same time, there are strong economies of scale effects in logistics, and thus also in e-tailing. Accordingly, it can be argued that competition in the B2C e-commerce market is strongly influenced by logistical scale.

This PhD thesis explores this interrelated set of topics through a combination of exploratory and confirmatory, qualitative and quantitative, empirical and mathematical research. The perspectives of customers as well as e-tailers, marketplace operators and third-party sellers are considered. Outsourcing to logistics service providers, transaction costs and possible regulation of the B2C e-commerce market are also discussed.

Contents

List of Figures

List of Tables

Abbreviations

(s, Q)-model	= Reorder point, order quantity model
3PL	= Third-party logistics
3P seller	= Third-party seller
AISeL	= Association for Information Systems eLibrary
B2B	= Business-to-Business
B2C	= Business-to-Consumer
BIBD	= Balanced incomplete block design
BWS	= Best-worst scaling
C2C	= Consumer-to-Consumer
CA	= Continuous approximation
CFI	= Comparative fit index
CFS	= Contingent free shipping
CI	= Confidence interval
CR	= Critical ratio
CV	= Coefficient of variation
EOQ	= Economic order quantity
E-tailer	= B2C e-commerce retailer
FBA	= Fulfillment by Amazon
FBS	= Fulfillment by Seller
FTC	= Federal Trade Commission

GMV	= Gross merchandise volume
HHI	= Herfindahl-Hirschman Index
IoT	= Internet of Things
LIP	= Location-inventory problem
LowerCB	= Lower confidence bound
LP	= Loyalty program
LSP	= Logistics service provider
ME.1 and ME.2	= Model extension case 1 and model extension case 2
MFC	= Multi-channel fulfillment
MFS	= Membership-based free shipping
n.d.	= No date
Nash-EQ	= Nash equilibrium
ODW	= On-demand warehousing
RMSEA	= Root mean square error of approximation
S.O.C.	= Second order condition
SCM	= Supply chain management
SCV	= Squared coefficient of variation
SEM	= Structural equation modeling
SRMR	= Standardized root mean squared residual
TLI	= Tucker-Lewis index

On B2C E-commerce Logistics, Economies of Scale, Marketplace Platforms, and the Difficult Competitive Position of Smaller Retailers

Part A: Synopsis

1. Introduction: An overview of the papers in this PhD thesis

In 2005, *Amazon* introduced its *Prime* subscription, which offers unlimited two-day delivery in the United States when buying from *Amazon*. In 2015, *Amazon* began offering free same-day or even two-hour delivery to *Prime* subscribers in select metropolitan areas. Today, *Prime* is available in more than 20 countries and serves as the nexus of *Amazon's* business-to-consumer (B2C) retail strategy. Numerous studies have shown that customers value fast delivery times (e.g., Gupta et al., 2004; Hsiao, 2009; Marino et al., 2018; Gawor & Hoberg, 2019). There are also indications that fast delivery lead times reduce the likelihood of product returns (Asdecker & Karl, 2018, p. 45). Accordingly, *Amazon's* strategy has been very successful, and *Amazon* has grown to become the largest B2C e-commerce retailer in the world. At the same time, *Amazon* has evolved from a pure B2C e-commerce retailer (e-tailer) and is now also a marketplace operator and a logistics service provider.

Because of *Amazon's* popularity, it is attractive for other e-tailers (so called third-party sellers) to offer products on *Amazon's* marketplace, and *Amazon* encourages this because it receives commission fee revenue from each third-party sale. In fact, it is not at all uncommon for both *Amazon* itself and one or more third-party sellers to offer an identical product on the *Amazon marketplace*. This creates an interesting dynamic in which *Amazon* and the third-party sellers compete with each other, but also cooperate with each other (also known as *coopetition*).

Through programs such as *Fulfillment by Amazon* (FBA), e-tailers can take advantage of *Amazon's* superior logistics when selling products on *Amazon's* marketplace, and even when selling products through other channels (such as their own website). This logistics outsourcing is attractive to smaller e-tailers, because offers with FBA are eligible for free fast delivery as part of the *Prime* subscription, and because smaller e-tailers, on their own, simply cannot offer the speed and quality of logistics that *Amazon* provides. Both the trend toward ever-faster delivery times and the success of combining marketplace platforms with logistics services can be seen in many parts of the world (e.g., Zalando, 2024a, in Germany, and JD.com, 2024, in China).

We can also observe, that the B2C e-commerce market is rather concentrated, with only a handful of e-tailers/marketplaces generating the majority of B2C e-commerce revenues. It is likely that B2C e-commerce logistics plays an important role in this context. Logistics is one of the most important, if not the most important, part of an e-tailer's value chain. Procurement, inbound logistics, inventory holding, picking, packing, and delivery often account for the majority of an e-tailer's costs and the fulfillment process is also one of the few processes that are visible to the customer[1]. At the same time, B2C e-commerce logistics benefits greatly from economies of scale. The largest e-tailers can offer the fastest delivery speeds while having relatively low logistics costs. B2C e-commerce is one of the few industries where logistics performance is

[1] For example, the annual report of Zalando SE for 2023 (Zalando, 2024b, p. 167) states that the procurement of merchandise (cost of sales) accounts for about 61.8% of total costs and fulfillment for about 24.4%. This compares to only 7.5% for marketing and only 4.9% for administrative expenses.

at the core of marketing and therefore very important to an e-tailer's competitive position. Consequently, it is not very surprising that the B2C e-commerce market has developed the way it has. Smaller e-tailers can still find niches. However, as far as the majority of the B2C e-commerce business is concerned, they are at a severe disadvantage.

Several antitrust proceedings have accused the world's largest e-tailers of abusing their market power (e.g., Deng, 2023; European Commission, 2020; Federal Trade Commission, 2024). However, so far there has been relatively little research on the *reasons* for this market power. I hope that my PhD thesis contributes to filling this research gap. Comprehensive derivations of the various subtopic-specific research gaps, including related literature, can be found at the beginning of each paper. Additionally, the first two papers of this thesis, i.e., "*Current Trends in B2C E-Commerce Logistics – A Content Analysis*" and "*A theoretical framework for the logistics strategies of B2C e-commerce retailers based on current trends in the industry*" may serve as further justification why the topics described above are important.

In addition to advancing the current state of science, this PhD thesis may be of interest and use to e-tailers of all sizes (e.g., marketplace operators and third-party sellers) as well as antitrust regulators. **Figure 1** provides an overview of the different papers included in this thesis.

The research question of *Paper I* (Straubert et al., 2019a) "*Current Trends in B2C E-Commerce Logistics – A Content Analysis*" was *"What are the current trends in B2C e-commerce logistics?"*. We conducted a systematic content analysis of non-scientific, practice-oriented articles (grey literature) to identify the current trends in B2C e-commerce logistics. The trend towards faster deliveries was the most frequently mentioned trend, followed by more transparency across processes, logistics cooperation/outsourcing, and more urban warehouses.

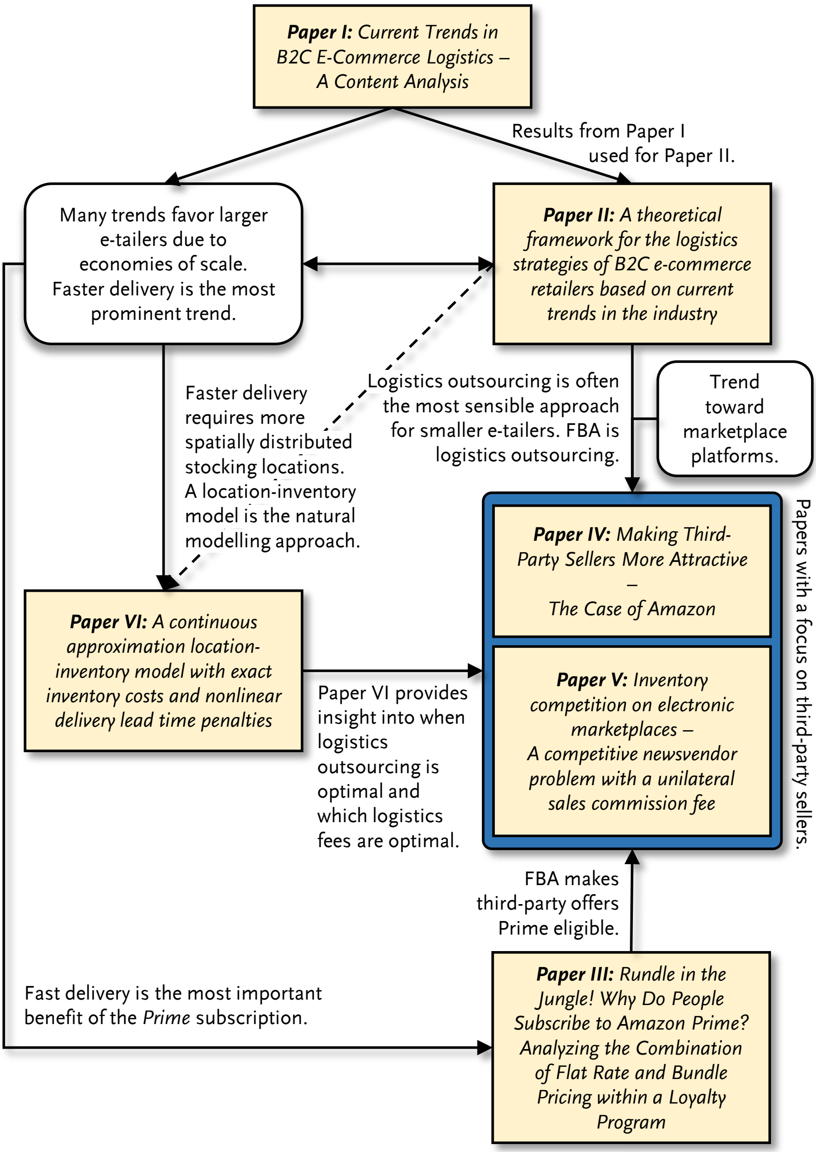


Figure 1 Overview of the topics and papers of this PhD thesis

The research questions of *Paper II* (Straubert, 2022) "*A theoretical framework for the logistics strategies of B2C e-commerce retailers based on current trends in the industry*" were *"Which logistics strategies can be identified based on the current trends in B2C e-commerce logistics?"* and *"How does differentiation via logistics services influence general competition in the B2C e-commerce market?"*. The goal of the paper was to develop a theoretical framework for logistics strategies in B2C e-commerce, emphasizing the importance of strategic positioning. The development of the framework was based on the current and future trends in B2C e-commerce logistics that were identified in *Paper I* and further discussed in a paper that I presented at the 7th IEEE International Conference on Advanced Logistics and Transport (Straubert et al., 2019b). The paper emphasizes that logistics plays a crucial role in B2C e-commerce, serving as a strategic value proposition for creating a competitive advantage, while also being a major operational cost driver. Combined with well-known models of competitive strategy, this reasoning led to a framework with the following three B2C e-commerce logistics strategies: *Logistics Industry Leaders*, *Logistics Efficiency Seekers*, and *Logistics Niche Concepts*. At the end of my study, I concluded that companies that are able to adopt a *Logistics Industry Leader* strategy are more likely to gain significant market share due to the importance of logistics services to customers and the strong economies of scale effects in logistics.

The main research question of *Paper III* (Straubert et al., 2024) "*Rundle in the Jungle! Why Do People Subscribe to Amazon Prime? Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program*" was *"Because of what benefits do people subscribe to Amazon Prime?"*. Based on a best-worst scaling discrete choice experiment, we concluded that delivery-related benefits are by far the most important benefits for *Prime* subscribers. In particular, free fast shipping seems to be only slightly less important than free shipping. This once again confirms that fast shipping is highly valued by customers in B2C e-commerce.

This result is also important to many other e-tailers because *Amazon* is the largest marketplace operator in the world (Pool, 2024). If a third-party seller uses *Amazon's Fulfillment by Amazon* program, its offers are

Prime eligible and therefore more attractive to *Prime* subscribers. These interrelationships were the focus of *Paper IV* (Straubert et al., 2023) *"Making Third-Party Sellers More Attractive—The Case of Amazon"*, which explored the role of FBA (i.e., logistics outsourcing) in *Amazon's* marketplace strategy. The primary research question for this paper was *"How much do customers like to buy from the different seller types?"*. Focusing on logistics (because of FBA) and the trust in the different seller types, we also asked, *"What determines how much customers like to buy from the different seller types?"*. Using a survey and structural equation modeling we found that *Amazon's Prime* logo, when combined with a customer's *Prime* subscription, significantly boosts trust in third-party seller offers, thereby increasing their sales. In part, this is due to the perception that *Amazon's* logistics are fast and reliable. Consequently, we also found that free fast shipping, enabled by FBA and the *Prime* subscription, has become a key customer expectation. Both results implicitly pressure third-party sellers into using FBA, which in turn generates additional logistics revenue for *Amazon*. One could argue that *Amazon's* strategy is not only about how to make third-party offers more attractive but also about how to make *non-Prime* offers less attractive.

Another important mechanism in B2C e-commerce marketplaces is that third-party sellers pay a commission to the marketplace operator when they sell something on the marketplace. Focusing on these commission payments and inventory management, we investigated the coopetition between a marketplace operator and its third-party sellers in *Paper V* (Straubert & Sucky, 2023) "*Inventory competition on electronic marketplaces – A competitive newsvendor problem with a unilateral sales commission fee*". When a third-party seller uses FBA, one of the few logistical decisions left to the third-party seller is how much of a product to stock and offer. If both the marketplace operator and a third-party seller offer the same or substitutable products, there is direct competition and the quantities offered by the competing party should ideally be taken into account when making inventory management decisions. In *Paper V*, we provide solutions for the noncooperative Nash equilibrium and the optimal cooperative quantities (centralized solution). From a competition perspective, it is noteworthy that the commission makes it optimal for the marketplace operator to shift part of its demand risk to third-party sellers. The commission puts the marketplace operator in a very comfortable position.

However, e-tailers do not necessarily have to offer their goods on a marketplace. The question therefore arises as to whether the competitive position of small e-tailers is also weak if they use logistics outsourcing but do not operate within a marketplace service. The mathematical model in *Paper VI* (Straubert, 2024) "*A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties*" is valid for a case where the e-tailer outsources all logistics processes except inventory management. Outsourcing the ownership of products is very complex, which is why it is rarely done in practice. Therefore, in many cases, the model represents the maximum realistic level of logistics outsourcing. But even then, *Paper VI* shows how difficult the competitive position of small e-tailers is. To achieve ever-faster delivery lead times, e-tailers must store their products in more and more spatially distributed warehouses, so that they move closer to customers. This inventory decentralization, however, increases inventory holding, replenishment, and stockout-related costs. Thus, there is a trade-off between shorter transport distances to customers through inventory decentralization and lower inventory management costs through inventory centralization. I proved the quasi-convexity of the model from *Paper VI* and discussed some general properties of this trade-off. One of the many findings of this paper is that *Amazon's* strategy of offering faster and faster delivery lead times is especially powerful when it changes customers' general expectations of what a normal delivery lead time is. While the logistics costs associated with very fast delivery lead times are painful but manageable for large e-tailers, small e-tailers are put in an exponentially worse competitive position, either because they have to spend much more on logistics to achieve the same service level, or because customers do not like to buy from an online shop with slow delivery lead times.

In the following, I will now define and introduce some of the most important terms and concepts of this PhD thesis, such as B2C e-commerce and electronic marketplaces. I will discuss the extraordinary importance of logistics for the B2C e-commerce market, before taking a closer look at the economies of scale effects that are present in B2C e-commerce logistics. These economies of scale explain many phenomena in the B2C e-commerce market. The synopsis will conclude with a broader scientific perspective on this PhD thesis, its limitations, and possible future research.

2. B2C e-commerce and the importance of logistics

2.1. Defining B2C e-commerce

B2C e-commerce is a relatively broad term. **Figure 2** illustrates the focus of this PhD thesis within the broad field of e-commerce. As the term *e-commerce* already suggests, this dissertation is about trading through an electronic interface. This means that the seller and the buyer are usually geographically separated. Business-to-consumer (B2C) means that a business sells to consumers. A typical characteristic of such a business relationship is that the business sells a product not just to one customer, but to many different customers. In addition, most of this thesis focuses on the trade of physical products. Physical products can be stored (inventories) and require physical logistics (e.g., shipping). Furthermore, this thesis is primarily about retailing. This means that

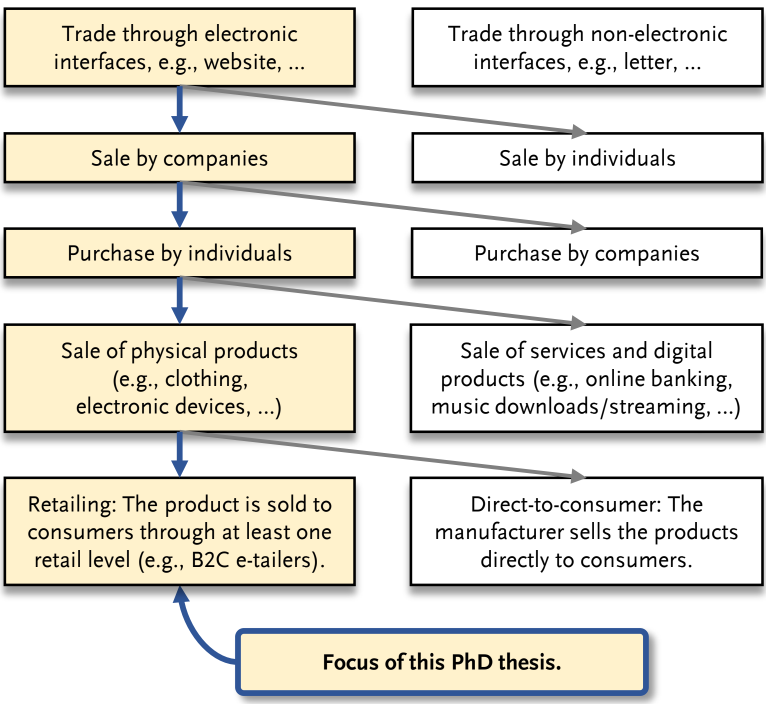


Figure 2 Focus of this PhD thesis within the world of e-commerce

instead of selling a product directly to a consumer, a manufacturer sells its product to a wholesaler or retailer. The final retailer ultimately sells the product to a consumer.

Retailers provide several valuable services and are therefore often used by manufacturers to distribute their products. Already in 1918, Karl Oberparleiter described six valuable functions of retailing that are still very relevant today (Peterson & Balasubramanian, 2002). According to Oberparleiter (1918), retail bridges the distance between manufacturers and consumers in the following ways:

- **The spatial function** – Often the production of a good does not take place at the location where it is consumed. The product must somehow get from the manufacturer to the customer. A retailer can help make this process more efficient.
- **The temporal function** – Often a good is produced before it is consumed. The product must be stored to bridge this temporal gap. A retailer can help make this process more efficient.
- **The quantity function** – A retailer can be useful when it comes to exploiting economies of scale in the spatial function and the temporal function. Additionally, a retailer can also be useful when production takes place in large lots and consumption in small lots, or vice versa. Retailers can help make the physical change from a production unit to a sales unit more efficient.
- **The quality function** (which arguably should be called the **assortment function**) – Through product selection and assortment, retailers can help make quality differences between products and within a batch of a product more transparent. This helps to better match the customer's quality requirements with the quality offered by different manufacturers or different batches of a product. Product assortments also have practical benefits for customers, as they can search for and buy several related products in one place, which adds value to the offering (economies of scope).
- **The cultural function** (in subsequent publications, Oberparleiter (1955) integrated this function into what he called the **advertisement/marketing function**) – Our markets are imperfect in part

because perfect information does not exist. Therefore, products must be advertised in order to maximize profits. Retailers are well-positioned to advertise as they tend to know their customers well (e.g., their culture, which may be foreign to a spatially distant manufacturer).

- **The credit function –** If there is a demand for a product but the manufacturer of the product does not have the money or motivation to produce it, or if there is demand for a product but some customers do not have the money to buy it, a retailer can help by giving loans to the manufacturer or the customers. A retailer can be an efficient lender because a retailer often knows the products, and its customers and manufacturers, better than other potential lenders.

Since the focus of this PhD thesis is on B2C e-commerce logistics, it is primarily concerned with the spatial function, the temporal function, and the quantity function of retailing. In general, retailing is to a large extent logistics. Comparing Oberparleiter's functions of retailing with the so-called *Seven Rights*[2], a frequently used definition of logistics, makes this more apparent. Logistics aims to provide:

- the right product
- to the right place
- at the right time
- in the right quantity
- in the right quality/condition
- to the right customer
- at the right cost

Oberparleiter (1955) also defined the risks involved in retailing (see also Zitzmann, 2018, pp. 50–63). This PhD thesis focuses on *demand risk*, which is probably the most important risk associated with the spatial,

[2] The *Seven Rights* definition is often attributed to Dr. Edward G. Plowman and dated to the beginning of the second half of the 20th century. However, the exact origins of the definition have been lost over time. The similarities between the *Seven Rights* definition of logistics and the functions of retailing according to Oberparleiter (1918) are remarkable.

temporal, and quantity functions of retailing. The demand for a product is uncertain. Due to its stochastic nature, demand can be unevenly distributed in space and time. Customers in region *A* may have a high demand for a product and therefore the product is out of stock in the warehouse serving that region. This leads to shortage costs (e.g., lost sales, dissatisfied customers). At the same time, customers in region *B* may have a low demand for the product and therefore the warehouse serving that region has much of the product in stock. This results in high inventory holding costs. The goal is to have neither too much nor too little in stock. This is often a difficult task. However, compared to manufacturers, retailers may be better at anticipating demand, as they often know the end markets better than the manufacturers. Therefore, the existence of demand risk provides further justification for the existence of retail trade.

Lastly, it should be noted that B2C e-commerce often involves not only the sale of products, but also the handling of product returns (Asdecker, 2014). Although some of the insights from this thesis are also relevant to product returns, they are largely ignored in this PhD thesis. The product returns process is very important for an e-tailer, and it has some interlinkages with other processes (e.g., inventory management), however, by and large, it is a stand-alone process. This is also evident when looking at the value chain of an e-tailer, which I will discuss in the next section.

2.2. The value chain of a B2C e-commerce retailer and the extraordinary importance of logistics within B2C e-commerce

In order to understand a company's business, it can be useful to look at the individual activities in the company's value chain. **Figure 3** shows the value chain of an archetypal e-tailer. The value chain depicted in this figure is based on the generic value chain by Porter (2004a, p. 37), and was adapted to fit an archetypal e-tailer. In practice, e-tailers may have additional businesses and value-adding processes. For the sake of brevity, these are ignored here.

In contrast to the generic value chain by Porter (2004a, p. 37), *Procurement of goods* is depicted as a primary activity in **Figure 3**. Indeed, it can be argued that for any manufacturing company, the purchase of raw materials and intermediate products should be considered a primary activity, since these products flow directly into the input-output system where they are transformed into the product sold to customers. This argument is even stronger in the context of retailing because a retailer usually makes little or no changes to the merchandise that it procures. Instead, the input-output process is primarily of a temporal and spatial nature. Sourcing goods from manufacturers or wholesalers is part of a retailer's value proposition (also recall the six functions of retailing detailed in **Section 2.1**). Along the value chain, products age, and change location from the manufacturer to the retailer and then to the customer.

Storing products incurs inventory holding costs for the retailer, as warehouse space is used, capital is tied up and products typically depreciate as they age. Optimizing the trade-off between larger procurement lot sizes (and thus lower procurement costs) and lower inventory levels (and thus lower holding costs) is at the core of inventory management (Hadley & Whitin, 1963, pp. 10–17). The model presented in *Paper VI* takes this trade-off into account. The newsvendor model in *Paper V* emphasizes the fact that products depreciate over time, e.g., at the end of a selling season, and that it is therefore important not to buy too much of a product.

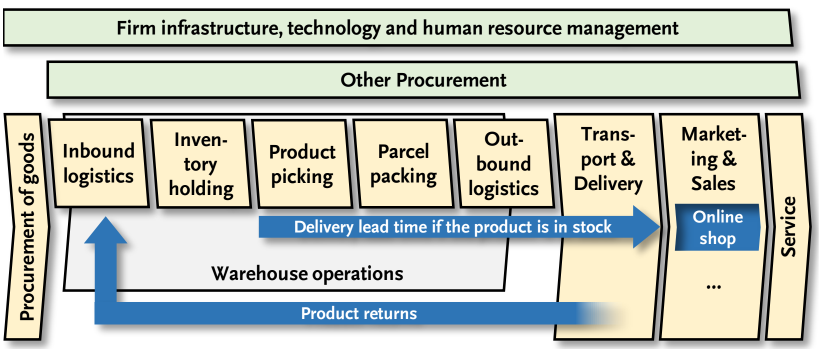


Figure 3 The value chain of an archetypal B2C e-commerce retailer

When a customer decides to buy a product, it is picked, packed, and shipped to the customer, for example to their home or to a parcel locker. A customer therefore not only buys the product, but also the spatial relocation (transport) of the product to a location of their choice. Also note that customers do not buy just any shipping. They often buy/expect timely delivery. After all, what good is it if an e-tailer has a product in stock right now but only ships it after a long delay? In *Paper I*, we identified fast delivery lead times as the most important trend in B2C e-commerce logistics.

In *Paper I*, we also identified *more smaller warehouses closer to customers* and *logistics outsourcing/cooperation* as important trends. It is obvious that goods must be physically stored closer to customers if very short delivery lead times are to be achieved. However, using many smaller warehouses reduces economies of scale. Since the curve of typical economies of scale effects is convex, large e-tailers can cope with such inventory decentralization much better than smaller e-tailers (the loss of economies of scale due to inventory decentralization would be much greater for smaller e-tailers). Indeed, in 2023, *Amazon* split its US fulfillment network from a single network into eight independent networks. Their goal is to store as many products as possible in all eight distribution regions so that *Amazon's* delivery lead times can become even faster. This change seems to be working out well for *Amazon*. In their 2023 third-quarter earnings release, they say (Amazon, 2023a):

> *"The benefits of moving from a single national fulfillment network in the U.S. to eight distinct regions are exceeding our optimistic expectations, and perhaps most importantly, putting us on pace to deliver the fastest delivery speeds for Prime customers in our 29-year history. [...]"*

Smaller e-tailers struggle to build such capabilities. In *Paper II*, I argued that the economies of scale effects in B2C e-commerce logistics are likely to lead to an increasingly unfavorable competitive situation for smaller e-tailers. In *Paper II*, I also argued that smaller e-tailers should probably try to remedy this situation by cooperating with each other, either directly or indirectly (e.g., outsourcing to a logistics service provider) so that their scale disadvantage is reduced.

In fact, as already mentioned in the introduction and researched in *Paper IV*, *Amazon* itself offers e-tailers the possibility of outsourcing their logistics through its *Fulfillment by Amazon* service. Considering the value chain (**Figure 3**), it is interesting to note that *Amazon* handles so much of the value chain that a third-party seller that uses FBA only has two primary activities: *Procurement of goods* and *Service* (although part of the service is also being handled by *Amazon*). Therefore, from a strategic point of view, an e-tailer that uses FBA increases its chances of success if it excels in the area of procurement, for example by offering products that are not offered by many other e-tailers or by offering products that are often out of stock at other e-tailers.

Coming back to the differences between the value chains of manufacturing companies and B2C e-commerce retailers, it is also noteworthy that e-tailers often cannot adequately differentiate their business from the competition on the basis of the products they offer. While a manufacturer can market the quality of the craftsmanship of its products (i.e., the quality of the primary production activities in its input-output system), in retailing it is often the case that identical goods are offered by multiple retailers. In the case of retailing, production is replaced by logistics as one of the most important primary activities. It stands to reason, then, that e-tailers should emphasize the quality of their logistics as part of their marketing. And indeed, with the *Amazon Prime* subscription and the famous *Prime* logo, *Amazon* has created a program that specifically combines the promise of fast, high-quality logistics with low costs. A powerful combination that we researched in *Papers III* and *IV*.

This special characteristic of B2C e-commerce is also evident when looking at a classic framework from marketing theory: the four Ps of the marketing mix.

The term *marketing mix* describes the idea that a company should ideally coordinate its product, pricing, distribution, and communication policies in an integrated manner (Diller et al., 2011, p. 36). In the English literature, these four levers are often referred to as the four Ps: product, price, place, and promotion (Kotler et al., 2024, p. 74). The typical marketing mix assumes a situation where the *place* lever describes the access to the products. The easier the access is (e.g., multiple sales channels, more points of sale, better logistics), the larger the potential market, but also the higher the costs and the greater the risk of brand dilution. Thus, from a typical point of view, *product* and *place* are two different levers. However, this is not true in the context of B2C e-commerce. In B2C e-commerce, the access to the merchandise (e.g., the online shop and

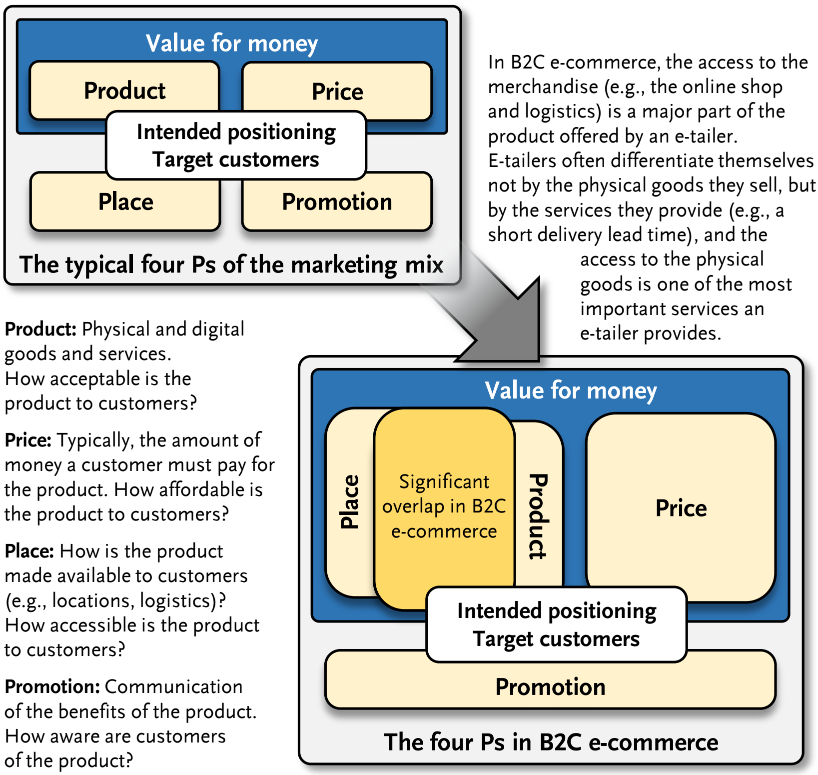


Figure 4 The four Ps (adapted from Kotler et al., 2024, pp. 74–75)

logistics) is a major part of the product offered by an e-tailer. It is therefore not very useful to separate the *product* and *place* levers, as there is a large overlap between them. If the *product* and *place* levers are understood to be largely the same in B2C e-commerce, it is also logical to consider the *place* lever as an important part of the *value-for-money* ratio. It is always important to consider both sides of the value-for-money ratio so that one can determine which side is the better lever to increase the ratio (Diller et al., 2011, p. 292). Applied to the B2C e-commerce context, this means that an e-tailer can lower its prices (the *price* lever) and/or improve access to its merchandise (the *place/product* lever), for example through faster delivery lead times. This circumstance, the typical four Ps of the marketing mix, and the corresponding B2C e-commerce perspective are illustrated in **Figure 4**. This corresponds to the classic trade-off in inventory management, where a company can either reduce costs or improve service levels (i.e., product availability). In practice, we see that the large e-tailers tend to choose to improve their logistics rather than reduce costs/prices. I hope that this PhD thesis will shed some light on why this might be.

2.3. Parallels between classical mail-order business and modern B2C e-commerce

A comparison of the value chains of an archetypal e-tailer (**Figure 3**) with those of a classical mail-order business reveals that the two are largely identical. The only major difference is that e-commerce customers order via electronic interfaces (typically an online shop) while in the classical mail-order business, customers typically order via letter. It could therefore be argued that there should be many similarities between these two related industries, particularly with regard to logistics. This is indeed the case, and I would like to illustrate this with a historical comparison of two companies. The mail-order pioneer *Sears, Roebuck and Company*, whose origins date back to 1886, was once the largest mail-order company and later the largest retailer in the United States. The B2C e-commerce pioneer *Amazon* is currently the largest e-tailer and the second largest retailer in the United States. In the following section, I will elaborate on the remarkably similar success stories and business models of these two companies. Many of the problems and solutions of today's e-tailing were already relevant in the mail-order business over 100 years ago.

The initial strategy of *Richard Sears*, the first president of *Sears*, can be described as growing fast while offering good quality for the lowest prices (Tedlow, 1990, pp. 265–266). The strategy of *Amazon* was and still is very similar.

Sears accomplished his objective through a combination of apt procurement and marketing. A key challenge was to build trust. In the early days of mail-order shopping, there was a lack of trust in this novel way of shopping. We observed similar trust issues during the nascent phases of B2C e-commerce. To build trust, *Sears* offered customer-friendly policies, which were advertised with eye-catching slogans such as: *"Satisfaction guaranteed or your money back"* and *"Send No Money"* (Tedlow, 1990, p. 272; Emmet and Jeuck, 1950, pp. 74, 87). The latter refers to *Sears' Cash on Delivery* policy, which allowed customers to order from *Sears* without having to pay upfront. Over time, both services became widespread and still exist today in e-tailing, more or less unchanged. While the trust issue in mail-order retailing has received some scientific attention in the past (McCorkle, 1990), it was not until the rise of B2C e-commerce that trust in (electronic) mail-order has received the attention it deserves (e.g., Gefen, 2000). *Paper IV* of this dissertation explored the issue of trust in the context of the *Amazon marketplace* platform with its third-party sellers. In particular, it highlighted the role of fast, high-quality logistics as an important component in building trust.

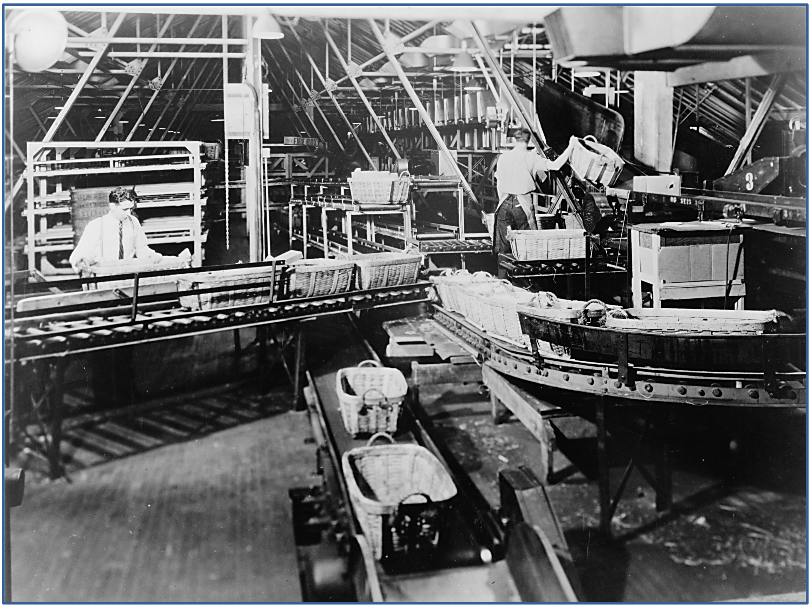
Indeed, fast, high-quality logistics was the one area where *Richard Sears* failed. This destroyed some of the trust that he had built up. His relentless push for more sales increasingly pushed the company into operational chaos. A recollection of historian *Florence Kiper Frank* about what *Julius Rosenwald*, a part-owner of the company, encountered when he joined *Sears* (Emmet and Jeuck, 1950, p. 130):

"When Rosenwald came to work in the morning he would find Sears, absorbedly writing at his desk in the large room that bulged with scattered heaps of merchandise. Boxes of watches, piles of clothing, samples of groceries in corners, files of correspondence, grimy with dust, on the floor. And in the midst of it all, that obsessed fiction writer, spinning more fancies to bring in more orders, more merchandise, more confusion. [...]

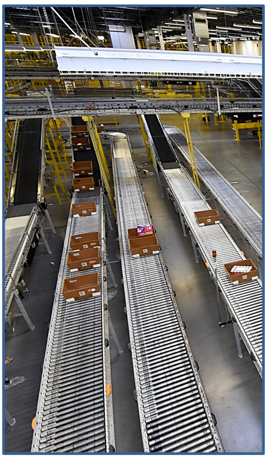
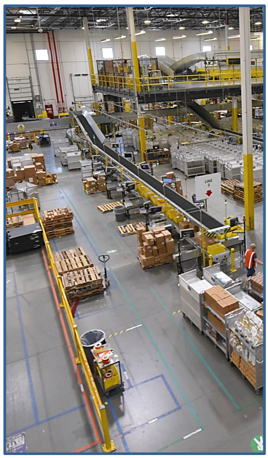
He found a harassed correspondence department doing its poor best with all manner of inquiries and complaints. [...] Still the orders poured in. Faster than the factories could supply the goods, faster than they could be cleared through warehouses and shipping rooms. Departments fell behind – thirty days, sixty days, sometimes three and four months. The panting executives were out-stripped, and the order they sought to impose seemed an unattainable dream. The most pressing problem was the problem of space. Month after month the cry firm would add to its offices and shipping rooms, but the Mad Hatter's of "No room! No room!" became a chronic plaint."

Julius Rosenwald managed to bring some order to the chaos and eventually replaced *Richard Sears* as president of the company in 1908. *Otto Doering*, vice president of operations, eventually led Sars to operational excellence. In 1906 the company entered its new *Merchandise Building* in Chicago and around the same time a so-called *schedule system* was devised by *Doering* and his lieutenants. The new Merchandise Building was proudly described in the 1905 catalog (Emmet and Jeuck, 1950, pp. 132–133):

"Miles of railroad run lengthwise through, in and around this building for the receiving, moving and forwarding of merchandise: elevators, mechanical conveyors, endless chains, moving sidewalks, gravity chutes, apparatus and conveyors, pneumatic tubes and every known mechanical appliance for reducing labor, for the working out of economy and dispatch is to be utilized here in our great Works."



Interior view of Sears's mail-order plant in Chicago, merchandise building, shipping court; circa 1920s; Retouched; Photographer unknown; Source: Library of Congress (n.d.)

Interior views of Amazon's fulfillment center in Cecil County, Maryland; September 2017; Retouched; Photographer: Joe Andrucyk; Sources: Maryland GovPics (2017a, b and c)

Figure 5 Cutting-edge warehouse fulfillment technology; a century apart

Figure 5 shows an illustrative comparison between *Sears'* Merchandise Building and a modern *Amazon* fulfillment center. This cutting-edge warehouse fulfillment technology, in combination with the schedule system, enabled *Sears* to fulfill (pick, pack, and hand over to shipping) most orders within 24 hours or 48 hours. By 1926, this was reportedly improved to 10 hours for about half the orders and at most 24 hours for about 99 percent of all orders (Emmet and Jeuck, 1950, p. 299). It is remarkable that a century ago, *Sears* was able to offer warehouse fulfillment times similar to those offered by *Amazon* today.

On the flip side, this is not to say that *Sears* didn't suffer from (supposedly modern) problems even back then. The advent of the US parcel post in 1913 brought with it an order and returns volume problem. Because shipping of parcel post eligible items became very cheap, customers ordered smaller quantities more often (Emmet and Jeuck, 1950, p. 298), a phenomenon which we discuss in our *Paper III* in the context of the *Amazon Prime* service. Reportedly, the number of orders *Sears* received increased fivefold in the year after the US parcel post was introduced. The returns ratio also increased. This was further aggravated by the fact that *Sears* paid both the delivery to the customer and back to *Sears* in case of returns.

It becomes clear that the current discussion about the benefits and drawbacks of free returns is by no means new. However, thanks to its highly successful schedule system, *Sears* was able to manage these problems, and, in the spring of 1929, *Sears* announced through its general catalog (Emmet and Jeuck, 1950, p. 451):

> *"We Pay the Postage! We Pay the Postage! We Pay the Postage!*
>
> *Sears, Roebuck and Co. announce with the issuance of this catalog the greatest single forward step in mail-order merchandising since the establishment of the Parcel Post. [...] Every article in this catalog which can be conveniently sent by parcel post will be shipped to you POSTAGE PREPAID. [...] Remember that fact when comparing our prices with others. Think of the convenience – NO ADDITIONAL POSTAGE TO FIGURE."*

Emmet and Jeuck (1950, p. 451) further elaborate that:

> *"The first inside page of the spring 1929 book asserted that the company had worked for years to make possible prepayment of parcel-post charges and had been estopped only by not having enough mail-order plants to blanket the nation. The editorial declared that establishment of ten mail-order branch plants had increased total sales and made possible operating economies which would allow Sears to absorb "a large part of the parcel post charges." The catalog asserted, opposite a map of the United States showing the ten mail-order plants and additional warehouses, "Our prices are the lowest in America!" A letter signed by General Wood ("My PLEDGE to our customers") stressed over and over the theme of "Service–Quality–Savings." Each of these three points was still further developed on the facing page; under "Service" on that page was the assertion that 99 out of 100 orders received at all mail-order plants were shipped in less than twenty-four hours."*

The fact that *Sears* was prominently advertising the performance of its logistics system even back then further supports the motivation of this dissertation. As argued in *Papers I* and *II*, logistics plays such an important role in the mail-order business that it can be an integral part of corporate strategy.

Moreover, the catalog text quoted above, which emphasized the spatially distributed warehouse and fulfillment network, shows that the executives at *Sears* were aware of the trade-offs faced in a location-inventory problem (LIP). *Paper VI* of this PhD thesis contains a LIP model that fits modern e-tailers, but apparently would also have been appropriate for mail-order companies a century ago.

After *Sears* announced that it would pay all postage, even if the item was not returned, customers ordered even smaller quantities more often. Reportedly, the average value per order dropped by about 17% (Emmet and Jeuck, 1950, p. 452). Today, we are sensitized to the fact that this behavior is not only costly but also bad for the environment (see also our

Paper III). At the time, the economic costs were enough of an argument for *Sears* to discontinue free delivery by the fall of 1933. The experiment failed, and even today, hardly any (electronic) mail-order company offers unconditional free delivery. Most e-tailers use some form of minimum order value as a condition for free delivery, and other e-tailers, such as *Amazon* with its *Prime* subscription, offer membership-based free shipping.

It becomes evident that the traditional mail-order business has many similarities with the electronic mail-order business (i.e., B2C e-commerce). Many of the logistical problems and solutions are the same. *Jeff Bezos*, the founder of *Amazon*, had recognized the importance of logistics to his business from the very beginning. In general, *Amazon* seems to have learned many lessons from the pioneering days of mail-order business.

However, not much research concerning traditional mail-order operations exists. This is probably because the peak of the classic mail-order business was before the emergence of operations research science. For *Sears* and other mail-order businesses, the heyday was over after the automobile became more common. Mail-order back then was especially attractive for geographically isolated customers such as farmers. Once car ownership became widespread, brick-and-mortar stores could be reached within a reasonable time. As a result, mail ordering lost importance and then stagnated as a retail niche for many decades. It was neither particularly important nor new and exciting. Only with the advent and rapid growth of B2C e-commerce was it that the academic community began to take a renewed interest in many of the old mail-order operations topics. This dissertation is an example of this renewed interest.

However, not everything is old. There are also some innovations in B2C e-commerce that have been made possible by the Internet and the resulting simplified automation and communication. One important new development are electronic marketplace platforms on which many different third-party sellers offer their goods. In this context, our *Paper V* presents a model of a new form of coopetition that arises between some marketplace operators and the third-party sellers that are active on their marketplace platforms.

2.4. Key figures and characteristics of the B2C e-commerce market

The B2C e-commerce market has grown rapidly in recent years and already accounts for a considerable share of retail trade. Unfortunately, there is no reliable information on the market shares of e-tailers. Even when statistics are available from more reputable sources, it is often unclear what these statistics include (e.g., physical, digital products, services, marketplace revenues?). However, while no definitive statements can be made, the information that is available should be sufficient to provide a rough overview of the B2C e-commerce market. **Figure 6** provides some information on the markets in Germany and the US.

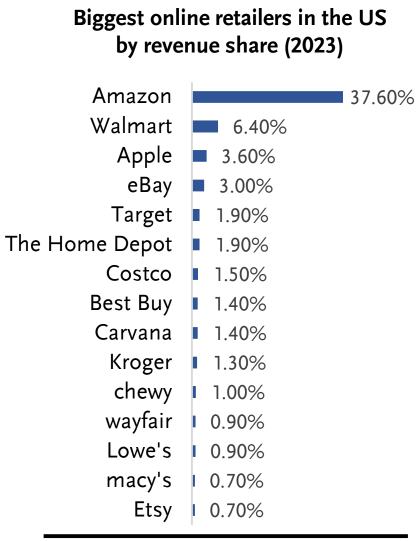
Overall, *Amazon* is living up to its reputation as a leader in B2C e-commerce. *Amazon* has a very strong competitive position in the US and Germany. However, its market shares do not indicate that *Amazon* has exorbitant market power in these countries[3]. **Figure 6** shows that the market share of the top 10 online stores in Germany increased significantly in the years up to 2015. Since then, the top 10 market share has more or less stagnated. This suggests that while the B2C e-commerce market experienced significant market concentration during its initial growth phase, this trend toward greater market concentration now seems to have slowed (at least in Germany).

[3] The Antitrust Division of the US Department of Justice uses the Herfindahl-Hirschman Index (HHI) for measuring market concentration. The Department of Justice (2024) specifies that the *"agencies generally consider markets in which the HHI is between 1,000 and 1,800 points to be moderately concentrated, and consider markets in which the HHI is in excess of 1,800 points to be highly concentrated"*. If we calculate the HHI based on the 15 market share values in **Figure 6** for Germany and the US, we get an HHI of ~512 for the B2C e-commerce market in Germany and an HHI of ~1520 for the B2C e-commerce market in the US. As an example, the calculation for the US market is as follows: $37.6^2 + 6.4^2 + 3.6^2 + 3^2 + 2 * 1.9^2 + 1.5^2 + 2 * 1.4^2 + 1.3^2 + 1^2 + 2 * 0.9^2 + 2 * 0.7^2 + (35.8/0.7) * 0.7^2 = 1520.42$ These are mathematical upper bounds. If the market shares of all e-tailers in the B2C e-commerce market were known, the HHI would be a little bit lower.

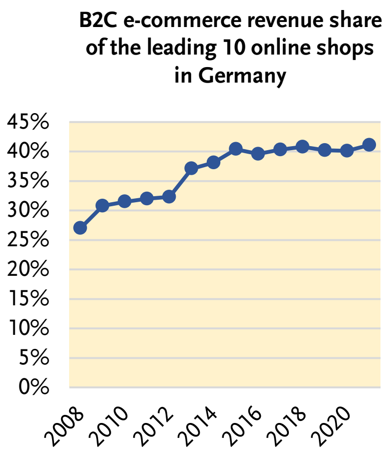
This market development appears to be in line with Carroll's (1985) *resource partitioning model*, which posits that an industry dominated by economies of scale will evolve so that there are only a few generalist firms (e.g., *Amazon* and *Otto* in Germany) and many smaller, but healthy specialist firms. Consequently, the Federal Trade Commission (2024) (FTC) does not accuse *Amazon* of having a monopoly position in the B2C e-commerce market in general, but only in the *superstore* segment (as the FTC calls it) and in the online marketplace segment (more on electronic marketplaces in **Section 2.5** below).

Indeed, such a perspective on the B2C e-commerce market can be quite reasonable. According to Carrol (1985, p. 1272), *"Resource partitioning makes markets in equilibrium appear as though generalist and specialist organizations operate in entirely distinct resource spaces"*, that is, there is only subdued competition between generalist and specialist firms, and the generalist oligopoly (or monopoly) has a very comfortable competitive position. While this is not necessarily a problem for specialist e-tailers, it is potentially detrimental to consumer welfare, and it also becomes a problem when a specialist e-tailer or a new market entrant tries to penetrate the so-called *big middle,* which is often the most obvious option for more revenue share in retailing (Levy et al., 2005). In **Figure 6** we see that in Germany, *Amazon* has about twice the revenue scale of the *Otto Group*, the only other general merchandise retailer of significant size (which controls several other online shops in addition to *otto.de,* such as *bonprix.de, aboutyou.de,* and *baur.de*). And the *Otto Group,* in turn, has about twice the revenue scale of the *MediaMarktSaturn Retail Group* (a consumer electronics specialist). If we assume that revenue earned is also more or less indicative of the number of items sold, this difference in revenue also translates into an equivalent difference in logistical scale. In **Section 3**, I will give some examples of what *'twice as big'* means in terms of economies of scale in B2C e-commerce logistics.

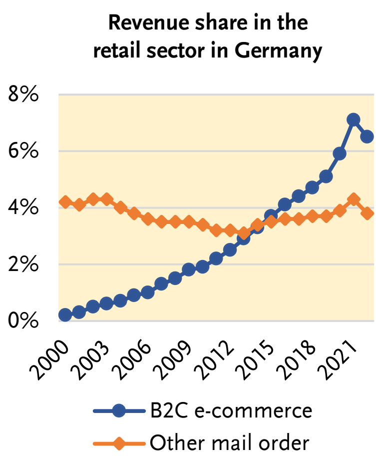
Source: EHI Retail Institute, 2022a, as cited in Statista, 2022a, in combination with EHI Retail Institute, 2022b.



Source: eMarketer, 2023



Source: EHI Retail Institute, 2022b, as cited in Statista, 2022b.



Source: IfH Köln, 2023, as cited in Statista, 2023a.

Figure 6 Statistics about the B2C e-commerce markets in Germany and the US

Sources: Amazon, 2023b, as cited in Statista, 2023b; and Janski, 2021.

Figure 7 Locations of Amazon's fulfillment centers in Germany

Amazon has invested heavily in its logistics network right from the start. In 2017, Zhang et al. (2019) collected data on the promised delivery lead times of *Amazon* and competing e-tailers in the US market. They found that *Amazon*, on average, promised a delivery lead time of 1.92 days, while competing e-tailers promised, on average, a lead time of 4.81 days. In Germany, *Amazon* now operates 20 fulfillment centers (i.e., primary warehouses). Their locations are shown in **Figure 7**. *Amazon* is building an increasingly dense logistics network in order to be able to offer increasingly faster delivery lead times.

Interestingly, Berlin and its surrounding region do not currently have a fulfillment center. Until mid-2023, *Amazon* operated a fulfillment center in Brieselang, Potsdam, just outside of Berlin. *Amazon* justified the closure of this fulfillment center by saying that the building is outdated. One can only speculate whether the closure is also related to the fact that *Amazon* has in recent years built three new fulfillment centers in Poland, close to the German border and right next to major freeways leading to Berlin (see **Figure 7**; Janski, 2021). However, based on the insights from my PhD thesis, I believe that in the long run, the advantage of lower wages in Poland will not outweigh the value of faster delivery through spatially closer warehousing in Germany.

Despite the density of *Amazon's* logistics network, some product categories remain challenging for *Amazon* (and other e-tailers as well). Not every product category is equally suitable for B2C e-commerce. Grocery stores, for example, are difficult to replace with mail-order retailing. Recently, more and more e-tailers have entered the grocery delivery market, but with little success so far. Many grocery items require refrigeration, which makes logistics comparatively expensive. Moreover, customers usually need to be present during delivery, which is inconvenient. In addition, the density of brick-and-mortar grocery stores is very high. This means that customers often only need to travel short distances to meet their consumption needs. This is a major hurdle for e-tailers entering the market. Logistics, and in particular the delivery lead time, is often a decisive factor in why a product category succeeds or fails in mail order versus brick-and-mortar retail (Forman et al., 2009).

A recent development are so-called *on-demand delivery services* (e.g., the companies *Flink* and *Gorillas*), which utilize many small hyper-local grocery warehouses and two-wheeled delivery drivers to be able to serve their customers within a short delivery lead time. Such a business model certainly presents unique challenges, but at its core there is still the typical trade-off between inventory management efficiency and fast delivery lead times that is so important in B2C e-commerce in general. Indeed, I would argue that the location-inventory model in my *Paper VI* is very well suited for an application in the on-demand delivery industry. The success of on-demand delivery services depends primarily on their scale. Only when their operational scale exceeds a critical threshold can these services compete with brick-and-mortar grocery stores in terms of cost.

Another phenomenon in B2C e-commerce that is strongly dominated by economies of scale is the rise of marketplace platforms. I will discuss the importance of these platforms for the B2C e-commerce market in the following section.

2.5. The trend toward marketplace platforms

The B2C e-commerce marketplace model is successful. The largest e-tailers all operate a marketplace. But not all marketplaces are created equal. A distinction must be made between pure and hybrid marketplaces (Hagiu & Wright, 2013). In a pure marketplace model, the marketplace provider only provides the infrastructure to match supply from third-party sellers with demand from customers. In a hybrid marketplace model, the marketplace provider is also a first-party seller on its own marketplace.

The hybrid marketplace model offers many advantages to the marketplace providers, including an information advantage over third-party sellers, the ability to prioritize its own offerings in the online store, and the ability to reserve the most profitable products for its own e-tailing operations. But it does not even have to be self-preferencing. Often, marketplace customers simply prefer to buy from the marketplace provider rather than from unknown third-party sellers, as we showed in our

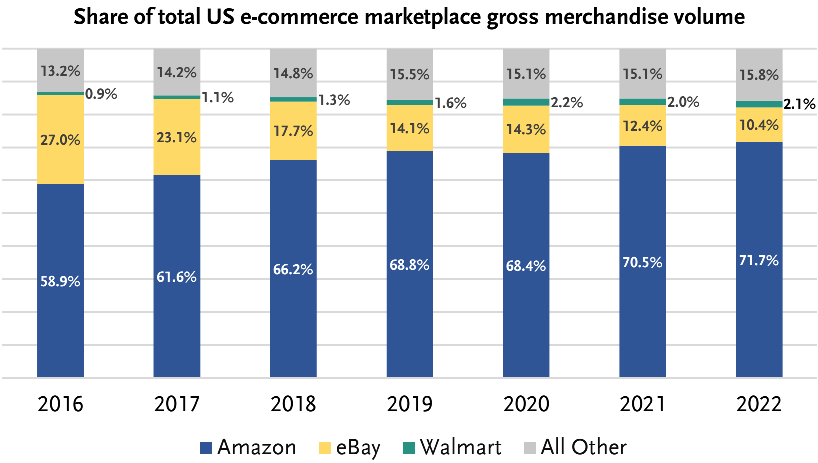
Paper IV. In *Paper V*, we showed what this advantage means for the marketplace providers in the context of inventory management.

However, a marketplace provider should not give itself too much of an advantage, or else the marketplace will become unattractive to third-party sellers. It is a delicate balancing act of coopetition, which ideally should keep itself in check. Nevertheless, the hybrid marketplace model does become a problem for competition in B2C e-commerce if one marketplace has such a high market share that the e-tailers cannot afford not to offer their goods on that marketplace, even if the conditions for them are very poor. The Federal Trade Commission (2024) argues that in the US, *Amazon* has monopoly power in the market for online marketplace services. They cite data (see **Figure 8**) from *eMarketer Insider Intelligence*, which estimates that *Amazon* had a ~72% market share of the e-commerce marketplace industry in 2022 (Federal Trade Commission, 2024, p. 70).

Given the so-called *flywheel effect*, it is not surprising that the market for online marketplace services converges to an oligopoly or even a monopoly. The more customers a marketplace has, the more attractive it is to third-party sellers. And the more third-party sellers a marketplace has, the broader the assortment offered and the lower the prices (due to increased competition among third-party sellers). And the broader the assortment and the lower the prices, the more attractive the marketplace is for customers. Given the flywheel effect, it also makes sense that many marketplaces have emerged from large e-tailers (e.g., the marketplaces of *Amazon*, *Walmart*, *Otto*, *Zalando*, *MediaMarktSaturn*, *JD.com*, ...). Only if an e-tailer's own online shop is sufficiently successful can it persuade third-party sellers to participate in a new marketplace. A direct entry into the online marketplace segment is likely to be very difficult.

Online marketplaces are similar to price comparison websites (e.g., *Google Shopping* and *idealo.de* in Germany). In general, the Internet makes it easy to find and compare offers from different e-tailers, and both marketplaces and price comparison websites have further strengthened this advantage of B2C e-commerce over brick-and-mortar retail. That is, reducing the customer's search costs is a value proposition of both marketplaces and price comparison sites. As different e-tailers often offer the same or similar products, competition on marketplaces and price comparison websites is tough and often based on hard offer characteristics such as price and delivery lead time. When shopping on a marketplace or a price comparison website, the customer does not experience the retailers' websites, at least not initially, and therefore is not/less influenced by the shop design or the curated assortment of an e-tailer. In terms of soft differentiation of the offers, only a few features remain, such as a star rating of the e-tailer (see also my *Paper II* for this and the following paragraph).

As small and large e-tailers compete directly on marketplaces and price comparison websites, and because the competition on these websites is much more direct, based on hard offer characteristics such as prices and delivery lead times, large e-tailers have an outsized advantage on these websites due to economies of scale that lower costs and/or result in faster delivery lead times. If the trend toward more sales via marketplaces



Source: eMarketer Insider Intelligence, n.d.,
as cited in Federal Trade Commission, 2024, p. 70.

Figure 8 Statistics about the US market for online marketplace services continues, it is likely that logistics, with its hard, easily measurable service characteristics (e.g., delivery lead times, lenient and easy returns) will become even more important in B2C e-commerce. In addition, if an e-tailer is out of stock (zero units in inventory), the barrier to simply ordering the product from a competing e-tailer on the marketplace is very low. This means that a customer is probably less likely to wait for the product to become available again at the e-tailer without stock. This means that an out-of-stock situation would be more likely to result in a lost sale. This is another reason why logistics, in this case inventory management, would become more important. As already mentioned in **Section 2.2** (p. 37), small e-tailers should therefore offer products that are not easily substitutable. This is another theoretical argument why in the future the B2C e-commerce market is likely to be increasingly divided into a few large generalist e-tailers and many small specialist e-tailers.

Looking at the B2C e-commerce market, it appears that online marketplaces are significantly more successful than price comparison websites. Although online marketplaces and price comparison websites are similar, it can be argued that marketplaces offer critical value propositions to customers that price comparison websites cannot provide. When shopping on a price comparison website, if you want to take a closer look at an offer, you will be redirected to the online stores of the various e-tailers. Conversely, when you shop at a marketplace, you stay on the marketplace website, which is more convenient. In the case of a price comparison website, the purchase contract and payment are processed via the individual e-tailers. In the case of a marketplace, the purchase contract and payment are usually processed via the marketplace provider, which is perceived as more convenient and secure. That is, the lack of trust in unknown e-tailers is alleviated by the marketplace model and its services (Pavlou & Gefen, 2004).

Most online marketplaces are not as simple as an ordinary weekly farmers' market, where the marketplace is essentially just that, a place for vendors to offer their goods. Instead, an online marketplace can be understood as a platform within a business ecosystem that offers complementary services as part of its value proposition (Felch & Sucky, 2023;

Felch et al., 2022; Goertler et al., 2023). Arguably, some of the most important complementary services of B2C e-commerce marketplaces are logistics services, such as *Fulfillment by Amazon*. Several other major online marketplace providers also offer similar fulfillment services (e.g., *Walmart, Zalando, Otto, JD.com*). These fulfillment services stand for fast and reliable logistics. If a third-party seller uses these services, this is usually clearly highlighted on the marketplace. As we showed in our *Paper IV*, these fulfillment services are an important tool for increasing customers' trust in third-party offerings and *Amazon* has perfected this with its *Prime* subscription (see also our *Paper III*). And because many marketplace providers also *logistically operate* their own marketplace fulfillment services, their logistical scale also benefits from third-party offerings that use these fulfillment services, making them even more competitive. Hybrid marketplace providers can thus increase their logistical scale and make their direct competitors (i.e., other e-tailers) pay for it.

Ivanov et al. (2022) identified *Fulfillment by Amazon* as a stepping stone toward a platform-based supply chain-as-a-service system. Papert et al. (2024, p. 4982) identified platform-based supply chain management (SCM) as a future evolution of the SCM concept. They specify the objective of platform-based SCM as the "creation of structural flexibility or adaptability [...]". Similarly, Ivanov et al. (2022) emphasize that dynamic outsourcing and contracting are critical issues in a platform-based supply chain-as-a-service system. Services such as FBA provide dynamic outsourcing and contracting possibilities. While large e-tailers increasingly build their own logistics platforms, small e-tailers outsource and contract logistics platforms from other e-tailers or third-party logistics service providers to be able to compete as small, specialized e-tailers. Small e-tailers often prefer plug-and-play platform solutions (i.e., supply chain-as-a-service; easy outsourcing and contracting). IT infrastructure and business management platforms, such as the one offered by the company *Shopify*, and logistics outsourcing platforms, such as the one offered by the company *Flexport*, seem to be particularly important in this context. Combining these platforms makes it possible to manage two of the most important parts of an e-tailer's value chain (i.e., the online shop and logistics; see **Figure 3**) in an efficient and integrated manner. This is probably the most viable way for small, specialized e-tailers to remain attractive in the competitive B2C e-commerce market.

3. Economies of scale in B2C e-commerce logistics

As already mentioned, economies of scale play an important role in B2C e-commerce, not only in logistics (IT infrastructure, for example, also often benefits greatly from economies of scale) but especially in logistics. Economies of scale are also an important topic for this PhD thesis.

3.1. Defining economies of scale

Despite the importance of economies of scale and the attention given to the topic by many highly esteemed people in the field of economics, there is still no common understanding of what is meant by the term *economies of scale*. Some scholars define the term narrowly, focusing primarily on the indivisibility of fixed costs. These fixed costs (e.g., research and development costs and machinery/equipment costs) are spread over more units, the more units are produced, i.e., the bigger the scale of the output. Other scholars define the term much more broadly. Probably the most thorough discussions of this topic to date have been given by Robinson (1931), Pratten and Dean (1965), and Silberston (1972). Typically, the term *economies of scale* refers to a reduction in unit costs with increasing scale. However, there are questions about what is meant by *unit* and *scale* (i.e., "scale of what?"; Silberston, 1972, p. 371).

Notably, the classical economic literature often considered the costs per *unit produced*, whereas management science would seek to minimize the costs per *unit sold*. Consequently, management science would also consider penalty costs, for example, due to customer dissatisfaction. While this aspect is often neglected in the economics literature, it is very important in B2C e-commerce logistics.

What is meant by *scale* depends on the concrete context. In the following sections, it will become clear that also in B2C e-commerce logistics, different economies of scale are based on different quantities. For example, the scale could be the demand an e-tailer receives for a product per unit of time, or, in the context of a warehouse, the total number of parcels packed per unit of time.

In this PhD thesis, the term *economies of scale* is used very broadly and incorporates many different effects. Over the years, various terms have been used (sometimes inconsistently) to describe different types of economies of scale. Examples include economies of massed reserves (e.g., Mulligan, 1983; Robinson, 1931), which are known as *risk pooling* in operations research (e.g., Berman et al., 2011), economies of increased dimensions (e.g., Pratten & Dean, 1965), economies of specialization (e.g., Pratten & Dean, 1965), economies of scope (e.g., Panzar & Willig, 1981; Teece, 1980), economies of density (e.g., Caves et al., 1984; Harris, 1977), economies of agglomeration (e.g., Goldstein & Gronberg, 1984), the learning effect/curve (e.g., Pratten & Dean, 1965; Wright, 1936; Yelle, 1979), and the network effect (e.g., Katz & Shapiro 1994). I refrain from making such differentiations.

When talking about economies of scale, it is important to remember that there are also *diseconomies* of scale, such as coordination costs. These diseconomies of scale are one of the reasons why there is an upper limit to the optimal size of a firm (Bain, 1954; Robinson, 1934; Stigler, 1958). Diseconomies of scale also exist in B2C e-commerce logistics, but they appear to be comparatively weak and will therefore not be discussed in detail below. In logistics, the general rule is that the bigger the scale (e.g., the more demand), the better. In this context, it is important to differentiate between the logistical processes within a company and the company as a whole[4]. A freight carrier or an e-tailer, when viewed in their entirety, may well be subject to significant diseconomies of scale even at moderate firm sizes. Nevertheless, the logistics processes within

[4] It is difficult to measure economies of scale in logistics companies using conventional methods, because effects such as risk pooling and economies of density, which are otherwise rather exotic in economics literature, play a decisive role in logistics companies (Oum & Zhang, 1997; Xu et al., 1994). Xu et al. (1994, p. 282), found that *"Previous econometric investigations of size effects have failed to allow output characteristics to vary as firm size increases. [...] Our results indicate that larger firms do not have a cost advantage due to size directly, but that size indirectly influences the operating characteristics of the carriers in such a way that their unit costs are lowered"*. The following sections may be of value in this regard, as I discuss the operating characteristics of B2C e-commerce logistics and how they are affected by greater scale.

these companies are likely to experience significant diseconomies of scale only at much bigger scales, if at all.

The purpose of the following sections is to introduce the reader to some of the most important economies of scale in B2C e-commerce logistics. However, this introduction is certainly not comprehensive. For example, there are obvious economies of dimensions when building a new warehouse (a building requires disproportionately less wall length the larger the area of the building becomes). Instead, the focus will be on the *operational processes* that are typically associated with B2C e-commerce logistics, i.e., merchandise procurement, inbound logistics, inventory holding, picking and packing, outbound logistics, and delivery (see **Figure 3**).

3.2. Scale effects in inventory management

Papers V and *VI* both feature an inventory management model. In *Paper V*, two e-tailers (e.g., *Amazon* and a third-party seller on *Amazon's* marketplace) compete within a newsvendor model. It is an educated guess that *Amazon*, as the larger e-tailer and marketplace provider, usually enjoys more scale advantages. In this section I will give a brief overview of the scale effects that exist within a newsvendor model.

In the problem of *Paper VI*, an e-tailer must decide how many stocking locations to use for a product. An e-tailer with more demand will optimally use more stocking locations than a smaller e-tailer with less demand. This decision problem is directly affected by the economies of scale within inventory management. In this section I will therefore also give a brief overview of the scale effects that exist within a continuous review, order quantity inventory model ((s, Q)-model) such as the one used in *Paper VI*.

However, before I discuss these two types of models, I will first start with the economic order quantity model (EOQ-model), arguably the simplest inventory management model. While the newsvendor model and the (s, Q)-model take into account the stochastic uncertainty of customer demand, the EOQ-model assumes that demand is deterministic.

Nevertheless, even within a deterministic model, powerful economies of scale effects exist.

3.2.1. The economic order quantity model

The EOQ-model assumes that demand is deterministic and stationary. This means that customers always order the same quantity of the product per period, indefinitely, over and over again. The period length can be selected as desired. One example would be that customers order exactly 14 units of the product every week (or 2 units per day, or 28 units every two weeks, ...). This implicitly assumes that the demand is evenly distributed over time. Furthermore, we will simplify the problem by imposing a continuous space. Often, demand is discrete in nature. For example, a customer can order only 1, 2, 3, ... chairs, but not half a chair. By allowing for fractional demand, we make the model mathematically simpler without losing much generality.

Moreover, it is also assumed that the replenishment lead time (the time between ordering from a supplier and restocking the items) is deterministic, for example, always 3 days. For the sake of exposition, and without loss of generality, we will restrict ourselves in the following to the case of instant replenishments, that is, no replenishment lead time.

If neither the demand nor the replenishment is uncertain, it is optimal to carry no safety stock. Furthermore, it is assumed that intentional backordering is not allowed, that is, every demand is instantly fulfilled. Thus, the inventory level will always go down to exactly zero units, at which point a replenishment shipment will arrive, thus raising the available inventory to some level. This system is depicted in **Figure 9**.

The goal of the EOQ problem is to determine the optimal replenishment order quantity (the economic order quantity) x^* that will be reordered when the inventory level decreases to zero. Consistent with the processes found in the value chain of an e-tailer (see **Figure 3**), the EOQ-model balances two types of costs. Fixed replenishment costs are incurred every time a replenishment order is placed and received. And inventory holding costs for every piece stored per time unit. These two costs are minimized to find the economic order quantity. The higher the fixed replenishment costs, the higher the EOQ because then the high reorder costs are spread across more pieces of the product. The higher the inventory holding costs, the smaller the EOQ because then the average inventory level is reduced (see **Figure 9**).

Decision variable (to be optimized):

- $x \in \mathbb{R}^{+}$ = the order quantity
  (e.g., 200 pieces per replenishment order)

Parameters (given):

- $D \in \mathbb{R}^{+}$ = the demand rate of the product
  (e.g., 14 units per week)
- $R \in \mathbb{R}^{+}$ = the costs per replenishment of the product
  (e.g., 100€ per replenishment)
- $h \in \mathbb{R}^{+}$ = the inventory holding costs per piece stored per time unit (e.g., 0.50€ per piece per week stored)

For the sake of brevity, I would like to refer the reader to Sucky (2022, pp. 223–236) for the complete mathematical model. Solving the model yields the following economic order quantity:

$$x^{*} = \sqrt{\frac{2 * D * R}{h}}$$

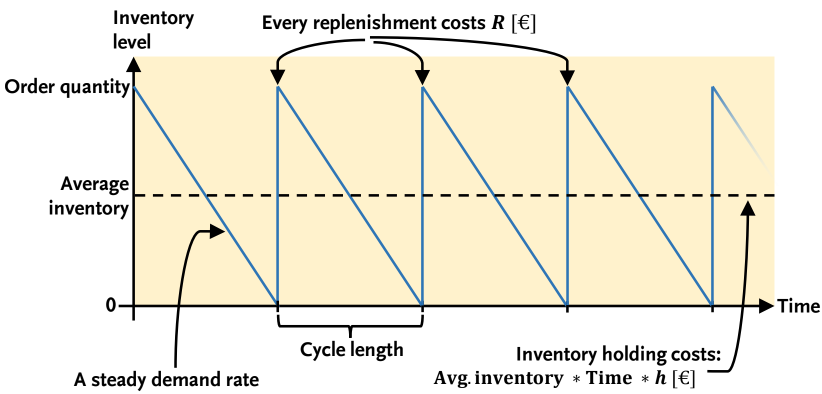


Figure 9 Basic properties of the economic order quantity model

The optimal costs per piece (c_p^*) are given by:

$$c_p^* = \sqrt{\frac{2 * h * R}{D}}$$

We can now observe the following. When the demand rate (D) increases by a factor of $k \in \mathbb{R}^+$, then the economic order quantity increases by a factor of $\sqrt{\frac{2*k*D*R}{h}} / \sqrt{\frac{2*D*R}{h}} = \sqrt{k}$. For example, in case of a doubling of the demand rate ($k = 2$), the economic order quantity only increases by a factor of $\sqrt{2} \approx 1.4142$. The same is true for the average inventory level.

The optimal avg. costs per piece, on the other hand, decreases by a factor of $\sqrt{\frac{2*h*R}{k*D}} / \sqrt{\frac{2*h*R}{D}} = \sqrt{\frac{1}{k}}$. For example, in case of a doubling of the demand rate ($k = 2$), the optimal avg. costs per piece decrease by a factor of $\sqrt{½} \approx 0.7071$. The same is true for the reorder and inventory holding costs. The cycle length also shortens by a factor of $\sqrt{1/k}$.

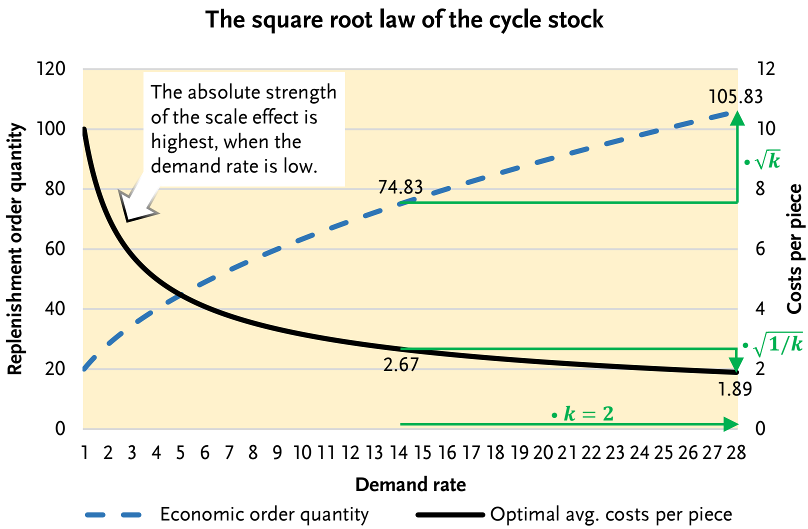
It may also be noted that when the demand rate goes toward infinity, the average inventory replenishment and holding costs per piece go toward zero. Conversely, when the demand rate goes toward zero, then the costs per piece go towards infinity. **Figure 10** exemplifies this. These economies of scale are called the square root law of the cycle stock. The absolute cost effect due to the square root law of the cycle stock depends on the replenishment and inventory holding cost factors (R and h). That is, the more costly the replenishment process of a product, or the more costly the storage of a product, for example, because it needs to be cooled, takes up a lot of space, or ties up a lot of capital, the higher the absolute economies of scale. Thus, according to the EOQ model, smaller e-tailers should preferably offer products that are easy to source (low R), inexpensive, and easy to store (low h).

It is also interesting to note that, due to the convex form of the curve, the absolute scale effect is greater when the demand rate is low. Small e-tailers therefore benefit (or suffer, depending on the viewpoint) disproportionately from the square root law of the cycle stock.

The economic order quantity model shows that there are strong deterministic economies of scale in B2C e-commerce logistics, even considering only the most basic processes. Because of their very basic nature they exist in many different inventory management systems. Similar economies of scale exist when demand fluctuates stochastically, as is almost always the case in B2C e-commerce.

3.2.2. The newsvendor model

The newsvendor model is a stochastic inventory management model with only one selling period. Before the selling period, retailers must decide on how much of a product they want to order. It is assumed that no additional units can be reordered during the selling period. All ordered units arrive before the selling period and are sold during the selling period. Demand for the product during the selling period is uncertain, that is, the retailer must order without knowing the exact future



Numerical example: $R = 100$, $h = 0.50$

Figure 10 Economies of scale within the economic order quantity model demand. Demand may fluctuate arbitrarily. The expected demand could be high or low, it could be relatively certain (low standard deviation) or uncertain (high standard deviation). The retailer must estimate the future demand and base its decision on the estimated demand distribution. This system is depicted in **Figure 11**.

Decision variable (to be optimized):

- $x \in \mathbb{R}^+$ = the order quantity (e.g., 200 pieces)

At the end of the selling period, the retailer either has leftover stock because it ordered more units before the selling period than were sold during the selling period, or a shortage because the order quantity was not sufficient, and the retailer went out of stock at some point during the selling period.

Furthermore, it is assumed that the product loses a significant portion of its value after the selling period. More specifically, it is assumed that the

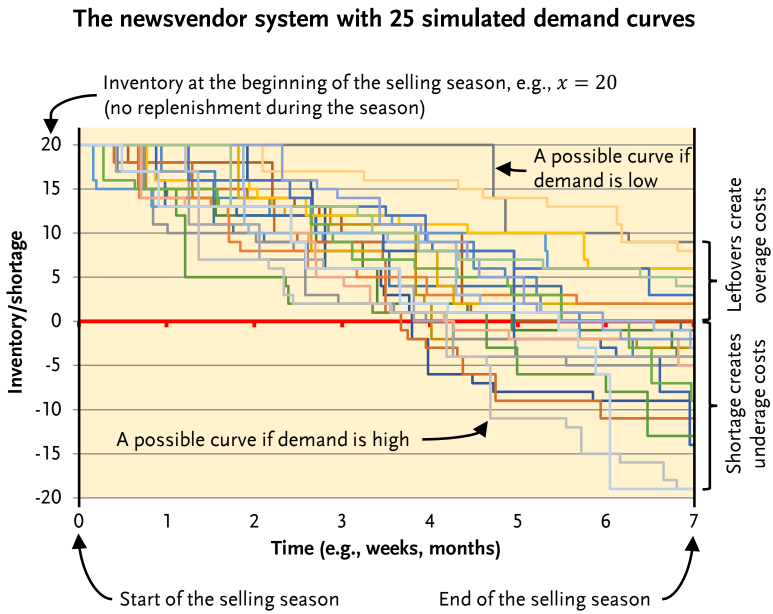


Figure 11 Basic properties of the newsvendor model

selling price drops below the purchase price. Therefore, leftover stock creates overage costs.

A shortage on the other hand creates underage costs because the retailer could have sold more units if it had ordered more units. The underlying assumption in this case is that the selling price during the selling period is greater than the purchase price.

In a surprisingly large number of situations (for example in fashion retailing), the newsvendor model is a pretty good approximation of reality. Moreover, because of its simplicity, it is also often used to demonstrate the behavioral dynamics of systems with uncertainty (e.g., Dobhan & Oberlaender, 2013). In *Paper V* of this PhD thesis, we used a newsvendor model to describe inventory competition dynamics on electronic marketplaces.

Parameters of the newsvendor model (given):

- $c \in \mathbb{R}^+$ = the purchase price (e.g., 10€ per piece)
- $r \in \mathbb{R}^+$ = the selling price during the selling period (e.g., 30€ per piece)
- $v \in \mathbb{R}$ = the selling price (or salvage cost) after the selling period (e.g., 5€ per piece, or -3€ per piece in case of salvage costs)

Restrictions:

- $r > c$ (so that it is profitable to retail the product at all)
- $c > v$ (so that each piece leftover after the selling period costs money)

Defined quantities:

- $c_o = c - v$ = the overage costs
- $c_u = r - c$ = the underage costs

The goal is to minimize the sum of the expected overage- and underage costs. For the sake of brevity, I would like to refer the reader to Thonemann (2015, pp. 270–272) for the complete mathematical model. Solving the model yields the following optimal critical ratio:

$$F(x^*) = \frac{c_u}{c_o + c_u} = CR$$

, with:

- $0 \leq F(\square) \leq 1$ = the cumulative distribution function of the expected demand distribution

The critical ratio ($CR = \frac{c_u}{c_o+c_u}$) takes values between $0 < CR < 1$. It can be interpreted as a percentage (e.g., $CR = 0.8 = 80\%$). The formula $F(x^*) = \frac{c_u}{c_o+c_u}$ states that the order quantity x should be chosen so that with a certain probability, which is determined by the CR, no shortage will occur during the selling period. **Figure 12** contains a graphical example.

Economies of scale arise in the newsvendor model when two, at least partially, statistically independent demand flows are pooled, or, formulated differently, when the intensity of a demand stream increases and the additional demand is, at least partially, statistically independent from the other demand. Excluding negative correlations, the so-called *risk pooling* effect is strongest when all demand streams are completely uncorrelated from each other. That is, when the behavior of one customer does not influence the behavior of another customer.

The mathematical reason for the beneficial risk-pooling effect lies in the subadditivity of the square root, which is used to convert the variance of a distribution σ^2 (e.g., 49) into the standard deviation σ (e.g., 7). When two statistically independent distributions are pooled (a so-called *convolution* of two probability distributions), the variance of the pooled distribution is the sum of the variances of the individual distributions ($\sigma^2_{\text{pooled}} = \sigma^2_{\text{dist1}} + \sigma^2_{\text{dist2}}$; e.g., $98 = 49 + 49$). The subadditivity of the square root then results in the standard deviation of the pooled distribution $\sigma_{\text{pooled}} = \sqrt{\sigma^2_{\text{pooled}}}$ (e.g., $9.8995 = \sqrt{98}$) being smaller in relative terms. In the example from **Figure 12**: $\frac{9.8995}{40} < \frac{7}{20}$

The following may be a more tangible explanation: It is relatively improbable that the intensities of all demand flows are high (or low) at the same time. Taken together, the different intensities of the demand flows therefore often cancel each other out, and, as a result, the sum of all demand fluctuates less in relative terms.

Figure 12 exemplifies this with a concrete numerical example. It is evident, that the pooled distribution must have a mean of $\mu = 40$, which is just the sum of the means of the two unpooled distributions. Because we pool two identical distributions, the new mean is therefore twice as high

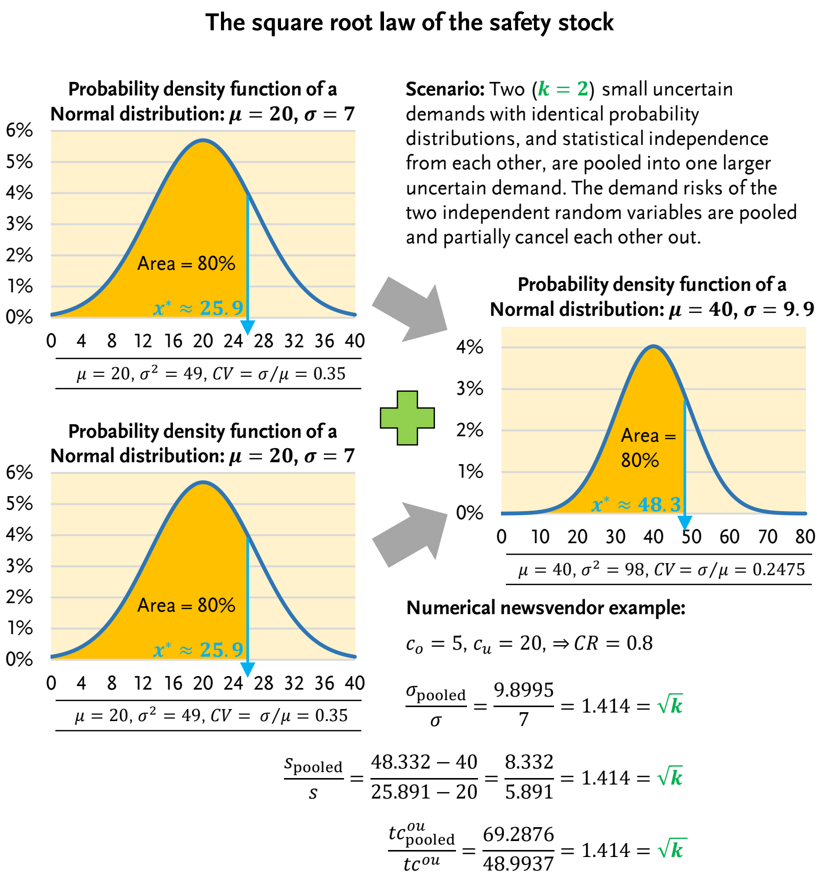


Figure 12 Economies of scale within the newsvendor model

as the means of the two unpooled distributions. The variance is also twice as high ($\sigma^2_{\text{pooled}} = 98$). The standard deviation, however, is not twice as high. With $\sigma_{\text{pooled}} = 9.8995$, it is only 1.414 times as high. Recall, that $\sqrt{2} = 1.414$ and that $\sigma_{\text{pooled}} = \sqrt{\sigma^2_{\text{pooled}}}$. This is the subadditivity effect of the square root. Therefore, the coefficient of variation (CV) also decreased from $CV = 0.35$ to $CV_{\text{pooled}} = CV/\sqrt{2} = 0.24749$. A lower coefficient of variation means that the distribution of the expected demand has become more certain in relative terms. Graphically, this is reflected in the fact that the distribution has become steeper and narrower.

The more certain the demand is, the better a retailer can plan and the (relatively) lower the estimated costs of over- and understocking. Compare, for example, the optimal order quantity of 25.9 pieces in the unpooled case, with the optimal order quantity of 48.3 in the pooled case. In one case, the retailer should order 25.9 pieces for an expected demand of 20 pieces (a safety stock of $s = 5.9$ pieces), and in the other case, the retailer should order 48.3 pieces for an expected demand of 40 pieces (a safety stock of $s = 8.3$ pieces). The amount of safety stock is directly related to the coefficient of variation and therefore also to the square root law ($8.3/5.9 \approx \sqrt{2}$). This is why the risk-pooling effect is often also called the square root law of the safety stock. Ideally, in a world with no demand uncertainty, the retailer would order 20 pieces for a deterministic demand of 20 pieces. In the newsvendor model, demand uncertainty is the only reason why a retailer should order more (or less) than the expected demand. The more demand uncertainty there is, the more likely it is that under- or overage costs will occur. Thus, demand uncertainty is always bad. This is also known as the cost of uncertainty (Schlaifer, 1959, pp. 256–257). Indeed, the optimal total estimated under- and overage costs (tc^{ou}) also only increase by a factor of $\sqrt{2}$ when the demand rate doubles (assuming no correlation between the demand streams).

Similar to the economic order quantity model, the optimal average estimated over- and underage costs per piece sold decrease to zero with increasing demand. Or conversely, they increase to infinity, as demand decreases to zero. The absolute cost effect due to the risk-pooling depends on the over- and underage cost factors (c_o and c_u) and the coefficient of variation ($CV = \sigma/\mu$) of the demand distribution. According to the newsvendor model, smaller e-tailers should focus their product portfolio on products with relatively certain demand and low overage cost factors.

3.2.3. The continuous review, order quantity (s, Q)-model

The continuous review, order quantity (s, Q)-model combines both the square root law of the cycle stock and the square root law of the safety stock. It is a more complex but also a more realistic model that approximates the practice of inventory management well. While the newsvendor model has one selling period, there are no periods in this model. The product is simply sold until it gets delisted. The inventory level of the product is continuously monitored (reviewed) while customers order the product. Once the inventory of the product falls under a so-called reorder point s (e.g., 80 pieces), a replenishment order of size Q (e.g., 1000 pieces) is started. After a replenishment lead time ℓ (e.g., one week) the replenishment order is ready to be sold, and the inventory level increases. Similar to the economic order quantity model, the inventory level plotted against time creates a sawtooth pattern. This sawtooth pattern is visualized in **Figure 13**. However, this sawtooth pattern is not as regular as in the EOQ model. This is because the (s, Q)-model takes into account that customers can order at random. During some periods customers order a lot of the product and during other periods demand is low.

For the following numerical examples, it is assumed that demand is a Poisson stream, that is, the expected demand per time unit follows a Poisson distribution with intensity D. Furthermore, a backordering system is considered. This means that in the case of a stockout, that is the real on-hand inventory is zero, customers still order but are backordered.

These backorders must wait until replenishments arrive. Thus, the delivery lead time is prolonged in these cases (also see **Figure 3**). As this is undesirable, any extension of the delivery lead time due to backordering

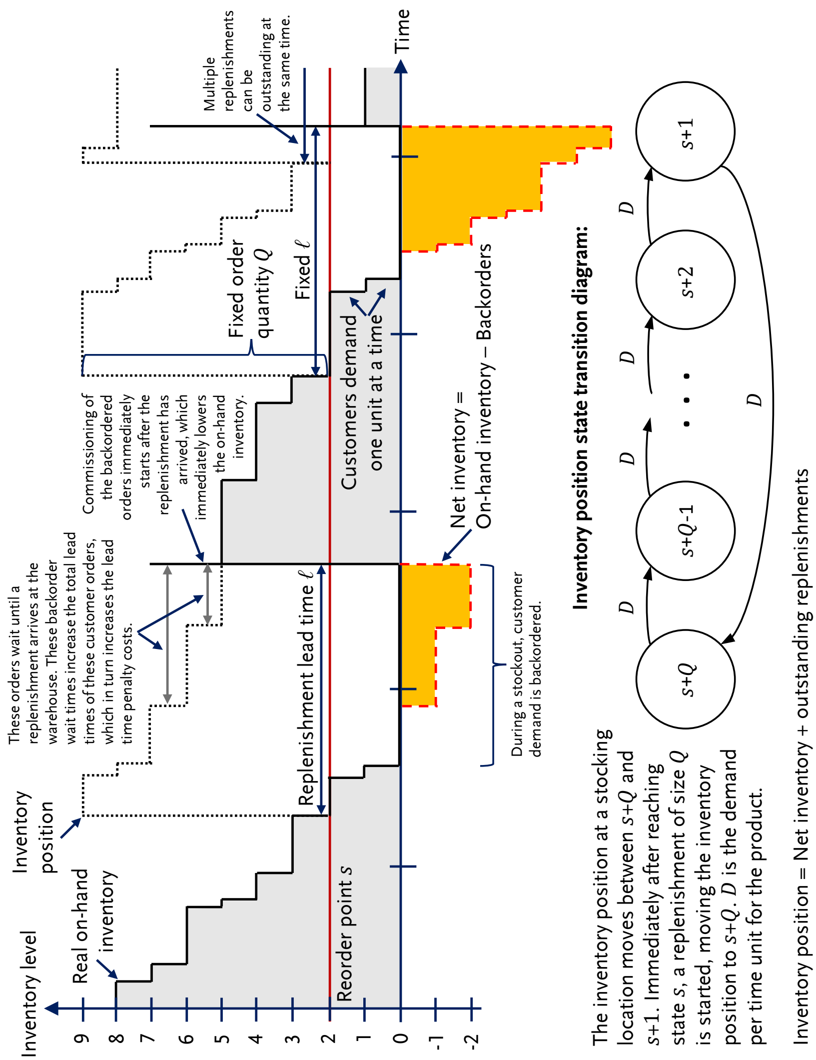


Figure 13 Basic properties of the (s, Q)-model (see also Paper VI)

is subject to penalty costs b. This is the same model I used in my *Paper VI*.

Decision variables (to be optimized):

- $s \in \mathrm{Z}$ = the reorder point (e.g., 80 pieces)
- $Q = 1,2,3, \dots$ = the order quantity (e.g., 1000 pieces)

Parameters of the (s, Q)-model (given):

- $D \in \mathbb{R}^+$ = the probabilistic demand rate for the product (e.g., on average, 14 units per week)
- $R \in \mathbb{R}^+$ = the costs per replenishment of the product (e.g., 100€ per replenishment)
- $h \in \mathbb{R}^+$ = the inventory holding costs per piece stored per time unit (e.g., 0.50€ per piece per week stored)
- $b \in \mathbb{R}^+$ = the penalty costs per piece backordered per time unit (e.g., 1€ per piece per day backordered)
- $\ell \in \mathbb{R}^+$ = the replenishment lead time (e.g., one week)

The goal is to minimize the sum of the expected replenishment, inventory holding, and backorder penalty costs. The trade-off between the replenishment costs and the inventory holding costs is similar to that in the economic order quantity model. The higher Q, the fewer replenishment orders are necessary, but also the higher the average inventory level (i.e., the higher the cycle stock). Additionally, the (s, Q)-model incorporates a trade-off between inventory holding and the risk of backorders. Due to the uncertainty of the demand, it is impossible to eliminate the risk of backorders completely. It can, however, be reduced by raising the reorder point s (which can be interpreted as the safety stock) and the order quantity Q. Economies of scale in the form of the square root law of the cycle stock and the square root law of the safety stock make the balancing of these two trade-offs more efficient.

For the sake of brevity, I would like to refer the reader to Hadley & Whitin (1963, pp. 181–188) for the complete mathematical model. The model cannot be solved analytically in closed form. An algorithm is needed to find the minimum of the convex objective function (Federgruen & Zheng, 1992; Rubalski, 1972). A numerical example of the economies of scale within the (s, Q)-model is shown in **Figure 14**. Since the (s, Q)-model incorporates the concept of risk pooling into the economic order quantity model, it is not surprising that the costs within the (s, Q)-model also behave according to the square root law.

Similar to the economic order quantity model and the newsvendor model, the average inventory and stockout-related costs also decrease to zero with increasing demand. Or conversely, they increase to infinity, as demand decreases to zero. The absolute strength of the economies of scale depends on the replenishment and inventory holding cost factors (R and h) and the backorder time penalty cost factor (b). That is, the more time-critical a product is (i.e., the higher b is), for example, because customers usually need the product on short notice, the higher the absolute economies of scale. Thus, in addition to the learnings from the EOQ model, the (s, Q)-model also indicates that smaller e-tailers should focus on products that are not very time-critical.

Note, that, while the (s, Q)-model has uncertain demand, it assumes that the demand always follows a Poisson distribution. Compared to the newsvendor model, the variability of demand is therefore rather restricted. While an uncorrelated order stream always follows a Poisson distribution, it is, nevertheless, possible that a customer may order two, three, or more pieces of a product per order. In such a case, the number of pieces ordered per time unit would follow a so-called compound Poisson distribution (Axsäter, 2015, pp. 65–67). Because of the varying order sizes, a compound Poisson distribution has a higher variance than an equivalent Poisson distribution with the same demand rate. And the absolute economies of scale are higher the more the order sizes vary. Equivalent to the newsvendor model, we can therefore conclude that smaller e-tailers should preferably offer products that customers typically only buy one piece at a time.

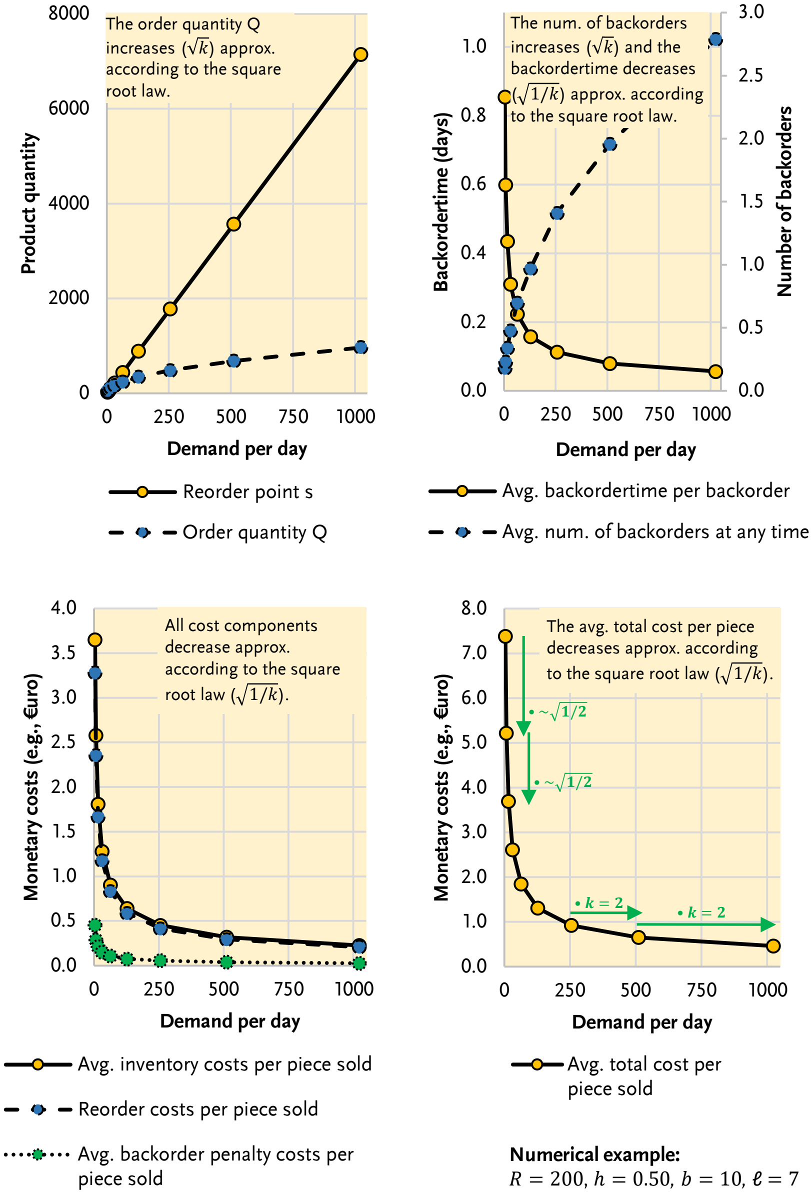


Figure 14 Economies of scale within the (s, Q)-model

Probably less known is the fact that the economies of scale within the (s, Q)-model can be even stronger when the model is subject to a maximum backordertime per backorder constraint. Backorder time costs no longer exist in this modified model (i.e., $b = 0$) as the service level constraint has taken its place. Sometimes it is difficult to determine a sensible backorder time penalty cost factor b because the effect of a backorder depends on customer behavior. Often it is easier to set a service level target, e.g., not being out of stock more often than 5% of the time. This target could be based on the market average service level. A company may choose to perform above, below, or in line with the average. There are several types of service levels. Some are more appropriate than others, depending on the context. One of the most sensible service performance indicators is the average backordertime per backorder. Every backorder is bad for the customer, but a backorder that waits a long time is much worse than a backorder that only lasts a short time. Therefore, the average backordertime per backorder should not be too long, which is why an e-tailer may set a constraint, e.g., ≤1 day. The algorithm that solves this modified (s, Q)-model has to adhere to this constraint while minimizing the sum of replenishment and inventory holding costs. Such optimization is only possible when the average backordertime per backorder is jointly convex in s and Q, which I proved, probably for the first time ever, in my *Paper VI*.

We can observe in **Figure 14** how the average backordertime per backorder decreases according to the square root law. This is in addition to the decrease in costs. An increase in demand therefore simultaneously results in an increase in service levels and a decrease in the average costs per order. It is therefore reasonable to expect that, if the average backordertime per backorder is kept constant, the cost decrease could be even greater. And indeed, this is the case. Depending on the demand intensity an increase in demand can lead to a cost reduction of down to $\sqrt{1/k} * \sqrt{1/k} = 1/k$. That is, if demand doubles ($k = 2$), the average costs per order can decrease by a factor of down to $1/k = 0.5$ (compared to $\sqrt{1/k} = 0.7071$), while the average backordertime per backorder stays the same. This reflects one of the most basic trade-offs when it comes to economies of scale. As demand increases, a company can either reduce costs and maintain the service level, maintain costs and improve the service level, or choose a middle ground. That is why the relative cost decrease is even stronger (down to $1/k$ vs. $\sqrt{1/k}$) when the average backordertime per backorder, i.e., the service level, is kept constant.

3.3. Scale effects in warehouse operations

When a customer buys a product, it is picked from storage and packed into a parcel that is then shipped to the customer (see **Figure 3**). Due to the uncertainty involved in the picking and packing processes it is possible to benefit from economies of scale. Firstly, the flow of customer orders is random. This means that sometimes many customers order in a short period of time and at other times only a few orders arrive. Each order represents at least one picking and packing task. Secondly, the picking and packing tasks also require different lengths of time, for example depending on where the products are stored, how easy they are to pack, and which other products are also to be included in the parcel.

As a result of these uncertainties, picking and packing queues form. The picking queue is just a list of unfulfilled picking tasks. The packing queue on the other hand is a queue of physical products waiting to be packed. From a spatial perspective, the packing queue is therefore more problematic than the picking queue. However, both queues extend the delivery lead time, making them both undesirable. In the following, I will explain how economies of scale help to reduce queues and wait times without increasing costs. Large e-tailers have a huge queuing advantage over smaller e-tailers, especially when fast delivery lead times are required, which we had identified as an important trend in our *Paper I*. In *Papers I* and *II*, we also found that logistics outsourcing is one of the more promising ways for smaller e-tailers to alleviate their scale disadvantage. *Fulfillment by Amazon*, which we researched in *Paper IV*, is one possible outsourcing model. With FBA, *Amazon* offers other e-tailers a service that covers all physical processes from storage to shipping, while ownership of the merchandise remains with the e-tailer. The FBA service is therefore primarily a service for the outsourcing of physical processes that enables e-tailers to offer fast delivery lead times similar to those of *Amazon*. Queueing theory can explain why this type of outsourcing may be sensible.

Figure 15 visualizes some basic properties of queueing models. A queueing model consists of two components: the queue and the servers. Servers could, for example, be robots that pick products or packing stations in which employees pack products. Multiple servers (e.g., packing stations) can work concurrently and every server has approximately the same average process time (e.g., the time needed to pack a parcel). The packing time typically varies from parcel to parcel and follows some probability distribution (e.g., the exponential distribution).

If one or more servers are currently idle, incoming packing tasks (i.e., orders that have been picked), will not wait in the queue but are sent immediately to one of the free servers. If, however, every server is currently occupied, packing tasks must wait in the queue until one of the servers has finished their current task. The queueing discipline is usually first-come first-serve, meaning that the task that has waited the longest is the task that is next in line for fulfillment.

The incoming stream of tasks is typically also random. In the case of B2C e-commerce, it is evident that the arrival of customer orders is mostly random. The packing stations also experience a random task stream, as the customer order stream passes through a picking system with random processing times. Although this changes the randomness of the stream, i.e., the probability distribution, the stream is still random.

Parameters of the queueing model (given):

- $\lambda \in \mathbb{R}^+$ = the probabilistic demand rate (e.g., a Poisson distribution with an average of 50 new tasks per hour)
- $\mu \in \mathbb{R}^+$ = the probabilistic processing rate of one server (e.g., a Poisson distribution with an average of 20 product picks per hour, i.e., a picker may pick an average of 20 products per hour)

- $m = 1,2,3, \dots$ = the number of servers that can work concurrently (e.g., three packing stations)

Restriction:

- $m * \mu > \lambda$ (so that the total processing capacity is greater than the demand rate; otherwise, the queue would grow indefinitely)

Due to the randomness, both in processing times and the arrival process of new tasks, the server system is sometimes idle and sometimes over-

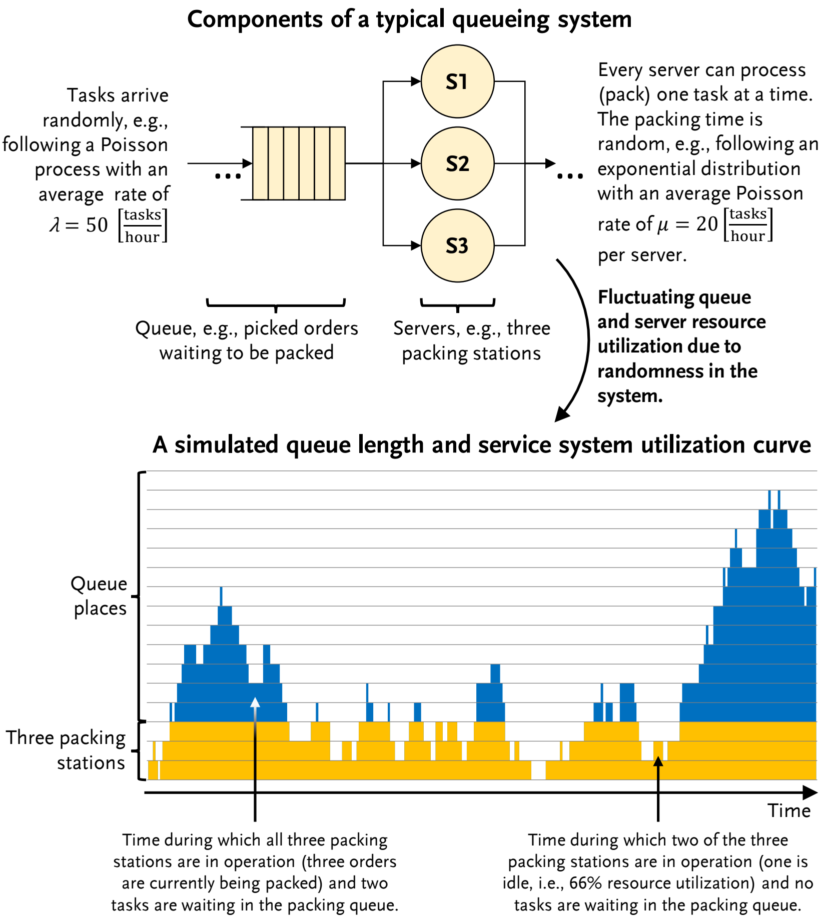


Figure 15 Basic properties of queueing models

whelmed, resulting in a queue. A queueing model has several key performance measures:

- $0 < \alpha < 1$ = the average utilization of the servers (e.g., on average, the servers are occupied 95.24% of the time)
- L_Q = the average queue length (e.g., on average, 18.58 tasks are waiting)
- $L = L_Q + L_S$ = the average number of tasks in the system, with $L_S \leq m$ = the average number of tasks being processed concurrently
- T_Q = the average wait time in the queue (e.g., on average, tasks are waiting 9.29 minutes)
- $T_S = 1/\mu$ = the average processing time (e.g., if, on average, a server can process $\mu = 10$ tasks per hour, the server needs, on average, 1/10 hours = 6 minutes per task)
- $T = T_Q + T_S$ = the average total time in the system (e.g., 9.29 minutes waiting + 6 minutes processing = 15.29 minutes in the system)

Little's Law states that (Little, 1961):

$$L_Q = T_Q * \lambda \text{ and } L_S = T_S * \lambda \text{ and } L = T * \lambda$$

Note, that an average wait time of 9.29 minutes could mean that some tasks may wait more than an hour while other tasks do not wait at all (see also **Figure 15**). Also note, that these are long-run averages based on the assumption of stationary probability distributions. If the queue is emptied regularly, for example because hardly any orders arrive overnight (that is, if the probability distribution is not stationary), these average values will not manifest. However, with the trend towards ever shorter delivery lead times, the relevant time windows are becoming shorter, making the assumption of a stationary probability distribution within these shorter time windows more realistic.

In order to keep this introduction to queueing theory brief, I will not specify a cost minimization objective function for the queueing model. Instead, I want to focus on how the model behaves when certain input parameters are changed, with the goal of revealing the economies of scale that exist in queueing models. I will demonstrate how the service measures improve due to economies of scale. As usual, a company can then choose to benefit from the improved performance or to reduce costs and offer the same level of performance as before.

First, observe that the average utilization of the servers is simply calculated by relating the average demand rate to the average total processing rate of the system:

$$\alpha = \frac{\lambda}{m * \mu}$$

This means that the average utilization is always identical, regardless of how the problem is scaled. If the demand rate numerator λ doubles and the total processing rate denominator $m * \mu$ also doubles (for example by doubling the number of servers m or by doubling the processing rate per server μ), the average utilization does not change. Therefore, the average utilization is not subject to economies of scale. Nevertheless, the average utilization is of course important for a company's profitability. Ideally, an e-tailer wants to have a high utilization so that the investment costs of, for example, a picking robot or a packing station, are spread over many orders. Unfortunately, as we will see in the following (**Figure 16**), the average queue length L_Q and the average wait time in the queue T_Q grow exponentially toward infinity the higher the average utilization.

While the average utilization does not benefit from economies of scale, the average wait time *does* benefit from economies of scale. Therefore, when a large e-tailer increases its average utilization, the absolute increase in the average wait time is not as severe as when a small e-tailer increases its average utilization. This creates a competitive advantage for larger e-tailers because they can run a higher utilization while offering the same service level as smaller e-tailers.

One of the simplest queueing models is a model where both the demand rate (with mean λ) and the processing rate (with mean μ) follow a Poisson distribution and one server exists ($m = 1$). For this so-called M/M/1-

model it is possible to show that the average wait time in the queue is (Hopp & Spearman, 2008, p. 287):

$$T_Q^{MM1} = \left(\frac{\alpha}{1-\alpha}\right) * \frac{1}{\mu}$$

Suppose now that the demand rate λ doubles ($k = 2$). Because $m = 1$ is fixed in this model, the only available scaling option is a doubling of the processing rate μ. As explained above, when λ and μ double, α stays the same. But the term $\frac{1}{\mu_{\text{new}}} = \frac{1}{k*\mu_{\text{old}}}$ decreases by the factor $1/k = 0.5$, and due to the multiplication of the two terms $\frac{\alpha}{1-\alpha} * \frac{1}{\mu}$, the average wait time also decreases by the factor $1/k = 0.5$. Note, that this scale effect simply exists because the orders in the queue are processed twice as fast. **Figure 16** illustrates this model dynamic. The average queue length L_Q does not change because the demand rate λ also doubles.

In practice, the demand rate and especially the processing rate do not always follow a Poisson distribution. For the so-called G/G/1-model, which allows for arbitrary (general) probability distributions, the following good approximation for the average wait time in the queue is available (Hopp & Spearman, 2008, p. 288):

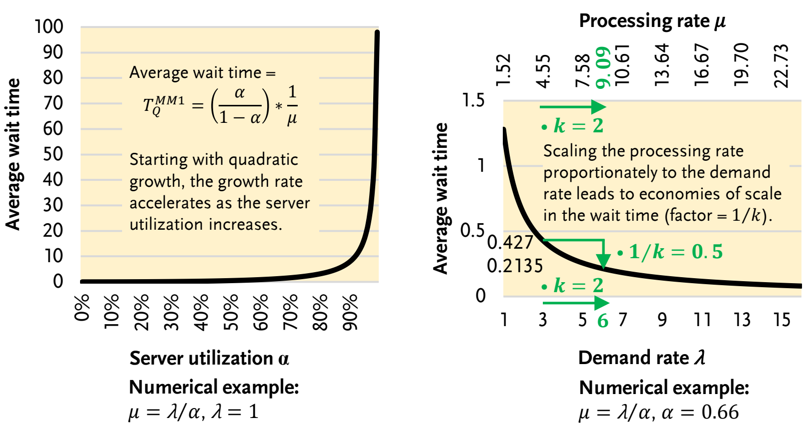


Figure 16 Economies of scale within queueing models

$$T_Q^{GG1} = \left(\frac{SCV_{\text{demand}} + SCV_{\text{processing}}}{2}\right) * \left(\frac{\alpha}{1-\alpha}\right) * \frac{1}{\mu}$$

Since this is only an approximation that performs better in some parameterizations than in others, no generally valid statements can be derived from this formula. Nevertheless, the approximation is often very good, especially when the wait times are high and therefore important.

SCV denotes the squared coefficient of variation and is defined as follows:

$$SCV_{\text{demand}} = \frac{\sigma^2_{\text{demand}}}{\lambda} \qquad SCV_{\text{processing}} = \frac{\sigma^2_{\text{processing}}}{\mu}$$

, with σ^2 being the variance of the probability distributions.

Therefore, according to this approximation, the average wait time increases the higher the dispersion (SCV) of the probability distributions of the demand rate and the processing rate. Interestingly, and unlike the coefficient of variation CV (see **Subsection 3.2.2**), the SCV is not affected by scale. This is of course assuming statistical independence between the demand arrival/processing events. Given statistical independence, the SCV is always the same, no matter how high the demand/processing rate is, because the variance σ^2 grows proportionately to the mean (λ or μ; means and variances are additive). However, if the events were (partially) correlated with each other, diseconomies of scale would arise, which would in part negate the economies of scale due to faster processing times. However, in order to keep this overview brief, I maintain the assumption of statistical independence in the following.

In practice, it is not always realistic that only one arbitrarily fast server exists. Instead, it is often the case that several (slower) servers work in parallel. This has advantages and disadvantages. It is possible to approximate the so-called G/G/m-model, in which more than one server $m = 1,2,3, \ldots$ may be used (see Hopp & Spearman, 2008, p. 291). However, in order to make this comparison only as complex as necessary, I will limit myself to a few general characteristics of the G/G/m-model.

Since the number of servers is not limited to one server, it is possible to scale the G/G/m-model by changing the average processing rate μ or by changing the number of servers m. Concerning the average wait time T_Q^{GGm} it can be shown that changing the scale of the system by changing the number of servers m, creates stronger economies of scale than in the case of a scaling with the processing rate μ. More specifically, the economies of scale get stronger, the smaller α is and the higher m is. The more servers there are and the lower their utilization, the more likely it is that a new incoming task can be processed immediately and does not have to wait.

Thus, only considering the average wait time T_Q^{GGm} , increasing the number of servers m is better than increasing the processing rate μ. However, if we consider the average total time in the system ($T^{GGm} = T_Q^{GGm} + 1/\mu$), scaling with the processing rate μ is the better option. Depending on the context either T_Q^{GGm} or T^{GGm} is the more important performance measure and either m or μ is the better scaling option. For the picking and packing operations of e-tailers, the average total time in the system T^{GGm} is probably more important.

There are also differences between the average number of tasks in the system (L) and the average queue length (L_Q), depending on whether the number of servers m or the processing rate μ increases. The measures L and L_Q are important if an e-tailer has problems due to lack of space. Unfortunately, no, or only marginal economies of scale exist for L and L_Q. Scaling with μ does not change L_Q at all. Scaling with m does reduce the average queue length L_Q, but more servers also take up more space. The only impactful way to reduce L is to decrease the average utilization α. For the sake of brevity, however, I will refrain from a more in-depth analysis.

While there are hardly any economies of scale in L_Q, all queueing models exhibit strong economies of scale in T_Q. Given the importance of the delivery lead time, of which T_Q is a part, we, therefore, can conclude, that yet another important part of an e-tailer's value chain benefits directly and significantly from logistical scale. Note, that these economies of scale are not limited to the demand of a single e-tailer but are based on the total throughput in a warehouse. Thus, based on the economies of scale described above, an outsourcing of warehouse operations seems desirable, as a warehouse can store and fulfill products from many different e-tailers.

3.4. Scale effects in transport operations

The last remaining logistical step in the value chain of an e-tailer is to ship the parcels to the customers (see **Figure 3**; ignoring a potential product return). Shipping often has two phases. In the first phase, mid-to-long haul transportation takes place to bring the parcels close to the customers. In the second phase, the parcels are delivered to the customers via a locally limited delivery route (last-mile delivery). **Figure 17** visualizes these two phases and their respective challenges.

3.4.1. Scale effects in mid-to-long haul transportation to the delivery regions

Transportation to the delivery regions costs time and money. These costs depend largely on the distances between the warehouses and the customers. The only way to reduce these distances is to use more stocking locations spread out over the market area. For example, an e-tailer could decide to use eight stocking locations spread across the US instead of one central warehouse for the entire US. Indeed, this is more or less what *Amazon* did in 2023 (Amazon, 2023a). In the following, I would like to give a rough estimate of the distance (and time) savings that are possible by doing this.

The effective savings depend on the shape of the market area, the locations of the warehouses, and how the customers are distributed within the market area. The term market area usually refers to a country or a toll-free trade region. Sometimes market areas cannot be clearly delineated, and cross-border trade further clouds the calculation. But for many market areas in the world, the following approximation is adequate to get a rough idea of what economies of scale are possible.

We assume that the total market area of size A is in the shape of a hexagon, a circle, or some other regular shape. If we store a product at two stocking locations, the market area (for this product) is split into two smaller market areas, each with size $A/2$, and both in the same shape as the total market area (e.g., a hexagon). This is of course impossible. But the distortions caused by this simplification can often be neglected. We also assume that the stocking locations are always located in the center of the market areas. Moreover, demand is assumed to be uniformly spread across the market areas. In reality, more densely populated areas (e.g., cities) with a higher demand per area exist. However, these demand pockets are often spread across the market areas, which means that the assumption of spatial uniformity of demand only becomes a problem when the market areas become small (that is, in cases of many stocking locations).

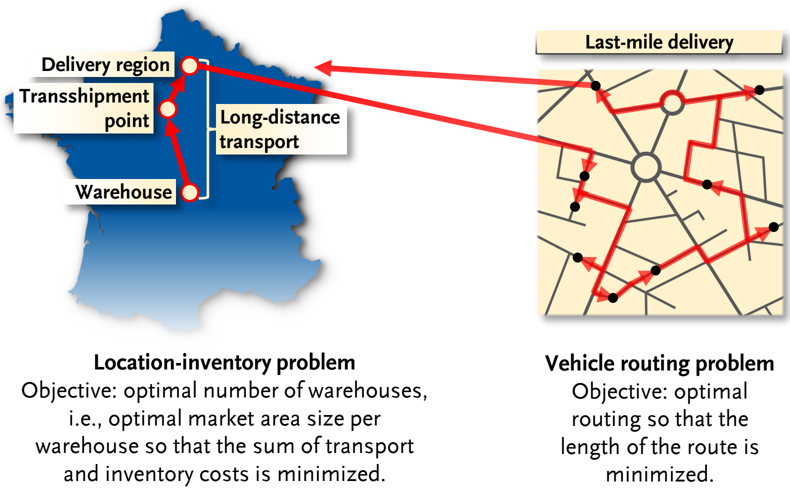
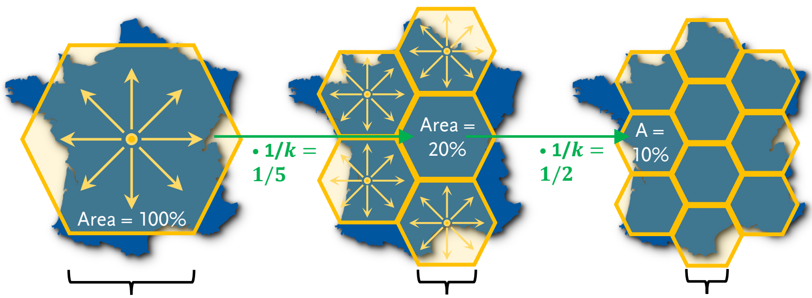


Figure 17 Two phases of product shipping

Figure 18 exemplifies such a model using a hexagon as the market area shape. Erlenkotter (1989, p. 54) found that the choice of shape is not overly important because every simple market area shape produces more or less the same results. For the hexagon shape, it can be shown that the radius r_u decreases by a factor of $\sqrt{1/k}$, when the number of stocking locations, i.e., the number of market areas, increases by factor k (or, to put it the other way round, when the area $A = \frac{3*\sqrt{3}}{2} * r_u^2$ of the market areas decreases by a factor of $1/k$). Furthermore, it is possible to show that the average transport distance from the stocking locations to every point within the market areas is given by (Erlenkotter, 1989, p. 54):

$$\bar{d} = r_u * \frac{4 + 3 * \ln(3)}{12}$$

The square root law of the transport distance



$r_u^{100\%} = 1.5cm$ $\quad r_u^{20\%} = \frac{r_u^{100\%}}{\sqrt{k}} = \frac{r_u^{100\%}}{\sqrt{5}} = 0.67cm$ $\quad r_u^{10\%} = \frac{r_u^{20\%}}{\sqrt{k}} = \frac{r_u^{20\%}}{\sqrt{2}} = 0.47cm$

Average transport distance from the center to every point within the area is given by: $\bar{d} = r_u * \frac{4+3*\ln(3)}{12}$

$\bar{d}^{100\%} = 0.91cm$ $\quad \bar{d}^{20\%} = \frac{\bar{d}^{100\%}}{\sqrt{5}} = 0.41cm$ $\quad \bar{d}^{10\%} = \frac{\bar{d}^{20\%}}{\sqrt{2}} = 0.29cm$

The square root law is valid for many different stylized distance models.

Figure 18 The square root law of the transport distance

This formula corresponds with the assumption that the demand is uniformly distributed across the market areas. The term $\frac{4+3*\ln(3)}{12}$ is just a constant. Therefore, the average transport distance $\bar{d}$ decreases by a factor of $\sqrt{1/k}$ when the number of stocking locations increases by factor k. Thus, we see a square root law of the transport distance within the market area model. Similar to the other models in this section, the distance between stocking locations (i.e., warehouses) and customers goes to zero the more spatially distributed stocking locations an e-tailer uses.

At the same time, however, there is a maximum reasonable number of stocking locations. Recall from **Section 3.2** ("Scale effects in inventory management") that the inventory management costs are dependent on the demand per stocking location. If the number of stocking locations is increased in order to reduce transport distances, the demand rate per stocking location decreases accordingly and inventory management costs increase. This trade-off is known as the location-inventory problem. In my *Paper VI*, I developed, solved, and discussed a LIP that includes economies of distance (long trips are more efficient than short trips) and a nonlinear perception of delivery lead time by customers.

When the total demand rate for a product increases, the demand rate per stocking location also increases proportionately (all else equal). An e-tailer can then decide to sacrifice some of the economies of scale due to this higher demand rate by increasing the number of stocking locations for the product. The e-tailer would forego some inventory management scale effects. But on the other side of the trade-off, the e-tailer would reduce the average transport distances to the delivery regions. Due to the slopes of these two effects, it is optimal when the e-tailer sacrifices some of the increased demand rate per stocking location in favor of more stocking locations. I call the cost reduction that results from optimizing the number of stocking locations the LIP effect, and showed in *Paper VI* that this effect also has a positive impact when replenishment costs (R) and inventory holding costs (h) decrease (see **Section 3.2.3** and Straubert, 2024, pp. 14–16).

3.4.2. Scale effects in last-mile delivery

After a parcel has arrived in the delivery region of its recipient, it is typically delivered via a delivery route together with other parcels destined for other customers in the vicinity. This problem is known as the vehicle routing problem (Braekers et al., 2016).

A delivery route is more efficient the closer the customer locations are to each other. For example, if all customers lived in a single high-rise building, the driver would only have to make one stop and all parcels would be delivered. In a rural area with few customers, however, a delivery route often requires long distances between delivery stops. The same difference often occurs when comparing e-tailers with high demand and e-tailers with low demand. The more demand, i.e., the more customers an e-tailer has, the denser the customer locations within an area. That is, the distance to the nearest other customer (mathematically called the nearest neighbor) is shorter the more customers order from an e-tailer. **Figure 19** exemplifies this.

Once again, the square root law can be observed. If the density of customers doubles ($k = 2$), then the average distance to the next customer decreases by a factor of $\sqrt{1/k} = \sqrt{1/2}$. The average route length is reduced by the same factor. This square root law of the route distance is

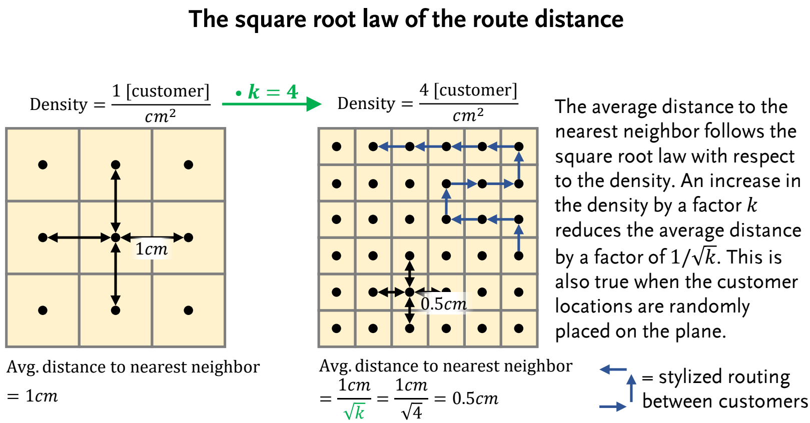


Figure 19 The square root law of the route distance

valid when customer locations are randomly distributed over the area and customer orders occur independently of each other. The reality is of course more complicated (density pockets, one-way streets, ...) and the factor $\sqrt{1/k}$ is not always achieved. Nevertheless, it is valid as a rough guideline (Boyer et al., 2009; Punakivi et al., 2001).

Both the mid-to-long haul transport to the delivery region and the last-mile delivery are often outsourced to logistics service providers (e.g., *DHL*, *FedEx*, ...). This makes sense because from a physical point of view, it is largely irrelevant which package and which customer belongs to which e-tailer. A delivery driver can easily deliver a parcel to a customer of e-tailer A, and then in the next step deliver another parcel to a customer of e-tailer B. By bundling the demand of multiple e-tailers within the logistics service provider's processes, all parties benefit from greater economies of scale. It also makes sense that e-tailers with more demand get better conditions from logistics service providers than e-tailers with less demand (Sully, 2020). After all, the loss of process efficiency would be greater if the larger e-tailer were to withdraw from the collaboration.

3.5. Summary of the scale effects

I hope that the above subsections have adequately demonstrated that there are strong logistical economies of scale in almost every part of an e-tailer's value chain. As mentioned earlier, logistics plays a very important role in B2C e-commerce. Consequently, understanding the logistical economies of scale at play is imperative when analyzing competition in B2C e-commerce. More so than in many other industries.

With respect to competition in B2C e-commerce, concrete recommendations for e-tailers can be derived on the basis of the logistical economies of scale effects described above. Considering only these effects, small e-tailers should prefer to offer products that have low fixed replenishment costs, low inventory holding costs (see **Subsection 3.2.1**; the economic order quantity), low overage costs, and relative certain demand (see **Subsection 3.2.2**; the newsvendor model). Furthermore, **Subsection 3.2.3** (the (s, Q)-model) showed that small e-tailers should prefer to offer products for which a fast delivery lead time is not important. This conclusion is supported by the queueing models in **Subsection 3.3** and the market area and routing models in **Subsection 3.4**. These recommendations are based on the argument that the logistical economies of scale are particularly important when the logistics costs are high relative to other costs, such as the purchase price. Many logistics costs [per unit sold] decrease to zero as demand increases.

In this context, it is probably pertinent to also briefly discuss the words *small* and *large* in the terms *small e-tailer* and *large e-tailer*. At no point in my dissertation have I defined what a small e-tailer is and what a large e-tailer is. This is intentional, as I am less interested in the absolute size of e-tailers than in the size differences between e-tailers. In other words, when I talk about small e-tailers vs. large e-tailers, it could mean comparing e-tailers with 100 vs. 1000 units of demand per year or comparing e-tailers with 4000 vs. 8000 units of demand per year. The greater the relative difference in size between e-tailers, the greater the difference in economies of scale. This is not to say that the absolute size of an e-tailer is not important. It certainly is, and I have discussed some effects related to absolute size in the previous subsections. Nevertheless, I would argue that it is not very useful to introduce a rigid size classification for this dissertation.

Finally, it is worth reiterating that the terms *demand* and *scale* can mean different things at different stages of the value chain. The economies of scale within a delivery network depend on the number of deliveries. The economies of scale within a warehouse depend on the products stored and the orders and parcels fulfilled within that specific warehouse (i.e., the throughput of a warehouse). And in inventory management, the economies of scale depend on the demand for a product per stocking location. Consequently, the respective scales often have different magnitudes depending on the process considered. The number of deliveries is probably higher than the throughput of a warehouse which is probably higher than the demand for a product at a warehouse. It is an educated guess that this is one of the reasons why outsourcing of the delivery process is standard, outsourcing of warehouse operations is common but not as widespread, and outsourcing of product ownership (inventory management) is rare.

It is no coincidence that the mathematical models of this PhD thesis (i.e., the newsvendor model in *Paper V* and the location-inventory problem in *Paper VI*) are inventory management models. Since the inventory management of a product is usually the responsibility of the respective e-tailer, it is a rather isolated problem, and mathematical models are therefore able to capture the essence of inventory management problems well. In comparison, outsourcing decisions, such as the decision to use *Fulfillment by Amazon*, i.e., to outsource warehouse operations, are more multifaceted. Consequently, we researched the topic of FBA from a higher-level strategic perspective in *Paper IV*, leveraging insights from a customer survey. Nevertheless, mathematical models and economies of scale, such as the queueing effects presented in **Subsection 3.3**, can contribute to a better understanding of why *Fulfillment by Amazon* is often attractive to third-party sellers.

4. About the methodological and theoretical background of this thesis

It is questionable whether *the* science of “business administration” still exists today (if it ever did). Over the course of many decades, the individual disciplines of business administration, such as logistics, marketing, finance, human resource management, and so on, have increasingly developed in isolation from one another. In addition, business administration has many connections with related sciences such as micro- and macroeconomics, sociology, and psychology (Diefenbach, 2003, pp. 21–28; Schreyögg, 2007). A PhD thesis cannot do justice to all these fields of research. Nevertheless, it can be argued that this dissertation is at least somewhat interdisciplinary in that it touches on several different areas and their interrelationships. **Figure 20** provides an overview of which disciplines of business administration and economics are particularly relevant to this PhD thesis.

Paper III, with its focus on *Amazon Prime*, and *Paper IV*, with its focus on FBA and trust in B2C e-commerce, are about how logistics can be used in marketing. They primarily concern the demand/customer side of the B2C e-commerce market. Free, fast, high-quality shipping is often

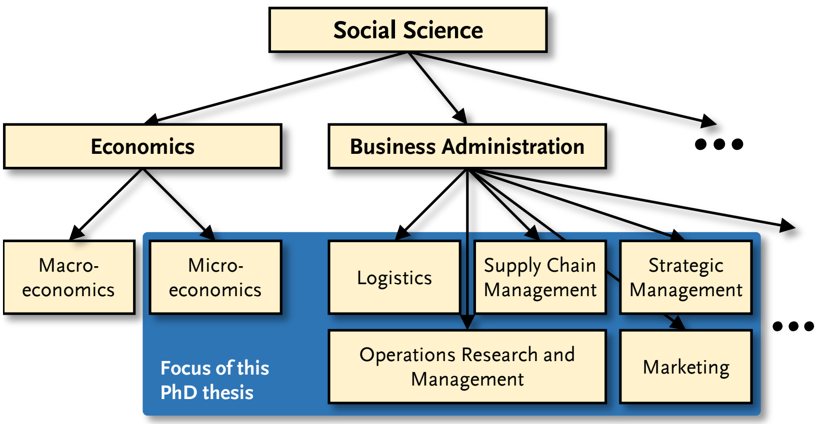


Figure 20 Different scientific disciplines relevant to this thesis

utilized by e-tailers to enhance their value proposition and can be an important part of an e-tailer's marketing strategy.

Papers V and *VI*, on the other hand, concern the supply side of the B2C e-commerce market. Shipping is not cheap, and the faster the delivery lead times, the more expensive B2C e-commerce becomes. In this context, economies of scale in logistics are very important. In their general form, economies of scale are often analyzed and discussed in microeconomics. More detailed mathematical models of procurement, inventory holding, and delivery processes are usually found in the operations research/management literature. The multi-echelon nature of vertical procurement, delivery, and horizontal coopetition are typical supply chain management topics.

Papers I and *II* take a more holistic viewpoint that combines the supply and the demand side into a discussion of trends and strategies in B2C e-commerce logistics. These papers therefore tangent the strategic management discipline of business administration. The economies of scale in B2C e-commerce logistics are so strong that there is a serious threat of even greater market concentration (which already can be regarded as elevated). In this respect, this thesis is also relevant to the antitrust law considerations that are being raised worldwide against the largest e-tailers. Research on market concentration and antitrust law is traditionally part of microeconomics.

In addition to the functional perspective, there is also an institutional perspective. For obvious reasons, it can be assumed that this thesis will be of potential use primarily to the retail industry, especially B2C e-commerce, and the logistics service provider industry.

The diversity of scientific disciplines represented in this thesis is also reflected in the diversity of scientific methods used in the various papers (**Figure 21** provides an overview).

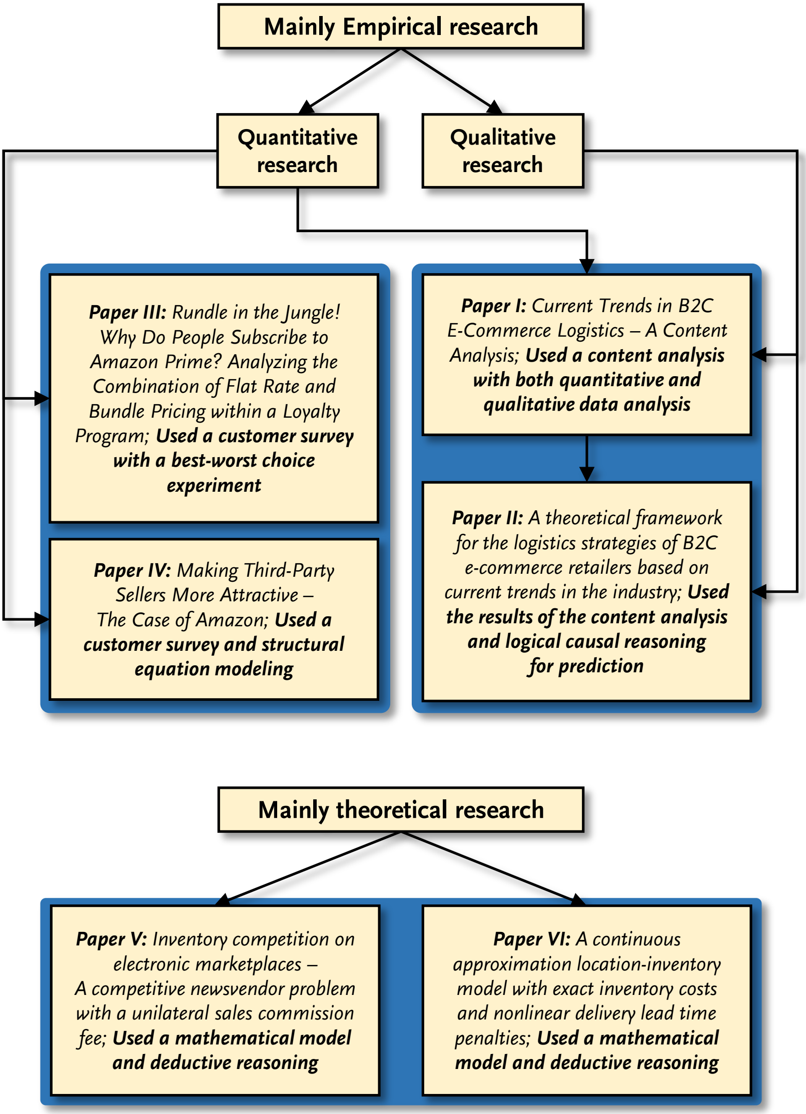


Figure 21 Overview of the different scientific methods used in this thesis

Since the topics of *Papers III* and *IV* are about *Amazon's* customers, it made sense to use customer surveys for this research. More specifically, we used customer surveys for empirical quantitative research. This type of research is characterized by the fact that the analysis of the collected empirical data is predominantly performed using statistical methods. *Papers III* and *IV* embrace the fact that the value of a product depends on the subjective evaluations of customers. It can therefore be argued, that surveying many different customers to gain an impression of the various customer valuations is a natural and appropriate scientific method for this line of research (Cooper and Schindler, 2014, pp. 124–139). For the sake of brevity, the reader is referred to the actual papers in which our research process is described in more detail.

Paper I and especially *Paper II* are more empirical qualitative in nature. The goal for *Paper I* was to understand how industry experts see the future of B2C e-commerce logistics. To this end, articles of various industry experts (i.e., text) were analyzed both quantitatively and qualitatively. Since these texts were not written according to accepted guidelines like we are used to in science, we were faced with a rather heterogeneous set of articles. For such cases, a content analysis is the scientific method of choice for transforming this complexity in presentation and communication into a unified analysis (Krippendorff, 2019). In addition to important descriptive results, we also aimed to create a rich picture of the various trends in B2C e-commerce logistics and their interrelationships. This systems science perspective (Jackson, 2019) was continued in *Paper II*. Methodologically, *Paper II* mainly uses non-deductive logical reasoning to combine the results from *Paper I*, the economies of scale effects in B2C e-commerce logistics, and strategic management frameworks such as the famous theory of Porter's generic competitive strategies (Porter, 2004b, pp. 34–46). More specifically, the logical reasoning in *Paper II* can be classified as analogical reasoning and as a prediction based on probabilistic cause-and-effect relationships (Hitchcock, 2021). The analogical reasoning part of the paper is based on the assumption that generic strategy frameworks are also valid for B2C e-commerce (logistics). This assumption seems reasonable, as the B2C e-commerce sector, with its similarities to traditional retailing, does not seem to be a particularly exotic industry. These strategy frameworks in combination with the economies of scale effects in B2C e-commerce logistics were then used for causal reasoning. *Paper II* argues that common-effect relationships and a reinforcing feedback loop lead to growing market shares for the largest e-tailers (see also Oliva et al., 2003).

While the reasoning in *Paper II* is largely theoretical, the reasoning itself is based on empirical data. This is different for *Papers V* and *VI*. These works are almost entirely theoretical. Both papers use mathematical models and deductive reasoning based on these models. *Paper V* features a game theoretical model where the optimal policies of multiple interacting players are calculated. The model in *Paper VI* is an advanced location-inventory optimization. Both papers, especially *Paper VI*, make extensive use of probability theory. Obviously, it must always be kept in mind that these mathematical models represent reality in a simplified way. It may also be noted that the game theoretical model in *Paper V* implicitly assumes rational actors by default. Since e-tailers are firms and not individuals, the assumption of rational actors is at least somewhat justified. Nevertheless, it may be noted that the model can also be useful if the behavioral policies of the opposing players are unknown.

This dissertation has a focus on B2C e-commerce logistics. Nevertheless, it is reasonable to assume that many of the produced insights and ideas are also relevant to the overall competitive strategy of an e-tailer. For example, *Paper II* argued only from a classical *external market positioning viewpoint* (e.g., Porter, 2004b) and did not consider the *internal resource-based view* (e.g., Barney, 1991). This is justified by the fact that many e-tailers offer the same merchandise, sourced from the same suppliers, and delivered by the same logistics service providers. Moreover, the mathematical mechanisms behind the economies of scale in B2C e-commerce logistics are the same for all e-tailers. Thus, an e-tailer is likely to have fewer unique and valuable resources compared to other firms in other industries.

To nevertheless reconcile the resource-based view with this line of thinking, one could say that the scale of large e-tailers is their unique resource that cannot be imitated by smaller e-tailers. Their scale allows them to use their other, common, imitable resources more efficiently.

From this point of view, it could be argued that the *resource advantage theory* is a good fit. Indeed, Hunt and Morgan's (1995, p. 7) *competitive position matrix* posits that firms can have a competitive advantage either by producing the same value as other firms with lower resource costs or by producing more value than other firms with the same resource costs. This is consistent with the idea that economies of scale allow large e-tailers to use their resources more efficiently. Hunt (2000, p. 138) views competition *"[...] as a constant struggle among firms for comparative advantages in resources that will yield marketplace positions of competitive advantage [...]"*. In the context of B2C e-commerce, this would mean that e-tailers should be in a constant struggle to grow so that they have more of the valuable resource *'size'*. However, both viewpoints are in themselves rather trivial, long-established economic insights that do not distinguish the resource advantage theory from other theories. What practical implications can be derived from the resource advantage theory that cannot be derived from basic economic theory?

In any case, it can also be questioned whether *'size'* is really an internal resource or rather an external resource. After all, the maximum possible size of a firm is determined by the total addressable market (i.e., *size*), which typically has an externally determined upper limit. In this sense, the size of a market can be understood as the sum of all externally available resources within that market (similar to the available food supply in the context of ecosystems). And the market, i.e., the externally available resources, i.e., *size*, is divided among the competing companies.

Following this line of thought, one could argue that Carrol's (1985) *resource partitioning model* is a good fit. This model posits that a mass market that is dominated by economies of scale will evolve into a state where there are only a few generalist firms, many specialist firms, and not much in between. Generalist firms aim for economies of scale and concentrate on the market's center to avoid the costs of specialization. In doing so, they abandon the market's periphery, which in turn can be served by specialist firms (see also Levy et al., 2005; Williams, 2009). An interesting implication of the resource partitioning model is that the higher the market concentration in the generalist submarket, the more market periphery is available for specialist firms. Translated to the B2C e-commerce market, this would mean that market conditions for smaller specialized e-tailers would be best if there was only one large general merchandise e-tailer in the market. Due to the ineluctable nature of mathematical economies of scale effects, the large general merchandise e-tailer would have a relatively comfortable foothold, while competition among the many smaller specialized e-tailers would be a constant struggle. Indeed, we seem to be experiencing this in the B2C e-commerce market (recall **Subsection 2.4**).

5. Limitations and outlook

Although this thesis can be considered extensive and interdisciplinary, it is also clear that the research presented has its limitations and that many interesting and important issues could not be covered within the limits of six papers.

Many of the limitations and weaknesses, as well as potential avenues for further research, have already been mentioned in the six papers. Therefore, this section will not focus so much on these specific limitations of the papers but rather aims to paint a bigger picture of what worthwhile future research might be able to fill the more conceptual gaps left open by this thesis. I would argue that there are two issues of great importance to society that require further investigation. The first is whether or not the largest e-tailers, such as *Amazon*, are too big and powerful relative to their competitors. Assuming for a moment, that the answer to this question is yes, then the next question is how to resolve this market imbalance.

One remedy would be to impose stark regulation or to break up the largest e-tailers. The idea behind this type of solution is to decrease the performance of the largest e-tailers so that they would compete on an equal (lower) footing with less performant smaller e-tailers. Such a solution seems to be undesirable because it effectively reduces consumer welfare. In the second half of this section, I will take a closer look at possible regulation of the B2C e-commerce industry.

Ideally, however, it would instead be better if smaller e-tailers could easily cooperate with each other and thereby profit, at least to some extent, from the same logistical economies of scale as large e-tailers. However, such collaboration is likely to be difficult. The question of how small e-tailers can cooperate efficiently and effectively in logistics is a very worthwhile topic for future research. In the following section I would like to present some initial, hopefully useful, thoughts on this.

5.1. Transaction costs, economies of scale, and outsourcing of logistics processes

In exploring logistical cooperation between e-tailers, one might consider Ronald Coase's (1937) transaction cost theory (see also Williamson, 2008). Our markets are not frictionless. There are costs involved such as search costs, bargaining costs, control costs, and many more. Some authors prefer to distinguish between transaction costs and information costs. Transaction costs in the narrower sense are the costs of establishing and enforcing property rights (Allen, 1991). In any case, the rationale is that information barriers within a firm are low and that it is easy to establish and enforce property rights within a legal entity such as a firm. On average, internal transaction costs are much lower than the external transaction costs that arise between two or more firms. These thoughts and theories are also relevant to the matter at hand, particularly with respect to economies of scale in B2C e-commerce and the possibility of logistics outsourcing. That is, in order to foster collaboration among multiple e-tailers, it is crucial to focus on minimizing the transaction costs associated with the sharing of logistics resources. Probably the most promising means to achieve this is to use intermediaries, that is, logistics service providers (LSPs) who provide services such as inventory storage (warehousing), order picking, packing, delivery, and maybe even procurement and ownership of products (i.e., inventory management).

More detailed discussions are left to future research; however, the following comments may be helpful as a starting point. (1) In practice, we see that outsourcing of the delivery process is widespread. Even the largest e-tailers still use (at least in part) parcel services such as *USPS* and *DHL*. (2) Outsourcing of warehouse activities, such as inventory holding, and order picking and packing, is less common but still occurs quite frequently. Examples of this are *Fulfillment by Amazon* and the *Shopify Fulfillment Network* (Shopify, 2024). (3) Outsourcing of the procurement and ownership of products, on the other hand, is very rare, and when it does occur, it is usually in a very specific form (e.g., dropshipping from overseas). These observations can be explained as follows.

(1) It would be very costly for small e-tailers to operate their own delivery networks because the delivery drivers would have to drive long distances between customers. At the same time, it is very easy to outsource parcel delivery and thus to achieve economies of scale by pooling the delivery demand of many e-tailers. From a transaction cost perspective, one simply hands the labeled parcel to the LSP and everything else is done by the LSP. There are only a few touchpoints between the e-tailer and the LSP and the transaction costs are therefore low.

(2) Warehouse operations outsourcing is more complex. The e-tailers must send their products to the warehouses. The warehouses store them and only retrieve them when an order for the product is placed. If an order is placed, it is first registered at the respective e-tailer and then forwarded to the LSP. Moreover, the parcel may be customized for the e-tailer (e.g., with a flyer or branded boxes). Compared to delivery networks, the e-tailer and the LSP have more touchpoints and more customized processes, and, therefore, also higher transaction costs. Compared to a delivery network, which profits from all customer orders within the network, the scale within a warehouse is often smaller. Thus, warehouse outsourcing is more expensive from a transaction cost perspective and has less scale potential.

(3) It gets even worse when looking at the procurement and ownership of products (i.e., inventory management). While a warehouse may fulfill a million or even more orders per year, the demand for any single product is usually much lower, and not every e-tailer offers every product. This means that the number of e-tailers that can cooperate at the product level is typically smaller than the number of e-tailers that can cooperate within delivery or warehousing. Procurement and ownership of products also directly relate to the costs of establishing and enforcing property rights. What if the LSP has ordered too little of a product and compensates this by reselling the product only to certain preferred e-tailers? Such business relationships require a lot of contracts and rules and have correspondingly high transaction costs.

In summary, we see that logistical cooperation/outsourcing at different stages of an e-tailer's value chain has inherently different potential for economies of scale. Moreover, transaction costs also appear to be highest where there is least potential for economies of scale. Based on the above, using a LSP (i.e., outsourcing) makes the most sense for parcel delivery, less so for warehousing, and even less so for product ownership.

This is also reflected in the fees charged by LSPs. (1) Parcel delivery services usually only charge one fee [per parcel]. (2) LSPs who provide warehousing services often charge three kinds of fees. An inbound logistics fee, an inventory holding fee, and a fulfillment fee. *Paper VI* of this thesis already contains some discussion of these three fees. Future research could expand on the optimality or suboptimality of such a fee structure. (3) Outsourcing of procurement and product ownership requires yet another set of fees. Following the reasoning above, these fees probably need to be even more complex. Whether this is indeed the case, and which fees are optimal, is certainly a worthwhile topic for future research. An obvious hypothesis is that in such a model no longer only service fees make sense, but also penalty fees. Since the e-tailers may have better information/forecasts about future demand, information sharing between the e-tailers and the LSP would make sense. But then the question arises of how to prevent deliberate misinformation by the e-tailers (e.g., deliberate overestimation of demand). This is a typical, but very complex, supply chain management problem.

Advances in digital technology could potentially solve some of these problems and thus reduce transaction costs. If the physical world can be reliably tracked by sensors, and this data can be easily stored and traced in the digital world (i.e., the Internet of Things; Fleisch, 2010), there is a chance that information barriers will be removed and that establishing and enforcing property rights, even across multiple companies, will be reliable and easy. However, there is still a long way to go. The complexity of a well-designed Internet of Things (IoT) ecosystem should not be underestimated, as Papert and Pflaum (2017) demonstrated in their study. Moreover, Papert and Pflaum (2017, p. 184), like many other studies, identified the *"issue of openness"* (i.e., open interfaces, open-source code, trust, and cooperation). That is, before the IoT can help to reduce information barriers, it must first become open itself. There is still much research needed on how a well-functioning, open IoT ecosystem, despite its complexity, can be achieved in practice. Addressing this research gap is important given the profound potential of the IoT to significantly reduce transaction costs across many sectors of the economy.

But even if all of the above transaction complexities did not exist, it may still be the case that e-tailers do not want to cooperate with competing e-tailers. Depending on how customers react to lower prices and better service, it may be better for e-tailers not to cooperate. When e-tailers cooperate with each other, some e-tailers may benefit more from economies of scale than others. Some e-tailers may even be worse off. These differences must be compensated for by clever allocation mechanisms that distribute the benefits of cooperation fairly. Nevertheless, sometimes cooperation is simply not the rational choice. There is already some research on this topic (Dror and Hartman, 2011; Paterson et al., 2011). However, the game theoretic modeling of these systems is extremely complex, and there are only a few definitive or even approximative insights on when cooperation is worthwhile for which e-tailer and when it is not.

5.2. To regulate or not to regulate

If it turns out that cooperation among small e-tailers is not feasible, then we should either regulate the dominant e-tailers or accept their dominance. A significant number of people, also in politics, are of the opinion that the dominant e-tailers should be regulated. However, it is possible, that a few large e-tailers with strong economies of scale generate higher consumer welfare than if there were many smaller e-tailers instead. A so-called *natural monopoly* does not have to be bad, especially if the market is contestable (Mosca, 2008; Baumol et al., 1982). Given the frictions in our markets, some degree of centralization (i.e., market concentration) is always desirable. The stronger the economies of scale, the more centralization is optimal. It could therefore also be the best course of action to accept the B2C e-commerce market as it currently is.

Ideally, we want our policymakers to make informed decisions based on a sound understanding of the various economic forces at play. Therefore, more research is needed on the optimal level of (de-)centralization. If this research is to produce valid results, it must integrate insights from logistics and operations research, such as the models in **Section 3**.

Certainly, one possible result of this research could be that regulation is the best way forward. If the regulatory route is taken, the concept of *slow logistics* could be used as food for thought. At the core of the slow logistics concept is a long (i.e., slow) delivery lead time (Wiese, 2017). Already in 2017, *Amazon* promised to deliver most items within 1–2 days (Zhang et al., 2019). *Amazon's* logistics has probably gotten even faster since then. A regulatory measure could force e-tailers not to offer delivery lead times faster than a set amount of time, e.g., four days.

I will not go into the details of what such a regulation would look like (although the devil is often in the details), and the legal feasibility of a regulation that forces slow logistics cannot be assessed here. Nevertheless, I want to point out some consequences of such a regulatory approach.

Note that almost all economies of scale are based on a scale quantity per unit of time (e.g., orders/parcels/product demand per picking/packing/delivery lead time). A slower delivery lead time would increase these quantities on several levels and thus lead to more economies of scale. And because economies of scale are stronger the smaller an e-tailer is, slower delivery lead times would reduce the inequality between small and large e-tailers (see **Figure 22**).

In light of this insight, it is also not surprising why *Amazon*, as the largest e-tailer, is pursuing ever faster delivery lead times. A large e-tailer, which is on the flatter part of the economies of scale curve, has to increase costs only a little bit to achieve faster delivery lead times. Smaller e-tailers, which are on the steeper part of the economies of scale curve, on the other hand, would either incur a hefty cost increase if they would try to offer the same fast delivery lead times. Or they would have to settle for slower delivery lead times, but would then be perceived by many customers as a poor alternative to the large e-tailer. As long as the value/utility that customers derive from a faster delivery lead time exceeds the cost increase due to reduced economies of scale, e-tailers will work towards faster delivery. The trend toward faster delivery lead times is also likely to shift customers' expectations of what a normal delivery lead time is. Due to the convex form of the economies of scale curve, small e-tailers face disproportionate utility losses (or cost increases) due to this dynamic and are increasingly forced out of the market.

A legally mandated slow delivery lead time would weaken this powerful competitive threat of ever-faster delivery. In addition, slower delivery lead times would have clear logistical advantages that go beyond purely financial considerations:

- Order bundling would be improved on multiple levels. This would lead to fewer (bigger) parcels and fewer transports within the delivery network, but at the same time also denser delivery routes. Orders could be accumulated over several days so that the optimization of delivery routes (i.e., minimizing the distances between stops) is easier. Denser delivery routes and fewer transports would not only have positive cost effects but would also be good for the environment.
- A temporal risk pooling effect could be exploited within the delivery and warehousing operations. Personnel headcount and

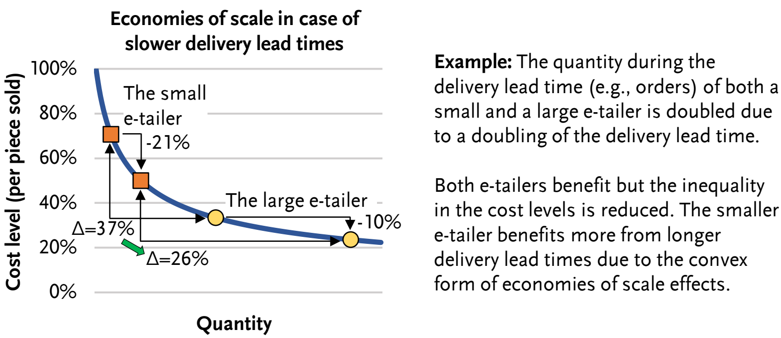


Figure 22 The equalizing effect of slow logistics

night shifts could be reduced. Over the long delivery lead time, there would be enough buffer time so that uneven workloads could smoothed by postponing work. If an order cannot be picked and packed or delivered today, the order would simply be postponed until the next day.

- Less warehouses would be needed because the warehouses would not have to be geographically close to customers. There would be a centralization of the warehousing infrastructure and the remaining warehouses would be larger. This in turn would lead to further economies of scale in warehouse operations and inventory management.
- The only logistical disadvantage for e-tailers would be an increase in work-in-progress. Thus, larger buffer storage areas would be required and inventory holding costs would increase due to the larger buffer inventories. However, the inventory holding reduction due to fewer stocking locations (i.e., inventory centralization; see the bullet point above) would likely be stronger than the inventory holding increase due to larger buffer stocks.

In summary, costs would fall and many e-tailers could become more profitable because the fierce competition observed in B2C e-commerce is potentially weakened through the legally mandated poor logistics service. However, slow logistics could also potentially lower consumer welfare. As many customers value fast delivery, the proposed regulation would not only destroy freedom of choice (private customers would not be allowed to choose express delivery) but would probably also lead to an overproprotional loss of utility in exchange for underproportional cost savings. Furthermore, the competitive position of the B2C e-commerce sector in general would worsen compared to brick-and-mortar retail[5].

[5] Note, that a considerable number of fellow citizens would probably perceive this as an additional benefit of such a regulation. Indeed, as a historical parallel, it is worth noting that the introduction of US parcel post was met with very strong opposition. In the years 1910 to 1912, many witnesses came before congressional hearings to denounce *"the pend-*

The rural population would be more affected by the regulation than the urban population, which has many brick-and-mortar stores available nearby. In any case, slow logistics would not eliminate all economies of scale advantages of large e-tailers. It would only weaken them. Moreover, logistical economies of scale are not the only economies of scale in B2C e-commerce. Logistics is certainly very important, but large e-tailers also enjoy scale advantages in other areas, for example within their IT infrastructure. These non-logistical economies of scale are likely to be diminished only slightly, if at all, by longer delivery lead times. Thus, slow logistics may not be the best solution or may not even be a good solution.

Nevertheless, as long as the logistics perspective is neglected in the discussion on antitrust regulation, economic reality will continue to benefit large e-tailers. In this respect, it is commendable that both the investigation by the European Commission (2020) and the complaint by the Federal Trade Commission (2024) correctly identified that the *Fulfillment by Amazon* service plays an important part in *Amazon's* business model (see also our *Papers III* and *IV*). A closer look at these antitrust proceedings, however, gives the impression that the *reasons* for the competitive advantages of large e-tailers are hardly analyzed and considered in the economic debate on antitrust regulation in B2C e-commerce. Case in point, the above-cited FTC complaint against *Amazon* argues that (Federal Trade Commission, 2024, pp. 44–45):

> *"Structural and direct evidence show that Amazon has monopoly power in two markets: (1) the online superstore market and (2) the market for online marketplace services [...]. The structural evidence of monopoly power in both markets includes Amazon's dominant market shares and the presence of significant barriers to entry, including powerful network effects and strong economies of scale."*

ing bills as class legislation designed to subsidize the mail-order houses at the expense of local merchants and thus to kill off the latter group." (Emmet and Jeuck, 1950, p. 187).

That is, strong economies of scale are one of the two main arguments for why *Amazon* allegedly has durable monopoly power. It is therefore only natural to check what else the complaint has to say about economies of scale. Unfortunately, there is not much to find. Notably, the term *economies of scale* is used only four times, the term *scale economies* is used only seven times, and the terms *logistic** are used only six times in the entire ~170-page long complaint. At one point, the complaint ignores any complexity in business and simply asserts (Federal Trade Commission, 2024, p. 118):

> *"As the sheer size of Amazon's fulfillment operations suggests, the online retail fulfillment services market benefits from economies of scale."*

While I agree with the conclusion (i.e., B2C e-commerce logistics benefits greatly from economies of scale), the theoretical underpinning of this statement by the FTC is very weak. Also recall that, on average, the stronger the economies of scale in an industry, the more optimal a monopoly is in that industry. And accordingly, consumer welfare may decrease if those economies of scale are reduced by regulation. In other words, by raising the issue of strong economies of scale, the FTC may actually be arguing against itself. The complaint further states that (Federal Trade Commission, 2024, p. 118):

> *"Online retail fulfillment service providers can ship products faster and cheaper when they can place products closer to the end-consumer by having a large network of fulfillment centers. These speed and cost savings may be shared with shoppers via faster deliveries and cheaper products."*

This statement directly relates to my *Paper VI*. However, the complaint does not go into further detail and therefore fails to capture the essence of location-inventory problems. Just because a retail fulfillment service provider has *"a large network of fulfillment centers", it* does not mean that it is economical (*"cost savings"*) to *"place products closer to the end-consumer"*. This is only economical if demand for the products is sufficiently high. This logical error is unfortunate because the concepts of inventory (de-)centralization and demand risk pooling are alluded to elsewhere in the complaint (Federal Trade Commission, 2024, p. 115). Although this is just a single example, it nonetheless raises the question of how the FTC can determine consumer harm if it does not understand the underlying economic models?

I would argue that one must first thoroughly understand why competitive advantages exist, to then be able to design sensible antitrust regulation based on these insights. The lack of such a thorough understanding seems to be a shortcoming of the current debate. That is why I am advocating for more management science expertise in the economic debate about antitrust regulation. To this end, I hope that my PhD thesis has provided some valuable contributions to the understanding of the B2C e-commerce market, especially from a logistics perspective.

References used in the synopsis

Allen, D. W. (1991). What are transaction costs? *Research in Law and Economics, 14*, 1–18.

Amazon. (2023a, October 26). *Amazon.com Announces Third Quarter Results*. https://ir.aboutamazon.com/news-release/news-release-details/2023/Amazon.com-Announces-Third-Quarter-Results/

Amazon. (2023b). Unspecified source.

Asdecker, B. (2014). *Retourenmanagement im Versandhandel: Theoretische und empirisch fundierte Gestaltungsalternativen für das Management von Retouren*. University of Bamberg Press.

Asdecker, B., & Karl, D. (2018). Big data analytics in returns management – Are complex techniques necessary to forecast consumer returns properly? *Proceedings of the 2nd International Conference on Advanced Research Methods and Analytics*, 39–46. http://dx.doi.org/10.4995/CARMA2018.2018.8303

Axsäter, S. (2015). *Inventory Control* (3rd ed.). Springer.

Bain, J. S. (1954). Economies of Scale, Concentration, and the Condition of Entry in Twenty Manufacturing Industries. *The American Economic Review*, *44*(1), 15–39.

Barney, J. (1991). Firm Resources and Sustained Competitive Advantage. *Journal of Management*, *17*(1), 99–120. https://doi.org/10.1177/014920639101700108

Baumol, W., Panzar, J. C., & Willig, R. D. (1982). *Contestable Markets and the Theory of Industry Structure*. Harcourt Brace Jovanovich, Inc.

Berman, O., Krass, D., & Mahdi Tajbakhsh, M. (2011). On the Benefits of Risk Pooling in Inventory Management. *Production and Operations Management*, *20*(1), 57–71. https://doi.org/10.1111/j.1937-5956.2010.01134.x

Boyer, K. K., Prud'homme, A. M., & Chung, W. (2009). The last mile challenge: evaluating the effects of customer density and delivery window patterns. *Journal of Business Logistics*, *30*(1), 185–201. https://doi.org/10.1002/j.2158-1592.2009.tb00104.x

Braekers, K., Ramaekers, K., & Van Nieuwenhuyse, I. (2016). The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, *99*, 300–313. https://doi.org/10.1016/j.cie.2015.12.007

Caves, D. W., Christensen, L. R., & Tretheway, M. W. (1984). Economies of Density versus Economies of Scale: Why Trunk and Local Service Airline Costs Differ. *The RAND Journal of Economics, 15*(4), 471–489. https://doi.org/10.2307/2555519

Carroll, G. R. (1985). Concentration and Specialization: Dynamics of Niche Width in Populations of Organizations. *American Journal of Sociology, 90*(6), 1262–1283. https://doi.org/10.1086/228210

Coase, R. H. (1937). The nature of the firm. *Economica, 4*(16), 386–405. https://doi.org/10.1111/j.1468-0335.1937.tb00002.x

Cooper, R. C., & Schindler, P., S. (2014). *Business Research Methods* (12th international edition). McGraw Hill.

Deng, I. (2023, December 29). *Chinese e-commerce giant JD.com wins antitrust lawsuit against Alibaba, which was ordered by a Beijing court to pay US$141 million in damages. South China Morning Post.* Retrieved March 18, 2024, from https://www.scmp.com/tech/big-tech/article/3246715/chinese-e-commerce-giant-jdcom-wins-antitrust-lawsuit-against-alibaba-which-was-ordered-beijing

Department of Justice. (2024, January 17). *Herfindahl-Hirschman Index. Antitrust Division of the U.S. Department of Justice.* Retrieved March 26, 2024, from https://www.justice.gov/atr/herfindahl-hirschman-index

Diefenbach, T. (2003). *Kritik und Neukonzeption der allgemeinen Betriebswirtschaftslehre auf sozialwissenschaftlicher Basis.* Deutscher Universitäts-Verlag.

Diller, H., Fürst, A., & Ivens, B. (2011). *Grundprinzipien des Marketing* (3rd ed.). GIM-Verlag.

Dobhan, A., & Oberlaender, M. (2013). Hybrid contracting within multi-location networks. *International Journal of Production Economics, 143*(2), 612–619. https://doi.org/10.1016/j.ijpe.2012.07.015

Dror, M., & Hartman, B. C. (2011). Survey of cooperative inventory games and extensions. *Journal of the Operational Research Society, 62,* 565–580. https://doi.org/10.1057/jors.2010.65

EHI Retail Institute. (2022a). *E-Commerce Markt Deutschland 2022.* ecommerceDB.com.

EHI Retail Institute. (2022b). *E-Commerce Markt Deutschland 2022.*

eMarketer. (2023, February). *Top 15 US Retail Ecommerce Companies, by Sales, 2023 & 2024*. Retrieved March 26, 2024, from https://www.emarketer.com/content/target-carvana-lowe-s-moving-up-us-ecommerce-sales-ranks

Emmet, B., & Jeuck, J. E. (1950). *Catalogues and Counters: A History of Sears, Roebuck and Company*. University of Chicago Press.

Erlenkotter, D. (1989). The General Optimal Market Area Model. *Annals of Operations Research, 18*, 45–70. https://doi.org/10.1007/BF02097795

European Commission. (2020, November 10). *Antitrust: Commission sends Statement of Objections to Amazon for the use of non-public independent seller data and opens second investigation into its e-commerce business practices*. Retrieved March 18, 2024, from https://ec.europa.eu/commission/presscorner/detail/en/ip_20_2077

Federal Trade Commission. (2024, March 15). *Amazon.com, Inc. (Amazon eCommerce), Amended Complaint*. Retrieved March 18, 2024, from https://www.ftc.gov/legal-library/browse/cases-proceedings/1910129-1910130-amazoncom-inc-amazon-ecommerce

Federgruen, A. & Zheng, Y.-S. (1992). An efficient algorithm for computing an optimal (r, Q) policy in continuous review stochastic inventory systems. *Operations Research, 40*(4), 808–813. https://doi.org/10.1287/opre.40.4.808

Felch, V., & Sucky, E. (2023). In search of a consensus definition of business ecosystems: a qualitative study. *Journal of Modelling in Management, 18*(6), 1834–1857. https://doi.org/10.1108/JM2-09-2021-0240

Felch, V., Sucky, E., & Schwartz, L. (2022). Business Ecosystem: More Than a New Name for Supply Chain? In M. Bilgin, H. Danis, E. Demir, & A. Zaremba (Eds.), *Eurasian Business and Economics Perspectives: Proceedings of the 34th Eurasia Business and Economics Society Conference* (pp. 121–146). Springer.

Fleisch, E. (2010). What is the Internet of Things? An Economic Perspective. *Economics, Management, and Financial Markets, 5*(2), 125–157.

Forman, C., Ghose, A., & Goldfarb, A. (2009). Competition Between Local and Electronic Markets: How the Benefit of Buying Online Depends on Where You Live. *Management Science, 55*(1), 47–57. https://doi.org/10.1287/mnsc.1080.0932

Gawor, T., & Hoberg, K. (2019). Customers' valuation of time and convenience in e-fulfillment. *International Journal of Physical Distribution & Logistics Management*, *49*(1), 75–98. https://doi.org/10.1108/IJPDLM-09-2017-0275

Gefen, D. (2000). E-commerce: the role of familiarity and trust. *Omega*, *28*(6), 725–737. https://doi.org/10.1016/S0305-0483(00)00021-9

Goertler, T., Papert, M., & Schmidt, M. (2023). Designing Digital Platforms: A Synthetization of Relevant Design Topics for Business Models from a Literature Review. *Proceedings of the 56th Hawaii International Conference on System Sciences, USA*, 4515– 4524.

Goldstein, G. S., & Gronberg, T. J. (1984). Economies of Scope and Economies of Agglomeration. *Journal of Urban Economics*, *16*(1), 91–104. https://doi.org/10.1016/0094-1190(84)90052-4

Gupta, A., Su, B. C., & Walter, Z. (2004). An Empirical Study of Consumer Switching from Traditional to Electronic Channels: A Purchase-Decision Process Perspective. *International Journal of Electronic Commerce*, *8*(3), 131–161. https://doi.org/10.1080/10864415.2004.11044302

Hadley, G. F. & Whitin, T. M. (1963). *Analysis of Inventory Systems*. Prentice-Hall.

Hagiu, A., & Wright, J. (2013). Do You Really Want to Be an eBay? *Harvard Business Review*, *91*(3), 102–108.

Harris, R. G. (1977). Economies of Traffic Density in the Rail Freight Industry. *The Bell Journal of Economics*, *8*(2), 556–564. https://doi.org/10.2307/3003304

Hitchcock, C. (2021). Probabilistic Causation. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Retrieved December 12, 2023, from https://plato.stanford.edu/archives/spr2021/entries/causation-probabilistic

Hopp, W. J. & Spearman, M. L. (2008). *Factory Physics* (3rd international edition). McGraw-Hill.

Hsiao, M.-H. (2009). Shopping mode choice: Physical store shopping versus e-shopping. *Transportation Research Part E: Logistics and Transportation Review*, *45*(1), 86–95. https://doi.org/10.1016/j.tre.2008.06.002

Hunt, S. D. (2000). *A General Theory of Competition: Resources, Competences, Productivity, Economic Growth*. SAGE.

Hunt, S. D., & Morgan, R. M. (1995). The Comparative Advantage Theory of Competition. *Journal of Marketing, 59*(2), 1–15. https://doi.org/10.1177/002224299505900201

IfH Köln. (2023). HDE - Zahlenspiegel 2023.

Ivanov, D., Dolgui, A., & Sokolov, B. (2022). Cloud supply chain: Integrating Industry 4.0 and digital platforms in the “Supply Chain-as-a-Service”. *Transportation Research Part E: Logistics and Transportation Review, 160*, 102676. https://doi.org/10.1016/j.tre.2022.102676

Jackson, M. C. (2019). *Critical systems thinking and the management of complexity*. John Wiley & Sons.

Janski, A. (2021, January 21). *Weiteres Zentrum von Amazon in Grenznähe zu Deutschland*. infoPOL.PRESS. Retrieved March 26, 2023, from https://www.infopol.press/weiteres-zentrum-von-amazon-in-grenznaehe-zu-deutschland/

JD.com. (2024). *How to Partner with JD.com*. https://corporate.jd.com/forPartners

Katz, M. L., & Shapiro, C. (1994). Systems Competition and Network Effects. *Journal of Economic Perspectives, 8*(2), 93–115. https://doi.org/10.1257/jep.8.2.93

Kotler, P., Armstrong, G., & Balasubramanian, S. (2024). Principles of Marketing (19th ed.). Pearson.

Krippendorff, K. (2019). *Content analysis: An introduction to its methodology* (4th ed.). SAGE.

Levy, M., Grewal, D., Peterson, R. A., & Connolly, B. (2005). The concept of the “Big Middle”. *Journal of Retailing, 81*(2), 83–88. https://doi.org/10.1016/j.jretai.2005.04.001

Library of Congress. (n.d.). *Photographic copy of retouched photograph*. www.loc.gov/pictures/item/il0737.photos.048605p

Little, J. D. C. (1961). A Proof for the Queuing Formula: L = λW. *Operations Research, 9*(3), 383–387. https://doi.org/10.1287/opre.9.3.383

McCorkle, D. E. (1990). The role of perceived risk in mail order catalog shopping. *Journal of Direct Marketing, 4*(4), 26–35. https://doi.org/10.1002/dir.4000040406

Marino, G., Zotteri, G., & Montagna, F. (2018). Consumer sensitivity to delivery lead time: a furniture retail case. *International Journal of Physical Distribution & Logistics Management, 48*(6), 610–629. https://doi.org/10.1108/IJPDLM-01-2017-0030

Maryland GovPics. (2017a, September 18). *Amazon Tour*. Flickr. Retrieved September 7, 2023, Licensed CC BY 2.0, from www.flickr.com/photos/mdgovpics/36490655883

Maryland GovPics. (2017b, September 18). *Amazon Tour*. Flickr. Retrieved September 7, 2023, Licensed CC BY 2.0, from www.flickr.com/photos/mdgovpics/36467011734

Maryland GovPics. (2017c, September 18). *Amazon Tour*. Flickr. Retrieved September 7, 2023, Licensed CC BY 2.0, from www.flickr.com/photos/mdgovpics/36906693900

Mosca, M. (2008). On the origins of the concept of natural monopoly: Economies of scale and competition. *The European Journal of the History of Economic Thought, 15*(2), 317–353. https://doi.org/10.1080/09672560802037623

Mulligan, J. G. (1983). The economies of massed reserves. *The American Economic Review, 73*(4), 725–734.

Oberparleiter, K. (1918). *Die Funktionen des Handels*. Verlag der K. K. Exportakademie.

Oberparleiter, K. (1955). *Funktionen und Risiken des Warenhandels* (2nd ed.). Springer.

Oliva, R., Sterman, J. D., & Giese, M. (2003). Limits to growth in the new economy: exploring the 'get big fast' strategy in e-commerce. *System Dynamics Review: The Journal of the System Dynamics Society, 19*(2), 83–117. https://doi.org/10.1002/sdr.271

Oum, T. H., & Zhang, Y. (1997). A note on scale economies in transport. *Journal of Transport Economics and Policy, 31*(3), 309–315.

Panzar, J. C., & Willig, R. D. (1981). Economies of scope. *The American Economic Review, 71*(2), 268–272.

Papert, M., Hamper, A., Hoßbach-Zimmermann, N., Schuster, T., Buehler, L., & Pflaum, A. (2024). Towards the data-driven circular and embedded supply chain: Considerations from an ICT perspective. *Proceedings of the 57th Hawaii International Conference on System Sciences, USA*, 4975– 4984.

Papert, M., & Pflaum, A. (2017). Development of an Ecosystem Model for the Realization of Internet of Things (IoT) Services in Supply Chain Management. *Electronic Markets, 27*, 175–189. https://doi.org/10.1007/s12525-017-0251-8

Paterson, C., Kiesmüller, G., Teunter, R., & Glazebrook, K. (2011). Inventory models with lateral transshipments: A review. *European Journal of Operational Research, 210*(2), 125–136. https://doi.org/10.1016/j.ejor.2010.05.048

Pavlou, P. A., & Gefen, D. (2004). Building Effective Online Marketplaces with Institution-Based Trust. *Information Systems Research, 15*(1), 37–59. https://doi.org/10.1287/isre.1040.0015

Peterson, R. A., & Balasubramanian, S. (2002). Retailing in the 21st century: reflections and prologue to research. *Journal of Retailing, 78*(1), 9–16. https://doi.org/10.1016/S0022-4359(01)00062-8

Pool, J. (2024, March 6). *The World's Top Online Marketplaces 2023*. Webretailer. Retrieved March 18, 2024, from https://www.webretailer.com/marketplaces-worldwide/online-marketplaces/

Porter, M. E. (2004a). *Competitive Advantage: Creating and Sustaining Superior Performance* (1st Free Press export edition). Free Press.

Porter, M. E. (2004b). *Competitive strategy: techniques for analyzing industries and competitors* (1st Free Press export edition). Free Press.

Punakivi, M., Yrjölä, H., & Holmström, J. (2001). Solving the last mile issue: reception box or delivery box? *International Journal of Physical Distribution & Logistics Management, 31*(6), 427–439. https://doi.org/10.1108/09600030110399423

Pratten, C., & Dean, R. M. (1965). *The economics of Large-scale Production in British Industry: An Introductory Study. Department of Applied Economics Occasional Papers 3*. Cambridge University Press.

Robinson, E. A. G. (1931). *The Structure of Competitive Industry*. Nisbet & Company Limited.

Robinson, E. A. G. (1934). The Problem of Management and the Size of Firms. *The Economic Journal, 44*(174), 242–257. https://doi.org/10.2307/2224765

Rubalski, G. B. (1972). Calculation of optimum parameters in an inventory control problem. *Engineering Cybernetics, 10*(2), 182–187.

Schlaifer, R. (1959). *Probability and Statistics for Business Decisions: An Introduction to Managerial Economics Under Uncertainty*. McGraw-Hill.

Schreyögg, G. (2007). Betriebswirtschaftslehre nur noch als Etikett? Betriebswirtschaftslehre zwischen Übernahme und Zersplitterung. *Schmalenbachs Zeitschrift für betriebswirtschaftliche Forschung, 59*, 140–159. https://doi.org/10.1007/BF03373736

Shopify. (2024). *Shopify Fulfillment Network: Scale your business with fast, easy, and affordable fulfillment*. Retrieved March 19, 2024, from https://www.shopify.com/fulfillment

Silberston, A. (1972). Economies of Scale in Theory and Practice. *The Economic Journal, 82*(325), 369–391. https://doi.org/10.2307/2229943

Statista. (2022a, October). *Revenue of the 100 most successful online shops in Germany in 2021*. Retrieved March 26, 2024, from https://www.statista.com/statistics/450288/leading-100-online-shops-by-revenue-germany/

Statista. (2022b, October). *Revenue share of the leading 10 online shops in the total e-commerce revenue in Germany from 2008 to 2021*. Retrieved March 26, 2024, from https://www.statista.com/statistics/450367/leading-online-shops-e-commerce-revenue-share-germany/

Statista. (2023a, August). *Marktanteil von Versand- und Online-Handel im Einzelhandel in Deutschland in den Jahren 2000 bis 2022*. Retrieved March 26, 2024, from https://de.statista.com/statistik/daten/studie/452891/umfrage/marktanteil-des-interaktiven-handels-im-einzelhandel-in-deutschland/

Statista. (2023b, August). *Amazons Logistik-Netz*. Retrieved March 26, 2024, from https://de.statista.com/infografik/30720/anzahl-der-amazon-logistikzentren-in-deutschland-und-jahr-der-eroeffnung/

Stigler, G. J. (1958). The Economies of Scale. *The Journal of Law and Economics, 1*, 54–71. https://doi.org/10.1086/466541

Straubert, C., Asdecker, B., & Zitzmann, I. (2019a). Current Trends in B2C E-Commerce Logistics–A Content Analysis. In C. Bierwirth, T. Kirschstein, & D. Sackmann (Eds.), *Logistics Management: Lecture Notes in Logistics* (pp. 123–140). Springer. https//doi.org/10.1007/978-3-030-29821-0_9

Straubert, C., Asdecker, B. & Zitzmann, I. (2019b). A Content Analysis on Current Technological Trends in B2C E-Commerce Logistics. *Proceedings of the 7th IEEE International Conference on Advanced Logistics and Transport*, 117–122.

Straubert, C. (2022). A theoretical framework for the logistics strategies of B2C e-commerce retailers based on current trends in the industry. *International Journal of Shipping and Transport Logistics, 14*(1-2), 78–93. https://doi.org/10.1504/IJSTL.2022.120673

Straubert, C. (2024). A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties. *International Journal of Production Economics, 268,* 1–25. https://doi.org/10.1016/j.ijpe.2023.109092

Straubert, C., & Sucky, E. (2023). Inventory competition on electronic marketplaces–A competitive newsvendor problem with a unilateral sales commission fee. *European Journal of Operational Research, 309*(2), 656–670. https://doi.org/10.1016/j.ejor.2023.02.002

Straubert, C., Sucky, E., & Altewischer, D. (2024). Rundle in the Jungle! Why Do People Subscribe to Amazon Prime? Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program. *Proceedings of the 57th Hawaii International Conference on System Sciences, USA*, 4393–4402.

Straubert, C., Sucky, E., Felch, V., Karl, D., & Altewischer, D. (2023). Making Third-Party Sellers More Attractive—The Case of Amazon. *Proceedings of the 56th Hawaii International Conference on System Sciences, USA*, 3838–3847.

Sucky, E. (2022). *Supply Chain Management*. Kohlhammer.

Sully, E. (2020, September 22). *Newly released documents show Amazon is a $3.9 billion-per-year customer the Post Office can't afford to lose.* Business Insider. https://www.businessinsider.com/amazon-brings-in-nearly-4-billion-annual-revenue-usps-report-2020-9

Tedlow, R. S. (1990). *New and improved: The story of mass marketing in America*. Harvard Business School Press.

Teece, D. J. (1980). Economies of scope and the scope of the enterprise. *Journal of Economic Behavior and Organization, 1*(3), 223–247. https://doi.org/10.1016/0167-2681(80)90002-5

Thonemann, U. (2015). *Operations Management: Konzepte, Methoden und Anwendungen* (3rd ed.). Pearson.

Wiese, J. (2017). *Slow Logistics: eine simulationsgestützte Analyse der ökonomischen und ökologischen Potentiale der Sendungsbündelung.* University of Bamberg Press.

Williams, D. E. (2009). The evolution of e-tailing. *The International Review of Retail, Distribution and Consumer Research, 19*(3), 219–249. https://doi.org/10.1080/09593960903233657

Williamson, O. E. (2008). Outsourcing: Transaction Cost Economics and Supply Chain Management. *Journal of Supply Chain Management, 44*(2), 5–16. https://doi.org/10.1111/j.1745-493X.2008.00051.x

Wright, T. P. (1936). Factors affecting the cost of airplanes. *Journal of the Aeronautical Sciences, 3*(4), 122–128. https://doi.org/10.2514/8.155

Xu, K., Windle, R., Grimm, C., & Corsi, T. (1994). Re-evaluating returns to scale in transport. *Journal of Transport Economics and Policy, 28*(3), 275–286.

Yelle, L. E. (1979). The learning curve: Historical review and comprehensive survey. *Decision Sciences, 10*(2), 302–328. https://doi.org/10.1111/j.1540-5915.1979.tb00026.x

Zalando. (2024a). *Zalando Fulfillment Solutions.* https://corporate.zalando.com/de/marken-handelspartner/plattformdienste/zalando-logistics-solutions/zalando-fulfillment-solutions

Zalando. (2024b, March 13). *Annual Report 2023.* https://corporate.zalando.com/en/investor-relations/annual-report-2023

Zhang, J., Onal, S., Das, R., Helminsky, A., & Das, S. (2019). Fulfilment time performance of online retailers–an empirical analysis. *International Journal of Retail & Distribution Management, 47*(5), 493–510. https://doi.org/10.1108/IJRDM-10-2017-0237

Zitzmann, I. (2018). *Supply Chain-Flexibilität zur Bewältigung von Unsicherheiten: Taktisch-operative Potenzialplanung zur Schaffung von Robustheit, Resilienz und Agilität.* University of Bamberg Press.

Part B: Papers

Paper I: Current Trends in B2C E-Commerce Logistics – A Content Analysis

Christian Straubert, Dr. Björn Asdecker and Dr. Immanuel Zitzmann

Abstract. In the B2C e-commerce market, logistics plays a crucial role, not only as a cost factor, but also as a major success factor for the firms. It is therefore important for e-tailers to know which trends in B2C e-commerce logistics are considered to be the way forward. This also has implications for the directions of future research.

We present a systematic content analysis of 87 non-scientific, practice-oriented articles published on the internet. This results in a comprehensive overview of the trends currently being discussed in B2C e-commerce logistics. An additional correlation/association analysis reveals important relationships between the trends.

A total of 36 trends were identified. Overall, the trend towards faster deliveries was most frequently mentioned in the articles. Followed by more transparency across the processes (track-and-trace), logistics cooperation/outsourcing and more (smaller) urban warehouses.

To the best of our knowledge, this is the first scientific paper that systematically examines current trends in B2C e-commerce logistics practice. The results of this article can serve as an impetus for a variety of research questions, some of which we will touch upon in the course of this article.

Keywords: e-commerce logistics, future, trend, content analysis

Reference: Straubert, C., Asdecker, B., & Zitzmann, I. (2019a). Current Trends in B2C E-Commerce Logistics–A Content Analysis. In C. Bierwirth, T. Kirschstein, & D. Sackmann (Eds.), Logistics Management: Lecture Notes in Logistics (pp. 123–140). Springer. doi.org/10.1007/978-3-030-29821-0_9

I.1. Introduction

The B2C e-commerce sector has been growing significantly over the recent years and in the search for ways to differentiate their service, many e-tailers identified logistics as a central part of their value proposition to the customer. The delivery of the ordered goods is becoming faster (a recent trend being same-day delivery) and more delivery options (parcel lockers, time-window delivery) are available.

However, this trend comes at a cost. For the year 2016 (the last time the industry leader Amazon.com, Inc. published its net shipping cost) AMAZON.COM (2017) reported that its shipping costs (~12% of all operating expenses) were roughly 1.78 times higher than its shipping revenues (delivery fees). In addition, there are fulfillment costs (storage, packaging, etc.) which make the whole logistics operations even more expensive.

In every recent report, Amazon.com, Inc. explicitly discusses logistics as a major risk but also as a major success factor for the company. China's largest e-commerce firm JD.com, Inc. names similar risks and places an additional emphasis on the strategic role of logistics for its business model. Otto GmbH & Co KG, Germany's biggest e-tailer, is no different. This is illustrated by the following selected quotes:

> *"We expect our cost of shipping to continue to increase to the extent our customers accept and use our shipping offers at an increasing rate, we reduce shipping rates, we use more expensive shipping methods, and we offer additional services."*
>
> *"If we do not adequately predict customer demand or otherwise optimize and operate our fulfillment network and data centers successfully, it could result in excess or insufficient fulfillment [...] In addition, a failure to optimize inventory in our fulfilment network will increase our net shipping cost by requiring long-zone or partial shipments."*

(AMAZON.COM, 2018, p. 27, 37)

"We have adopted shipping policies that do not necessarily pass the full cost of shipping on to our customers. We also have adopted customer-friendly return and exchange policies that make it convenient and easy for customers to change their minds after completing purchases. [...] These policies improve customers' shopping experience and promote customer loyalty, which in turn help us acquire and retain customers."

"We deliver a majority of the orders directly to customers ourselves, and therefore our customers interact with delivery personnel more often than with any other representatives of our company. [...] We believe that our professionally trained delivery personnel are important in helping us to shape customer experience and distinguish ourselves from our competitors. [...]"

(JD.com, 2018, p. 21, 63)

"The logistics area occupies a key position within the Otto Group. Highly advanced processes and systems are employed [...]. However, modified business models for intra-Group customers are leading to new logistical challenges. [...] For this reason, logistical and IT changes must be made to existing systems [...]."

"WE MUST DELIVER! The customers increasingly want to determine exactly how, when and where they receive their parcels. Logistics providers additionally have to deal with debates around congestion and poor air quality in cities; zero-emission transport and even car-free cities are no longer theoretical concepts."

(Otto GmbH & Co KG, 2018, p. 105, 40)

Based on these statements it becomes evident that logistics plays a major role for e-commerce firms. The logistics services offered by an e-tailer are essential to their business model and long-term survival. Logistics is treated as a strategic value proposition, which is used to create a competitive advantage. At the same time, logistics is a major operational cost driver. E-tailers have to decide which logistics strategy they want to pursue. Which delivery options should they offer at which prices? Which trends can and which trends cannot be ignored? Currently, only very limited scientific literature exists that covers this topic. Indeed, to the best of our knowledge, a comprehensive discussion does not exist. This paper is intended to close one part of this research gap by using a content analysis of various sources (articles, white papers) published on the internet. The goal is to have a comprehensive overview of what trends are currently being discussed. This also has implications for the scientific community in that it indicates which trends should be researched, as there is a need in practice. Thus, our research question is:

- What are the current trends in B2C e-commerce logistics?

I.2. Existing Literature

Discussions about the future of logistics have a tradition. There are some contributions concerning the general future of logistics (e.g., Ballou, 2007). Other papers focus on more specific topics such as Industry 4.0 (e.g., Hofmann & Rüsch, 2017), Reverse Logistics (e.g., Govindan et al., 2015) or City Logistics (e.g., Taniguchi et al., 2014). However, many papers about the general future of logistics are rather subjective and only very few papers explicitly discuss the future of B2C e-commerce logistics.

Gracht and Darkow (2010) used a comprehensive Delphi study in order to identify probable trends for the logistics services industry. Their research questions are not e-commerce specific and rather general: "How will the macro-environment (political/ legal, economic, socio-cultural, and technological structure) of the logistics services industry change by 2025?" and "How will the micro-environment (industrial structure) of the logistics services industry change by 2025?" (Gracht & Darkow, 2010, p. 46). Two of the identified scenarios may be of interest in the context of B2C e-commerce: "Customer demands for convenience, simplicity, promptness, and flexibility have turned logistics into a decisive success factor for customer retention" and "Alternative distribution networks have been established in the CEP-market (courier, express, parcel). Petrol stations, kiosks, and local public transport are increasingly used for pickup and delivery of parcels." (Gracht & Darkow, 2010, p. 54).

Bask et al. (2012) performed a literature review of e-commerce logistics research and discuss topics for future research. Their focus is B2C, B2B and C2C e-commerce. Some findings are of relevance to the paper at hand. For example, delivery times play a decisive role in the choice between the online channel and brick-and-mortar retail (Bask et al., 2012, p. 13). However, Bask et al. (2012) found that only a few scientific articles discuss the matter hand (i.e., B2C e-commerce logistics): "Based on the review, however, it seems that the logistics solutions have not been as extensively analyzed as could be expected, given that physical delivery is a key issue in successful e-commerce. In many cases, e-commerce was approached from the information systems or marketing points of view, and thus not many specific logistics solutions for e-commerce deliveries were proposed or analyzed." (Bask et al., 2012, p. 10).

Based on the literature search we conclude that there is a research gap with regard to B2C e-commerce logistics. Papers such as Ballou (2007) are rather abstract, while the paper at hand is much more specific but comprehensive at the same time. Gracht and Darkow (2010) is a methodically sound Delphi study but not about e-commerce. Bask et al. (2012) has a rather similar objective compared to the paper at hand but is based on scientific literature. Our paper is predominantly practice-oriented. Bask et al. (2012) emphasize that the scientific literature is lacking in the area of e-commerce logistics.

I.3. Methodology

In order to identify the trends, we used a 'Problem-Driven Content Analysis' (Krippendorff, 2013, pp. 357–370). Besides the fact that there is very little scientific literature on this topic, we decided to use mostly non-scientific online articles as source material because the topic is practice-oriented, and the internet is known as a fast, responsive publication channel.

Although the content analysis method has rarely been used in logistics research, it fits the purpose of this study. Mir et al. (2018, p. 166), who did a review of content analysis in logistics research, explicitly state: *“quantitative content analysis methods may be utilized to examine the latest trends in last mile delivery such as Amazon’s intentions to use drones for order fulfillment.”*

Krippendorff (2013, p. 358), named nine general steps to consider for a methodologically sound, problem-driven content analysis:

1. Formulating research questions
2. Ascertaining stable correlations
3. Locating relevant texts
4. Defining and identifying relevant units in texts
5. Sampling these units of texts
6. Developing coding categories and recording instructions
7. Selecting an analytical procedure
8. Adopting standards
9. Allocating resources

Due to the nature of our analysis some steps are of lesser/minor relevance. We do not need to ’ascertain stable correlations’ because we are mainly interested in what is said literally. Furthermore (similar to Spens & Kovács, 2006), we can mostly omit the steps ’Defining and identifying relevant units in texts’ and ’Sampling these units of texts’. Most of the publications found on the Internet are short articles that can be read in their entirety. We split the articles so that each paragraph, heading and bullet point is a text unit. During the recording process all these text units were coded one by one. After the recording we mainly used tabulations and statistical tests on correlation/differences for the ’analytical procedure’. The remaining five steps suggested by Krippendorff (2013) are described in the following:

Formulating research questions – Our Content Analysis follows the broad research question: ’Which current B2C e-commerce logistics trends are mentioned on the internet?’

Locating relevant texts – We chose the three search engines 'www.google.com', 'www.bing.com' and 'www.duckduckgo.com' (a meta search engine that does not personalize the search results) to find the relevant texts. In order to find the best search queries we conducted a pre-test with several different search queries such as "future e-commere supply chain", "developments B2C e-commere logistics" or "trends e-tail logistics". For the final search, we used the following search queries because they yielded by far the best search results:

"future e-commerce logistics"
"trends e-commerce logistics"

For each search engine and query, we checked every search result until we reached a point at which 20 consecutive search results were irrelevant. We performed the search between the 2018-08-29 and the 2018-09-04. We recorded only texts that are written from the viewpoint of the most mature e-commerce markets in the world (Europe, China, North America, ...). Furthermore, we only considered publications dating from 2014 or later in order to focus on sources that are more likely to describe current trends in B2C e-commerce logistics and because the search restricted to those years yielded enough results. It was therefore not necessary to search further in the past. After the final recording, we calculated the *Pearson correlation coefficients* for the trend-year combinations and found no clear correlations between the year of the source and the trends. This suggests that the articles from 2014 are not outdated.

During the screening process of the query results we paid attention if a text prominently featured an aspect of the two themes 'future/trends of e-commerce' and/or 'future/ trends of e-commerce logistics' either in the title, abstract or introductory paragraph. If a text was deemed relevant for further analysis, it was read (in most cases in its entirety). If the text indeed contained content about the current trends in B2C e-commerce logistics, it was logged in a spreadsheet that we used during the screening process. We used the following definition of B2C e-commerce logistics during this step and the actual recording process: The physical scope of B2C e-commerce logistics is every touch-point the finished product has after it was ordered by the customer. In addition to the physical interactions (warehouse, delivery, ...), the information flows associated with these interactions and the configuration of the network/logistics market are also included. We considered a topic to be a trend if the author explicitly mentions that it will become more important/increase in the future or if the topic/technology/method did not exist or was less prevalent in the past.

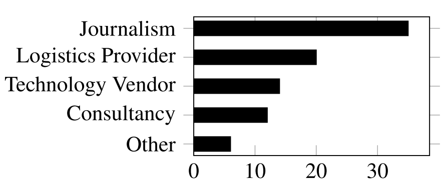


Figure I.1
# of texts by type of source

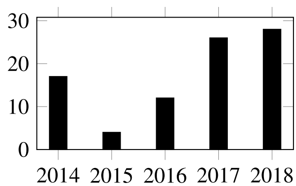


Figure I.2
# of texts by year

In total 87 texts were recorded. **Figure I.1** shows the distribution of the texts by type of source (e.g., Journalism) and **Figure I.2** shows the number of texts aggregated by year. It is noteworthy that relatively few relevant articles were found for the year 2015 (especially compared to the year 2014). If one assumes that the number of hits decreases from year to year, the further one goes into the past (because the search engines find older articles to be less relevant), it is surprising that so many sources were found for the year 2014 compared to so few for 2015. However, this outlier for 2014 can be explained by a largescale study by DHL which was published in 2014. In this study, four different scenarios are presented, which are considered separately in the content analysis. In addition, the study contains eight expert interviews, which are also separately included in the content analysis.

Developing coding categories and recording instructions – The coding process itself was challenging. It is an explorative content analysis. This means that the codes/trends are not known at the beginning of the analysis. Also many trends/concepts are named in multiple sources but with slightly different terminology (e.g., 'autonomous shipping vehicles' vs.

’self-driving shipping vehicles’). Therefore, in a first phase, two coders A and B separately looked at all text units and tried to identify all mentioned trends. Then the two coders harmonized the terminology and created a codebook. Particular care was taken to ensure that the codes are exhaustive and mutual exclusive. The codebook (including coding instructions) was iteratively refined based on a test data sample until a satisfactory inter-coder reliability was achieved (Krippendorff, 2013, p. 130).

Adopting standards and Allocating resources – In order to minimize a bias that could occur if all coders were very familiar with the codebook (Krippendorff, 2013, p. 131) coder B who took part in the first phase was swapped with a third coder C who was not involved in the process of creating the codebook. Coder C was trained on a small test data sample (5 articles). In the second phase coder A and coder C coded all text units based on the codebook created in the first phase. This coding process was difficult because many different trends were identified in the first phase and often several different trends were mentioned in one text unit. In order to make the large number of codes easier to handle, a hierarchy system was used (Krippendorff, 2013, p. 135).

Table I.1 contains the ’Agreement Rate’ (Matching Recordings / All Recordings) and the inter-coder reliability scores ’Scott’s Pi’, ’Cohen’s Kappa’ and ’Krippendorff’s Alpha’. All values fall within ranges that are regarded as acceptable or good (Neuendorf, 2002, p. 143). Whether a certain amount of inter-coder unreliability can be accepted is always dependent on the matter at hand and the research goals. Since our main research goal is to identify the current trends in B2C e-commerce logistics we can accept a moderate level of unreliability because the results are not invalidated if, for example, the actual count of a trend is 60 times versus a recorded count of 70 times. All disagreements were discussed between coder A and coder C until a consensus decision was reached (similar to Mir et al., 2018; Seuring, 2008).

Score	Value	LowerCB 95%	Upper Confidence Bound 95%
Agreement Rate	0.754		
Cohen's Kappa	0.672	0.653	0.691
Scott's Pi	0.672	0.652	0.691
Krippendorff's Alpha	0.672		

Table I.1 Inter-Coder Reliability Scores

It is always difficult to make predictions about the future. This is no different in our case either. Nevertheless, the case can be made, that if different people agree on predictions, it is more likely that these predictions come true (Rowe & Wright, 2001, p. 128). Furthermore, it stands to be reasoned that if the people who made the predictions can influence the subject matter, that the predictions become a self-fulfilling prophecy. In our case, the people who made the predictions are journalists, logistics/technology providers and consultants, all of whom have the ability to influence the matter at hand. The journalists influence the public. The logistics and technology providers influence the e-commerce firms through their products. And the consultants influence the e-commerce firms because they ask the consultants for advice. The nature of our analysis is rather straightforward and therefore less vulnerable in general. The use of three different search engines makes it probable that most of the relevant texts have been identified. Therefore, we are confident that our analysis, which we present in the following, is valid.

I.4. Results of the Content Analysis

Table I.2 shows how often the trends were recorded. For better readability some trends that were mentioned only a few times are not included in the table. We grouped the trends under headings like 'Analyics/Software' and 'Types of Delivery' for a better understanding of common themes. Please note that one article can contain multiple trends.

Trend	# of articles	% of articles
Faster Delivery	72	83%
Generally Faster Delivery	59	68%
Same-Day Delivery	34	39%
One- /Two-Hour Delivery	15	17%
Types Of Delivery	52	60%
Click-and-Collect/Parcel Locker	28	32%
Drone Delivery	24	28%
General More Delivery Options/Variety	18	21%
Time Window Delivery	9	10%
More Delivery Hours	7	8%
Anywhere/Any-Place Delivery	5	6%
Delivery Characteristics	50	57%
Green Logistics	17	20%
Free/Low Price Delivery	16	18%
Easier/Free Returns, More Returns	16	18%
Value Added Services/Quality in Delivery	13	15%
Difficult Deliveries (Food Delivery, ...)	11	13%
Bundling, City Logistics Concepts	9	10%
Delivery Fee Subscriptions	5	6%
Periodic Deliveries (e.g. 10th of each month)	3	3%
Analytics/Software	49	56%
Data/Operations/Process Transparency	36	41%
More/Better Software/Analytics	25	29%
Big Data/Business Intelligence	19	22%
Machine Learning/Artificial Intelligence	18	21%
Predictive Shipping/Logistics	10	11%
Automation, Technology in Warehouse/Delivery	41	47%
Warehouse/Fulfillment Automation	20	23%
Better Technology in Warehouse/Fulfillment	17	20%
Automated Delivery Vehicles	17	20%
General Automation	14	16%
Better Technology in Delivery	10	11%
Logistics Market/Infrastructure	66	76%
Logistics Outsourcing/Logistics Cooperation	32	37%
(Smaller) Regional/Urban Warehouses	30	34%
Multi-Channel/Omni-Channel	28	32%
Logistics Insourcing	24	28%
Logistics Marketplaces, Crowd Logistics	20	23%
International/Cross-Border Logistics	16	18%
Bigger Warehouse/Fulfillment Infrastructure	7	8%
Mobile Warehouses (Store on Wheels, ...)	5	6%

Table I.2 Number of times a trend was named (# of articles)

Trend 1	Trend 2	# of articles	% of articles	Cramer's V
Faster Delivery	(Smaller) Regional/ UrbanWarehouses	29	33%	0.257*
Data/Operations/ Process Transparency	Logistics Outsourcing/ Logistics Cooperation	20	23%	0.322**
Data/Operations/ Process Transparency	More/Better Software/ Analytics	18	21%	0.391***
(Smaller) Regional/ Urban Warehouses	Logistics Outsourcing/ Logistics Cooperation	17	20%	0.295*
(Smaller) Regional/ Urban Warehouses	Click-and-Collect/ Parcel Locker	16	18%	0.325**
Multi-Channel/Omni-Channel	Logistics Outsourcing/ Logistics Cooperation	16	18%	0.287*
Data/Operations/ Process Transparency	Logistics Marketplaces, Crowd Logistics	16	18%	0.426***
(Smaller) Regional/ Urban Warehouses	Multi-Channel/ Omni-Channel	15	17%	0.272*
Data/Operations/ Process Transparency	Warehouse/Fulfillment Automation	14	16%	0.314**
(Smaller) Regional/ Urban Warehouses	More/Better Software/ Analytics	14	16%	0.284*
Data/Operations/ Process Transparency	Better Technology in Warehouse/Fulfillment	14	16%	0.407***
Multi-Channel/ Omni-Channel	Click-and-Collect/ Parcel Locker	14	16%	0.259*

*/**/*** Significant on 95%, 99% or 99.9% level respectively

Table I.3 Significant associations between trends – often occurring combinations in the articles

We also performed an additional correlation (or rather association) analysis, the results of which can be found in **Table I.3**. We first counted how often a pair of trends was found within the articles. Reading example: The first row in **Table I.3** means that 29 articles contained the trend *'Faster Delivery'* as well as the trend *'(Smaller) Regional/Urban Warehouses'*. Then we calculated Cramér's V, a measure of association between the two trends (nominal variables; trend mentioned: yes/no), for each trend pair. We only report trend pairs that are significant on at least a 95% confidence level. We restricted our analysis to trend pairs that were found in at least 13 articles (~15% of all articles) in order to ensure the generalizability of the results. All reported trend pairs have a moderate (0.20 to <0.40) or relatively strong association (0.40 to <0.60) (Rea & Parker, 2014, p. 219). We examined each association by studying the underlying set of articles. In the following discussion, the results of the statistical analysis will be underpinned and substantiated by selected quotes from the recorded texts.

Overall, the trend towards faster deliveries was most frequently mentioned. *Same-Day Delivery* or even *One-/Two Hour Delivery* was often explicitly mentioned as the goal while *Slower Delivery* (which is not listed in the table) was only mentioned three times. This addresses one of the last advantages of brick-and-mortar retail: instant gratification. The customers want fast delivery (at least as an option) and while it makes the delivery more expensive it also provides the opportunity for the e-tailers to shift revenue from traditional brick-and-mortar retail towards e-commerce. This aspect is also one of the findings of the literature review performed by Bask et al. (2012) (see **Section I.2**), which indicates that this is still one of the most important trends.

However, the last mile was and is a problem that will only become more acute due to increased demands in terms of speed and flexibility. Some articles predict that the very costly *Time-Window Delivery* option will become more popular. Other articles say that there will be more hours of operations (e.g., delivery on Sundays or in the early morning, late evening) and that it will be possible to receive the parcel anywhere. This increases the costs for the last mile, but *Drone Delivery*, *Click-and-*

Collect/Pick-Up and Parcel Lockers might be viable solutions to this problem. Drones are mostly automated, not bound to the (possibly congested) road network and can travel in linear distance. This greatly reduces personnel costs and can shorten delivery times. *Click-and-Collect* or *Parcel Lockers* basically outsource the burden of the last mile to the customer. But at the same time they provide added value to those customers who want to decide when to receive their parcel. Not listed in the table are *In-Car Delivery* and *In-Home Delivery* which were named only in two or one article/s respectively.

Damian Harrington, 2015, Colliers International

"As the essential element in improving urban logistics is to limit deliveries to the shortest route, e-commerce retailers have started to include smaller urban warehouses to shorten delivery routes and provide quick delivery services to online customers."

If the future indeed lies in very fast delivery it is necessary to build more (smaller) warehouses that are close to the urban demand. This is a clearly named trend that was mentioned in many articles and is emphasized by our statistic: the trend towards more *(smaller) regional/urban warehouses* was named in 34% of all articles compared to more *big warehouses* which was predicted in only 8% of all articles. In addition, the calculation of Cramér's V reveals that the association between the trend *'Faster Delivery'* and *'(Smaller) Regional/Urban Warehouses'* is indeed statistically significant. The next most frequent trend pair involving *'(Smaller) Regional/Urban Warehouses'* is with *'Logistics Outsourcing/ Logistics Cooperation'* highlighting that many e-commerce firms, especially the smaller ones, will struggle to establish their own network of regional/urban warehouses. This is exemplified by the following quote:

Michael Lierow et al., 2014, Oliver Wyman

"When it comes to innovative logistics solutions, pure-play SMEs may find introducing a physical means of differentiation such as same-day delivery tough to implement, as most have only one or a few centralized warehouses and thus cannot offer broad same-day delivery. Once same-day takes off

(and customers demand it), solutions could include developing more decentralized distribution centers or teaming up with other niche SMEs to set up local same-day warehouses for critical stock. Enabling IT won't be trivial [...]"

The above text excerpt also notes that there are new IT challenges associated with small urban warehouses. This association between the trends *'(Smaller) Regional/Urban Warehouses'* and *'More/Better Software/Analytics'* is also statistically significant in our sample.

We see great research potential in this area. Many small e-commerce firms struggle to compete with the big industry leaders such as Amazon.com. Some pundits even consider the market an oligopoly. Trends like faster delivery will make the competitive edge of the industry leaders even more pronounced. Small businesses will probably have no other choice than to cooperate with one and another, either through an intermediate 3PL provider or directly. Urban Warehouses have to be shared facilities in order to achieve the necessary throughput. But why stop there? Many small firms will also struggle to even have stock at every urban warehouse. They would have to increase their number of stock keeping units and suffer an increase in tied-up capital. It will therefore probably be necessary for small firms to cooperate with inventory sharing whenever possible and sensible. The necessary coordination is indeed not trivial and we are planning further research on this topic.

Brian Straight, 2018, for FreightWaves

"Approximately three-quarters of retailers (76%) currently use store inventory to fill online orders with 86% planning to institute buy online/pick up in store programs next year. Some retailers are retrofitting stores to double as online fulfillment centers and 70% of surveyed executives believe this trend will grow."

Some authors argue that the demand for big warehouses will increase or at least remain steady for the next years due to many traditional brick-and-mortar retailers who still do not have a comprehensive e-commerce strategy. But the shift towards a *multichannel* model is inevitable for many of these retailers because they would otherwise have trouble to survive the increased competition. However, the brick-and-mortar retailers are in a good position to make the shift because they already have a lot of *urban warehouses*: their stores. Several articles argue that these stores will become multi-use facilities: A traditional place to shop, a click-and-collect point and a mini-fulfillment center for urban demand. This was also a result of the Delphi study performed by Gracht and Darkow (2010) (see **Section I.2**).

But the shift from traditional retail towards a multi-channel model is not easy, which is why a lot of retailers have to rely on 3PL service providers, at least in the beginning.

Wenda Ma, 2017, HKTDC Research

“The retail industry is undergoing a structural change amid the proliferation of e-commerce – increasingly applying an online-to-offline model to their businesses. The new mandate for omni-retailers includes customer-centricity, digital fluency, and complete agility. Rather than developing their own logistics networks and delivery systems, relying on a 3PL provider’s extensive distribution and delivery networks might help retailers that are currently behind the curve to rapidly establish an online presence.”

Big industry leaders like Amazon.com on the other hand are increasingly insourcing their logistics operations. This gives them more control over the processes and provides them with an additional opportunity to differentiate themselves from the competition. Of course faster delivery is not the only viable strategy. Many authors also argue that *Cross-Border Logistics* will increase. This is a very natural trend (globalization) and the technical barriers (website) are low. Additionally, this strategy increases the economies of scale which make it possible to offer the products at a lower price (if the increased shipping costs do not overcompensate). Already today business models exist (e.g., AliExpress.com), where the products are directly shipped from China to customers in Europe or the USA.

Another widely discussed topic is the increased use of advanced *Software and Analytics*. In this theme the most frequent topic which was covered in 41% of all articles is *Process Transparency*, either with regard to the parcel (Track-and-Trace) or with regard to the internal processes (e.g., stock levels). Many articles featured popular topics such as *Big Data* and *Artificial Intelligence*. These technologies are trending in many areas and it is very likely that they will also play an important role in B2C e-commerce logistics. One particularly difficult application of these technologies is *Predictive Shipping/Logistics* which describes a process where selected products are picked from the warehouse shelf without a corresponding order. In some cases this product is also loaded onto a truck anticipating that some customer in the respective delivery area will order the product in the next hours. Not listed in the table is the *Blockchain* technology which was only named in two articles. All in all, *Software and Analytics* are classic enablers that help to implement other trends. Be it the coordination of different logistics service providers (freight brokerage), the coordination of many different independent 'Uber-like' courier drivers (who enable faster delivery to customers) via *Logistics Marketplaces*, or the increased use of (better) *Automation and Technology* in warehouse and delivery processes.

Charlie Hitt cited by John Schulz, 2018, for Logistics Management

"'The big differentiator is technology' Hitt explains. 'Everybody at XPO looks at technology as a leading driver as to what we do with customers. Technology manages workflows, directs the customer experience and provides visibility to all parties, customers and retailers.'"

Better Technology in Warehouses and Delivery like Radio-Frequency Identification and mobile devices connected to the internet/intranet is necessary in order to make the processes transparent. Other technology makes the pick and pack process faster (e.g., pick-by-light systems in warehouses) or help to increase the quality of the delivery process (e.g., sensors in the packages). The advantage of *Warehouse Automation* and *Automated Delivery Vehicles* is evident in that the technologies help to reduce the payroll costs and make the process less vulnerable to fluctuations in capacity and quality.

However, a more automated and therefore anonymous delivery process also has disadvantages. As JD.com puts it (see introduction above): *"[...] our customers interact with delivery personnel more often than with any other representatives of our company. [...] We believe that our professionally trained delivery personnel are important in helping us to shape customer experience and distinguish ourselves from our competitors."* Several articles talked about the trend towards increased *quality in delivery* (e.g., White Glove Delivery) to increase the value of the service and differentiate it from other more standard delivery experiences. Another possibility for differentiation are the return policies of the e-commerce firms. Several articles predict that the returns management will become even more important and an easy returns process can help to distinguish a firms' service offering from the competitors. One could argue that *Green Logistics* concepts also have the potential to make a delivery service special, but this is probably not the case in the future. While some articles see it as a way of differentiation, most articles simply state that this a general trend and many large e-tailers and logistics companies have programs to reduce the CO_2 output and air pollution. It will basically become obligatory and the only difference is the scope and number of measures taken. Another basic trend is that the *customer does not have to pay a delivery fee* or only has to pay a subsidized fee that is lower than the actual costs. Some firms use *Delivery Fee Subscriptions* (e.g., Amazon Prime) in order to provide the customer with unlimited delivery for a fixed fee. Delivery fee subscriptions also play an important role in the *difficult grocery e-commerce*. After the failure to establish grocery e-commerce during the dot-com bubble (e.g., 'Webvan') some authors think that the time (demand and technology) has now come for e-commerce to establish itself in the food sector. Other authors remain skeptical due to the immense costs associated with the delivery of refrigerated goods. However, it is a natural assumption (although not necessarily correct) that difficult deliveries (also furniture, luxury items, etc.) will increase as these areas still have a comparatively small e-commerce market share and a lot of start-ups are currently emerging who see their chance.

Looking at the big picture, one might ask whether it is possible to predict which trends are more likely to occur than others based on observations other than the mere frequency with which a trend was mentioned. Two possible considerations are:

1.) Some trends are ongoing, i.e., they existed in the past, but the change process is not yet finished (an example for this is the trend towards faster delivery). Other trends are mere predictions of changes that may occur in the future but are currently at an early stage or have not even been implemented outside of field trials. It can be assumed that trends of the first category are more likely to exist in the future than trends of the second category. For trends of the second category, uncertainty is greater due to a lack of actual experience.
2.) The identified trends can be split into actual goals and methods/technologies which help to achieve these goals. For example, faster delivery is an actual goal and smaller urban warehouses are a method/technology that enables the e-tailers to provide fast delivery because the inventory is stored close to the customer. Some of these relationships are also emphasized by the presented association analysis. It can be assumed that methods/technologies that help to achieve one or several goals without conflicting with the other goals have an increased chance of being successful. Lower-level goals such as faster delivery will be more successful if they support higher-level goals such as financial profit or customer satisfaction. The separation between goals and methods/technologies is gradual, since every method can be made a goal and lower-level goals can also be interpreted as methods to achieve high-level goals.

Figure I.3 illustrates these viewpoints with some of the most frequently mentioned trends. These two considerations may also be of value for the assessment of trends that do not appear in this study but may arise in the future. A theoretical framework that allows the assessment of (new) trends in B2C e-commerce logistics in general is a desirable goal for future research efforts.

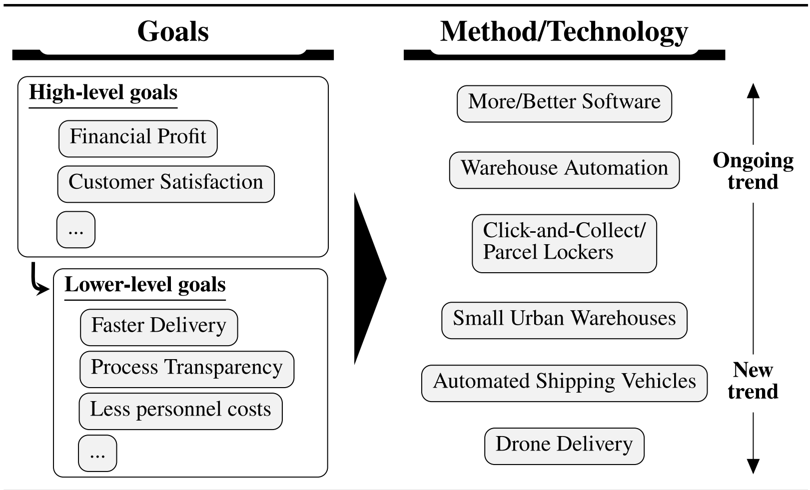


Figure I.3 Goals versus methods/technologies – A framework for the classification of trends

I.5. Conclusion and Outlook

The paper at hand incorporates both a scientific and a practical perspective. Our content analysis helps to put the ubiquitous discussions about the future of B2C e-commerce logistics on a sound footing that can be used as a basis for further research, or, if we look further into the future, as a snapshot in time that can serve as a basis for comparison. The results provide strong indications as to which trends are promising research areas and which are not. For example, almost no article mentions *In-Car Delivery*, *In-Home Delivery* and *Blockchain* as promising trends, suggesting that these concepts are a dead end.

A central finding of our study is that often faster delivery is considered the central goal for the future. Many of the other trends mentioned by the articles can be connected to this. Other central goals are transparency about the order/delivery status, more quality during the process, environmentally friendly operations and more service/convenience for the customer. It can be assumed that if a method/technology supports several of these goals without conflicting with the other goals or high-level objectives such as financial profit, it has an increased chance of being successful. Methods and technologies often mentioned are click-and-collect and parcel lockers, drone delivery, more and better software, warehouse automation, logistics outsourcing/insourcing (depending on the situation and also in combination with logistics/driver marketplaces), smaller urban warehouses and a multi-channel business model. Our association analysis emphasizes some of these relationships. For example, the most often occurring statistically significant trend pair is faster delivery together with smaller urban warehouses. If one wants to achieve short delivery times, it is physically necessary to keep the inventory close to the customer. Another viewpoint when evaluating the likelihood that a trend will occur is the age of said trend. If a trend exists for some time, but still has room for more change, it is a reasonable assumption that this trend will continue for the foreseeable future. Faster delivery is one example of an ongoing trend.

Basically, it is quite natural that companies try to reduce their costs and improve their service through, for example, more automation or better software, and thereby create a competitive advantage over their competitors. However, the methods employed can differ from firm to firm and our content analysis may shed some light onto which methods are currently considered to be the way forward.

Identifying the current trends in B2C e-commerce logistics and their relationships, however, is only the first step towards fully exploring the topic. Research is needed from multiple angles. Many of the trends have implications regarding the planning and execution of last mile delivery. Disrupting technologies such as drone delivery or automated delivery vehicles require other planning priorities than the traditional *Vehicle Routing Problem*. The industry leaders increasingly use advanced methods such as machine learning to better predict customer demand and the IT-systems of all firms are more and more interconnected. This opens up new possibilities like inventory sharing between multiple companies, which is often necessary to provide very fast delivery without exploding costs, but also has more generally the potential to increase welfare by making the economy more efficient.

The e-commerce firms have to decide how they use the information provided in this paper. As we said in the introduction of the paper: E-tailers have to decide which logistics strategy they want to pursue. Which delivery options should they offer at which prices? Which trends can be and which trends cannot be ignored based on their logistics strategy? Perhaps one answer to this lies in the classical competitive strategy theories by Porter (Porter, 1998). Or it is necessary to develop a B2C e-commerce specific framework (e.g., Ghezzi et al., 2012). In this paper we present a framework that categorizes the trends in *goals* and *methods/technologies*. This could help to better understand the relationships between the trends and be useful when drafting a logistics strategy. In any case, logistics is a central component of the competitive strategy of e-commerce firms and every firm needs a plan how to position itself in the market.

However, the paper has some limitations. Generally, predictions about the future are not deterministic but rather an indication. Moreover, is not entirely known how knowledgeable the authors of the sources used in the content analysis really are. Also the inter-coder reliability was not perfect. The paper presents a good starting point for complementary further research which could use a questionnaire or a Delphi study approach. In the course of this it might also be useful to consider different countries/regions or products groups (market characteristics) separately, as it can be assumed that customer preferences and B2C e-commerce logistics systems may be very different depending on the circumstances. Every market has different characteristics and for a more in-depth look it would be useful to understand these characteristics and correlate them with the different trends. A desirable goal of such further research could certainly be to understand more thoroughly why the trends mentioned in this paper are taking place.

We hope that this paper is useful in guiding researchers and practitioners alike. The B2C e-commerce market is huge and still growing. The underlying logistics and service processes directly impact our quality of life. Thus, the importance of research in this field and its application cannot be underestimated.

I. References

AMAZON.COM, INC. (2017). FORM 10-K - ANNUAL REPORT PURSUANT TO SECTION 13 OR 15(d) OF THE SECURITIES EXCHANGE ACT OF 1934 For the fiscal year ended December 31, 2016

AMAZON.COM, INC. (2018). FORM 10-Q - QUARTERLY REPORT PURSUANT TO SECTION 13 OR 15(d) OF THE SECURITIES EXCHANGE ACT OF 1934 For the quarterly period ended June 30, 2018

Ballou, R. (2007). The evolution and future of logistics and supply chain management. *European Business Review, 19*(4), 332–348. https://doi.org/10.1108/09555340710760152

Bask, A., Lipponen, M., & Tinnilä, M. (2012). E-Commerce Logistics. A Literature Research Review and Topics for Future Research. *International Journal of E-Services and Mobile Applications, 4*(3), 1–22. https://doi.org/10.4018/jesma.2012070101

Ghezzi, A., Mangiaracina, R., & Perego, A. (2012). Shaping the E-Commerce Logistics Strategy. A Decision Framework. *International Journal of Engineering Business Management, 4*(2), 1–13. https://doi.org/10.5772/51647

Govindan, K., Soleiman, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research, 240*(3), 603–626. https://doi.org/10.1016/j.ejor.2014.07.012

Gracht, H., & Darkow, I.L. (2010). Scenarios for the logistics services industry: A Delphi-based analysis for 2025. *International Journal of Production Economics, 127*(1), 46–59. https://doi.org/10.1016/j.ijpe.2010.04.013

Harrington, D. (2015). *E-Commerce and New Delivery Paradigms Shape Future of Global Logistics*. Retrieved February 26, 2019, from https://www.supplychain247.com/article/ecommerce_and_new_delivery_paradigms_shape_future_of_global_logistics

Hofmann, E., & Rüsch, M. (2017). Industry 4.0 and the current status as well as future prospects on logistics. *Computers in Industry*, *89*, 23–34. https://doi.org/10.1016/j.compind.2017.04.002

JD.com, Inc. (2018). FORM 20-F - ANNUAL REPORT PURSUANT TO SECTION 13 OR 15(d) OF THE SECURITIES EXCHANGE ACT OF 1934 For the fiscal year ended December 31, 2017

Krippendorff, K. (2013). *Content analysis. An introduction to its methodology*. SAGE.

Lierow, M., Janssen, S., & D'Inca, J. (2014). *Disruptive Logistics - The new frontier for ecommerce*. https://www.oliverwyman.com/content/dam/oliver-wyman/global/en/2014/sep/MUN-MKT20101-011_screen12.pdf

Ma, W. (2017). *E-commerce: The Logistics Industry*. Retrieved February 26, 2019, from http://hkmb.hktdc.com/en/1X0A9I7B/hktdc-research/E-commerce-The-Logistics-Industry

Mir, S., Lu, SH., Cantor, D., & Hofer, C. (2018). Content analysis in SCM research: Past uses and future research opportunities. *The International Journal of Logistics Management*, *29*(1), 152–190. https://doi.org/10.1108/IJLM-09-2016-0200

Neuendorf, K. (2002). *The Content Analysis Guidebook*. SAGE.

Otto GmbH & Co KG. (2018). ANNUAL REPORT 2017/18

Porter, M. (1998). *Competitive strategy. Techniques for analyzing industries and competitors*. Free Press.

Rea, L., & Parker, R. (2014). *Designing and conducting survey research. A comprehensive guide*. John Wiley & Sons.

Rowe, G., & Wright, G. (2001). Expert Opinions in Forecasting: The Role of the Delphi Technique. In J. S. Armstrong (Ed.), *Principles of Forecasting*. Springer.

Schulz, J. (2018). *Battle for the last mile. The e-commerce logistics revolution. Logistics Management*. https://www.logisticsmgmt.com/article/the_e_commerce_logistics_revolution

Seuring, S. (2008). Assessing the rigor of case study research in supply chain management. *Supply Chain Management: An International Journal, 13*(2), 128–137. https://doi.org/10.1108/13598540810860967

Spens, K., & Kovács, G. (2006). A content analysis of research approaches in logistics research. *International Journal of Physical Distribution & Logistics Management, 36*(5), 374–390. https://doi.org/10.1108/09600030610676259

Straight, B. (2018). *Survey finds manual processes still dominate e-commerce logistics*. Retrieved February 26, 2019, from https://www.freightwaves.com/news/technology/ecommerce-logistics-remains-a-manual-process

Taniguchi, E., Thompson, R., & Yamadaa, T. (2014). Recent Trends and Innovations in Modelling City Logistics. *Procedia - Social and Behavioral Sciences, 125*, 4–14. https://doi.org/10.1016/j.sbspro.2014.01.1451

Paper II:
A theoretical framework for the logistics strategies of B2C e-commerce retailers based on current trends in the industry

Christian Straubert

Abstract. In B2C e-commerce, logistics is highly visible and very important to customers. It is therefore important that e-tailers have a strategy for the type of logistics services they want to offer, i.e., how they position themselves compared to their competitors. This paper aims to develop a theoretical framework for logistics strategies within the B2C e-commerce market. The theoretical framework developed in this paper is based on two sources of information: extensive quantitative data concerning the current and future trends in B2C e-commerce logistics and well-known models concerning competitive strategy. We find that B2C e-commerce logistics can be categorised into three main strategies: the 'logistics industry leaders' strategy, the 'logistics efficiency seekers' strategy, and the 'logistics niche concepts' strategy. We argue that because of the strong economies of scale in B2C e-commerce, logistics industry leaders are in a disproportionately strong position, especially compared to logistics efficiency seekers. The emphasis on competitive advantages and market positioning based on the logistics services offered by e-tailers is a new perspective that has not yet been discussed in the literature.

Keywords: B2C e-commerce electric commerce, logistics strategy, theoretical framework, competition, trends

Reference: Straubert, C. (2022). A theoretical framework for the logistics strategies of B2C e-commerce retailers based on current trends in the industry. International Journal of Shipping and Transport Logistics, 14(1-2), 78–93. doi.org/10.1504/IJSTL.2022.120673

 This paper is a revised and expanded version of a paper entitled 'A content analysis on current technological trends in B2C e-commerce logistics' presented at the 7th IEEE International Conference on Advanced Logistics and Transport (ICALT), Marrakesh, Morocco 14–16 June 2019.

II.1. Introduction

The B2C e-commerce market is now over 25 years old. After a very volatile phase around the turn of the millennium (dot-com bubble), the B2C e-commerce market grew steadily and very strongly. Shopping over the internet has become a part of everyday life. Many issues that were highly relevant at the beginning of B2C e-commerce, such as the trust problem concerning the new way of shopping, are largely irrelevant today. Industry standards have been established, and consumer rights have been strengthened. This means that customers currently pay attention to fewer factors when shopping on the internet. Therefore, B2C e-commerce companies have very few opportunities for differentiation in this competitive market. The core of the e-tailer business model is that e-tailers sell goods produced by other companies. The object of desire and consumption is the same for a customer regardless of which e-tailer the customer buys the product from. It is also likely that the product will be available from more than one e-tailer, as the cost of stocking the product is rather low for e-tailers compared to traditional retailers.

The business model of an e-tailer consists of a product-service bundle. The physical products cannot be altered. However, the services can be influenced, and the logistics service is particularly suitable for differentiation in the market. The following article is based on this idea.

This article builds on an extensive content analysis of various practice-oriented sources that discuss current or future trends in B2C e-commerce logistics. In this content analysis, 87 sources were coded, and 36 different trends were identified (Straubert et al., 2019). In the following sections, these 36 trends are briefly discussed, and the codings are further summarised to illustrate the current trends in B2C e-commerce logistics. These trends are then linked to existing theories on business models and competition. The result is a theoretical framework containing various B2C e-commerce logistics strategies. Finally, the implications of these logistics strategies for the general B2C e-commerce market are briefly discussed. Based on this analysis, research questions for future research are derived.

Given this context, the article focuses on the following two research questions:

1. Which logistics strategies can be identified based on the current trends in B2C e-commerce logistics?
2. How does differentiation via logistics services influence general competition in the B2C e-commerce market?

Methodologically, the theory in this article is derived from two sources of information. One source is quantitative data from which the current trends in B2C e-commerce are deduced. The other source consists of well-known and accepted models of competitive strategies, namely, the works of Porter (1998) and Treacy and Wiersema (2006). These two sources are combined to create a theoretical framework for logistics strategies in B2C e-commerce. Subsequently, a brief analysis of what the identified logistics strategies mean for general competition in B2C e-commerce will be provided. This closing, forward-looking analysis utilises economics-and history-based arguments.

II.2. Current trends in B2C e-commerce logistics

The derivation of the current B2C e-commerce logistics trends is based on a content analysis conducted in 2019 (Straubert et al., 2019). The content analysis used various practice-oriented sources that discuss current and future trends in B2C e-commerce logistics. In this content analysis, 36 different trends were coded, and those codes were further aggregated for this article. For the second coding level, we grouped the first-level codes into second-level themes. For example, the trends regarding faster delivery were grouped under the theme ‘faster delivery’, and the trends regarding the use of advanced software were grouped under the theme ‘more analytics’. We focused on designing the second-level themes in such a way that they are exhaustive and mutually exclusive (Krippendorff, 2013, p. 132). The results are displayed in **Figure II.1**.

The numbers represent the sum of the occurrences of the first-level trends under the corresponding theme.

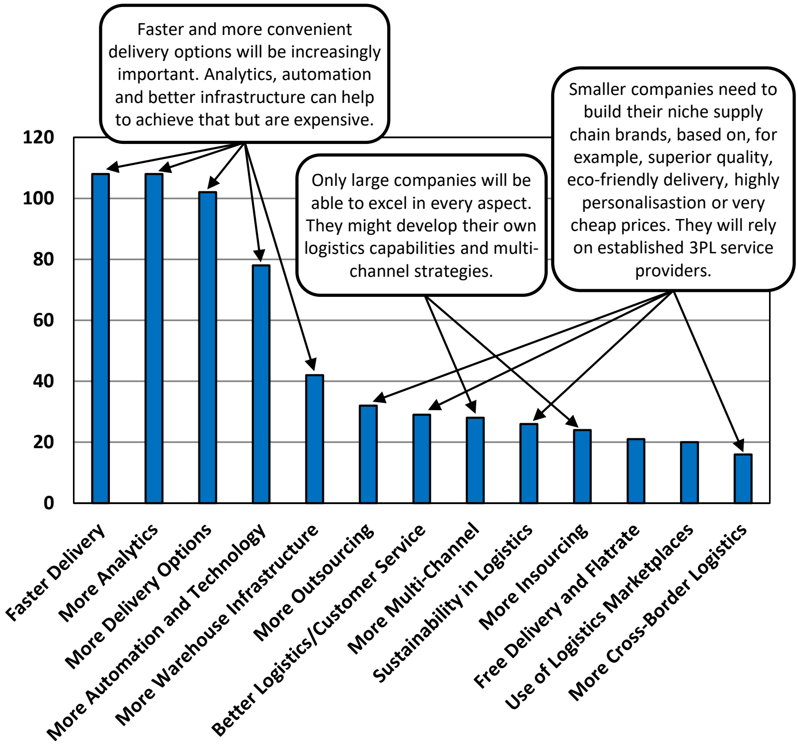


Figure II.1 Number of times a B2C e-commerce logistics theme was coded

The trend towards faster deliveries was mentioned most frequently. A fast delivery time is one of the performance features that are most important to customers. If an e-tailer is able to deliver products on the same day they were ordered or even within a few hours, it is able to provide near instant gratification, sometimes even faster than brick-and-mortar retailers (e.g., if the customer has to drive into town first). Therefore, very fast delivery erodes one of the biggest competitive advantages of brick-and-mortar retail compared to B2C e-commerce and is therefore particularly attractive to e-tailers (Gupta et al., 2004; Hsiao, 2009; Gawor & Hoberg, 2019).

However, very fast delivery is only possible if the products are stored close to the customer. Small regional or even urban warehouses are therefore required. This logic is confirmed by the content analysis. Thirty-four percent of the sources stated that there will be a growing number of small regional warehouses in the future. For large e-tailers and multichannel retailers (who can use their local stores as urban warehouses), this trend seems feasible. However, small e-tailers will have major problems establishing these small warehouses due to high rents and operating costs, on the one hand, and missing economies of scale, on the other hand.

However, customers are not always at home. This often leads to failed delivery attempts. A failed delivery attempt is not only expensive for the e-tailer but also prolongs the time until the customer holds the product in his or her hands. The sources included in the content analysis therefore often cite the use of click-and-collect solutions (pickup at a parcel shop or brick-and-mortar chain), parcel lockers and time window deliveries as promising trends. Additionally, the implementation of very advanced track-and-trace solutions is very often mentioned by the sources as a trend. A transparent delivery process is a good service and can help prepare customers for deliveries.

If e-tailers want to offer faster delivery times, more shipping options and transparent processes, they must consider how this can be done without significantly increasing costs. Advanced software and analytics can help and are often cited by the sources. One possibility for optimisation is, for example, the assortment decision for an e-tailer's urban warehouse, which, due to its size and cost structure, should probably not store every product offered by the e-tailer. Advanced algorithmic decision support, such as predictive picking and predictive shipping, can also help to reduce delivery times without large additional costs.

In addition, high degrees of automation in warehouses and delivery processes are often mentioned by the sources. Automated machines require large initial investments and high setup costs but are usually much cheaper than human labour during operations. In addition, the use of machines facilitates transparency in the processes, as the machines can be directly connected to software. Furthermore, machines work faster and can therefore contribute to shorter delivery times.

Large e-tailers can and probably want to develop and build these automated processes on their own (insourcing) to create a competitive advantage. Investments in automation, track-and-trace solutions, sophisticated algorithms or, for more forward-thinking e-tailers, drones and automated vehicles cost considerable amounts of money and require appropriate know-how (Straubert et al., 2019, p. 120). Smaller e-tailers will most likely not be able to develop and build these processes on their own and will have to outsource to external logistics providers if they want to offer logistical standards similar to those of large e-tailers. These two trends are also evident in the content analysis, as increasing both insourcing and outsourcing are often mentioned.

The B2C e-commerce market is becoming larger and more international, which makes the development of new business models possible. The analysed sources point out different possible approaches. If an e-tailer wants to differentiate itself logistically from the competition, it can achieve this through, for example, high-quality logistics (e.g., very good packaging, white-glove delivery). Alternatively, exactly the opposite can be achieved for a very streamlined but not particularly sophisticated logistics service, which makes a lower price possible. Some sources explicitly point to the trend towards international deliveries, including direct deliveries from overseas (e.g., China). Another frequently mentioned topic that lends itself to differentiation is the environmental impact of logistics. Some sources assume that no e-tailer can avoid this topic. On the other hand, the subject also creates an opportunity for a niche business model if an e-tailer wants to make its logistics very environmentally friendly.

In fact, in addition to the cost, the environmental impact of B2C e-commerce logistics (Mangiaracina et al., 2015) is the other main argument against a very fast delivery. Longer delivery times mean that customer orders can be bundled more effectively. Shorter delivery times have exactly the opposite effect. Less bundling can take place. In the case of a very fast delivery time of, for example, 1–2 hours, meaningful bundling can take place only very rarely. In light of increasing environmental awareness in the public debate, this is a strong argument against a very short delivery time.

Nevertheless, it cannot be denied that e-tailers have been offering increasingly shorter delivery times in recent years. The delivery time is a performance feature that is important to customers, and e-tailers would like to meet the demand. In addition, through very fast delivery, an e-tailer has the opportunity to be an alternative to brick-and-mortar retailers. Thus, there are good reasons for continuing the trend towards shorter delivery times, and very many of the sources studied believe this will be the case. Finally, yet importantly, a short delivery time can also help to differentiate oneself from competitors.

II.3. The significance of the current trends in B2C e-commerce logistics for the logistics strategies of e-tailers and the competitive landscape in general

There are basically only seven key elements of the e-commerce experience that are important to a customer: the price of the product the customer is shopping for, the website, the product assortment, the brand/company, the terms of the contract, customer service, and logistics:

- The price of the product is extremely important to the customer. As the product is the same regardless of which e-tailer it is shipped from, a higher price compared to that offered by a competitor is difficult to justify. Research shows that on meta search engines, the brand of the e-tailer, the price of the product and, to a lesser extent, the delivery time are the main factors that affect customer choice (Smith & Brynjolfsson, 2001).

- If customers use a meta search engine for their online shopping, the website and the product assortment are no longer differentiation criteria.
- The brands of an online shop are very important but at the same time the most intangible feature of an e-tailer. Furthermore, a brand has a limited half-life and must be constantly backed up by the corresponding service.
- Contract terms have become less important in recent years due to strengthened consumer rights.
- However, if the search is not performed via a meta search engine, the website of the online shop is perceived directly during the search-and-purchase process. A good design and functionality are therefore very important but somewhat subjective.
- Customer service and advice provided by the online shop can be important for a purchase decision but are generally difficult to measure and must be experienced before an assessment can be made.
- An assortment specially compiled by the e-tailer (e.g., an assortment geared towards gift ideas) can also serve as a means of differentiation from the competition. If customers are aware of the assortment orientation, this reduces search costs. However, this aspect is also difficult to measure.
- The logistics service offered by an e-tailer (delivery time, delivery flexibility, delivery transparency, etc.) occupies a special position among the areas mentioned. The logistics service offered is easily measurable and easily communicated to customers. The logistics service is therefore suitable as a clear distinguishing feature among competitors. A study of the B2C e-commerce market for electronic goods was able to measure a very strong and significant correlation between the logistics capabilities of an e-tailer and the success (profit, customer satisfaction, etc.) of the e-tailer (Cho et al., 2008). In addition, a significant strong negative correlation between logistics outsourcing and the success of the e-tailer was

measured. This means that, on average, e-tailers with good in-house logistics achieve the most success in the market. Furthermore, there are studies that confirm the competitive advantage provided by shorter delivery times: For the Taiwanese B2C e-commerce book market, experimental data (stated preference choice modelling) from 2002 were used to calculate an inflation-adjusted value of approximately US$1.61 per day of delivery time (Hsiao, 2009). This means that if an e-tailer has a delivery time that is one day longer than that of its competitors, the e-tailer would have to offer the book for US$1.61 less than the competition if it does not want to be rated comparatively worse by customers. Data from 2017 for the higher-priced US B2C e-commerce electronics market indicate even higher values. In a choice-based experiment with a subsequent conjoint analysis, the respondents valued one day less of delivery time at an average of US$3.84 after adjusting for inflation (Gawor & Hoberg, 2019). The rating is even higher for so-called lead-time shoppers. These customers place particular value on the time between ordering and receiving goods. For the randomly selected sample in the study, these shoppers composed just under 25% of the respondents and had an average inflation-adjusted valuation of approximately US$8.60 per day of delivery time. This is a remarkable valuation of delivery time. Admittedly, these are only two studies, and they determined their data with fictitious scenarios and online questioning tools. Nevertheless, these studies can serve as a good indication that an e-tailer is likely to achieve a concrete, monetarily comparable competitive advantage through shorter delivery times. Indeed, it is a logical assumption, as the distance to the store is also an important criterion in brick-and-mortar retail. For traditional brick-and-mortar retail, the same source computes an average valuation of approximately US$10.62 for one hour of travel time (e.g., return journey in the city).

This does not mean, however, that an e-tailer with comparatively poor logistics services cannot be successful in the market. A very good website with a specially curated product assortment, very good customer service or a business model that emphasises environmental sustainability can outweigh other weaknesses. Likewise, the e-tailer's administrative and logistics services could be organised so efficiently that a cost advantage over competitors is created and a lower price for its products is possible.

Generally, to survive in a market, a company needs a strategy/business model that creates a competitive advantage (Porter, 1998; Morris et al., 2005). If the business model is inferior, the company will eventually fail. The competitive advantage of a business model is generally dependent on the relationship between the value/utility of the product being offered and the price demanded for the product (Oster, 1999, p. 128; Besanko, 2013, pp. 294–307).

Traditionally, logistics is not regarded as a direct product characteristic. Instead, it plays a major role in an efficient value chain (Porter, 1985) and is therefore important for achieving low costs and good service. Hence, traditionally, superior logistics indirectly leads to competitive advantages. In the case of B2C e-commerce, we are confronted with a different situation. The delivery of goods is a major part of the service offered by e-tailers. In light of the absence of physical product differentiation, it is therefore evident that the logistics of an e-commerce company can be considered a central, primary aspect of its competitive strategy.

The existing literature on B2C e-commerce logistics strategies largely ignores this aspect. Instead, the focus lies on the process perspective and the back-end configuration (outsourcing vs. insourcing, etc.) (Ghezzi et al., 2012; Bask et al., 2012). Additionally, the well-known strategy research on the early e-commerce trend (during the dot-com bubble) misses this central role of logistics (Mahadevan, 2000; Bakos, 2001). However, since then (and largely because of the rise of e-commerce), the logistics industry has continuously become more competitive and innovative. Companies such as Amazon.com are successful, especially because their fulfilment services are very good and logistics is treated as a central aspect of their service offerings and competitive strategies.

Nevertheless, this is not to say that other parts of the e-commerce experience (e.g., the website, customer service, etc.) are not important. They are also very important. Therefore, we want to point out that the following thoughts about the future competitive landscape in B2C e-commerce are from a logistics perspective. This perspective is embedded in a larger overall picture, and for a successful business model, all parts must fit together (Osterwalder et al., 2005, p. 9).

Porter (1998) introduced a generic framework (known as 'Porter's generic strategies') for the categorisation of successful strategies. If a company chooses to only serve parts of a market, it pursues a 'focus' strategy. This is not the same as searching for a market niche but similar. If the company chooses to serve the whole market and to compete via price, it pursues a 'cost leadership' strategy. If the company serves the whole market and does not compete via price, it pursues a 'differentiation' strategy, i.e., the product/service must be different (better, depending on the customer) compared to other products.

Treacy and Wiersema (2006) ('the discipline of market leaders') presented a categorisation similar to *Porter's generic strategies* but slightly adapted to account for the modern environment. They proposed that a company must concentrate on either 'customer intimacy', 'operational excellence' or 'product leadership' while showing at least an acceptable performance in the other categories. The term 'customer intimacy' emphasises that not all customers have similar preferences. The company can choose to target specific customer groups, (e.g., groups that value eco-friendly businesses) and provide tailor-made solutions for the different groups. The term 'operational excellence' describes a strategy where the company does not offer complicated services or cutting edge technology. Instead, it tries to provide a basic (good) service for the lowest price possible. If competing via 'product leadership', on the other hand, the company tries to offer the best (generic) service/product for a fair price.

Many other sources use a categorisation similar to the two prominent examples mentioned above. Of course, dividing reality into three categories creates a simplifying model. Nevertheless, the categorisation mentioned above (although not without criticism) is widely regarded as a sensible division of the market. Combining these thoughts about business model categorisation and the future trends of B2C e-commerce logistics, we obtain a categorisation as shown in **Figure II.2**.

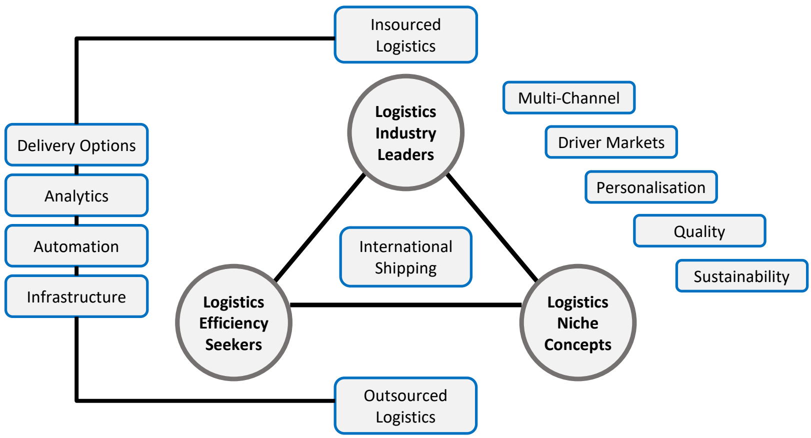


Figure II.2 The new competitive logistics landscape of B2C e-commerce logistics

- There are *'logistics industry leaders'*. These are large companies that have a considerable market share, a wide range of products and an extensive logistics infrastructure. Large parts of their logistics infrastructures are under their own control. Some industry leaders also have their own brick-and-mortar store networks. They offer a variety of fast and flexible delivery options at fair prices. They bind their customers in the long-term with subscriptions or bonus programmes. Each company has its own supply chain system, which it uses to differentiate itself from its major competitors. Examples of such companies are Amazon.com, Otto Group in Germany and JD.com in China.
- There are also *'logistics niche concepts'*. Usually smaller companies that focus on a small area of the market. The focus can be achieved through, for example, product assortment (e.g., luxury goods). The logistics service is aligned with the product range (e.g., through better packaging and white glove delivery). The

market focus can also be achieved through the supply chain itself. In line with the motto ‘buy the supply chain’, companies can, for example, specialize in environmentally friendly supply chains. The companies can develop a specialized ‘supply chain brand’, which helps to differentiate the overall branding of the company. The company HelloFresh SE is an example of a logistics niche concept. The company operates in the logistically very difficult food delivery business. Sophisticated packaging is necessary. Periodic deliveries and a limited assortment of predefined product baskets help to keep costs low.

- The third category contains the *‘logistics efficiency seekers’*. The logistics systems of these companies are less complex than those of the logistics industry leaders. Only the standard shipping methods provided by 3PL service providers are available. Since these companies cannot differentiate themselves from their competitors through complex supply chain systems or niche concepts, the aim of these companies is to achieve increasingly efficient and cost-effective logistics systems. This category is therefore very generic and contains many different types of companies. However, there are also companies that explicitly align their strategies towards logistical efficiency. In extreme forms, goods are shipped directly to customers from overseas, or customer orders are bundled into bulk orders (e.g., drop.com, AliExpress.com).

In addition to the three strategies, **Figure II.2** contains characteristics and services that are typical for the respective strategies. These characteristics and services are derived from the content analysis regarding the current and future trends in B2C e-commerce logistics (see above). The positioning of each characteristic and service within the figure implies the importance of the characteristic or service for the corresponding strategy:

- ‘International shipping’ can be used sensibly in all three strategies. Logistics industry leaders use international shipping to expand their product catalogues if local inventory is too costly. Logistics efficiency seekers generally use international shipping to save costs. They hold no or only limited local inventory because fast delivery is not as important. Logistics niche concepts use international shipping, for example, for made-to-order services.

- As mentioned above, logistics niche concepts differentiate themselves from their competition by offering special services, such as eco-friendly supply chains ('sustainability'), high quality in their fulfilment processes ('quality'), or personalised customer service and personalised products ('personalisation'). Logistics industry leaders can and will, of course, also offer all or some of these services, but these services are not at the core of their value propositions.
- 'Multi-channel' logistics will probably only be achieved by the logistics industry leaders. A 'multi-channel' fulfilment network is very complex and therefore not suitable for logistics efficiency seekers and logistics niche concepts.
- Logistics industry leaders use their own logistics networks ('insourced logistics'). Increasingly, they even use their own delivery networks with their own drivers. This is only possible because of their size. Nevertheless, they might find it difficult to react quickly in situations of demand spikes and thus may use 'driver markets' to supplement their own logistics capabilities.
- Logistics efficiency seekers and logistics niche concepts, on the other hand, are likely to use 'outsourced logistics'. Strategies geared towards efficiency strive for the lowest costs. Low costs in logistics are only possible with large fulfilment networks and their economies of scale. It is therefore logical to use third-party logistics providers who operate large networks for many customers. Logistics niche concepts are typically not geared towards huge transaction volumes. Thus, outsourced logistics is often the only viable strategy for logistics niche concepts to avoid very high logistics costs.
- Logistics industry leaders strive to provide the most versatile and innovative 'delivery options' to their customers, while logistics efficiency seekers only want to provide acceptable services for very

low prices. With large insourced logistics networks, it is easier to improve logistics services.

- Outsourced logistics systems therefore focus much more on large infrastructures, which provide the necessary economies of scale to be competitive in terms of logistics costs.
- Insourced logistics infrastructures are much more visible to e-tailers than infrastructures operated by third-party logistics service providers. E-tailers with insourced logistics therefore have an 'analytics' advantage. However, the topic of analytics is, of course, also important for outsourced logistics.
- 'Automation' is also important for both insourced logistics and outsourced logistics. However, third-party logistics service providers may not be as flexible as insourced logistics systems. Therefore, both strategies profit from the positive aspects of automation, but e-tailers with outsourced logistics may be disproportionately affected by the disadvantages of automation, such as less flexibility and reduced personalisation.

The three main logistics strategies are in competitive relationships. **Figure II.3** shows the main value propositions of the strategies and their competitive relationships exemplified by typical questions that can be asked regarding the competitive positioning of the e-tailers. Please note that these questions are only examples, and many other questions can be asked. The main value propositions are as follows:

'Logistics Industry Leaders'	►	'Superior overall service'
'Logistics Efficiency Seekers'	►	'Lower delivery fees'
'Logistics Niche Concepts'	►	'Special services/logistics'

An e-tailer with a logistics efficiency seekers strategy therefore might ask if its logistics costs are lower than the logistics costs of logistics niche concepts. If this is not the case, the e-tailer has failed and must take appropriate measures to reduce its logistics costs or change its logistics strategy. Concerning its relationship with logistics industry leaders, the e-tailer might ask if it is missing too many delivery options. Logistics efficiency seekers want to offer a very low price but must still be careful to offer at least a somewhat acceptable and modern service. Furthermore, logistics efficiency seekers are also in a competitive relationship among themselves. Assuming that they all have low costs and prices, logistics efficiency seekers do not necessarily compete among themselves on the basis of costs and prices. Instead, a customer who chooses between several logistics efficiency seekers might, in the absence of large price differences, consider other factors, such as the quality of the e-tailers' logistics services, when deciding which e-tailer to buy from.

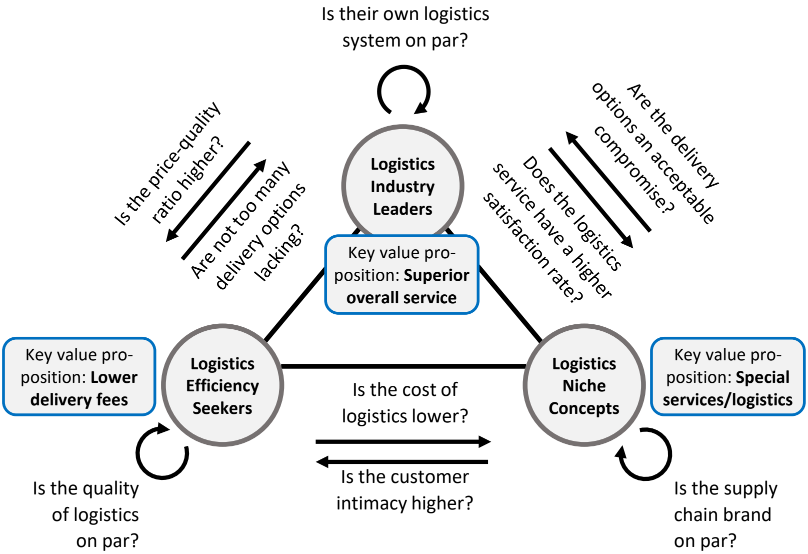


Figure II.3 The key value propositions of the three strategies and their relationships

It can be assumed that every B2C e-commerce company will decide which logistics strategy it pursues. Can an online shop that specializes in the delivery of curated apparel tailored to the individual tastes of its customer be successful and survive in the market without a logistics strategy? If the company is very good at what it is doing and can create a strong competitive advantage independent of its logistics operations, the answer is probably yes. Nevertheless, the e-tailer will likely opt for high-quality personalised packaging (niche concept) or will try to become more efficient in its basic logistics operations (efficiency seeker). This is not because those actions are the central aspects of its overall business model but because they help to create additional value or to lower costs.

In the B2C e-commerce market, however, there are special circumstances due to logistics. The logistics industry leader strategy has an inherent competitive advantage in regard to dominating the majority of the market.

The logistics niche concept strategy is by definition designed for niche business models. The logistics efficiency seeker strategy would theoretically be suitable to cover a significant market share but is—due to the economies of scale in logistics—at a strong disadvantage as soon as an e-tailer with a logistics industry leader strategy has a significant market share. The economies of scale in B2C e-commerce, especially in logistics, are very pronounced. For example, economies of scale are achieved through bulk discounts in purchasing and, due to larger lots, more efficient handling of incoming goods; through risk pooling in warehousing; and through denser warehouse networks in terms of delivery times.

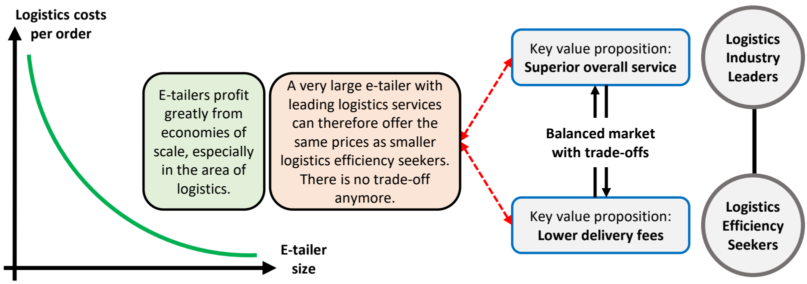


Figure II.4 The self-enforcing position of large e-tailers with leading logistics services

At the same time, logistics represents a very large part of the costs of e-tailers compared to other industries, and no differentiation can be made via the physical products being sold, as they are independent of the e-tailers. Therefore, once an e-tailer with a logistics industry leader strategy has a larger market share, that e-tailer is literally running away from the competition. Smaller competitors with logistics efficiency seeker strategies have problems offering lower prices than the market leader despite their efforts, as the market leader benefits from strong logistical economies of scale. In addition, the market leader's customers have little reason to switch to a competitor, as the market leader follows a logistics industry leader strategy, i.e., offers the best product/service bundle, at least in terms of logistics. This self-enforcing quality is summarised in **Figure II.4**.

Amazon has actually exploited this theoretically derived competitive advantage. Since its inception in 1994, Amazon has rarely made a substantial profit, and often even incurs a loss, with its B2C e-commerce business. In 1996, for example, Amazon generated $15.75 million in sales and incurred a $5.78 million loss. Instead, Amazon has focused on gaining market share through low prices and excellent service. This strategy has worked. Amazon has grown rapidly and has become dominant in important markets, such as the USA and Germany. Amazon has a clear industry-leading position in these countries, particularly in the area of logistics. Amazon is no longer incurring large losses with its B2C e-commerce business but rather generating small profits, and it still offers very competitive prices. Amazon recognised the importance of logistics for its business model early on. In 1999, one year before the dot-com bubble burst, Amazon's CEO Jeff Bezos explained the following in an interview (Lardner & Yang, 1999):

Interviewer: *"What's the biggest problem with E-commerce?"*

> Jeff Bezos: *"The logistics and the customer service--the nonglamorous parts of the business. A lot of these companies that are coming online spend all their money and effort building a beautiful Web site and then they can't get the stuff to the customers."*

In a TV interview that same year (CNBC, 1999), Jeff Bezos explained as follows:

> Jeff Bezos: *"Yeah, we have over 3000 employees and over 4 million square feet of distribution centre space, and those are things I am*

very, very proud of, because with that distribution centre space and half a dozen distribution centres around the country, it allows us to get product close to customers so that we can ship it to customers in a very timely way. Which improves customer service levels. That's what we are about. If there is one thing Amazon.com is about, its obsessive attention to customer experience. End to end. And that's whats those distributions centres are for."

Interviewer: *"So, you will open as many square feet of space, physical space as you have to, hire as many employees as you have to."*

Jeff Bezos: *"To service customers. Absolutely. And we will do it as rapidly as we can."*

Interviewer: *"That's a very cost-intense proposition."*

Jeff Bezos: *"Not compared to opening an equivalent network of retail stores. If you open a bunch of chain stores; look, when we open a distribution centre, we opening places that may have, you know, where we may pay 30 cent a square foot for a lease instead of paying 7 dollars a square foot, which you might pay in a high traffic retail area. So when you compare those things, they are not the same. You can't compare a big chain of retail stores to half a dozen distribution centres. It's just not. You know, it's bad math."*

[...]

"We are growing, you know, historically, very rapidly. We opening new product categories, we are expanding in new geographies; we have whole new business models with things like auctions. Now we think this is the last risky of the two approaches because scale is important in this business and it, you need scale also to offer the lowest prices and the best customer service to people. So scale is important to us. And we are going to go after that kind of scale, but it does mean that the executional challenges are huge. [...]"

These statements contrast with the scientific literature on competition in B2C e-commerce. The scientific articles on this topic are generally limited to the fields of marketing and information systems. In these articles, the word "logistic*" is often not even mentioned at all in the B2C e-commerce context. See, for example, Schmitz and Latzer (2002) and Ariguzo et al. (2006).

Amazon was successful with its logistics-driven strategy and is now in a very strong competitive position. In addition, Amazon has not relinquished its pioneering role in logistics services. Amazon recently started operating its own parcel lockers, it offers home delivery (Amazon Key) and it is testing 1–2 hour deliveries with its own vehicles and drivers. In addition, various other innovations, such as drone delivery, are being explored. However, Amazon is also active in the underlying logistics network. For example, Amazon Air is already competing with FedEx and UPS in air freight, and Amazon is now pushing into ocean freight as well.

Moreover, Amazon's competitive position is further strengthened by Amazon's platform character. In the USA, the United States Department of Justice (2019) announced in mid-2019 that it would review the market power of the leading online platforms (including Amazon). Through its *marketplace*, Amazon has a platform on which many of its direct competitors offer their goods for sale. This process has no implications for the logistics operations of Amazon but is nevertheless advantageous for Amazon because the sales information of its competitors can be used, and is already being used, by Amazon to optimise its own product assortment (U.S. House of Representatives, 2019).

This is already problematic from a competition point of view. However, in addition to the pure *marketplace*, there is also the option of *fulfillment by Amazon*. With this option, Amazon also takes over the logistics operations for the competitor, except for the purchase of goods. This reinforces Amazon's problematic competitive position. Amazon acts as a logistics service provider for its competitors, thereby strengthening its own logistics network. This enables Amazon to further expand its warehouse density and, at the same time, pass on some of the costs to its competitors.

II.4. Summary, limitations and outlook

The aim of this article was to develop a theoretical framework for the logistics strategies of modern B2C e-tailers. The result was a categorisation of three strategies: the logistics industry leader strategy, the logistics efficiency seeker strategy and the logistics niche concept strategy. Based on the current trends in B2C e-commerce logistics, it was explained why such a categorisation makes sense and what each strategy entails. Based on this theoretical framework, it was then argued what these logistics strategies mean for general competition in B2C e-commerce. We theorised that only companies with the logistics industry leader strategy have a chance to cover significant market shares in the long run. This is due to the importance of the logistics service to customers and the intense economies of scale that can be achieved in B2C e-commerce logistics. This creates a self-enforcing leading position for market leaders that provide good logistics services. In many other markets, this leading position is more vulnerable due to smaller economies of scale and the lesser importance of logistics.

For B2C e-commerce companies, these findings have the following implications: If an e-tailer is a logistics industry leader, it should strive to maintain its lead. Since logistics is such an important part of the value proposition in B2C e-commerce, this lead can definitely help to create an overall competitive edge. If an e-tailer uses a logistics niche concept, that e-tailer should align its logistics with its overall image. The e-tailer might even try to position its specific logistics and supply chain at the core of its brand in the sense of 'buy the supply chain, not just the product'. Logistics efficiency seekers, such as the logistics niche concepts, are unlikely to achieve large market shares. They might be content with covering the market for cheap supply chains (e.g., international shipping from China). However, if they want to achieve larger market shares, they will not be able to do so through their logistics rationalisation efforts (unlike in some other industries), as the large established companies are benefit disproportionately from economies of scale. Instead, they should try to offer acceptable logistics services and shine in other areas. Although logistics is very important in B2C e-commerce, there are also other ways to differentiate oneself from competition, for example, with a superior website or superior customer service. The logistics industry leaders are very difficult to challenge in the field of logistics. Instead, one should look for other areas where the barrier to entry is not so high.

However, the present article presents only a theory. Albeit well founded, this theory may or may not be true. Another limitation is that the article is largely based on a content analysis of the current trends in B2C e-commerce logistics. This content analysis has been supported and supplemented by other sources. Nevertheless, there is a risk that the content analysis cited has identified trends that will not occur. In this case, the theoretical framework for B2C e-commerce logistics strategies could also be partially unsuitable.

The theory developed should act as a cornerstone for further research. Therefore, the question arises, which research directions and research questions can be derived from it?

- An obvious research question relates to the possible niche concepts in B2C e-commerce logistics. Which niche concepts already exist? Which niche concepts are conceivable? How are the niche concepts perceived by customers? These questions could be answered with a market analysis and a questionnaire survey of customers.
- This article argued that an e-tailer with significant market share and a logistics industry leader strategy will outperform its competition. Because it has superior logistics service and, at the same time, economies of scale, it also offers very low prices. The company Amazon.com was cited as an example for the USA and Germany. It is therefore important to examine the following questions. Does Amazon have the best logistics service and very low prices in the USA and Germany compared to its competitors? How are these things perceived by customers? Tangible metrics such as product prices and delivery speed can be obtained by collecting data from e-tailers' websites and through mystery shop-

ping. For softer factors, different customers would have to be surveyed.

- If according to the theory presented, Amazon will become more powerful, the question of the competitive implications of B2C e-commerce logistics will arise. Although there are already investigations by the relevant authorities in the USA and Germany, these investigations refer to Amazon's platform, the *marketplace*. The role of logistics in the competition between e-tailers has been largely ignored until now. This article argues that B2C e-commerce logistics is indeed very important for competition in the B2C e-commerce market. There is a research deficit in this area.
- In addition, increasingly better e-commerce logistics is opening up new avenues for competition between brick-and-mortar retail and B2C e-commerce. If the logistics of e-tailers become faster and more convenient for customers, the competitive positions of brick-and-mortar retailers will be weakened. It is not clear to what extent a multi-channel model can bring success to large e-tailers. Ultimately, it is the customer who decides which channel to choose for his or her purchase. It is therefore important to better understand the customers. Such a study should include both customers who primarily shop locally and customers who primarily shop online.

We hope that this article can provide a basis and a starting point from which to explore these important questions. The B2C e-commerce market is growing and becoming increasingly important. We, as customers, are directly affected by the services of e-tailers, and research in this area is therefore important.

II. References

Ariguzo, G. C., Mallach, E. G. & White, D. S. (2006). The first decade of e-commerce. *International Journal of Business Information Systems, 1*(3), 239–255. https://doi.org/10.1504/IJBIS.2006.008598

Bakos, Y. (2001). The emerging landscape for retail e-commerce. *Journal of Economic Perspectives, 15*(1), 69–80.

Bask, A., Lipponen, M. & Tinnilä, M. (2012). E-commerce logistics: a literature research review and topics for future research. *International Journal of E-Services and Mobile Applications, 4*(3), 1–22. https://doi.org/10.4018/jesma.2012070101

Besanko, D. (2013). *Economics of Strategy* (6th ed.). Wiley.

Cho, J. J-K., Ozment, J. & Sink, H. (2008). Logistics capability, logistics outsourcing and firm performance in an e-commerce market. *International Journal of Physical Distribution and Logistics Management, 38*(5), 336–359. https://doi.org/10.1108/09600030810882825

CNBC (1999). *Jeff Bezos 1999 Interview on Amazon before the Dotcom Bubble Burst.* Retrieved March 15, 2021, from https://www.cnbc.com/2019/02/08/jeff-bezos-1999-interview-on-amazon-before-the-dotcombubble-burst.html

Gawor, T. & Hoberg, K. (2019). Customers' valuation of time and convenience in e-fulfillment. *International Journal of Physical Distribution and Logistics Management, 49*(1), 75–98. https://doi.org/10.1108/IJPDLM-09-2017-0275

Ghezzi, A., Mangiaracina, R. & Perego, A. (2012). Shaping the e-commerce logistics strategy: a decision framework. *International Journal of Engineering Business Management, 4*(2), 1–13. https://doi.org/10.5772/51647

Gupta, A., Su, B. C. & Walter, Z. (2004). An empirical study of consumer switching from traditional to electronic channels: a purchase-decision process perspective. *International Journal of Electronic Commerce, 8*(3), 131–161. https://doi.org/10.1080/10864415.2004.11044302

Hsiao, M. H. (2009). Shopping mode choice: physical store shopping versus e-shopping. *Transportation Research Part E: Logistics and Transportation Review, 45*(1), 86–95. https://doi.org/10.1016/j.tre.2008.06.002

Krippendorff, K. (2013). *Content Analysis: An Introduction to its Methodology* (3rd ed.). SAGE.

Lardner, J. & Yang, D. J. (1999). A Flea Market for Webheads. *U.S. News and World Report, 127*(14).

Mahadevan, B. (2000). Business models for internet-based e-commerce: an anatomy. *California Management Review, 42*(4), 55–69. https://doi.org/10.2307/41166053

Mangiaracina, R., Marchet, G., Perotti, S. & Tumino, A. (2015). A review of the environmental implications of B2C e-commerce: a logistics perspective. *International Journal of Physical Distribution and Logistics Management, 45*(6), 565–591. https://doi.org/10.1108/IJPDLM-06-2014-0133

Morris, M., Schindehutte, M. & Allen, J. (2005). The entrepreneur's business model: toward a unified perspective. *Journal of Business Research, 58*(6), 726–735. https://doi.org/10.1016/j.jbusres.2003.11.001

Oster, S. M. (1999). *Modern Competitive Analysis* (3rd ed.). Oxford Univ. Press.

Osterwalder, A., Pigneur, Y., & Tucci, C. L. (2005). Clarifying business models: origins, present, and future of the concept. *Communications of the Association for Information Systems, 16*(1), 1–25. https://doi.org/10.17705/1CAIS.01601

Porter, M. E. (1985). *Competitive Advantage: Creating and Sustaining Superior Performance*. Free Press.

Porter, M. E. (1998). *Competitive Strategy: Techniques for Analyzing Industries and Competitors*. Free Press.

Schmitz, S. W. & Latzer, M. (2002). Competition in B2C e-commerce: analytical issues and empirical evidence. *Electronic Markets, 12*(3), 163–174. https://doi.org/10.1080/101967802320245938

Smith, M. D. & Brynjolfsson, E. (2001). Consumer decision-making at an internet shopbot: brand still matters. *The Journal of Industrial Economics, 49*(4), 541–558. https://doi.org/10.1111/1467-6451.00162

Straubert, C., Asdecker, B. & Zitzmann, I. (2019). A content analysis on current technological trends in B2C e-commerce logistics. *Proceedings of the 7th IEEE International Conference on Advanced Logistics and Transport (ICALT)*, 117–122.

The United States Department of Justice (2019). *Justice Department Reviewing the Practices of Market-Leading Online Platforms. Review Focuses on Practices that Create or Maintain Structural Impediments to Greater Competition and User Benefits*. Retrieved March 15, 2021, from https://www.justice.gov/opa/pr/justice-department-reviewing-practices-market-leading-onlineplatforms

Treacy, M. & Wiersema, F. D. (2006). *The Discipline of Market Leaders: Choose your Customers, Narrow your Focus, Dominate your Market*. Basic Books.

U.S. House of Representatives (2019). *Responses to Questions for the Record Following the July 16, 2019, Hearing of the Subcommittee on Antitrust, Commercial, and Administrative Law, Committee on the Judiciary, Entitled 'Online Platforms and Market Power, Part 2: Innovation and Entrepreneurship'*. Retrieved March 15, 2021, from https://docs.house.gov/meetings/JU/JU05/20190716/109793/HHRG-116-JU05-20190716-SD038.pdf

Paper III:
Rundle in the Jungle!
Why Do People Subscribe to Amazon Prime?
Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program

Christian Straubert, Eric Sucky, Delia Altewischer

Abstract. We analyze the flat rate and bundle pricing nature of the Amazon Prime subscription. Based on two customer surveys (n=2062, n=906) and using a best–worst scaling (BWS) discrete choice experiment, we determine how important the different Prime benefits are to the subscribers. We perform a correlation and cluster analysis. We find mostly very weak correlations between the importance scores of the different Prime benefits. This is beneficial for a bundle pricing model such as Amazon Prime. We also find that the importance scores of the individual benefits are only weakly correlated with the actual usage rates of the benefits. This is beneficial for a flat rate pricing model such as Amazon Prime. Furthermore, we see indications customers value having various benefits included in Prime, even if they do not use them. We also discuss how Amazon Prime fits into the business strategy of Amazon.

Keywords: bundle pricing, flat rate pricing, subscription, loyalty program, B2C e-commerce

Reference: Straubert, C., Sucky, E., & Altewischer, D. (2024). Rundle in the Jungle! Why Do People Subscribe to Amazon Prime? Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program. Proceedings of the 57th Hawaii International Conference on System Sciences, USA, 4393–4402. https://hdl.handle.net/10125/106913

III.1. Introduction

In today's competitive environment, it is important for retailers to know how, where and when to engage consumers as they journey through the complex online-driven retail jungle (The Economist, 2017). In this article, we focus on the world's top-selling online retailer (Amazon.com) and its powerful tool in the battle for customers and market share, Amazon Prime. Amazon offers an ever-growing portfolio of complementary products and services so that the consumption of one product/service encourages the use of other products/services from the portfolio.

Amazon Prime is the consumer's ticket into this business ecosystem. Prime members pay an annual subscription fee to access benefits such as free two-day shipping, video and music streaming, gaming, photo storage, special discounts, and many more. Thus, Amazon Prime is a paid subscription program that bundles a range of products and services. Scott Galloway coined the term "Rundle" for bundled recurring revenue subscription models such as Amazon Prime (Galloway, 2020). These models have become increasingly popular in recent times and appear to be very successful. But what makes a 'Rundle' desirable to companies and their customers? We contribute to answering this question by investigating the Amazon Prime subscription, which is probably the world's most successful fee-based premium loyalty program.

Looking at the related literature (**Section III.2**), we note that while there is already research concerning the Prime subscription and similar programs, this research has not addressed the combination of bundle and flat rate pricing that can be found within Amazon Prime. These are two very important pricing strategies with distinctive economic features. Extant research, however, has often placed an emphasis on explaining customer behaviors from a psychological perspective and largely neglected an economic analysis and discussion. With this paper, we aim to contribute to closing this research gap.

In **Section III.3**, we explain the theoretical background of bundle and flat rate pricing models and derive concrete research questions and hypotheses. To answer our research questions and hypotheses, we conducted two customer surveys, which we will describe in **Subsection III.4.1**. In this section, we also explain the statistical methods that we used. Subsequently **Section III.4.2** contains the statistical results of our analyses. Finally, we discuss our results from a broader managerial viewpoint in **Section III.5** and close with **Section III.6**.

III.2. Related literature and research gaps

Extensive research exists on loyalty programs (LPs), which have long been used in a broad range of industries to reward repeat customers. Chen et al. (2021) provides an extensive literature review. Many empirical studies show positive effects of LPs ranging from increased purchase quantities and frequencies (e.g., Wang et al., 2016; Iyengar et al., 2022) to increased lifetime values, wallet shares (e.g., Leenheer, 2007; Gopalakrishnan et al., 2021) and an increase in customer satisfaction (e.g., Mimouni-Chaabane & Volle, 2010; Zakaria et al., 2014).

Much of the above research is about free LPs. Amazon Prime is a rather special LP because it is both a so-called fee-based LP and a subscription program. A fee-based LP requires an upfront payment to access the membership benefits. A membership fee can be viewed as an investment in future payoffs, and customers try to maximize their benefits so that the membership fee 'pays for itself'. In addition to rational considerations, many customers are also influenced by irrational biases. The subscription fee is held in a consumer's mental account and moves them to amortize the psychological burden of the cost (e.g., Thaler, 1980; Dick & Lord, 1998). This is widely known as the sunk cost effect and can lead to an increase in purchases to recoup the initial payment (Guo & Liu, 2023; Iyengar et al., 2022).

The most prominent benefit of the Amazon Prime subscription is Membership-based Free Shipping (MFS). A Prime subscriber pays the subscription fee and in return can order from Amazon without having to pay any further shipping fees. Guo & Liu, 2023 features a literature review about MFS. Sun et al. (2017, 2018) identified three effects associated with MFS: (i) a demand increase effect; (ii) a price increase effect, as higher prices can be realized for members; and adversely, (iii) a negative effect because nonmember demand decreases due to the higher prices.

Guo and Liu (2023) analyzed an B2C e-commerce retailer (e-tailer) with MFS and contingent free shipping (CFS). CFS means that nonmembers need to spend a minimum amount to be eligible for free shipping. The profitability of MFS is found to vary by customer segment. Some MFS customers are less profitable than CFS customers because they do not order more but merely order more often. However, MFS also increases the purchase and revenue contribution of many customers. Guo and Liu (2023) also found that the revenue increasing effect of MFS becomes stronger the longer customers have their subscription. The sunk cost effect, on the other hand, often weakens over time.

However, most research on MFS neglects the impact of program benefits other than free shipping. Literature on paid subscription programs that bundle multiple benefits, such as Amazon Prime, is very sparse.

Iyengar et al. (2022) studied a LP from a cosmetics retailer that bundled both MFS and special discounts. However, such a bundle is still very simple, and MFS was rather unimportant in their study context. Walsman and Dixon (2020) studied a LP from a hospitality firm with subscription fees ranging from $400 to $600. They drew a comparison with the Amazon Prime subscription and proposed that Prime subscribers predominantly use benefits that justify the subscription fee by providing clear savings. That is, free two-day shipping and Prime Video streaming. Free shipping provides a direct monetary saving and the monetary value of using Prime Video is quantifiable because the service can be easily compared with competing services such as Netflix.

Ashley et al. (2016), Krämer (2017) and Ramadan et al. (2021) explicitly studied the Amazon Prime subscription and are therefore closest to our research. The study by Ashley et al. (2016) found that customers who pay to be a member have more positive attitudes toward a loyalty program and value the benefits more favorably. Moreover, their results suggest that paid programs indeed increase net revenue (i.e., subscription fees + increased sales – waived shipping fees).

The study by Krämer (2017) confirmed stronger engagement and more intensive use when consumers subscribe to Amazon Prime. Krämer (2017) explained this behavior with the sunk cost effect and the taximeter effect. The taximeter effect is grounded in the theory of mental accounting (Thaler, 1985). Paying per use reduces the pleasure of consumption because consumers attribute the cost, and thus the pain, of paying for consumption at the time of use. In contrast, paying a flat rate decouples consumption from payment because the cost is mentally prepaid (Lambrecht & Skiera, 2006).

Ramadan et al. (2021) showed that programs such as Amazon Prime increase impulsive behavior and give shoppers a false sense of self-control. Amazon Prime members feel attached to the e-tailer both from a cognitive and emotional perspective, which reinforces their impulsive buying behavior.

In summary, research on fee-based LPs has mainly focused on explaining customer behavior based on psychological effects such as the sunk cost and the taximeter effect. An economic analysis both from the retailer's and customers' perspectives was often neglected. Moreover, almost no extant research on LPs addresses the combination of bundle and flat rate pricing. These are two very important pricing strategies. The economic theory behind bundle pricing is virtually absent from the existing literature. We aim to contribute to closing this research gap by examining the combination of bundle and flat rate pricing in the context of the Amazon Prime subscription and especially from an economical perspective.

III.3. Theoretical background, research questions and hypothesis development

In this section, we now take a closer look at the theoretical background of the central research topics of our paper (bundle pricing and flat rate pricing). This theoretical background guided us in the creation of the survey and its analysis.

The concept of bundle pricing is an important topic in both economics and marketing. Bundle pricing means that a company sells multiple products as a bundle with one price for the whole bundle. There are several biases that make bundle pricing attractive. According to prospect theory, for example, bundles are perceived as less costly (Stremersch & Tellis, 2002). In addition, customers seem to automatically infer savings from a bundle offer (Heeler et al., 2007).

However, bundle pricing can also be profitable if every customer acts completely rational. In economic theory, every customer has a reservation price for each component of a bundle (the highest price for which the customer would be willing to buy the product/service). Setting aside confounding effects, the reservation price of a bundle would simply be the sum of the reservation prices for the individual components. It can be shown mathematically that offering a bundle instead of selling the components individually can be profitable for the seller (in our case Amazon). Bundling tends to be more profitable the lower the correlation between the reservation prices of the individual components (Schmalensee, 1984; Hanson & Martin, 1990). An example of a negative correlation would be if customer 1 is willing to pay $15 for service A (e.g., a video streaming service) and $10 for service B (e.g., MFS), and another customer 2 is willing to pay $10 for service A and $15 for service B. If the services are sold separately, both services would be sold at a price of $10 to attract both customers, resulting in a maximum revenue of $40. In contrast, if services A and B are bundled and priced at $25, both customers would buy the bundle, and the total revenue would be $50, a 25% increase compared to the unbundled case.

Amazon decided to bundle many different benefits within their Prime subscription and its strategy is almost pure bundling. Only the Prime Video streaming service is available as a standalone subscription. All other benefits are exclusive to the Prime subscription. From a purely rational perspective, the profitability of this pricing strategy is therefore mainly dependent on how correlated the reservation prices of the Prime benefits are. In our paper, we focus on this fundamental effect.

Since it is questionable to ask survey participants directly for their reservation prices, we opted to use the importance of a Prime benefit as a proxy for the reservation prices and estimated the importance scores based on a choice-based experiment (more about this in the following **Section III.4**). Logically, the more important a benefit is to a customer, the higher the customer's reservation price should be for that benefit. We use this approach to answer the following research questions:

- **RQ1:** Because of what benefits do people subscribe to Amazon Prime?
- **RQ2:** How strong are the correlations between the importance scores of the individual Prime benefits?

Our digital appendix contains a table with all of the Prime benefits that we found. Note, that even a seemingly small service such as Amazon Music contains multiple smaller services, for example, music streaming and podcasts. One could argue that it is worthwhile to distinguish between these two because some customers might find podcast streaming very important but music streaming less important. For our survey, we had to decide on a subset of Prime benefits, and we had to combine some Prime benefits so that the survey did not become too long and tedious. From a pre-study, we knew that free (fast) shipping and Amazon Prime Video are very important to Prime subscribers. Therefore, we chose to prioritize these two benefits by aggregating them less than the other benefits. In total, we asked participants about 13 different benefits (listed in **Figure III.1.1**), covering almost all Prime benefits. Detailed descriptions, as shown to the participants in our second survey, can be found in the digital appendix.

Most of the 13 benefits are self-explanatory and a natural choice considering our research goals. However, two subtleties in our selection merit a brief explanation. Note that we asked survey participants how important "Free shipping" and "Free fast shipping" are to them. At first glance, one might assume that customers always prefer free fast shipping over free shipping. However, environmentally conscious people may not want to use fast shipping, and even people who like fast shipping could do without fast shipping but not without free shipping. A similar difference exists between the "Free on-demand video streaming (in general)" and the "Free on-demand streaming of exclusive movies/tv series" benefits. By breaking down the benefits, we can analyze which individual components are more or less important and how important the benefits are overall. It is important to keep this design in mind when interpreting our correlation analysis, as some of the benefits will be highly correlated simply because we have provoked this through our selection of the (sub-)benefits.

While the profitability of bundle pricing is closely related to the variability of the reservation prices for the individual components of a bundle, the profitability of flat rate pricing is mainly determined by the variability of the actual usage of the services and its valuation by customers (Lambrecht & Skiera, 2006). Customers who use a service a lot but only pay a fixed price (flat rate) are unprofitable because the variable costs that they create are higher than the fixed price. Customers who use the service seldomly, on the other hand, are profitable. There are also several biases which make flat rate pricing more attractive to customers such as the taximeter effect, the insurance effect and the overestimation effect (Lambrecht & Skiera, 2006).

At the same time, however, customer satisfaction with the Prime subscription is a key metric for Amazon. If every customer had the same valuation scale, customers who rarely use the Prime benefits would be less satisfied with the Prime subscription than customers who often use the benefits. For a successful application of flat rate pricing, it is therefore beneficial to Amazon if customers have different valuation scales. Usually, it would be ideal to have many customers who are satisfied with little usage. The Prime subscription, however, has the peculiarity that the frequent use of the free shipping benefit may be positive for Amazon if more gross profit is realized. Conversely, if customers merely ordered smaller quantities more frequently, Amazon's profit would decrease (Guo & Liu, 2023).

It stands to reason that, on average, customers who opt for a Prime subscription both order in smaller quantities and spend more money on Amazon. After all, free (fast) shipping is an important part of the value proposition of Amazon Prime. As outlined above, such behavior could have rational and irrational reasons. The only difference is that irrational biases would affect all Prime subscribers, while the rational behavior would be correlated with how important the free (fast) shipping service is to a Prime subscriber. Thus, we can formulate the following hypotheses:

- Customers with Prime order more frequently on Amazon (**H1.1**) and spend more on Amazon (**H1.2**).
- **H2:** Customers with Prime are more likely to order just a few items per order.
- **H3:** Both **H1** and **H2** are moderated by how important free (fast) shipping is to the respective Prime subscriber.
- **H4:** The relative effect of **H1.1** is stronger than the relative effect of **H1.2** (because Prime customer not only buy more on Amazon, but also in smaller order sizes).

To test these hypotheses, we asked in our first survey (which was answered by both customers with and without Prime) about the average online shopping frequency, on average, how much of the online shopping is done on Amazon and how much money the survey participants spend on Amazon, and how likely it is that they order 1, 2, 3, 4–5 and 6–10 items per order when shopping on Amazon.

However, such a comparison between customers with and without a Prime subscription is only possible for shopping on Amazon, as both types of customers can order from Amazon. Benefits such as music streaming are exclusive to Prime subscribers. We therefore asked in our second survey for a self-assessment (Likert scale) of the usage rates of the different Prime benefits. We also asked about the average Prime Video viewing hours per month. These additional questions allowed us to conduct several important analyses:

- **RQ3:** How strong is the correlation between the self-assessed usage rate of Prime Video (Likert scale) and the actual estimated usage (viewing hours)?

A correlation that is not very strong would be beneficial for Amazon's flat rate pricing strategy. Furthermore, it is known that a flat rate creates an insurance effect that is often irrationally highly valued by customers (Lambrecht & Skiera, 2006). For this bias, it is not even necessary that the subscribers think that they (will) use a benefit often. Part of the value of Amazon Prime would come from simply having the opportunity to use the benefits without extra costs. This suggests the following hypothesis, which we can test with our survey:

- **H5:** Self-assessed importance scores are higher than self-assessed usage rates.

For example, if Prime subscribers answer that they use a benefit only "Sometimes" but at the same time answer that they find the benefit "Very important", then this would indicate that the above effect is indeed present among Prime subscribers. However, it is important to note that our Likert scale for the importance of a benefit ranged from "Very important" to "Very unimportant", and our scale for the usage rates ranged from "Very often" to "Very rarely or never". The word "Unimportant" with its 'un'- prefix could be perceived more 'negatively' than a word such as "Rarely" which has no negating prefix. This could potentially create a bias.

Finally, we also asked about how satisfied the participants were with their Prime subscription (measured on a Likert scale: "Very satisfied", ..., "Very unsatisfied") This question enabled us to combine the economic theories behind bundle and flat rate pricing:

- **H6:** Customers who find many benefits important are more satisfied with their Prime subscription (because they have a high reservation price for the bundle, but pay the same price as everyone else, and therefore have a higher consumer surplus).
- **H7:** Customers who use many benefits are more satisfied with their Prime subscription (because they obtain much utility and

thus have higher reservation prices but must only pay the fixed flat rate price and therefore have a higher consumer surplus).

- **H8**: Taking **H6** and **H7** together explains the satisfaction with the Prime subscription better than either relationship on its own (because if having merely the option to use a benefit is valuable to customers, then the importance scores should contain partially different information than the usage scores).

III.4. The customer survey

III.4.1. Data collection, survey design and methods

We conducted our survey in May and June 2023 using the Sawtooth survey tool. To generate our survey responses, we used the Prolific panel, which is widely used by companies and in academia for marketing research (Eyal et al., 2022). We used a two-tier approach for our survey. For our first survey, we merely restricted the Panel to consumers in the United States. Therefore, the first sample consists of people with and without a Prime subscription. For our second survey, we invited the participants from our first survey who answered that they have a paid Prime subscription. Our digital appendix contains demographic statistics about both the first and the second sample. Due to the Prolific panel, we were able to cover a large portion of society representatively. Our digital appendix also contains further useful information, such as the questions that we asked in both surveys and additional statistics.

We were able to survey n=2200 people in our first survey and n=1021 in our second survey (complete answers). Both surveys contained multiple quality controls, such as comprehension and attention checks (instructional manipulation) and nonsensical answers. We removed all answer sets with poor quality and all sets from participants who clicked through the survey exceptionally quickly. After our quality checks, n=2062 and n=906 full answer sets remained.

We used the Sawtooth survey tool because it is the most mature software package available for best–worst scaling questionnaires. As stated in **Section III.3**, one of our primary goals is to determine how important the various Prime benefits are to Prime subscribers. For this purpose, we used a BWS discrete choice experiment. More precisely, we conducted a BWS case 1 experiment (the object case). The “objects” in our case are the different Prime benefits. This case of BWS can be used as an alternative to Likert scales and has become increasingly popular in recent years (Louviere et al., 2015, pp. 1–5, 14). That is, instead of asking “How important is the free shipping benefit to you?” and letting the participants answer on a Likert scale, the participants must click through several BWS questions. Note that in addition to our BWS experiment, we also asked Likert scale-type questions in our survey.

A BWS question consists of a set of items/objects. In our case, we showed 13 BWS questions, each with a set of four Prime benefits. Only looking at these four benefits (the benefits differed from question to question), the participants had to decide which one of the four they found most important and which one they found least important (our digital appendix contains an example). We generated many different randomized balanced incomplete block designs (BIBDs) with the R package “crossdes” and imported them into Sawtooth. Our BIBDs had the characteristic that over the 13 BWS questions, each benefit was shown in exactly four sets and co-occurred within these four sets exactly one time with every other benefit. Such a balanced design has the advantage that many known statistical methods can be used for estimating the importance scores and that these scores are more robust. The exact composition of the different sets was randomized, as was the display sequence of the different sets and the display position of the different items within the sets. To achieve an absolute scaling, we additionally opted for a direct-anchoring approach (Lattery, 2011). After the 13 BWS questions we asked for each of the 13 benefits whether the person finds the benefit actually important or not. The answers to these anchoring questions can then be combined with the ranking from the BWS questions to improve the ranking and to find a threshold under which a benefit is considered not important.

The statistical results, which we will discuss in the following, were generated using Sawtooth’s built-in analysis tool and double-checked with the R package “bwsTools”. The importance scores of the Prime benefits were estimated using hierarchical Bayes. The cluster analysis used a latent class multinomial logit model. The fitting process was started multiple times with random seeds to build confidence in the robustness of the identified customer groups.

Depending on the concrete analysis, we report two different scales for the importance scores. We report an anchored ratio scale ranging from min = 0 to max = 400 and an anchored interval scale ranging from min = -100 to max = 100. A ratio scale allows for statements such as ‘For the group of customers under consideration (e.g., the whole sample or only one person) the free shipping benefit is on average (avg. score of ~390) about 3.6 times as important as the free audio/music streaming benefit (avg. score of ~108)’. The interval scale is better suited for statements such as ‘Group 2 finds free on-demand streaming (avg. score of ~54) much more important than Group 1 (avg. score of ~13)’. Thus, the interval scale is also better suited for comparisons between individuals, that is, for regression analyses.

In addition to the statistical methods described above for analyzing the BWS data, we primarily used (multiple) linear regression and mean comparisons (e.g., u-tests) for the rest of our analyses.

III.4.2. Results

As an answer to our **RQ1**, we present in **Figures III.1** both the results of our BWS choice experiment and the answers to our Likert scale questions. For almost all respondents, free (fast) shipping is a very important Prime benefit. This suggests that the other benefits alone are not compelling enough to justify subscribing to Prime. However, they certainly make the Prime subscription as a whole more attractive. As expected, many survey participants also found free on-demand video streaming important, especially Amazon exclusive movies and tv series. On average, free live sport broadcasts, and kids specific content are less important. Free special shipping benefits, special discounts, free audio/music streaming and free e-books and audiobooks are also important to many customers. Free game(s) content, free photo storage and the free 7-day trial period are only important for a limited number of customers.

Nevertheless, although in the minority, there are special customer groups who found some of the less popular Prime benefits quite important. Young men, for example, find free games more important ($p<0.001$, $R^2=0.101$). Men are also more likely to state that they find live sport broadcasts important ($p<0.001$, $R^2=0.076$). Customers who use their Prime subscription together with children in their household find kids specific streaming content more important ($p<0.001$, $R^2=0.114$). More correlations based on demographics characteristics exist (albeit often less strong). The crucial takeaway is that different customers find different benefits important.

As an answer to our **RQ2**, we report a correlation matrix based on the anchored BWS scores (interval scale) in **Figure III.2.1**. Most of the correlations are very weak. Some are not significant. These nonsignificant correlations suggest that there is some potential for delineating customer groups. Ideal for Amazon's price bundling strategy would be a perfect negative correlation (r=-1). However, this is mathematically only possible in the case of two variables. We asked about 13 different benefits and therefore the lowest possible correlation, if all benefits were equally negatively correlated, is -1/12=-0.083. If two of the 13 benefits had a lower correlation, for example r=-0.5, this would mean that other benefits would have correlations higher than r=-0.083. Overall, many of the calculated correlations are significantly closer to r=-0.083 than to r=1. This means that it is likely that the economic conditions for bundle pricing to be profitable are good.

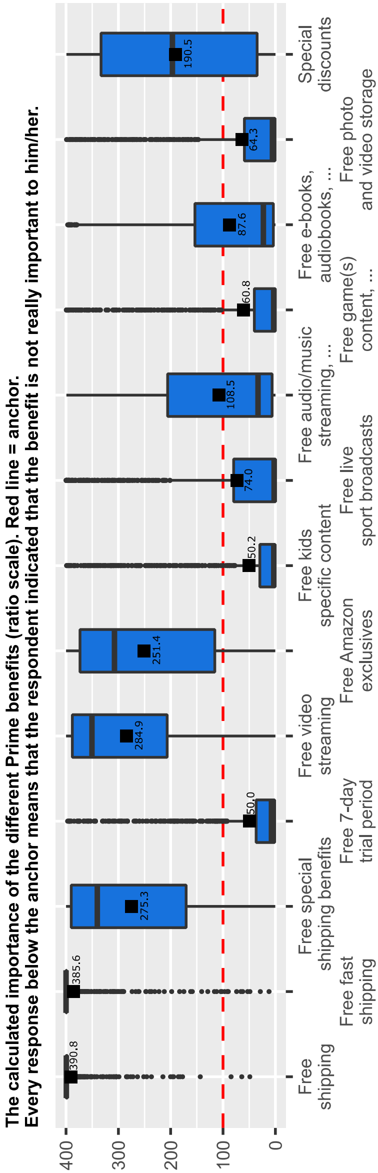


Figure III.1.1 The results of the BWS experiment and statistics about the Likert scale questions (part 1)

5 = "Very important" or "Very often"

Benefits	Self-reported importance of the benefits: 95% CI-	Mean	95% CI+	Hist.	Self-reported usage of the benefits: 95% CI-	Mean	95% CI+	Hist.	Actually important*:
Free shipping	4.82	4.85	4.88		4.73	4.76	4.79		99.45%
Free fast shipping	4.73	4.77	4.80		4.43	4.48	4.54		96.58%
Free special shipping benefits	3.96	4.02	4.08		3.22	3.31	3.39		73.29%
Free 7-day trial period	2.62	2.69	2.76		1.50	1.56	1.62		19.65%
Free video streaming	4.00	4.06	4.11		3.36	3.44	3.52		79.36%
Free Amazon exclusives	3.84	3.90	3.96		3.17	3.25	3.33		69.87%
Free kids specific content	2.37	2.44	2.52		1.72	1.79	1.85		19.09%
Free live sport broadcasts	2.37	2.46	2.55		1.71	1.78	1.86		25.17%
Free audio/music streaming, ...	2.99	3.06	3.13		2.13	2.21	2.29		40.73%
Free game(s) content, ...	2.44	2.52	2.60		1.74	1.82	1.90		23.07%
Free e-books, audiobooks, ...	2.92	2.99	3.06		2.06	2.13	2.20		37.97%
Free photo and video storage	2.56	2.64	2.71		1.68	1.76	1.84		25.28%
Special discounts	3.56	3.62	3.69		2.70	2.78	2.85		60.71%

* Percentage of participants who selected that the respective Amazon Prime benefit is **actually** important to them.

Figure III.1.2 The results of the BWS experiment and statistics about the Likert scale questions (part 2)

Benefits	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
1. Free shipping		0.66	0.33	0.01	0.12	0.12	0.18	0.29	0.32	0.12	0.27	0.22	0.14
2. Free fast shipping	***		0.46	0.2	-0.07	-0.05	0.2	0.22	0.17	0.25	0.16	0.21	0.19
3. Free special shipping benefits	***	***		0.29	-0.07	0.01	0.19	0.15	0.19	0.13	0.15	0.27	0.51
4. Free 7-day trial period		***	***		-0.02	0.04	0.04	0.04	0.18	0.05	0.2	0.21	0.33
5. Free video streaming	**					0.96	0.32	0.29	0.39	0.18	0.34	0.18	0.24
6. Free Amazon exclusives	**				***		0.31	0.31	0.43	0.2	0.34	0.21	0.26
7. Free kids specific content	***	***	***		***	***		0.15	0.35	0.2	0.22	0.18	0.14
8. Free live sport broadcasts	***	***	***		***	***	***		0.26	0.15	0.04	0.07	0.13
9. Free audio/music streaming, ...	***	***	***	***	***	***	***	***		0.2	0.45	0.41	0.19
10. Free game(s) content, ...	**	***	***		***	***	***	***	***		0.21	0.04	0.25
11. Free e-books, audiobooks, ...	***	***	***	***	***	***	***		***	***		0.29	0.33
12. Free photo and video storage	***	***	***	***	***	***	***		***		***		0.22
13. Special discounts	***	***	***	***	***	***	***	***	***	***	***	***	

Legend: *** = <0.001 significance level, ** = <0.01 significance level, * = <0.05 significance level

Figure III.2.1 The correlation matrix and a cluster analysis based on the BWS scores (part 1)

This can also be seen in our multinomial logit latent class cluster analysis. We report the results of the best fit on two clusters in **Figure III.2.2**. It is easy to see that Group 2 finds free on-demand video streaming more important than Group 1. Group 1 is more shopping oriented. However, the fit and explanatory value (e.g., Akaike Info Criterion, McFadden's R^2) is only marginally different between fittings on one, two, three or more clusters. Thus, the customer groups are not clearly delineated.

Given that almost every participant in our survey indicated that free (fast) shipping is important to them, it is not surprising that all of our hypotheses, **H1**, **H2**, **H3** and **H4**, are (partially) supported by the survey data (see also our digital appendix). In accordance with **H1**, customers with Prime indicated that they order on average 3.45 times on Amazon per month (95% CI: 3.27–3.64) vs. 1.27 times per month (95% CI: 1.16–1.39) for customers without Prime. A similar difference exists for the money spent online shopping on Amazon per month (mean: 133, 95% CI: 126–141 vs. mean: 60.6, 95% CI: 54.4–67). In support of **H2** we see

These scales are anchored →	**Interval scale:**		**Ratio scale:**	
Benefits	**Grp. 1**	**Grp. 2**	**Grp. 1**	**Grp. 2**
Free shipping	57	67	380	325
Free fast shipping	71	66	392	322
Free special shipping benefits	28	9	281	126
Free 7-day trial period	-15	-33	42	35
Free video streaming	13	54	183	289
Free Amazon exclusives	8	44	145	255
Free kids specific content	-24	-23	23	49
Free live sport broadcasts	-29	-6	16	84
Free audio/music streaming, ...	-8	1	62	103
Free game(s) content, ...	-23	-21	25	52
Free e-books, audiobooks, ...	-10	-11	55	71
Free photo and video storage	-17	-27	37	42
Special discounts	9	2	155	106

Interval: -100 to 100, Ratio: 0 to 400; Grp. 1 size: 62.9%, Grp. 2: 37.1%

The avg. max. membership probability is: 95.1%; McFadden's R^2: 0.374

Figure III.2.2 The correlation matrix and a cluster analysis based on the BWS scores (part 2) that respondents without a Prime subscription reported that they are most likely to order two items per order ($p<0.001$), compared to respondents with Prime, who are more or less equally likely to order one or two items order (see **Figure III.3**). We are not able to check **H3** because almost every survey participant found free (fast) shipping very important. These variables therefore have too little variance for any sensible analysis. On the other hand, **H4** is again clearly supported. Respondents with Prime estimate that they order 3.45/1.27 = 2.72 times as often on Amazon as respondents without Prime. However, they only estimate spending approximately 133/60.6 = 2.19 times as much money on Amazon. This is a significant difference ($p<0.001$).

Turning to **RQ3**, we can report that the self-assessed usage of Prime Video has only a moderate correlation with the estimated average number of viewing hours ($p<0.001$, $R^2=0.161$, see **Figure III.4**). This shows (at least for this benefit and our survey participants) that Prime customers are very heterogeneous in their assessments of what is frequent use. A person who answered that he or she uses free on-demand video streaming very often might mean an average of five or 15 hours per week. Such heterogeneity in customer perceptions is beneficial for a flat rate pricing strategy.

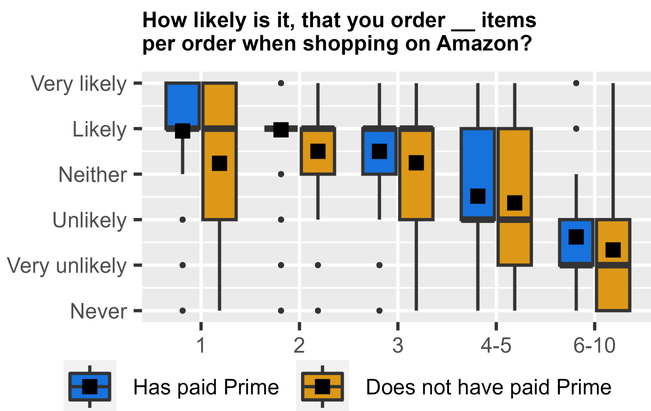


Figure III.3 Prime customers shop differently

The Likert scale responses are fairly consistent with the calculated anchored BWS scores. However, our survey participants seem to have had a tendency to rate benefits as more important on the Likert scale compared to when asked indirectly using the BWS questions. This tendency to find benefits important also becomes evident when looking at the differences between the self-reported importance scores and the self-reported usage scores (see **Figure III.1.2**). For every benefit, the mean importance score is higher than the mean usage score. This confirms **H5**. These scales cannot be compared 1:1; however, it is striking that, for example, only ~13% of the survey participants indicated that they use

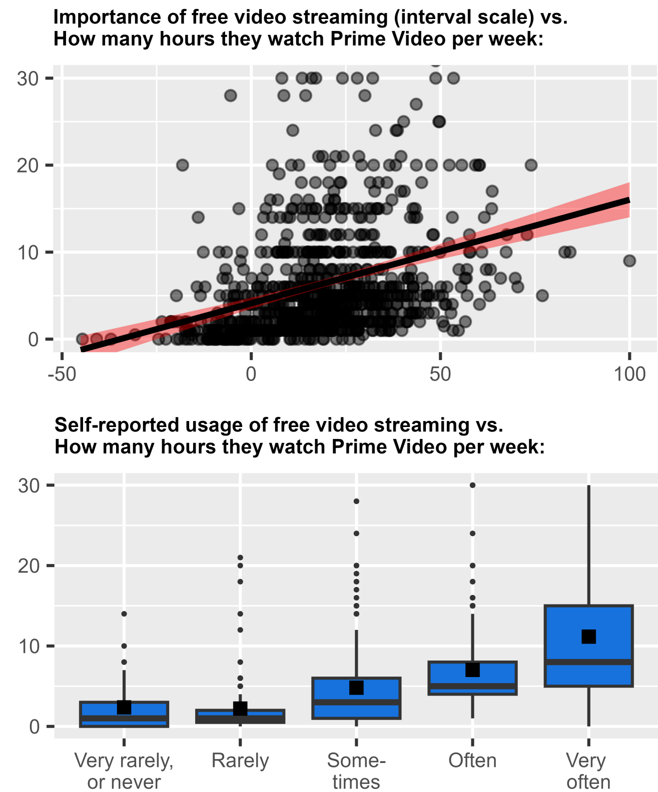


Figure III.4 "Very often" can mean something very different depending on the person free photo/video storage often or very often, but ~24% of the participants indicated that they find the free photo/video storage benefit important or very important. Similar comparisons can be made for many of the benefits. In particular, the benefits that are reportedly not used very often exhibit a large difference between the two scales ($p<0.001$, $R^2=0.659$).

Regarding the satisfaction of survey participants with their Prime subscription, we conducted multiple regression analyses. The importance scores explain 5.2% of the reported satisfaction (confirming **H6**), and a multiple regression against the usage scores explains 9.9% (confirming **H7**). The detailed results of the multiple regression analysis can be found in our digital appendix. A closer look reveals that the importance/usage of the free shipping benefit is the best predictor for satisfaction with the Prime subscription, followed by the free audio/music streaming benefit. All other importance/usage scores explain only little variance. A multiple regression analysis on both the importance and the usage scores explains 10.4% of the reported satisfaction (slightly confirming **H8**). This is just a small increase, which means that both the importance and usage scores contain much similar information. This is logical since both are closely related to the reservation prices of the participants (utility theorem). That the perceived usage explains more variance than the perceived importance also supports the intuition that the usage of benefits is more directly related to the utility received from the benefits (Goodman & Irmak, 2013). However, both the fact that the combined regression explains more variance, and the finding that the survey participants considered many benefits important even though they do not use them often, support the theory that Prime subscribers value simply having the option to use a service without having to pay extra.

III.5. Discussion and implications

Overall, we were able to confirm many of our hypotheses. It was our goal to look at the Prime subscription from a point of view that is more grounded in economic theories than is usually the case in other research about the Prime subscription. Indeed, based on our results, it is likely that classic economic theories have relevance for explaining the Prime subscription both from Amazon's and customers' perspectives. Given the enormous complexity of the Prime subscription, an explained variance of ~*10%*, for example, is more than significant. However, it is also clear that many other factors play a role in the perception and use of the Prime subscription.

Our correlation analysis revealed that the economic conditions for Amazon's bundle pricing strategy, while not perfect, are quite favorable overall. However, Amazon's strategy is not geared toward short-term profit. Primarily, Amazon wants to grow fast. A bundle pricing strategy has the advantage that the Prime subscription is very attractive for a wide range of heterogeneous customers (see also Stremersch & Tellis, 2002, p. 66). That the bundle pricing model also appears to be favorable from a short-term economic perspective creates a win–win situation for Amazon.

Membership-based Free Shipping also fits very well into the get big fast strategy. If, contrary to expectations, a customer who pays for MFS does not order much, then the flat rate revenue is higher than the variable costs, which is at least positive in the short-term. On the other hand, if a customer orders more due to MFS, this increases revenue long-term and therefore fulfills the primary goal of the strategy. Nevertheless, such a strategy can fail when customers do not spend more money overall, but instead, because of MFS, just order in smaller quantities. Then the e-tailer would not increase its revenue and even worse, would also have higher variable fulfillment costs. However, research has shown that MFS usually indeed leads to higher sales (see **Sections III.2** and **III.3**), and we also found indications for this in our survey data.

Jeff Bezos put it succinctly: "When we win a Golden Globe, it helps us sell more shoes" (Klatt, 2022). The idea behind this is clear. Benefits such as Prime Video make the Prime subscription more attractive, and since the Prime bundle always contains MFS, many customers will eventually order more from Amazon.

In addition to the economic perspective, it is also prudent to consider the ecological perspective. Our data shows that Prime customers are more likely to order only one or two items per order compared to customers without Prime. This can never be good for the environment. The tricky part, however, is that Amazon or any other e-tailer has no choice. The delivery process is a very important part of the value chain of an e-tailer, and free shipping is probably the best leverage point for a premium loyalty program. The whole value proposition of MFS is that the convenience of small orders does not cost more. Rational subscribers therefore always use this advantage; otherwise, the whole value proposition of MFS would be void. Interestingly, the Amazon Prime subscription, with its bundle of predominantly nondelivery-related benefits, has the potential to alleviate the pressure on subscribers to take advantage of MFS, since the subscription fee can be justified by using other, less environmentally harmful benefits. However, we have not seen any indication of this in our data.

Our results support the theory that Prime subscribers value simply having the option to use a benefit without extra costs. Moreover, past research has shown that customers often overestimate their future usage rates (Lambrecht & Skiera, 2006) and prefer multifeature products, especially if they have a hard time estimating their future usage rates (Goodman & Irmak, 2013). Both biases are beneficial to Amazon's strategy. Every usage of a benefit creates variable costs for Amazon. For a flat rate pricing strategy, it is, therefore, usually advantageous if customers use benefits only occasionally and are satisfied nonetheless. Additionally, Amazon bundles many different benefits and therefore exploits the bias that customers prefer multifeature products. Moreover, Amazon is in a special situation because a high usage rate of the free shipping benefit and the special discounts benefit are not necessarily bad for Amazon. This distinguishes the Prime subscription from the usual flat rate pricing models. High usage of these benefits can be a win–win for both Amazon and the Prime subscribers.

We conclude that the 'Rundle' strategy has many advantages because it exploits many rational and irrational, economic and psychological effects. However, it is by no means easy to implement. Only very large companies such as Amazon can create a product such as the Prime subscription on their own. Nevertheless, the 'Rundle' strategy offers so many advantages that it may be worthwhile for smaller companies to consider joining forces and creating a cross-company subscription program. Even Walmart, for example, despite being a very large company, has decided not to develop its own streaming service and has instead partnered with Paramount for its fee-based Walmart+ loyalty program. However, this dilutes the identity of the loyalty program. In addition, such cooperation is complicated by the fact that it is not always clear how subscription revenues should be distributed among the participating companies. These are important topics for future research.

III.6. Summary, limitations, and outlook

To the best of our knowledge, our survey is the first large scientific study to investigate the importance of the different Prime benefits. Our survey design allowed us to study the combination of bundle pricing and flat rate pricing, especially from an economic perspective.

We used a BWS discrete choice experiment to elicit the importance of the different Prime benefits from our survey participants. A correlation analysis of the computed importance scores showed that most correlations are very weak. Assuming rational customers, bundle pricing tends to be more profitable the lower these correlations. Thus, Amazon's bundling strategy is likely to be quite profitable compared to selling the Prime benefits individually.

Regarding the flat rate pricing component of the Prime subscription, we found that customers have very different perceptions of whether they use a benefit often or not. This means that Prime customers who rarely use a benefit may still have a high reservation price for this benefit. This is favorable for the profitability of Amazon's flat rate pricing strategy.

Usually, however, consumers do not behave in a fully rational manner. Fortunately for Amazon, many individuals are biased in favor of flat rates and bundle pricing. Regarding flat rate pricing, we found support for the theory that customers value simply having the option to use a service without paying extra. Creating value without increasing usage (i.e., variable costs) is very beneficial for the profitability of flat rate pricing.

In addition, not every company has the goal of maximizing profit in the short term. Amazon, for example, has a distinctly long-term strategy. In our discussion, we have argued that the Prime subscription with its combination of flat rate and bundle pricing fits very well into this long-term strategy.

Given our survey and the corresponding analyses, we are confident that we have made a significant contribution with this paper to the research about so-called ‘Rundles’, which are becoming increasingly important in business practice.

However, there are also limitations to our study. For example, the reported online shopping behavior and the reported viewing hours of Prime Video are based on estimates from the survey participants. These estimates are not perfect and can be biased. Nevertheless, most of our findings are so clear, that they remain valid, even if these estimates are moderately biased.

Due to the page limitations of this publication, we have also intentionally omitted some topics from our analysis. For example, potential interactions between hedonistic and utilitarian benefits. Furthermore, we combined individual Prime benefits such as e-books and audiobooks into category bundles. Amazon does the same on their website. One could say that the Amazon Prime subscription is being marketed as a bundle of bundles. Is this better than promoting all benefits individually? Many related topics are certainly sufficiently important to warrant future research.

III.7. Link to our digital appendix

dx.doi.org/10.6084/m9.figshare.23516244

III. References

Ashley, C., Gillespie, E. A., & Noble, S. M. (2016). The effect of loyalty program fees on program perceptions and engagement. *Journal of Business Research*, *69*(2), 964–973. https://doi.org/10.1016/j.jbusres.2015.09.001

Chen, Y., Mandler, T., & Meyer-Waarden, L. (2021). Three decades of research on loyalty programs: A literature review and future research agenda. *Journal of Business Research, 124,* 179–197. https://doi.org/10.1016/j.jbusres.2020.11.057

Dick, A. S., & Lord, K. R. (1998). The impact of membership fees on consumer attitude and choice. *Psychology & Marketing, 15*(1), 41–58. https://doi.org/10.1002/(SICI)1520-6793(199801)15:1<41::AID-MAR4>3.0.CO;2-N

Eyal, P., David, R., Andrew, G., Zak, E., & Ekaterina, D. (2022). Data quality of platforms and panels for online behavioral research. *Behavior Research Methods, 54,* 1643–1662. https://doi.org/10.3758/s13428-021-01694-3

Galloway, S. (2020). *Let's Get Ready to Rundle*. www.profgalloway.com/lets-get-ready-to-rundle/

Goodman, J. K., & Irmak, C. (2013). Having versus consuming: failure to estimate usage frequency makes consumers prefer multifeature products. *Journal of Marketing Research, 50*(1), 44–54. https://doi.org/10.1509/jmr.10.0396

Gopalakrishnan, A., Jiang, Z., Nevskaya, Y., & Thomadsen, R. (2021). Can non-tiered customer loyalty programs be profitable? *Marketing Science*, *40*(3), 508–526. https://doi.org/10.1287/mksc.2020.1268

Guo, F., & Liu, Y. (2023). The Effectiveness of Membership-based Free Shipping: An Empirical Investigation on Consumers' Purchase Behaviors and Revenue Contribution. *Journal of Marketing, 87*(6), 869–888. https://doi.org/10.1177/00222429231158116

Hanson, W., & Martin, R. K. (1990). Optimal bundle pricing. *Management Science,* *36*(2), 155–174. https://doi.org/10.1287/mnsc.36.2.155

Heeler, R. M., Nguyen, A., & Buff, C. (2007). Bundles = discount? Revisiting complex theories of bundle effects. *Journal of Product & Brand Management,* *16*(7), 492–500. https://doi.org/10.1108/10610420710834940

Iyengar, R., Park, Y. H., & Yu, Q. (2022). The impact of subscription programs on customer purchases. *Journal of Marketing Research, 59*(6), 1101–1119. https://doi.org/10.1177/00222437221080163

Klatt, T. (2022). The streaming industry and the great disruption: how winning a Golden Globe helps Amazon sell more shoes. *Media, Culture & Society, 44*(8), 1541–1558. https://doi.org/10.1177/01634437221104696

Krämer, A. (2017). Demystifying the "Sunk Cost Fallacy": When Considering Fixed Cost in Decision-Making is Reasonable. *Journal of Research in Marketing, 7*(1), 510–517. https://doi.org/10.17722/jorm.v7i1.701

Lambrecht, A., & Skiera, B. (2006). Paying too much and being happy about it: Existence, causes, and consequences of tariff-choice biases. *Journal of Marketing Research, 43*(2), 212–223. https://doi.org/10.1509/jmkr.43.2.212

Lattery, K. (2011). *Anchoring maximum difference scaling against a threshold–dual response and direct binary responses*. Sawtooth Software Technical Paper Library.

Leenheer, J., Van Heerde, H. J., Bijmolt, T. H., & Smidts, A. (2007). Do loyalty programs really enhance behavioral loyalty? An empirical analysis accounting for self-selecting members. *International Journal of Research in Marketing, 24*(1), 31–47. https://doi.org/10.1016/j.ijresmar.2006.10.005

Louviere, J. J., Flynn, T. N., & Marley, A. A. J. (2015). *Best-worst scaling: Theory, methods and applications*. Cambridge University Press.

Mimouni-Chaabane, A., & Volle, P. (2010). Perceived benefits of loyalty programs: Scale development and implications for relational strategies. *Journal of Business Research, 63*(1), 32–37. https://doi.org/10.1016/j.jbusres.2009.01.008

Ramadan, Z., Farah, M. F., & Bou Saada, R. (2021). Fooled in the relationship: How Amazon Prime members' sense of self-control counter-intuitively reinforces impulsive buying behavior. *Journal of Consumer Behaviour, 20*(6), 1497–1507. https://doi.org/10.1002/cb.1960

Schmalensee, R. (1984). Gaussian Demand and Commodity Bundling. *The Journal of Business, 57*(1), 211–230.

Stremersch, S., & Tellis, G. J. (2002). Strategic bundling of products and prices: A new synthesis for marketing. *Journal of Marketing, 66*(1), 55–72. https://doi.org/10.1509/jmkg.66.1.55.18455

Sun, G., Cavusoglu, H., & Raghunathan, S. (2017). An Analysis of Membership-Based Free Shipping Programs of Online Marketplaces. *ICIS 2017 Proceedings, South Korea*, 1–18.

Sun, G., Cavusoglu, H., & Raghunathan, S. (2018). Membership-Based Free Shipping Programs: A New Vehicle to Gain Competitive Advantage for Online Retailers? *ICIS 2018 Proceedings, USA*, 1–17.

Thaler, R. (1980). Toward a positive theory of consumer choice. *Journal of Economic Behavior & Organization, 1*(1), 39–60. https://doi.org/10.1016/0167-2681(80)90051-7

Thaler, R. (1985). Mental Accounting and Consumer Choice. *Marketing Science, 4*(3), 199–214. https://doi.org/10.1287/mksc.4.3.199

The Economist (2017). *The beast of Bentonville battles Amazon, the king of the e-commerce jungle.* www.economist.com/business/2017/12/07/the-beast-of-bentonville-battles-amazon-the-king-of-the-e-commerce-jungle

Walsman, M. C., & Dixon, M. J. (2020). Fee-based loyalty programs: An empirical investigation of benefit redemption behavior and its effects on loyalty. *Service Science, 12*(2-3), 100–118. https://doi.org/10.1287/serv.2020.0260

Wang, Y., Lewis, M., Cryder, C., & Sprigg, J. (2016). Enduring effects of goal achievement and failure within customer loyalty programs: A large-scale field experiment. *Marketing Science, 35*(4), 565–575. https://doi.org/10.1287/mksc.2015.0966

Zakaria, I., Rahman, B. A., Othman, A. K., Yunus, N. A. M., Dzulkipli, M. R., & Osman, M. A. F. (2014). The relationship between loyalty program, customer satisfaction and customer loyalty in retail industry: A case study. *Procedia - Social and Behavioral Sciences, 129*, 23–30. https://doi.org/10.1016/j.sbspro.2014.03.643

Paper IV: Making Third-Party Sellers More Attractive—The Case of Amazon

Christian Straubert, Eric Sucky,
Vanessa Felch, David Karl, Delia Altewischer

Abstract. We provide an analysis of third-party sellers on Amazon's online marketplace from a customer's viewpoint. While Amazon as a retailer sometimes directly competes with third-party sellers, Amazon is also interested in making the Amazon marketplace attractive for third-party sellers and making third-party sellers attractive to customers. Based on a large-scale survey (n=772) of Amazon customers in the U.S., we examine how much they like to buy from the different seller types (Amazon itself, third-party sellers with/without the Prime logo, i.e., with/without Fulfillment by Amazon). Among other results, we can show that the Prime logo on the seller side combined with a Prime subscription on the customer side significantly increases trust in a third-party seller, ultimately increasing third-party sales on Amazon's online marketplace. Furthermore, third-party sellers are implicitly incentivized to use the Fulfillment by Amazon service, which generates additional logistics service revenue for Amazon.

Keywords: B2C e-commerce, online marketplace, third-party seller, Amazon

Reference: Straubert, C., Sucky, E., Felch, V., Karl, D., & Altewischer, D. (2023). Making Third-Party Sellers More Attractive—The Case of Amazon. Proceedings of the 56th Hawaii International Conference on System Sciences, USA, 3838–3847. https://hdl.handle.net/10125/103100

IV.1. Introduction

In 2021, the total gross merchandise volume (GMV), including sales from Amazon itself and the marketplace, was more than $600 billion, adding nearly $120 billion in net growth. However, most of the growth came from third-party sellers, not Amazon as a retailer. Amazon's retail sales grew 14%, while the marketplace grew nearly 30%. Amazon's marketplace sales nearly doubled in two years, from $200 billion in 2019 to $390 billion in 2021, representing nearly 2/3 of Amazon's retail revenue. Ten years ago, in 2011, third-party revenue only accounted for approximately 38% of all sales (Kaziukėnas, 2022). Indeed, in the "2018 Letter to Shareholders", Amazon founder Jeff Bezos started with a list of the ever-growing share of third-party revenue and wrote (Bezos, 2019):

> "*We helped independent sellers compete against our first-party business by investing in and offering them the very best selling tools we could imagine and build. There are many such tools, [...]. But of great importance are Fulfillment by Amazon and the Prime membership program. In combination, these two programs meaningfully improved the customer experience of buying from independent sellers.*"

Amazon operates an online marketplace, where third-party sellers and Amazon itself offer products for sale. The relationship between Amazon and third-party sellers is ambivalent. On the one hand, Amazon has an incentive to offer products itself (i.e., to compete with third-party sellers) in order to capture the profit margins (Zhu & Liu, 2018). On the other hand, the attractiveness of the marketplace increases with the number of third-party sellers, a well-known part of the so-called Amazon flywheel. Additionally, Amazon generates profit through the sales commission fee that third-party sellers pay to Amazon. However, if customers do not like to buy from third-party sellers, a push for more third-party sales could backfire as a strategy. Therefore, Amazon is interested not only in the marketplace platform being attractive to third-party sellers but also in third-party sellers being attractive to customers.

The supposed ingenuity of combining the Fulfillment by Amazon (FBA) service and the Prime subscription is that Amazon not only increases the service quality of third-party sellers (3P sellers in the following) on Amazon's marketplace but also extracts more service revenue from them, making it optimal for Amazon to cede more of the overall demand to 3P sellers (because the opportunity costs are smaller). The FBA service is an optional service where 3P sellers can send products to Amazon fulfillment centers, and when a customer makes a purchase, Amazon handles packing, shipping, customer service, and returns for those orders for a fee that is paid by the 3P seller (in addition to the sales commission). 3P offers with FBA have Prime service available, and thus customers with a Prime subscription not only receive fast and free delivery when they buy from Amazon itself but also when they buy 3P offers with FBA. An intriguing hypothesis is that the combination of Prime subscription and FBA not only makes 3P offers with Prime service more attractive but possibly also makes 3P offers without Prime service less attractive, thus indirectly incentivizing 3P sellers toward the paid FBA service.

Our research is anchored in this context. The leading research questions were as follows:

- How much do customers like to buy from the different seller types (Amazon itself, 3P sellers with/without Prime/FBA)?
- What determines how much customers like to buy from the different seller types? We focus on the fulfillment processes (because of FBA) and the general trust in the different seller types (an important factor in B2C e-commerce in general).

We first present a systematic literature review of existing research addressing the Amazon marketplace. As we will show, not much research about these questions has been published. Following the literature review, we present the results of a survey of Amazon customers from the U.S. (n=772). Our survey provides insights into how important the FBA and Prime services are for the Amazon ecosystem.

IV.2. A systematic literature review

IV.2.1. Methodology

Our research focuses on Amazon's online marketplace. Thus, to identify relevant articles, we initially used the search string (Title = "Amazon" AND Abstract/key-words = ("marketplace" OR "fulfillment" OR "prime" OR "third party")) in four scientific databases, namely, Association for Information Systems eLibrary (AISeL), Business Source Ultimate via EBSCO, ScienceDirect (no title restriction), and Web of Science. Wildcard operators (*) were used where possible and sensible. The title restriction ensured that we only found articles explicitly focused on Amazon, similar to our study. The search was limited to articles in English published in scientific journals and conference proceedings. Altogether, 222 references were retrieved (see **Table IV.1**).

The initial search results were refined using the following criteria. In the first step, articles that were obviously not about the Amazon marketplace were excluded. These are, for example, articles about the biological ecosystem of the Amazon River and articles about the Amazon Mechanical Turk (MTurk) service. In the second step, we excluded articles that only discussed digital services or technical innovations—Amazon Prime Video, Amazon Web Services (AWS), Amazon Elastic Compute Cloud (EC2), Amazon Alexa, or Amazon Prime Air—and legal papers on Amazon's product liability. These criteria were checked against the articles' titles, abstracts, and keywords, narrowing the search results to 55 references. Next, we screened the articles based on their full text. Our intention was to identify articles that explicitly focused on the Amazon marketplace. Ultimately, 18 articles were classified as relevant.

Because we found relatively few articles relevant to our topic in our first literature search, we performed a second search for empirical research about Amazon in general with the following search string (Title = "Amazon" AND Abstract/keywords = ("questionnaire" OR "study" OR "experiment" OR "survey" OR "empiric" OR "case" OR "sample")) in the same four scientific databases mentioned above. We restricted this second search to articles published after 2009 because the B2C e-commerce market has changed considerably in recent years. Altogether, 1350 references were found in this second search (see **Table IV.1**). The search results were refined with a process similar to the first literature search. In total, 22 articles were classified as relevant. Both searches were conducted independently by two researchers from February to May 2022. The first search yielded 18 relevant references. This number, however, includes some duplicates across the four databases. After removing these duplicates, 15 relevant articles remained. The second search yielded 22 relevant references. After removing duplicates, a total of 13 remained. However, six of these 13 articles were already found in the first search. Thus, in total 22 relevant articles were found.

	Part I: Amazon marketplace and 3P sellers in general				**Part II: empirical research (e.g., surveys) regarding Amazon**			
Selection:	**# initial**	**# first**	**# second**	**# relevant**	**# initial**	**# first**	**# second**	**# relevant**
AISeL	23	15	9	6	23	12	8	8
Business Source Ultimate	39	35	17	4	136	22	15	9
ScienceDirect	96	19	7	1	165	9	4	1
Web of Science	64	42	22	7	1026	24	6	4
Subtotal	**222**	**111**	**55**	**18**	**1350**	**67**	**33**	**22**

Table IV.1 Search string results (incl. duplicates) for various databases and selection process

To date, there seem to be very few scientific studies that focus explicitly on the Amazon marketplace. This is surprising since the Amazon marketplace, as the largest B2C e-commerce marketplace, is very well suited as the subject of a case study. The small number of articles allows us to briefly address all the articles in the following. This is followed by a short discussion about how the existing studies relate to our research.

IV.2.2. Literature review results and discussion

Two articles addressed the relationship between the book market and Amazon: Tan et al. (2005) discuss the competitive implications of Amazon's marketplace for both the primary (selling new books) and secondary (reselling used books) book market. Chen (2008) argue that with the emergence of e-books, this competition is expanding to the digital channel.

Two articles are about the Amazon buy box: This box summarizes the most important information about a product offer (price, delivery time, ..., of the "best" offer) and is usually used by customers to place items in their shopping cart. The findings of Chen et al. (2016) indicate that the price of the offer is a crucial variable for winning the buy box. Gómez-Losada and Duch-Brown (2019) used a longitudinal approach and also found that the offer price plays an important role.

Two articles focused on product prices on Amazon's marketplace: Xu (2019) found that price promotions positively affect sales in the short term and can also have a positive long-term effect. Trenz and Veit (2012) investigated whether offers listed on price comparison (meta-search) websites affect sales on Amazon's marketplace. Depending on the product category, such a relationship can indeed be observed.

Eight articles researched customer reviews on Amazon: Ivanova et al. (2013) show that positive reviews have a greater influence on consumers' purchase intentions than negative reviews. Bao and Chang (2014) findings indicate that a positive feedback loop (including increased sales) can occur between traditional media, social media, and Amazon customer reviews. Khern-am-nuai et al. (2017) analyzed Amazon's Q&A system. They found that unanswered questions about a product negatively impact sales. Jeong (2021) showed how an analysis of customer reviews can be used to identify product characteristics that likely lead to good product reviews. Similarly, Huang et al. (2020) used product reviews to better predict consumer purchase preferences. Wu (2019) found that extrinsic motivation (e.g., status recognition) can crowd out intrinsic motivation and the enjoyment of writing reviews in various scenarios. Jabr (2022) developed, tested, and validated an approach to quantify the credibility of reviews. Jin et al. (2013) found that customers respond positively to price cuts when writing reviews.

One article (Cui et al., 2019) researched whether real-time information about the availability of goods can influence consumer buying behavior. Their findings indicate that a decrease in product availability (lower stock level) can have a signaling effect that increases sales.

Two articles used the so-called brand experience theory: Baswan and Farheen (2019) found that males and females perceive the Amazon brand differently. The emotional dimension of the brand is more important to females. Vakhariya (2020) identified important factors that influence the online shopping experience (namely, customer service, customer satisfaction, reliability, self-congruence, attractiveness, product variety, and affordability) and then compared the brand experience of Amazon to another online retailer.

Four articles addressed the relationship between Amazon and 3P sellers: Ritala et al. (2014) framed the Amazon marketplace as a coopetition-based business model. Coopetition refers to the phenomenon of simultaneous cooperation and competition (Nalebuff & Brandenburger, 1996). On the one hand, there is a collaboration with the 3P sellers by providing them the infrastructure and technical means to market their products online. On the other hand, as a retailer on the marketplace, Amazon directly competes with 3P sellers for customer orders (Ritala et al., 2014). The authors conclude that the Amazon marketplace is a win–win situation. Amazon can reduce operating costs because fewer products must be stored. In particular, however, Amazon can generate sales commission fees at negligible additional cost. On the other hand, the Amazon marketplace gives 3P sellers the opportunity to offer their products to millions of potential customers. Croitor and Werner (2021) investigated how "input control" (i.e., the mechanisms that screen 3P sellers before they can enter the Amazon marketplace) affects sellers' performance. For the Amazon marketplace, 3P sellers must, for example, prove the legality of their products, adhere to predefined product categories, and provide images that match specific attributes (Amazon, 2022). Based on survey results of 3P sellers on Amazon, Croitor and Werner (2021) conclude that 3P sellers have reduced motivation and thus a reduced performance if they find the input controls unfairly. Sun et al. (2020) analyze the choice between possible Fulfillment by Amazon (FBA) and Fulfillment by Seller (FBS). 3P sellers can fulfill demand through inventory stored in Amazon's distribution centers or through their own warehouse infrastructure. Using data from an e-retailer of wedding dresses in China to analyze the differences between these fulfillment options, the authors develop a decision model for choosing the right distribution channel based on predictive analytics. Zhu and Liu (2018) examined Amazon's entry patterns into 3P product spaces. If a 3P seller is successful with a product, Amazon may decide to offer the same or a similar product, potentially lowering the profit of the 3P seller. Using data from Amazon.com, the authors find that while Amazon is more likely to target successful product spaces, it is less likely to enter product spaces that require significant effort from the seller to grow. The authors recommend that 3P sellers should offer niche products on the marketplace and/or focus on products that require significant sales growth effort.

One article focused on the Amazon Prime subscription: It is well known that Prime members spend more money on Amazon.com than regular customers. However, the behavior and attitudes of shoppers in such programs have not yet been fully explored. Ramadan et al. (2021) show in their study that programs such as Amazon Prime reinforce impulsive behavior while giving shoppers a false sense of self-control.

Discussion: The literature review shows that our research questions have likely not been addressed in the literature thus far. Of the articles found, the ones about the relationship between Amazon and 3P sellers and the Amazon Prime subscription are probably closest to our research questions. Ramadan et al. (2021) found in their study that Prime subscribers, on average, feel emotionally more attached to Amazon than customers without a Prime subscription, which leads to more impulsive buying behavior from Prime subscribers on Amazon's marketplace. We also study the differences between customers with or without Prime subscriptions, but with a focus on the perceptual and behavioral differences depending on the different seller/offer types on Amazon's marketplace. Our study also has implications for coopetition between Amazon and 3P sellers (Ritala et al., 2014). A core hypothesis of our study is that the

Prime subscription combined with FBA makes 3P offers with FBA more attractive to customers. This seems (and probably is) favorable to 3P sellers, but this also means that 3P sellers have an incentive to use the paid FBA service, thus influencing models about the choice between FBA and FBS, such as the one from Sun et al. (2020).

IV.3. A Survey of Amazon customers

IV.3.1. Hypotheses underlying the models

While we can draw some inspiration from our literature review, it is also expedient to consider literature that does not explicitly focus on Amazon. There is already plenty of literature on B2C e-commerce in general and some literature on online marketplaces. For our survey, we note in particular that there is already well-cited and impactful research about trust in online retailing from the early 2000s (e.g., Gefen, 2000). It is generally accepted that trust plays an important role in customers' purchase intentions. Moreover, Gefen (2000) found that familiarity with an online retailer and its processes creates trust. It is easy to see how this logic could be applied to online marketplaces. However, although there are some similarities, the situation is also distinctively different. Many different 3P sellers are active on an online marketplace, and customers are often unfamiliar with these 3P sellers and do not build any lasting relationships with them. Thus, it was argued that in the case of online marketplaces, trust is created through so-called institutional mechanisms (Pavlou & Gefen, 2004). Institutional mechanisms are "soft" promises (e.g., the so-called Amazon A-to-Z guarantee) or "strong", legally binding mechanisms (e.g., all payments on Amazon.com are confidently processed by Amazon and not by 3P sellers) introduced by the marketplace operator and aimed at reducing risk and/or increasing trust. Several articles about this topic exist, predominantly about auction marketplaces such as eBay or the Amazon auction marketplace, which is no longer active (e.g., Pavlou & Gefen, 2004). To the best of our knowledge, there has been no such study (at least recently) that focuses on the Amazon retail marketplace and the differences in perception between Amazon itself and the 3P sellers on Amazon's online marketplace.

We largely follow the theory and logic used in Pavlou and Gefen (2004) for our survey and regression models and refer to their article for the relevant references. However, in addition to the trust differences between the different seller/offer types, our study has an additional focus on the combination of the Prime subscription and FBA. We, therefore, expand the theme of trust with questions about trust in the delivery and returns processes because, in the case of FBA, Amazon is responsible for these important processes (Nguyen et al., 2018).

Our model is logically relatively straightforward. We hypothesize that: Trust when buying from Amazon itself > Trust when buying from a 3P seller with Prime service > Trust when buying from a 3P seller without Prime service.

Note that we asked most questions in our survey two times, one time with "Prime service" and a second time with "shipped from Amazon itself". These two are almost always the same in practice because usually, the Prime logo is only awarded to offers shipped by Amazon (i.e., offers from Amazon itself or 3P seller offers with the FBA service). However, a customer without a Prime subscription is probably (we asked this in our survey) not paying much attention to the Prime logo/service and is instead paying more attention to whether a 3P offer is shipped from the 3P or Amazon.

Based on the existing research (Gefen, 2000; Pavlou & Gefen, 2004), we further assume that trust in the different buying options influences how much customers like to buy from the different sellers. Thus, we hypothesize the following: Like to buy from Amazon itself > Like to buy from a 3P with Prime service > Like to buy from a 3P without Prime service.

Note that other studies (e.g., Pavlou & Gefen, 2004) asked questions about a customer's purchase intention depending on the seller type. However, such a question is only of limited use in our context because, logically and ceteris paribus, a customer would always buy from their preferred seller type if possible. Questions about general purchase intentions are therefore not expedient. Instead, we used scenarios (e.g., Amazon itself does not offer the product) and asked the survey takers to estimate what they would do in such a scenario. There are more details about this at the end of this subsection.

However, other factors in addition to trust also influence how much a customer likes to buy from the different seller types. For our study, we focused on selected factors that are related to trust/familiarity and the Prime/FBA service.

Unlike 3P sellers, Amazon can build familiarity with customers (Ramadan et al., 2021). However, this familiarity could also be detrimental. Amazon is a very public company, and some people do not like Amazon as a company, perhaps because they have seen documentaries about demanding working conditions in Amazon's fulfillment centers. This could influence not only how much a customer likes to buy from Amazon itself but also how much a customer likes to buy from 3P sellers with Prime/FBA service or generally from the Amazon marketplace.

Amazon is generally known for its comparatively fast delivery times, and one of the most prominent features of the Prime subscription is that the fast "premium" shipping is free for offers with Prime service. Thus, we hypothesize that customers who like or need fast delivery would rather buy from Amazon itself or 3P sellers with Prime/FBA service.

Because fulfillment is so important in B2C e-commerce (Nguyen et al., 2018), we also asked how much customers trust the delivery and returns process when they order from a 3P seller with or without Prime/FBA service. The hypothesis is, of course, that trust in the different types of 3P offers is dependent on trust in the delivery/returns process. However, while the FBA service decreases the delivery/returns risk (Amazon also processes the returns for the 3P sellers when they use the FBA service), it is still possible that the 3Ps sell counterfeit products. Therefore, we also asked the customers about their trust in not receiving a counterfeit product when buying from a 3P seller. Logically, this trust should be largely independent of the Prime/FBA service.

We also asked the survey respondents whether they knew some of the institutional mechanisms of Amazon. We asked about the soft "Amazon A-to-Z guarantee" and the strong mechanism that all payments on the marketplace are exclusively processed by Amazon. We also asked whether the respondents trust that Amazon takes their side in a dispute with a 3P seller.

Amazon's strategy of using the Prime subscription and FBA service to generate additional service revenue and make 3P offers more trustworthy and attractive seems to be a good concept. However, it also makes the marketplace as a whole more complex. Some customers may find this exhausting and prefer a conventional online store instead. We asked about this in our survey.

Our model concludes with a question about what percentage of their online shopping the respondents do on Amazon.com and two scenarios about what they would do (on average) when Amazon itself does not offer a product (scenario 1) or when neither Amazon itself nor a 3P seller with Prime/FBA service offers the product (scenario 2). Do they think they would buy from a 3P seller with Prime/FBA service, from a 3P seller without Prime/FBA service, from another online shop, from a brick-and-mortar store, not buy at all, or choose another alternative? We hypothesize that the factors mentioned thus far directly or indirectly influence customer behavior with regard to these scenarios. The behavior indicated in these scenarios probably also influences what percentage of their online shopping the respondents do on Amazon.com. Last but not least, the answer to many of our questions probably also depends on whether the customer has a Prime subscription or not.

IV.3.2. Data collection and analysis methodology

The survey was conducted in May 2022 using the SurveyMonkey survey tool. The survey was targeted at Amazon customers from the U.S. using the SurveyMonkey Audience panel. This service is used both by companies, for example, for market research purposes, and by academia for research (e.g., Hall et al, 2017). A slight bias is introduced because the respondents answer many surveys online and are, therefore, probably more open to online shopping than the general population. However, this is not a problem per se because people who like to shop online are also the most important customer group for Amazon. The panel makes it possible to survey people from all age groups (≥18 years), income groups, and of all employment statuses. We used the census option, and therefore, our sample mirrors the actual USA census with some margin of error. For more details, see our digital appendix: → https://doi.org/10.6084/m9.figshare.20043131.v3

The digital appendix contains additional statistics, our models, and our survey questions. The survey consisted mainly of 7-point Likert scale questions (from strongly disagree to strongly agree), some percentage drop-downs and sliders, and single-choice questions (knowledge questions).

We were able to survey n=1070 people (complete answers) who answered that they had ordered something from Amazon at least once during the last 12 months (screening question). The SurveyMonkey Audience respondents are experienced survey takers and have a monetary incentive to speed through the survey. Thus, we excluded answers that were given faster than even an experienced survey taker could have done in a thoughtful manner. We also had some control questions, which we used to check the quality of the answers. After our quality checks, n=772 remained.

We use structural equation modeling (SEM) to test our hypotheses. Because of the complexity of our model and the number of questions necessary, we opted against a dedicated measurement model (confirmatory factor analysis). With the usual >3 questions per factor, we would have needed a prohibitively long survey. Instead, we decided that the structure of the model was more important. Furthermore, we do not have many opaque concepts, and the concept of trust has been researched in the past. To reduce the measurement error, we gave extra explanations, for example, regarding what we mean when we ask about “trust in a good experience when buying from ...”.

We used the R package "lavaan" for our SEM models. We tested the survey answers on normality using the "mardia" test, and the data clearly showed nonnormality. Therefore, we used the robust "MLMV" estimator (mean and variance corrections) for our models. This estimator often produces the most accurate results (Gao et al., 2020). However, it may be noted that our models also produce acceptable to good results with other robust or even nonrobust estimators. We split our hypothesized model into two SEM models for better readability. For the same reason, the models reported within this article contain only significant regression paths. The digital appendix also contains models with insignificant paths and one large model. However, the results do not differ much. We used SEM as a confirmatory hypothesis testing method, meaning that all paths reported have an underlying grounding in the logic developed in the previous subsection. **Figures IV.1** contain descriptive results of our survey. **Figures IV.2** and **IV.3** contain the SEM models and their fit estimates, indicating a good fit.

IV.3.3. Survey results

By and large, the data from our survey confirmed our hypotheses. Thus, we will not comment on every path modeled in the following and instead focus on highlights and perhaps less apparent results. Additionally, note that the demographics of the survey participants had only limited effects in our models. Only the age and gender of the respondents had some significant effects. For example, males seem to like Amazon more and tend to buy more often from Amazon, supporting the results of Baswan and Farheen (2019).

Results of model 1: This model (see **Figure IV.2**) contains paths leading from the 'Trust in ...' variables to the 'Percentage of online shopping on Amazon' variable. The model contains our "Trust in ..." and "Like to buy from ..." questions about 3P sellers with or without Prime service. However, a similar model with our questions regarding FBA can be constructed (→ digital appendix). Both models are similar; however, the model that focuses on the Prime service is more fitting for Prime subscribers, and a focus on FBA is more fitting for customers without a

Prime subscription, as these customers do not pay much attention to the Prime service (see **Figures IV.1**).

The model shows that trust in a good experience when buying from a certain seller type indeed has the strongest effect on how much the surveyed customers like to buy from these seller types. Furthermore, whether a customer has a Prime subscription or not has the second strongest effect for all three seller types in our model. If customers dislike Amazon as a company, this has, on average, a negative impact on both how much they like to buy from Amazon itself (strongly) and from 3Ps with Prime service available (weakly). This makes sense because Amazon is usually involved in Prime delivery through its FBA service. This result is rather important because it implies that Amazon must be careful not to spread its unpopularity (among some customers), through the FBA service, into 3P offers. It also becomes clear that some customers find the Amazon marketplace, with its different seller types, too complex and therefore tend to dislike ordering from 3P sellers. Additionally, note that 'Trust in a good experience when buying from Amazon itself' also affects 'Like to buy from 3Ps without Prime service available'. This is an indication that trust differences, in addition to absolute trust levels, influence how much customers like to buy from which seller type.

Similar effects can be observed for scenario 1. As a reminder, in scenario 1, we asked what the respondent would do (on average) when Amazon itself does not offer a product the respondent wants to buy. Also, note that the trust in the different seller types also directly affects the answers to the questions of scenario 1. Many of the variables have direct and indirect effects. The digital appendix contains the respective calculations. Finally, note that we also tested for moderation effects, especially dependent on whether a customer has a Prime subscription or not. However, few moderation effects exist. This also means that the negative effect of 'The Amazon marketplace is too complex' on 'S1: %Prob. buy from 3Ps with Prime/FBA' is largely independent of whether the respondent has a Prime subscription or not. This indicates that some Prime subscribers also prefer to buy solely from Amazon itself.

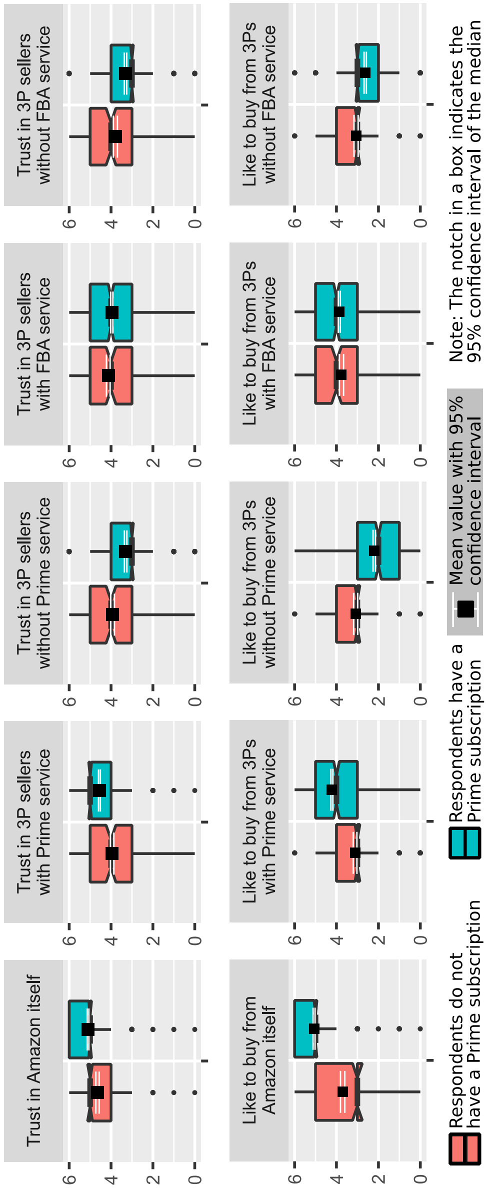


Figure IV.1.1 Descriptive statistics (part 1)

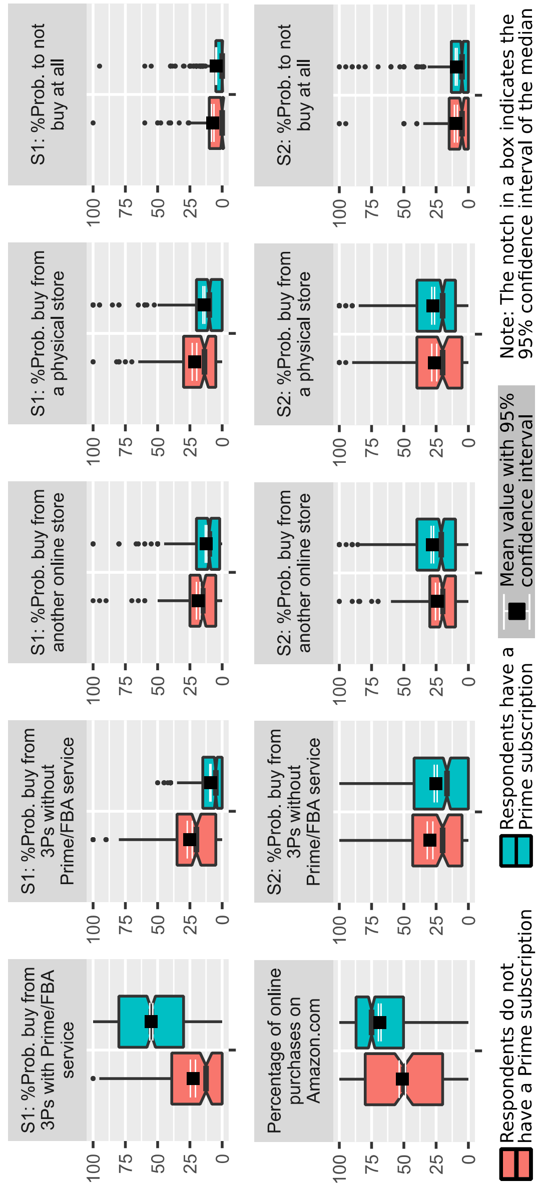


Figure IV.1.2 Descriptive statistics (part 2)

CI: Lower (-) and upper (+) bound of the 95% confidence interval Unless otherwise indicated: 0-6 Likert scale (0 = Strongly disagree, 6 = Strongly agree)	**Respondent has a Prime subcription**			**Respondent does not have a Prime sub.**		
Variable	**Mean**	**CI-**	**CI+**	**Mean**	**CI-**	**CI+**
Pays attention to whether an offer on Amazon.com has Prime service available or not	4.65	4.52	4.78	1.82	1.55	2.09
Pays attention to whether an offer is from Amazon itself or from a 3P seller	4.13	4.00	4.26	3.90	3.68	4.13
Pays attention to whether a 3P offer is shipped from the 3P or from Amazon itself	4.09	3.95	4.22	3.81	3.57	4.05
Fast delivery is important when buying online	4.76	4.66	4.85	3.98	3.78	4.18
With its different seller types, the Amazon marketplace is too complex	2.03	1.91	2.16	2.49	2.28	2.71
Trust in a good experience when buying from an online store in general	4.16	4.07	4.25	4.27	4.13	4.41
Trust in the delivery process when buying from a 3P with shipping from Amazon itself	4.57	4.48	4.67	4.54	4.39	4.69
Trust in the delivery process when buying from a 3P with shipping from the 3P	3.60	3.49	3.71	4.00	3.82	4.18
Trust in the returns process when buying from a 3P with shipping from Amazon itself	4.40	4.29	4.51	4.22	4.03	4.41
Trust in the returns process when buying from a 3P with shipping from the 3P	3.27	3.15	3.39	3.58	3.38	3.77
Trust that I will not receive fake products from 3P sellers	3.50	3.37	3.63	3.78	3.57	3.99
Trust that Amazon will take my side in a dispute with a 3P seller	4.39	4.27	4.51	4.26	4.07	4.45
Do not like Amazon as a company	2.01	1.85	2.18	2.52	2.23	2.81
Knows the "Amazon A-Z guarantee" (0 = no knowledge, 1 = knowledgable)	0.29	0.27	0.32	0.20	0.16	0.25
Knows that all payments are exclusively processed by Amazon (same scale as above)	0.46	0.42	0.49	0.39	0.33	0.45
Knows the details of the Fulfillment by Amazon service (same scale as above)	0.48	0.45	0.52	0.42	0.36	0.48
Knows the details of the Prime service (same scale as above)	0.88	0.86	0.90	0.77	0.73	0.82

Figure IV.1.3 Descriptive statistics (part 3)

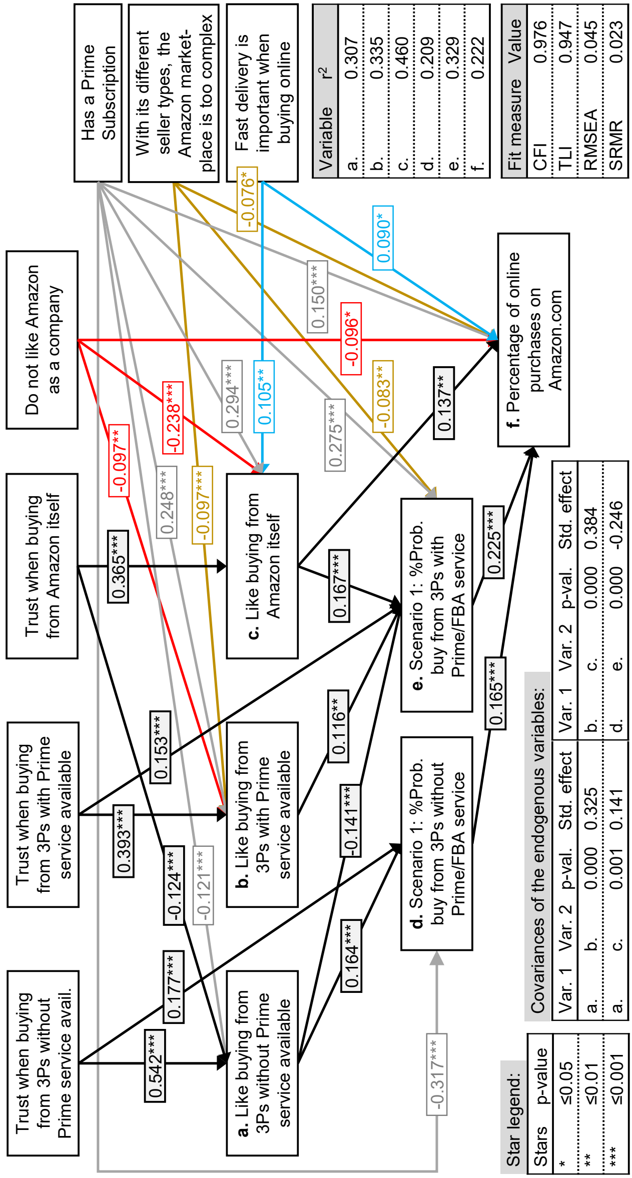
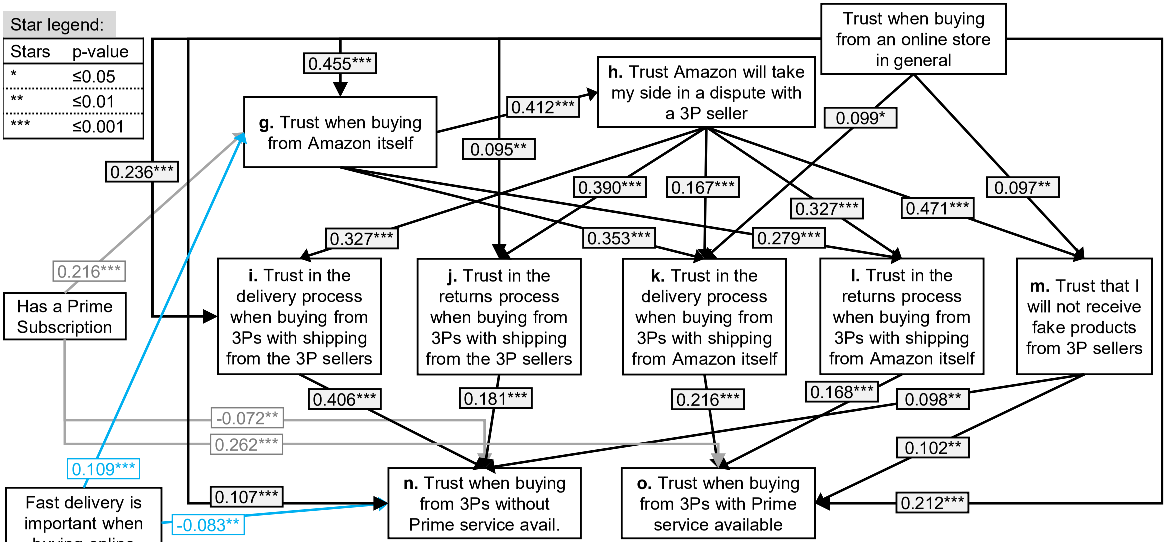


Figure IV.2 Path model 1 with significant correlations (standardized) and key estimates

Results of model 2: This model (see **Figure IV.3**) contains the paths leading to the different 'Trust in ...' Variables. The model paints a fairly clear picture. All three measured factors, 'delivery trust', 'returns trust', and 'product/counterfeit trust', are significant for trust in the different seller types (descending in importance). Furthermore, our second model shows that the trust in Amazon itself spreads through the FBA service into the 3P offers with Prime service. Amazon has excellent logistics, and the respondents seem to be aware of this. However, trust in Amazon's excellent fulfillment capabilities is not the only type of trust that spreads through the model. It is also evident that if the customers trust that Amazon takes their side in a dispute with a 3P seller, then this increases their trust in the Amazon marketplace as a whole.

We also tested a model that included our questions about how familiar customers are with the Amazon A-to-Z guarantee and the payment mechanism on Amazon. However, these variables have no significant effect, indicating that institutional mechanisms may be less important than previously thought. The very soft concept of trusting that the marketplace operator sides with the customer seems to be much more important.

In general, the customers of the Amazon marketplace do not seem to know much about the behind-the-scenes operations. **Figure IV.1.3** contains some descriptive statistics of the knowledge questions we asked (0 = no knowledge, 0.5 = some idea, 1 = knew the details). Only the features of the "premium" Prime shipping are known by the majority of the respondents. The FBA service is much less known. This seems to be a missed opportunity by Amazon, as many customers appreciate it when Amazon ships the goods. Amazon should therefore communicate better that the Prime service almost always means that Amazon ships the product.



Star legend:

Stars	p-value
*	≤0.05
**	≤0.01
***	≤0.001

Var.	g.	h.	i.	j.	k.	l.	m.	n.	o.
r^2	0.280	0.169	0.153	0.175	0.250	0.260	0.248	0.431	0.441

Covariances of the endogenous variables:

Var. 1	Var. 2	p-val.	Std. effect	Var. 1	Var. 2	p-val.	Std. effect	Var. 1	Var. 2	p-val.	Std. effect
g.	n.	0.000	0.151	i.	j.	0.000	0.561	m.	i.	0.000	0.352
g.	o.	0.000	0.486	k.	l.	0.000	0.444	m.	k.	0.000	0.165
n.	o.	0.000	0.332	k.	j.	0.000	0.267	m.	j.	0.000	0.343
i.	k.	0.000	0.467	j.	l.	0.000	0.403	m.	l.	0.019	0.100
i.	l.	0.000	0.298								

Fit measure	Value
CFI	0.980
TLI	0.947
RMSEA	0.035
SRMR	0.032

Figure IV.3 Path model 2 with significant correlations (standardized) and key estimates

Overall, the survey results permit the conclusion that the combination of the FBA service with its superior logistics and the Prime subscription (flat-rate premium shipping) is a very successful mechanism for Amazon. **Figure IV.1.1** shows that on average, customers with a Prime subscription like to buy from Amazon itself and 3P sellers with Prime service more than customers without a Prime subscription do. **Figure IV.2** shows that this leads to increased sales for the Amazon marketplace. Furthermore, Amazon receives additional revenue from 3P sellers because they need FBA for the Prime service.

IV.4. Conclusion, limitations, and outlook

Our systematic literature review revealed that not many studies exist that explicitly focus on the Amazon marketplace. While the coopetition between Amazon and 3P sellers was already identified as important (e.g., Ritala et al., 2014), no article we found had examined the relationship between the various 3P offer types, the Prime subscription, and the FBA service. In particular, until now, the well-studied theory around trust in B2C e-commerce does not seem to have been researched within this context.

Our survey confirmed that trust differences between the different seller types (Amazon itself, 3P offers with/without Prime/FBA service) indeed play a very important role in how much customers like buying from the Amazon marketplace. Interestingly, however, it seems that concrete institutional mechanisms, soft or hard, are not as important as previously thought (see, e.g., Pavlou & Gefen, 2004). Instead, the vague trust that Amazon takes the customer's side in a dispute with a 3P is probably more important for a customer when buying from 3P sellers.

Another important finding of our survey is that it is probably no longer sufficient to merely provide a general, vague feeling of trust in 3P sellers. Amazon has set new standards in fulfillment quality and especially delivery speed. For many customers, fast delivery is important (see also **Figure IV.1**). However, from a logistics standpoint, for many 3P sellers it is simply impossible to provide fast delivery. The FBA service, in combination with the Prime subscription, seems to be very well suited to solve several problems in this context. The FBA service enables 3P sellers to compete logistically with offers from larger retailers, making the Amazon marketplace more attractive for them. Moreover, it also increases general trust and delivery trust in particular. Last but not least, customers with a Prime subscription on average actually dislike buying offers without Prime service (see **Figure IV.1.1**). The 3P sellers, therefore, have a strong incentive to use FBA (and pay logistics fees to Amazon) to obtain the Prime logo/service for their offers.

Coming back to our research questions and the title of our article, the core question is, therefore perhaps, not only about how Amazon makes 3P offers more attractive but also about how Amazon makes certain 3P offers (those without Prime service) less attractive and thus incentivizes 3P sellers to use the paid FBA service. However, such a strategy can, of course, only work if there are enough offers with Prime service.

Certainly, our study also has limitations. For example, the percentage answers in our survey are self-reported estimates from memory or self-predicted behavior. This introduces random errors and perhaps biases. However, while it was not the goal of our survey, it may be noted that the r^2 values of our models are, despite these random errors, moderate to high, given that they predict human feelings, opinions, and behavior. Finally, it is worth mentioning that some of the concepts covered in our models are diluted by the "chicken or the egg" problem or other factors such as the Amazon Prime Video service, which is nowadays one of the more prominent reasons why Amazon customers have the Prime subscription. Does a customer who has a Prime subscription because of the streaming service also buy more from the Amazon marketplace? Future research could try to disentangle these effects.

IV. References

Amazon. (2022). *Category, Product, and Listing Restrictions.* https://sellercentral.amazon.com/gp/help/external/help.html?itemID=200301050&language=en_US& ref=efph_200301050_relt_G1801

Bao, T., & Chang, T. L. S. (2014). Why Amazon uses both the New York Times Best Seller List and customer reviews: An empirical study of multiplier effects on product sales from multiple earned media. *Decision Support Systems, 67,* 1–8. https://doi.org/10.1016/j.dss.2014.07.004

Baswan, T., & Fatima, F. (2019). A study on the relationship between gender preference and brand experience with reference to amazon brand. *IUP Journal of Brand Management, 16*(4), 64–77.

Bezos, J. (2019). *2018 Letter to Shareholders.* https://aboutamazon.com/news/company-news/2018-letter-to-shareholders

Chen, L. (2008). Analysis of Book Resale in Amazon Upgrade Framework: A Game-Theory Approach. *AMCIS 2008 Proceedings, Canada,* 1–10.

Chen, L., Mislove, A., & Wilson, C. (2016). An empirical analysis of algorithmic pricing on amazon marketplace. *Proceedings of the 25th international conference on World Wide Web, Canada*, 1339–1349. https://doi.org/10.1145/2872427.2883089

Croitor, E., & Werner, D. (2021). Exploring the Relationship between Perceived Input Control and Complementors' Perceived Performance: An Empirical Study on Amazon. *ECIS 2021 Research Papers, virtual,* 1–10.

Cui, R., Zhang, D. J., & Bassamboo, A. (2019). Learning from inventory availability information: Evidence from field experiments on amazon. *Management Science, 65*(3), 1216–1235. https://doi.org/10.1287/mnsc.2017.2950

Gao, C., Shi, D., & Maydeu-Olivares, A. (2020). Estimating the maximum likelihood root mean square error of approximation (RMSEA) with non-normal data: A Monte-Carlo study. *Structural Equation Modeling: A Multidisciplinary Journal, 27*(2), 192–201. https://doi.org/10.1080/10705511.2019.1637741

Gefen, D. (2000). E-commerce: the role of familiarity and trust. *Omega, 28*(6), 725–737. https://doi.org/10.1016/S0305-0483(00)00021-9

Gómez-Losada, A. & Duch-Brown, N. (2019). Competing for Amazon's Buy Box: A Machine-learning Approach. In W. Abramowicz & R. Corchuelo (Eds.), *Lecture Notes in Business Information Processing* (Vol. 373, pp. 445–456). Springer.

Hall, A., Towers, N., & Shaw, D. R. (2017). Understanding how millennial shoppers decide what to buy: Digitally connected unseen journeys. *International Journal of Retail & Distribution Management, 45*(5), 498–517. https://doi.org/10.1108/IJRDM-11-2016-0206

Huang, Y., Liu, H., Li, W., Wang, Z., Hu, X., & Wang, W. (2020). Lifestyles in Amazon: Evidence from online reviews enhanced recommender system. *International Journal of Market Research, 62*(6), 689–706. https://doi.org/10.1177/1470785319844146

Ivanova, O., Scholz, M., & Dorner, V. (2013). Does amazon scare off customers? The effect of negative spotlight reviews on purchase intention. *Wirtschaftsinformatik Proceedings 2013, Germany*, 23–37.

Jabr, W. (2022). Review credibility as a safeguard against fakery: the case of Amazon. *European Journal of Information Systems, 31*(4), 525–545. https://doi.org/10.1080/0960085X.2021.1886613

Jeong, J. (2021). Identifying Consumer Preferences From User-Generated Content on Amazon. Com by Leveraging Machine Learning. *IEEE Access, 9*, 147357–147396. https://doi.org/10.1109/ACCESS.2021.3123301

Jin, Y., Yang, S. B., Rhee, C., & Lee, K. Y. (2013). An exploratory study of the effects of price decreases on online product reviews: Focusing on Amazon's Kindle 2. *PACIS 2013 Proceedings, Korea.*

Kaziukėnas, J. (2022). *Marketplaces Year in Review 2021.* https://marketplacepulse.com/marketplaces-year-in-review-2021

Khern-am-nuai, W., Ghasemkhani, H., & Kannan, K. (2017). How questions and answers shape online marketplaces: The Case of Amazon answer. *Proceedings of the 50th Hawaii International Conference on System Sciences, USA*, 853–862.

Nalebuff, B. J. & Brandenburger, A. M. (1996). *Co-opetition*. Crown Business.

Nguyen, D. H., de Leeuw, S., & Dullaert, W. E. (2018). Consumer behaviour and order fulfilment in online retailing: A systematic review. *International Journal of Management Reviews, 20*(2), 255–276. https://doi.org/10.1111/ijmr.12129

Pavlou, P. A., & Gefen, D. (2004). Building effective online marketplaces with institution-based trust. *Information Systems Research, 15*(1), 37–59. https://doi.org/10.1287/isre.1040.0015

Ramadan, Z., Farah, M. F., & Bou Saada, R. (2021). Fooled in the relationship: How Amazon Prime members' sense of self-control counter-intuitively reinforces impulsive buying behavior. *Journal of Consumer Behaviour, 20*(6), 1497–1507. https://doi.org/10.1002/cb.1960

Ritala, P., Golnam, A., & Wegmann, A. (2014). Coopetition-based business models: The case of Amazon.com. *Industrial marketing management, 43*(2), 236–249. https://doi.org/10.1016/j.indmarman.2013.11.005

Sun, L., Lyu, G., Yu, Y., & Teo, C. P. (2020). Fulfillment by Amazon versus fulfillment by seller: An interpretable risk-adjusted fulfillment model. *Naval Research Logistics (NRL), 67*(8), 627–645. https://doi.org/10.1002/nav.21897

Tan, C. H., Yang, X., Chan, Y. P., & Teo, H. H. (2005). An investigation of book market aggregation in Amazon. *AMCIS 2005 Proceedings, USA*, 10–17.

Trenz, M., & Veit, D. J. (2012). When does price transparency matter?-The case of Amazon. *AMCIS 2012 Proceedings,* USA. 1–9.

Vakhariya, S. (2020). A Study of Online Shopping Experience and Swaying Brand Preference Between Noon and Amazon in UAE. *South Asian Journal of Management, 27*(2), 84–112.

Wu, P. F. (2019). Motivation crowding in online product reviewing: A qualitative study of amazon reviewers. *Information & Management, 56*(8), 103163. https://doi.org/10.1016/j.im.2019.04.006

Xu, L. (2019). The Prevalence and Effectiveness of Price Promotion on Amazon. *International Journal of Business and Administrative Studies, 5*(4), 177–185. https://dx.doi.org/10.20469/ijbas.5.10001-4

Zhu, F., & Liu, Q. (2018). Competing with complementors: An empirical look at Amazon.com. *Strategic Management Journal, 39*(10), 2618–2642. https://doi.org/10.1002/smj.2932

Paper V:
Inventory competition on electronic marketplaces – A competitive newsvendor problem with a unilateral sales commission fee

Christian Straubert, Eric Sucky

Abstract. We present a competitive newsvendor problem with a sales commission paid by one newsvendor to another newsvendor when selling a product. Customer demand is stochastic, and customers individually decide with given probabilities from which newsvendor they want to order or whether they do not want to order at all. These probabilities can vary depending on which newsvendor is out of stock. The problem described arises when marketplace operators offer the same or substitutable products as third-party vendors and therefore directly compete with third-party vendors on their platform. Our research is motivated by practical cases such as the Amazon marketplace, where Amazon is the marketplace operator that sets the sales commission and, simultaneously, directly competes with third-party vendors. Both the marketplace operator and a competing third-party vendor should ideally account for the other party's behavior when deciding their order quantity. We calculate the noncooperative Nash equilibrium and the optimal cooperative order quantities (centralized solution). Additionally, we emphasize analyzing the effect of the sales commission on the optimal order quantities and the gross profits of the two parties.

Keywords: inventory, game theory, e-commerce, platform, newsvendor

Reference: Straubert, C., & Sucky, E. (2023). Inventory competition on electronic marketplaces–A competitive newsvendor problem with a unilateral sales commission fee. European Journal of Operational Research, 309(2), 656–670. doi.org/10.1016/j.ejor.2023.02.002

V.1. Introduction

B2C e-commerce has grown rapidly in recent decades and is still growing faster than brick-and-mortar retail. Therefore, it is becoming increasingly important to analyze and optimize the behavior of the players involved. While e-commerce shares many similarities with brick-and-mortar retailing, there are also some new phenomena in B2C e-commerce. An important phenomenon of B2C e-commerce is the success of so-called online marketplaces. In a classical online shop, the online retailer buys goods from manufacturers or wholesalers and sells them directly to customers through the online shop's website. In contrast, online marketplaces offer products and services from a wide range of retailers (third-party vendors) via the marketplace website, and the marketplace operator receives a commission every time a third-party vendor sells a product. In this context, an important distinction is necessary. There are marketplaces where the marketplace operator merely provides the marketplace infrastructure (website, IT, payments, etc.) and is therefore not directly competing with third-party vendors on the marketplace website. However, a second type of very successful marketplace operator exists in B2C e-commerce. For example, the largest B2C e-commerce company, Amazon, operates an electronic marketplace, where third-party vendors and Amazon itself offer products for sale. Moreover, it is not uncommon for substitutable or even identical products to be offered by both third-party vendors and Amazon. In these cases, Amazon directly competes with third-party vendors while, as the marketplace operator, also setting the rules of the marketplace (e.g., commissions, search results, delivery flat rates, website design, logos). This creates a new form of retail competition (Ryan et al., 2012; Zhu & Liu, 2018). The obvious implication is that the marketplace operator has an exploitable advantage.

Our paper focuses on this new form of horizontal competition from an inventory management perspective. In B2C e-commerce of physical products, the retailer typically has to buy products based on forecasted demand before selling them. If the ordered products are not sold within a certain timeframe, they waste warehousing space and often lose a significant portion of their value (e.g., seasonal, perishable goods, and fashion products). Inventory risks are one of the most important risks in retailing, as they are at the core of the value chain of an online retailer. To model this inventory risk in the abovementioned context, we consider two newsvendors (companies): a marketplace operator, who is also a retailer on its own platform, and a third-party vendor on the platform. Both the marketplace operator and the third-party vendor sell an identical or substitutable product for an exogenously given market price. They either act independently (decentralized) by optimizing their own goals while accounting for the rational behavior of the other player (noncooperative game) or collude to attempt to create a stable cooperative optimum (centralized cooperative game). Customer demand is modeled as a stochastic stream in the sense that customers arrive one by one, for example, following a Poisson process, and then decide with given probabilities whether to order from the marketplace operator, from the third-party vendor or to not order at all. These probabilities can differ depending on which player is out of stock.

We especially focus on the commission, which can be freely set by the marketplace operator and is paid by the third-party vendor for each sale of the product. As the marketplace operator profits from the commission, it will, generally speaking, want to order/offer fewer quantities of the product and thereby pass part of the inventory risk onto the third-party vendor. However, the amount of commission that needs to be paid when selling on a marketplace also determines how attractive it is for a third-party vendor to offer the product on the respective marketplace. A higher commission generally leads to a lower order/offer quantity by the third-party vendor and could lead to situations where the third-party vendor would not offer the product on the marketplace at all. The newsvendors must account for and balance these effects when setting their order quantity. The special attention to the commission paid by the third-party vendor distinguishes this paper from the existing literature. Furthermore, to the best of our knowledge, the presented game-theoretical model that includes this commission has not yet been published. In the following section, we will go into more detail on how this paper relates to the literature on competitive newsvendor games. There-after, we introduce two model formulations (a base case model and a generalizing extension). We also present several numerical examples and comparisons and discuss managerial implications.

V.2. Related literature

The newsvendor problem, as one of the classical problems in the literature on inventory management (Arrow et al., 1951), is a single-product decision problem, where the decision maker places an inventory order in advance of a single selling season with uncertain demand. The inventory quantity x is ordered at the beginning of (or before) a single sales period for a fixed price per unit c. Sales of the product occur during the single period at unit price r. If too many units were ordered some units are left over at the end of the period and are salvaged for a per unit revenue of v, with $v<c$ (overage costs). If too few units were ordered, lost sales occur (underage costs). The goal is to determine the optimal order amount x^* given these two opposing effects. After optimizing either the expected profit or minimizing the expected over- and underage costs, the following unique solution is obtained:

(1) $$x^* = F^{-1}\left(\frac{r-c}{r-v}\right).$$

For detailed reviews of the newsvendor problem and its extensions, we refer to Khouja (1999) and Qin et al. (2011). The classical newsvendor problem involves a single decision maker and ignores the decisions of other competitors. A number of papers subsequently extended the classical newsvendor model to a noncooperative game (see, for example, Lariviere, 1999; Cachon, 2003). Here, a rough distinction can be made among (1) vertical competition (e.g., Li & Liu, 2020), (2) dual channel competition (e.g., Cui et al., 2007), and (3) horizontal competition (e.g., Netessine & Rudi, 2003) (see **Figure V.1**).

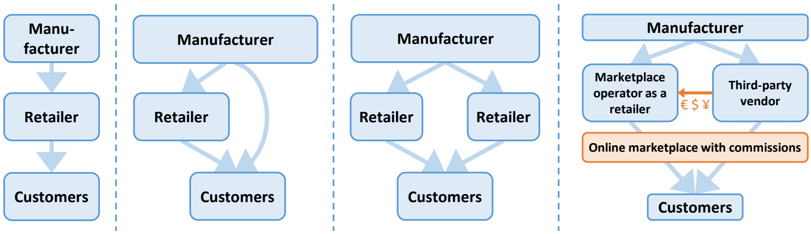


Figure V.1 Schematic diagram of vertical competition, dual channel competition, and horizontal competition

Our problem setting is a case of horizontal competition in the same supply chain echelon. In horizontal newsvendor competition, two or more newsvendors must decide their inventory quantities before selling in a joint demand market. A joint demand market can be found when multiple newsvendors offer the same product or when newsvendors offer different but substitutable products and customers are free to switch between the offers. Several works have addressed this problem. Silbermayr (2020) provides a rather up-to-date review of the research field of newsvendor games with horizontal inventory interactions. In contrast to the single-newsvendor problem, horizontally competing newsvendors face a more complex environment, because they have to consider the order quantities of the competitors when setting their own order quantity.

Parlar (1988) was the first to analyze a horizontal newsvendor game. He assumed that two newsvendors have independent demand and a deterministic portion of one newsvendor's unsatisfied demand is passed on to the other newsvendor if the latter has excess inventory. The existence and uniqueness of the Nash equilibrium is proved. It is also shown that the aggregate profits of two competing newsvendors are less than the profits they would have earned if they had cooperated. Wang and Parlar (1994) extend the model of Parlar (1988) to the case of three newsvendors. They also show that the optimal order quantity in their noncooperative game is higher than in the single-newsvendor model (an effect known as "overstocking"), because each retailer maintains some inventory for possible substitution of other retailers' demand. Lippman and

McCardle (1997) analyze a similar but more general version of the model. They consider four prespecified 'splitting rules' for allocating initial and excess demand. Lippman and McCardle (1997) remark that their "most general result is that if all excess demand is reallocated, i.e., there is perfect substitutability, then competition never leads to a decrease in industry inventory" (p. 54). Mahajan and van Ryzin (2001) expand the model to the case of n newsvendors. They model demand as a stochastic sequence of heterogeneous customers and show for their model that the overstocking effect (which Lippman and McCardle (1997) had shown for $n=2$) extends to the oligopoly case ($n \geq 3$). Netessine and Rudi (2003) provide a comparison between the centralized (cooperative) and the decentralized (competitive) solution to a model similar to Wang and Parlar (1994) but for n newsvendors. They again find that the newsvendors behave suboptimally under competition and usually overstock compared to the centralized solution. We will also present solutions for both the noncooperative case and the cooperative case of our model and compare them. Many extensions and adaptations of the basic models mentioned above exist (e.g., Netessine and Shumsky (2005), who apply the model to an airline revenue management context). For a more comprehensive overview of the research stream, we would like to refer the reader to Silbermayr (2020). We extend the abovementioned research stream by introducing a hitherto not considered planning parameter: a sales commission that is paid by one newsvendor A to another newsvendor B when selling a product. Furthermore, we embed our model into the highly relevant context of intra-platform competition in online marketplaces.

Generally, our model is similar to the models cited above. Rule 3 in Lippman and McCardle (1997) is close to our case. They split their model into two phases. The "initial allocation" phase, where demand is allocated to the two newsvendors, and the "reallocation" phase, which comes into play when one of the two newsvendors becomes out-of-stock. Some portion ($\leq 100\%$) of the leftover demand from the out-of-stock newsvendor is then assigned to the other newsvendor. Our model has a similar structure. Rule 3 in Lippman and McCardle (1997) describes a situation where, during the “initial allocation” stage, each customer demand is assigned to either newsvendor A or newsvendor B with certain probabilities. This includes the case where all the initial demand is assigned to A with 100% probability and 100% of the leftover demand (if A becomes out-of-stock) is reallocated to B. This corresponds to our “base case model” described in **Section V.3**. The generalization of this, which allows for probabilities other than 100%/0%, is similar to our model extension in **Section V.4**. However, our model extension is more general because it allows for a situation where an out-of-stock event at one newsvendor also influences the “initial allocation” quantity at the other newsvendor (similar to the stochastic demand stream in Mahajan and van Ryzin (2001)). Furthermore, neither Lippman and McCardle (1997) nor Mahajan and van Ryzin (2001) consider a unilateral sales commission. We will show that the inclusion of such a commission creates new dynamics in the model and leads to different Nash equilibria, which are not yet covered in existing research. In particular, it is often optimal for newsvendor A to reduce its inventory risk (by reducing its order quantity) because, to a certain extent, excess demand is transferred to and fulfilled by B, for which A receives commission revenue. Indeed, in our model, it is often the case that competition leads to understocking compared to the centralized optimum. Moreover, Lippman and McCardle (1997) and Mahajan and van Ryzin (2001) restrict themselves to a mathematical treatment of the competitive case. We additionally derive the cooperative optimum and discuss both solutions also from a managerial perspective within the context of online marketplaces.

Note that competitive newsvendor models with transhipments and transhipment fees are somewhat similar to but also distinctively different from a sales commission. In a transhipment model, a newsvendor without stock pays transhipment fees to a newsvendor that still has stock. In our model, the sales commission is paid by newsvendor B to the competing newsvendor A while the paying newsvendor B still has stock. Following Silbermayr (2020), one could say that similar to many transhipment models, our model has an ex ante contract in the form of the sales commission but customer-driven demand interaction between the newsvendors. The sales commission plays a crucial role in our model, as it is set ex ante by one newsvendor but heavily influences the behavior of both competing newsvendors. This not only creates new mathematical results but is also highly relevant for a better understanding of the competitive behavior in one of the most important and successful business models, the platform economy. In this regard, our paper is also related to the literature on intraplatform competition (e.g., Ryan et al., 2012; Cao & He, 2016; Zhu & Liu, 2018), but we are not aware of any existing research that has modeled such competition with a newsvendor model.

Finally, it is also worthwhile to consider research beyond purely rational and profit-maximizing newsvendors. For example, risk-averse newsvendors (e.g., Liu et al., 2013; Wu et al., 2014) or overconfident newsvendors (e.g., Li et al., 2017) are recognized in the literature. Risk-aversion could stem from a cognitive bias, or it can have quite logical financial reasons because less risky (i.e., less volatile) revenue streams are often preferred. Zhao and Zhao (2016), for example, performed an experiment for high- and low-margin products. Their results indicate that participants were more risk-averse in the low-margin product case but overstocked in the high-margin product case. This is potentially relevant within our context because the marketplace operator A might want to reduce its inventory risk and instead prefers less risky commission revenue from the third-party vendor B. A prerequisite for this is of course that B is not significantly understocking. This is especially interesting in relation to the commission fee that can be freely set by the marketplace operator. From a managerial viewpoint, the marketplace operator faces a tradeoff between short-term commission revenue maximization (with a high commission fee) and successful business relationships with third-party vendors in the long term (with a low commission fee). It is likely that the success of an online marketplace depends on how fair the third-party vendors consider the amount of sales commission they have to pay. Fairness concerns have been discussed in the literature but predominantly with regard to the relationship between a wholesaler and one or multiple retailers (e.g., Cui et al., 2007). In our case, it is interesting that the power imbalance exists on the same supply chain echelon (see also Zhu & Liu, 2018). One retailer (the marketplace operator) has power over another retailer (the third-party vendor), a constellation that is normally not observed. While this paper makes the typical assumption of rational, profit-maximizing newsvendors, it is useful to not lose sight of the managerial and human context of our model, and we will touch upon these issues in the course of our paper.

V.3. Base case model

There are two competing newsvendors, both of which offer a product for sale:

- The marketplace operator *A*, which also sells the product itself.
- A third-party vendor *B* that offers the product on *A*'s marketplace.

In our base case model, customers order first from the marketplace operator and only order from the third-party vendor if the marketplace operator is out of stock. Expanding on the notation in our introduction, we define the following decision variables:

x_A, x_B	Order quantities [units] of *A* and *B*.

The following model parameters:

$r=r_A=r_B$	Sales prices r per unit sold [monetary unit] of *A* and *B*.
$c_A<r$ and $c_B<r$	Purchase cost c per unit ordered [monetary unit] of *A* and *B*.
$v_A<c_A$ and $v_B<c_B$	Salvaging revenue v per unit salvaged [monetary unit] of *A* and *B*.
$p<r-c_B$ and $p<r-c_A$	Sales commission p [monetary unit] that is paid by the third-party vendor *B* to the marketplace operator *A* when *B* sells a unit of the product on *A*'s marketplace.

And the following defined quantities:

$x = x_A + x_B$	Quantity [units] available on the marketplace at the beginning of the period.
$c_{oA} = c_A - v_A$	Overage cost of *A* [monetary unit] (cost of overestimating demand by one unit).
$c_{uA1} = r - c_A - p$	Underage cost type 1 of *A* [monetary unit] (cost of underestimating demand by one unit) if *A* is out of stock but *B* still has units in inventory.
$c_{uA2} = r - c_A$	Underage cost type 2 of *A* [monetary unit] if both *A* and *B* are out of stock.
$c_{oB} = c_B - v_B$	Overage cost of *B* [monetary unit]
$c_{uB} = r - c_B - p$	Underage cost of *B* [monetary unit]

Please also note that we assume that $r = r_A = r_B$ (the sales prices quoted by *A* and *B* are identical). In practice, it is possible that the sales prices differ. However, the offers on the marketplace are competing not only against each other but also with offers in other e-commerce shops. It is logical to assume a functioning market and, thus, that a market equilibrium price exists, around which the various offers are positioned. Therefore, this assumption should be fulfilled, at least approximately, in many real-world cases. Nevertheless, it could be worthwhile to investigate a model with a price-dependent demand curve in future research.

Furthermore, we assume that the third-party vendor has a dedicated stock for the marketplace. This could for example be the case because maintaining a presence on several marketplaces and/or operating a separate online shop incurs high administrative costs. Moreover, several marketplaces offer fulfillment services, such as “Fulfillment by Amazon” (FBA), where third-party vendors send products to an Amazon warehouse for storage (Amazon, 2022a). These units, stored by Amazon, are often dedicated stock for the Amazon marketplace. The third-party vendors choose FBA because Amazon also ships the products from its warehouse and the FBA third-party offers therefore obtain the valuable

"Prime" logo, indicating fast and free delivery for "Amazon Prime" customers.

Note that the commission fee (which is usually a percentage of the sales price) could also incorporate additional costs, for example, the FBA fulfillment fee that Amazon charges if a third-party vendor uses FBA. However, in this article, we assume for ease of exposition that such additional services are offered at cost with equalizing effects. This means that if third-party vendors use the fulfillment service of the marketplace operator, they pay a fulfillment fee but also save the same amount in own fulfillment costs. Furthermore, the marketplace operator has additional fulfillment costs for each product sold but receives the same amount in fulfillment fees from the third-party vendors. Thus, we assume that the decision between using and not using the fulfillment service of the marketplace operator has no direct gross profit effects for either the marketplace operator or the third-party vendor. This can indeed be observed in practice. Amazon for example offers very competitive fulfillment fees and wants FBA to be an attractive service for third-party vendors.

In examining the offers on online marketplaces, one will find that some offers exactly fulfill our model's prerequisites (e.g., a product is offered for an identical or very similar price both by the marketplace operator and a third-party vendor with a dedicated stock for the marketplace). However, it is also true that such offers, although regularly observed, are not the norm. As we will see in the course of this article, the third-party vendors are often at a considerable disadvantage compared to the marketplace operator. Note that our model not only explains what the optimal strategies are but also why some constellations are not observed in practice because the conditions are too unfavorable for the third-party vendors. Therefore, the models not only reveal the optimal strategies for given platform rules but can also provide valuable insights into how to design the rules of a marketplace such that the platform is attractive to third-party vendors. Furthermore, our model is of course also a good fit for substitutable products, which are a typical assumption in the literature on newsvendor competition (e.g., Parlar, 1988; Lippman &

McCardle, 1997; Netessine & Rudi, 2003) and can be regularly observed in practice.

In our base case model, the following five cases are possible, which are summarized in **Figure V.2**. Cases 1 and 2 can be combined into $b \leq x_A < x$. If the realized demand (b) falls into this case, the cost functions of the respective newsvendor model are:

$K(x_A) = c_{oA} \cdot (x_A - b)$ for the marketplace operator A

$K(x_B) = c_{oB} \cdot x_B$ for the third-party vendor B

Cases 3 and 4 can be combined into $x_A < b \leq x$, with:

$K(x_A) = c_{uA1} \cdot (b - x_A)$ for the marketplace operator A

$K(x_B) = c_{oB} \cdot (x_A + x_B - b)$ for the third-party vendor B

Finally, case 5 can be combined into $b > x$, with:

$K(x_A) = c_{uA1} \cdot x_B + c_{uA2} \cdot (b - x_A - x_B)$ for the marketplace operator A

$K(x_B) = c_{uB} \cdot (b - x_A - x_B)$ for the third-party vendor B

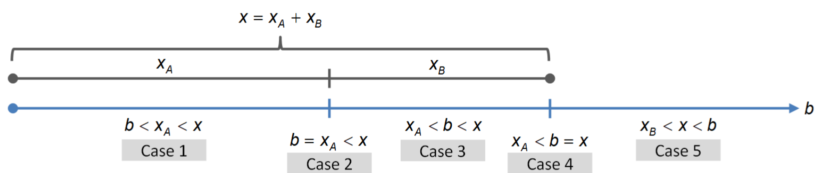


Figure V.2 Cases of the base case model depending on the realized demand (b) and the order quantities

Both companies want to minimize their expected cost. For the marketplace operator A, this is:

(2)

$$Min\ E[K_A(x_A)] = c_{oA} \int_{b=0}^{x_A} (x_A - b) f(b) db + c_{uA1} \int_{b=x_A}^{x_A+x_B} (b - x_A) f(b) db + \int_{b=x_A+x_B}^{\infty} \left[c_{uA1} \cdot x_B + c_{uA2} \cdot (b - x_A - x_B) \right] f(b) db$$

For the third-party vendor B, this is:

(3)

$$Min\ E[K_B(x_B)] = c_{oB} \int_{b=0}^{x_A} x_B f(b) db + c_{oB} \int_{b=x_A}^{x_A+x_B} (x_A + x_B - b) f(b) db + c_{uB} \int_{b=x_A+x_B}^{\infty} (b - x_A - x_B) f(b) db$$

Analogous to the solution procedure of the classic newsvendor problem, the minima of these convex functions can be found using the Leibniz integral rule. For the complete calculations of the first- and second-order conditions, we refer the reader to **Appendix V.A.1**. The optimal solution x_A^* for the order quantity of the marketplace operator A is given by:

(4)

$$F(x_A^*) = \frac{c_{uA2} - p \cdot F(x_A^* + x_B)}{c_{oA} + c_{uA1}} = \frac{c_{uA2} - p \cdot F(x_A^* + x_B)}{c_{oA} + c_{uA2} - p} = \frac{r - c_A - p \cdot F(x_A^* + x_B)}{r - v_A - p}, \text{ if } x_B > 0$$

$$
(5) \qquad x_A^* = \begin{cases} F^{-1}\left(\dfrac{r-c_A-p\cdot F(x_A^*+x_B)}{r-v_A-p}\right) & \text{if } x_B>0 \\ F^{-1}\left(\dfrac{r-c_A}{r-v_A}\right) & \text{else} \end{cases}
$$

The optimal solution x_B^* for the order quantity of the third-party vendor *B* is:

$$
(6) \qquad F\left(x_A+x_B^*\right)=\frac{c_{uB}}{c_{oB}+c_{uB}}=\frac{r-c_B-p}{c_{oB}+r-c_B-p}=\frac{r-c_B-p}{r-v_B-p}
$$

$$
(7) \qquad x_B^*=\max\left\{0,\ F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-x_A\right\}
$$

As expected, both the optimal policy of the marketplace operator and the optimal policy of the third-party vendor depend on the order quantity of the respective competitor. In the following subsection, we will determine the noncooperative Nash equilibrium. However, before we do so, it is expedient to discuss some observations concerning the individual reaction functions. A crucial part of our model is the commission p paid by the third-party vendor to the marketplace operator following a successful sale on the marketplace. If no commission existed, Equation 5 would reduce to the critical ratio of the classic newsvendor model. That is, the marketplace operator *A* would order a quantity based on the classic newsvendor model without accounting for the order quantity of the third-party vendor *B*. This is logical in our base case model because customers always first buy from *A* as long as *A* still has stock on hand. Thus, without having anything to gain from the nonexistent commission, the optimal policy matches the classic newsvendor model. However, if a commission exists, the optimal policy differs from the classic newsvendor model. Logic dictates that the critical ratio of *A* has to decrease with an increasing commission p. The marketplace operator *A* can pass some of the inventory risk onto the third-party vendor *B*. The higher the commission, the more attractive is an inventory risk shift toward B. In its extreme, if the commission were equal to or greater than the contribution margin of A ($p \geq r - c_A$) and if the third-party vendor were willing to order a substantial volume ($F(x_B) \to 1$), the marketplace operator A would order nothing because it would be optimal to completely shift the inventory risk onto B. Mathematically, this can also be seen in Equation 2. The underage cost of type 1 ($c_{uA1} = r - c_A - p$) decreases with an increasing commission p. This is the only effect of p in Equation 2 c_{uA1} appears in the second and third terms of Equation 2. Thus, the cost weights of the second and third cases decrease. The optimal reaction to this decreasing cost weight is to increase the probability weight of these cases (and/or to decrease the probability weight of the first case). The only way to do so is to decrease the variable x_A. In other words, if the underage cost decreases, it is optimal to plan for a higher expected shortage amount. However, beyond that, little can be said about the general behavior of the formula, since the optimal solution recursively depends on the case-specific demand distribution. Nevertheless, the critical ratio formula itself, as the classical newsvendor model, is independent of any demand distribution, which is a very powerful result.

The critical ratio of the third-party vendor B is similar to the classic newsvendor model with the additional influence of the commission p. If the marketplace operator A were to order zero units, then the third-party vendor B would order up to its critical ratio. The critical ratio tends toward zero with an increasing commission p. At some point, the commission would exceed the contribution margin, and every sale would mean a loss for B. If A orders some units, these units reduce the order quantity of B in a straightforward 1:1 ratio. Recall that in our base case model, customers order first from the marketplace operator A and only order from the third-party vendor B if A is out of stock. If A were to order the same number of or more units than indicated by the critical ratio of B, then B would order nothing because the probability that customers demand $x_A + 1$ units is so low that B expects a loss if it were to order an additional unit of the product. Therefore, for both A and B to order some units, at least either $c_B < c_A$ or $v_B > v_A$ must be true. That is, the third-party vendor B either has to have a lower purchase price or a higher salvage revenue per unit. Although an artifact of the rather restrictive assumptions of our base case model (which we will relax in later sections), this is an important insight, as it directly relates to the question under which conditions it is reasonable for third-party vendors to do business in an e-commerce marketplace such as Amazon.

Figure V.3 visualizes a numerical example of a product where the third-party vendor B has lower costs and a higher salvage value per unit. The commission varies between 5 and 25% of the sales price (r). For comparison, the commission on the Amazon marketplace ranges from 8% to 20% depending on the product category and is often 15% (Amazon, 2022a). The cost/salvage value parameters of the example are quite realistic for a setting where the third-party vendor B specializes in the product category. However, even in such a setting favoring the third-party vendor, B can often only extract a limited amount of gross profit from potential marketplace participation in our base case model. Furthermore, the commission p has a rather subdued effect. The model is largely dominated by the order quantity of the marketplace operator A. If all customers first order from the marketplace operator, A can capture the demand that will likely occur. The third-party vendor B is left with insecure demand and is not willing to order a large quantity of the product.

If the third-party vendor B increases its order quantity, the optimal order quantity of A slightly decreases. Because of the increased commission payments from B, A is willing to cede some of the more unsecured demand. In our base case model, this commission-based effect is the only effect on the optimal order quantity of A.

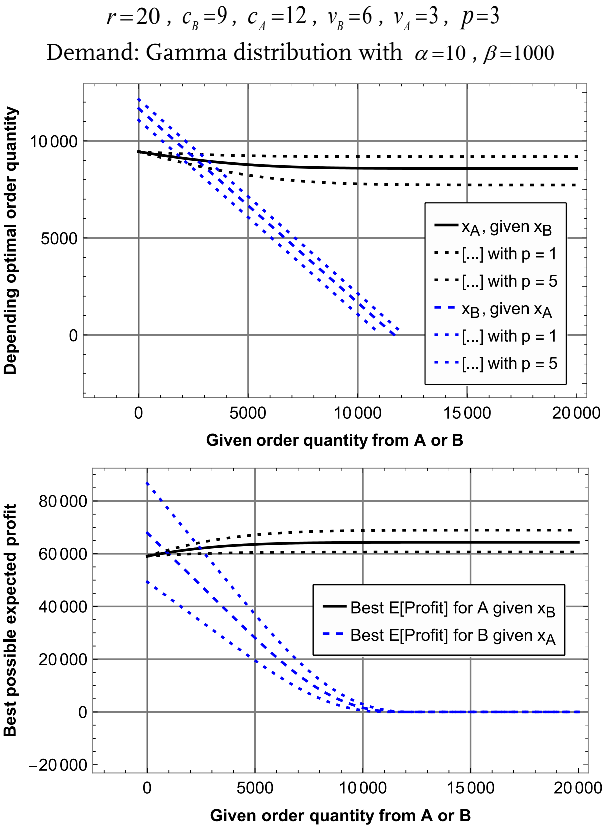


Figure V.3 Numerical example 1: the reaction functions

V.3.1. Base case model – noncooperative Nash equilibrium

As mentioned in the previous section, the optimal order quantity of the marketplace operator *A* depends on the order quantity of the third-party vendor *B* and vice versa. Concerning the noncooperative behavior of the two parties *A* and *B*, three cases are conceivable. Both parties simultaneously determine their order quantities (Nash equilibrium), *A* sets its order quantity first, and *B* reacts to *A*'s order quantity (*A* is the Stackelberg leader), or *B* sets its order quantity first, and *A* reacts (*B* is the Stackelberg leader). Both parties simultaneously choosing an order quantity is the case typically considered in the literature on competing newsvendors (Serin, 2007). In the case of electronic marketplaces, this situation occurs for example when the marketplace operator and the third-party vendor can estimate from experience that for a popular product category, the marketplace operator and one or more third-party vendors will simultaneously sell the same or substitutable products on the platform. Concerning the other two cases (*A* or *B* is the Stackelberg leader), it is possible to show that the Stackelberg equilibrium generally does differ from the Nash equilibrium. However, the Stackelberg solutions gravitate around the Nash equilibrium, and the Nash equilibrium is therefore arguably more important for explaining the model behavior, which is why our paper focuses on the typically assumed simultaneous decision. Nevertheless, the Stackelberg version of our model definitely merits a more detailed investigation in future research, especially considering potential information asymmetries between the players.

In a noncooperative setting, *A* and *B* have to estimate the demand distribution and assume values for c_B, v_B and c_A, v_A , respectively. If they cannot estimate the other party's cost/revenue structure reasonably well, the optimal order quantity might be missed significantly. The most likely information asymmetries are demand information asymmetry (Jiang et al., 2011) and cost information asymmetry (Güler et al., 2018). A marketplace operator typically has more demand information than a third-party vendor and can therefore more accurately estimate the demand distribution. Since the newsvendors in our model sell the same product or very similar substitutable products, it is reasonable to assume that the unit costs (c_A, c_B) and salvage prices (v_A, v_B) do not differ substantially between the two newsvendors. Nevertheless, neither the marketplace operator nor the third-party vendor usually has accurate information about the other player's parameters. Depending on how much information is known or can be reasonably estimated, different techniques such as absolute regret minimization (Jiang et al., 2011), a maximum entropy approach (Andersson et al., 2013) or probability distributions for the cost and salvage parameters (Güler et al., 2018) may be used to elevate the information asymmetry problem. Alternatively, the parties could share their information with each other. However, the parties are in competition and therefore usually do not want to share their information. However, there are models that show that information sharing can be beneficial for some or all parties, including in an online marketplace context (Li et al., 2021; Zha et al., 2023). Nevertheless, these are multi-echelon models in which the manufacturer plays a central role. Future research could investigate whether similar effects can be found in our model. This question is especially intriguing in our context because the marketplace operator usually not only has more information but also more power and could perhaps design the (information sharing) rules of the marketplace in a way that benefits some or all parties. However, this would be an extension. Since the problem discussed in our paper (newsvendor competition with a unilateral sales commission) has not yet been studied, we focus on the basics and therefore resort to the typical assumption of perfect information in the following.

The Nash equilibrium is (assuming that $x_B^* > 0$; see also **Appendix V.A.2**):

(8) $$x_A^* = F^{-1}\left(\frac{r - c_A - p \cdot \left(\frac{r - c_B - p}{r - v_B - p}\right)}{r - v_A - p}\right)$$

(9) $$x_B^* = F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) - F^{-1}\left(\frac{r-c_A-p\cdot\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$$

Again, the critical ratio of the third-party vendor *B* plays a crucial role. The marketplace operator *A* knows that a logically behaving *B* would always order $x_B^* = \max\left\{0,\ F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) - x_A\right\}$.

Figure V.4 contains the Nash equilibrium for the already introduced numerical example. Although the third-party vendor *B* optimally orders a sizeable amount (2644 units), it can only generate a rather limited amount of gross profit (4284). This was to be expected because the demand that the third-party vendor *B* can service is less likely to occur than the demand that is serviced by the marketplace operator *A*. **Figure V.4** also provides insight into the optimal commission p. Generally, the marketplace operator *A* is free to set the commission as high as it wants. However, it is not optimal to do so. In our example, it would be optimal for *A* to set the commission $p = 5.93$. This would correspond to 29.65% of the sales price (r). However, as mentioned in the previous section, the optimal profit of *A* is rather insensitive to the commission in our base case model. If the marketplace operator wished to facilitate the participation of third-party vendors on its platform, it could lower the commission and would only suffer a limited decrease in gross profit. However, this is in part an artifact of our restrictive assumptions in the base case model. The model behavior is different in our more realistic model extensions, where the commission fee becomes more important for the gross profits of *A* and *B*.

Parameters used for the example:

$r = 20$, $c_B = 9$, $c_A = 12$, $v_B = 6$, $v_A = 3$, $p = 3$

Demand: Gamma distribution with $\alpha = 10$, $\beta = 1000$

Nash-EQ E[Profit A]: 62,209, Nash-EQ E[Profit B]: 4,284

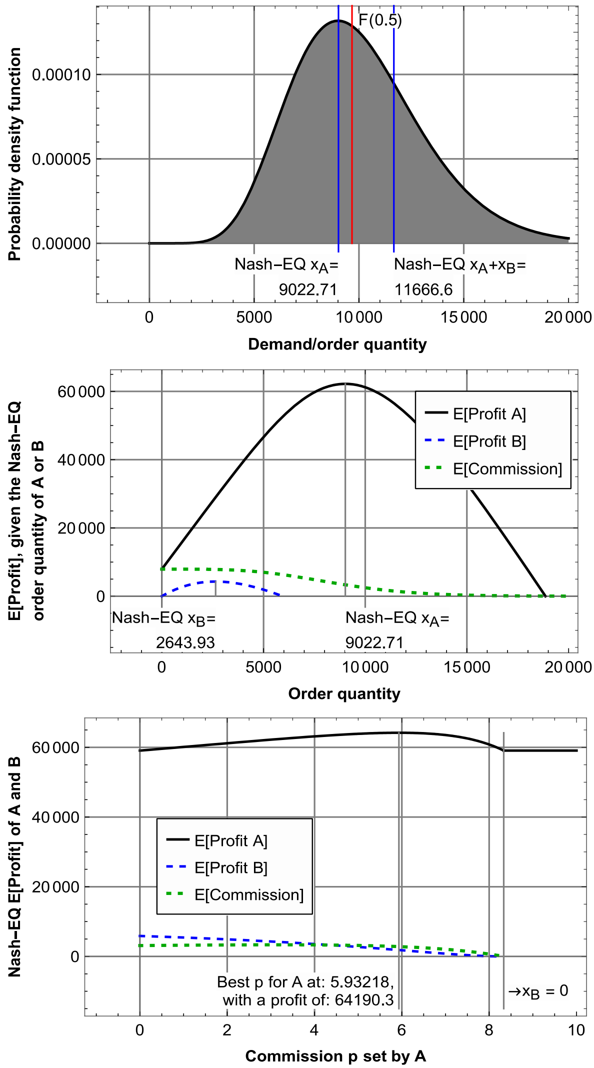


Figure V.4 Numerical example 1: the Nash equilibrium and the optimal commission *p*

V.3.2. Base case model – cooperative optimum

For the cooperative optimum, which leads to the lowest possible underage/overage costs, it is expedient to deviate from the typical cost minimization and to instead adopt a gross profit maximization approach.

The commission p is no longer part of the model, as it is merely a redistribution mechanism within the system. What B has to pay in commission, A obtains as additional gross profit. The gross profit function can be maximized using the Leibniz integral rule (first-order condition) and the eigenvalues/principal minors of the Hessian matrix (second-order condition).

(10)

$$Max\ E[\pi(x_A,x_B)] = \int_{b=0}^{x_A} \begin{bmatrix} rb - c_A x_A - c_B x_B + \\ v_A(x_A - b) + v_B x_B \end{bmatrix} f(b)db$$
$$+ \int_{b=x_A}^{x_A+x_B} \begin{bmatrix} rb - c_A x_A - c_B x_B + \\ v_B(x_A + x_B - b) \end{bmatrix} f(b)db$$
$$+ \int_{b=x_A+x_B}^{\infty} \left[r(x_A + x_B) - c_A x_A - c_B x_B \right] f(b)db$$

It follows that (**Appendix V.A.3** contains the complete mathematical derivation):

(11) $$x_A^* = \begin{cases} F^{-1}\left(\dfrac{c_B - c_A}{v_B - v_A}\right) & \text{if } v_B > v_A \text{ (S.O.C.) and } \\ & 0 < \dfrac{c_B - c_A}{v_B - v_A} \leq \dfrac{r - c_B}{r - v_B} \ (\rightarrow x_B^* \geq 0) \\ 0 \text{ or } F^{-1}\left(\dfrac{r - c_A}{r - v_A}\right) & \text{else} \end{cases}$$

$$(12)\qquad x_B^* = \begin{cases} F^{-1}\left(\dfrac{r-c_B}{r-v_B}\right) - x_A^* & \text{if } 0 \le F^{-1}\left(\dfrac{r-c_B}{r-v_B}\right) - x_A^* \\ 0 & \text{else} \end{cases}$$

Note that the following system of inequalities cannot be true: $\frac{r-c_A}{r-v_A} < \frac{c_B-c_A}{v_B-v_A} \le \frac{r-c_B}{r-v_B}$. Therefore, the condition $\frac{c_B-c_A}{v_B-v_A} \le \frac{r-c_B}{r-v_B}$ (see Equation 11) also automatically ensures that $\frac{c_B-c_A}{v_B-v_A} \le \frac{r-c_A}{r-v_A}$. This means, that if $v_B > v_A$ (S.O.C.) and $x_B^* \ge 0$ are fulfilled, then A orders $x_A^* = F^{-1}\left(\frac{c_B-c_A}{v_B-v_A}\right) \le F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$. In our base case model, it is never optimal for A and B to order more than their critical ratios $F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$ and $F^{-1}\left(\frac{r-c_B}{r-v_B}\right)$. In many cases, it is possible to be more precise than "0 or $F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$" (see Equation 11). However, there are numerous special cases involved, and we therefore refrain from listing them here. The policy of the cooperative optimum is highly volatile by nature. Depending on the cost/revenue parameters, it is often optimal that either only A or B orders. A more detailed discussion can be found in **Appendix V.A.3**. Generally, one can simply compare the two cases where either only A or only B orders. Indeed, one case exists (→ saddle point within the domain $x_A > 0 \wedge x_B > 0$) where such a comparison cannot be avoided.

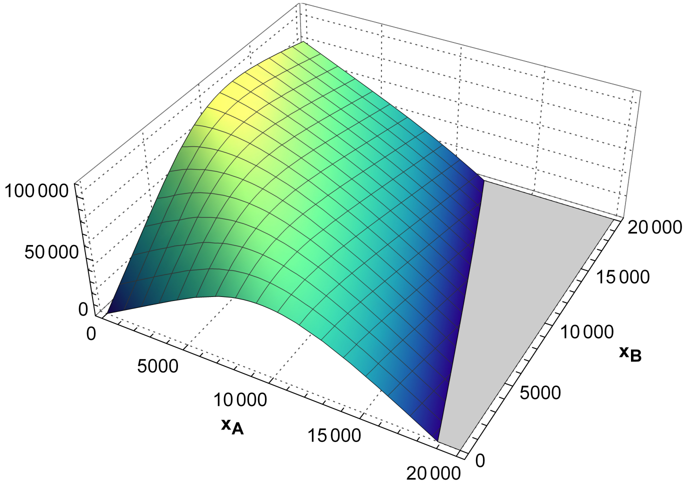
Figure V.5 shows the cooperative optimum for our initial numerical example 1 (see **Figure V.4** for the noncooperative Nash equilibrium) and for another numerical example 2. In the second numerical example, the marketplace operator A now has lower costs than the third-party vendor B. In practice, this could be the case because A receives rebates due to its high order volume. The salvage value, on the other hand, is much higher for B. In practice, this could, for example, be the case when the third-party vendor is running a lucrative liquidating business.

In our initial example 1, it is optimal that B should order everything. This is logical because B has lower costs and a higher salvage value. In the second numerical example, it is optimal for A and B to order roughly equal amounts of the product. The marketplace operator A, with its slightly lower costs but very small salvage value compared to B, should only service the most likely demand. In our base case model, the third-party vendor B only receives demand after A is out of stock. However, because B has a much higher salvage, it is optimal when B services much of the "leftover" distribution function.

Since there are only two parties involved in our model, the fair distribution of cooperation gains is rather straightforward. In **Table V.1**, we present the Shapley values, but other solution concepts are of course possible. Mathematically, the sum of the gross profits must be higher in the case of the centralized (cooperative) solution. Furthermore, because the commission p does not constrain the profitable upper bound in the cooperative case, it is evident that, from the customers' perspective, the α-service level in the cooperative case is always at least as high as in the noncooperative case. Therefore, in our model, cooperation between A and B leads to a win–win–win situation, since both parties A and B and the customers benefit from cooperation. Additionally, note that we do not assume that the parties cooperate in the procurement or salvaging of the products. Furthermore, in our base case model, customers still only buy from B if A is out of stock. The ownership of the two inventories of A and B is still separated between the two parties. The barriers to such cooperation are therefore rather low.

$r = 20$, $c_B = 9$, $v_B = 6$, Demand: Gamma distribution with $\alpha = 10$, $\beta = 1000$

Cooperative profit with: $c_A = 12$ and $v_A = 3$



Cooperative profit with: $c_A = 8$ and $v_A = 1$

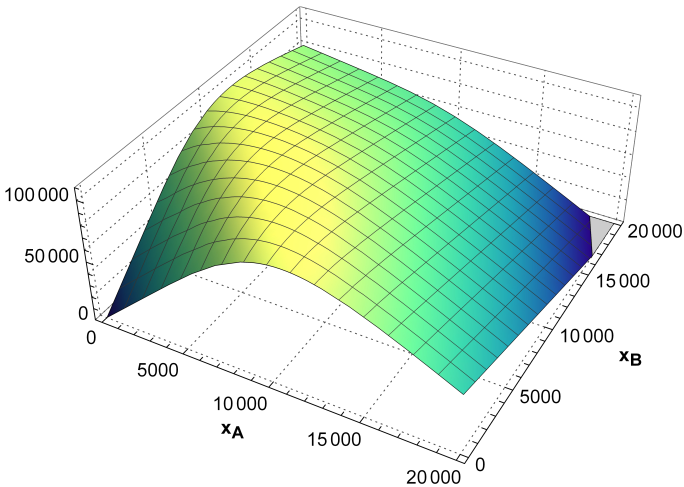


Figure V.5 Numerical example 1 and 2: the cooperative optimum

Given or expected values	Numerical example 1	Numerical example 2
Identical parameters	$r=20$, $c_B=9$, $v_B=6$, $p=3$	
Different parameters	$c_A=12$, $v_A=3$	$c_A=8$, $v_A=1$
Nash equilibrium	$x_A^{*Nash}=9023$, $x_B^{*Nash}=2644$	$x_A^{*Nash}=10592$, $x_B^{*Nash}=1074$
Nash-EQ total gross profit	66493	98534
Nash-EQ gross profits of A/B	Profit A = 62209 Profit B = 4284	Profit A = 97885 Profit B = 649
α-service level ($x_A^{*Nash}+x_B^{*Nash}$)	72.73%	72.73%
Cooperative optimum	$x_A^{*Coop}=0$, $x_B^{*Coop}=12336$	$x_A^{*Coop}=7289$, $x_B^{*Coop}=5047$
Coop. opt. total gross profit	96175	102191
Coop. opt. gross profits of A/B	Profit A = 28539 Profit B = 67636	Profit A = 90067 Profit B = 12124
Shapley values	Payoff A = 77050 Payoff B = 19125	Payoff A = 99714 Payoff B = 2477
α-service level ($x_A^{*Coop}+x_B^{*Coop}$)	78.57%	78.57%

Table V.1 Numerical comparisons between the noncooperative and cooperative cases

V.4. Model extension

V.4.1. Model description

The following extension models a situation where with certain probabilities, customers order from the marketplace operator or the third-party vendor. Similarly, if either is out of stock, customers order from the other only with a certain probability. We introduce the following four new h_i -parameters ($i=1,2,3,4$):

$0 \le h_1 \le 1$	Probability that a customer orders from the third-party vendor *B* while both *A* and *B* still have stock.
$0 \le h_2 \le 1$	Probability that a customer orders from *B* when *A* is out of stock.
$0 \le h_3 \le 1$	Probability that a customer orders from the marketplace operator *A* while both *A* and *B* still have stock.
$0 \le h_4 \le 1$	Probability that a customer orders from *A* when *B* is out of stock.

With the condition that:

$h_1 + h_3 \le 1$	A customer cannot order from both *A* and *B* simultaneously. Some customers may not want to order from either *A* or *B* (then $h_1 + h_3 < 1$).

These parameters may be exogenously given or depend on the price. For this article, we assume identical sales prices $r = r_A = r_B$ offered by *A* and *B* and therefore no price-sensitive demand. Future research could investigate price-sensitive demand (i.e., $h_i\left(r_A, r_B\right)$). In this article, we focus on non-price competition. It is well known that price competition and non-price competition are closely interrelated (Stigler, 1968; Spence, 1977). If a company has very good non-price variables (such as image, trust, or customer service) compared with its competitors, then the company receives a considerable demand even with market average sales prices (because customers evaluate the price-service bundles), or the company could increase prices and receive market average demand with high sales prices (Brynjolfsson & Smith, 2000). We concentrate on the former case. It is for example known that trust in an online retailer is very important for the purchase intention of customers (Gefen, 2000). Customers may trust a marketplace operator (e.g., Amazon) more than they trust third-party vendors (Pavlou & Gefen, 2004). This would lead to a high h_3 and a low h_1. On the other hand, it may be the case that some customers want to support small third-party vendors (vs. the "unsympathetic behemoth" Amazon) for ideological reasons. This would lead in contrast to a high h_1 and a low h_3. The average behavior/preferences across all potential customers are the basis for the exogenously given h_i-parameters.

Thus, the four h_i-parameters ($i=1,2,3,4$) have to be estimated by both the marketplace operator *A* and the third-party vendor *B*. This certainly makes the model less robust because *A's* and/or *B's* predictions could be inaccurate. However, non-price variables such as the image of or the trust in a company are more or less constant in the short to medium term. This makes estimating the h_i-parameters much easier because the predictions can be based on experience. Of course, the marketplace operator has substantially more experience and thus yet another advantage over the third-party vendors. In the long term, it is possible that the h_i-parameters are endogenously dependent on x_A, x_B because different order quantities mean different stockout probabilities (service-levels). We delve into somewhat more detail on this topic in **Section V.5**.

In any case, we would contend that the following model extension is valuable even with more or less inexact and differing estimates. If the parties are uncertain about their estimate, they can always calculate with more conservative estimates. Sensitivity analyses with different h_i-values can also be performed. This is not possible in our base case model.

The exact version of our model extension cannot, to the best of our knowledge, be formulated in a closed form. However, the following approximation produced excellent results in our tests. Because customers now potentially order from both *A* and *B* from the outset, the following case distinction must be made:

- Case ME.1, where $h_3 x_B \geq h_1 x_A$ $\rightarrow$ In this case, it is expected that the stock of *A* is depleted earlier than the stock of *B*.
- Case ME.2, where $h_3 x_B < h_1 x_A$ $\rightarrow$ In this case, it is expected that the stock of *B* is depleted earlier than the stock of *A*.

Note that our approximate models are based on these expected (!) thresholds of the demand phases (integral limits). The actual demand process, however, follows a multinomial distribution. Moreover, the distribution of the second demand phase depends on the distribution of the first phase, and there is a hard cutoff point between these phases: the moment where either *A* or *B* sells its last unit in stock. For the exact model, it would therefore be necessary to calculate the complete probability tree. In contrast, the integral limits in our approximate models are the expected means, and we ignore that there is a more or less significant probability that, for example, *B's* stock will be depleted far earlier than *A's* stock, although the mean values would indicate otherwise. However, we simulated different examples with different order quantities for *A* and *B* and found that if the demand is not very low, the approximation quality of the following formulas is almost always excellent (<0.3% deviation between simulated and calculated mean in the case of an expected gamma distributed demand with $\alpha = 10$, $\beta = 1000$). This is because the expected timeline, i.e., when is *A* or *B* out of stock, closely matches the observed real timelines (the normal distribution of the timeline deviations is very steep). The following analytical results from our approximate model can therefore be treated as quasi-exact for many realistic situations. Note that the approximation quality is worst around the crossover quantities between the two cases ME.1 and ME.2. However, the absolute differences are usually negligible. In our tests, we observed that the approximation quality starts to become noticeably worse in case of sales period demand levels lower than 100 units.

V.4.2. Mathematical results

Given the above-mentioned expected thresholds, one can formulate the approximate gross profit functions. The integral limits now incorporate the h_i-parameters. For example, in the case of $h_3 x_B \geq h_1 x_A$ for A:

(13)

$$
\begin{aligned}
E[\pi_A(x_A)] = & \int_{b=0}^{\frac{x_A}{h_3}} \left[r h_3 b - c_A x_A + v_A(x_A - h_3 b) + p h_1 b \right] f(b)db \\
& + \int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)} \left[\begin{array}{l} r x_A - c_A x_A + \\ p\left(\dfrac{h_1 x_A}{h_3}\right) + p h_2 \left(b - \dfrac{x_A}{h_3}\right) \end{array} \right] f(b)db \\
& + \int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)}^{\infty} \left[r x_A - c_A x_A + p x_B \right] f(b)db
\end{aligned}
$$

The other gross profit functions and the respective optimality conditions can be found in **Appendix V.A.4**. The following optimal order quantities (reaction functions) are obtained.

For A and B in the case of ME.1, where $h_3 x_B \geq h_1 x_A$ (it is expected that the stock of A is depleted earlier than the stock of B):

(14)

$$x_A^* = \begin{cases} h_3 F^{-1}\left(\dfrac{r - c_A - p\left(\dfrac{h_2 - h_1}{h_3}\right) F\left(\dfrac{x_A^*}{h_3} + \dfrac{1}{h_2}\left(x_B - \dfrac{h_1 x_A^*}{h_3} \right)\right)}{r - v_A - p\left(\dfrac{h_2 - h_1}{h_3}\right)} \right) & \\ \qquad \text{if} \left[\begin{array}{l} \left(v_A - r + p\left(\dfrac{h_2 - h_1}{h_3}\right)\right) f\left(\dfrac{x_A^*}{h_3}\right) - \\ p\left(\dfrac{h_2 - h_1}{h_3}\right) f\left(\dfrac{x_A^*}{h_3} + \dfrac{1}{h_2}\left(x_B - \dfrac{h_1 x_A^*}{h_3}\right)\right) \end{array} \right] < 0 & \\ 0 & \text{else} \end{cases}$$

(15) $$x_B^* = \max\left\{ 0,\ h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) - \left(\frac{h_2 - h_1}{h_3} \right) x_A \right\}$$

For A and B in the case of ME.2, where $h_3 x_B < h_1 x_A$ (it is expected that the stock of B is depleted earlier than the stock of A):

(16) $$x_A^* = \max\left\{ 0,\ h_4 F^{-1}\left(\frac{r - c_A}{r - v_A} \right) - \left(\frac{h_4 - h_3}{h_1} \right) x_B \right\}$$

(17) $$x_B^* = h_1 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right)$$

Compared to the result from our base case model, the reaction functions are similar but now incorporate the h_i-parameters. The term $h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) - \left(\frac{h_2 - h_1}{h_3} \right) x_A$, for example, can be derived in the following way. The order quantity of A is sold on average after x_A / h_3 (potential) customers. Until the stock of A is sold, some of these customers already order from B with a probability of h_1. Thus, for this first phase, B plans with an effective demand of $h_1\left(x_A/h_3\right)$ for its stock. After A's stock is sold out, customers order from B with a probability of h_2. However, B only wants to cover uncertain demand up to its critical ratio of $F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$. Thus, B wants to stock $h_2\left[F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-\frac{x_A}{h_3}\right]+h_1\frac{x_A}{h_3}$, which is equal to $h_2F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A$. The other reaction functions can be derived in a similar fashion.

Note that all the optimal solutions above are constrained by the conditions of the cases of ME.1 $h_3x_B\geq h_1x_A$ and ME.2 $h_3x_B<h_1x_A$. If either x_A or x_B is given, A or B can, within one case, only increase its order quantity until the own critical ratio or the case condition is reached. Depending on the given x_A or x_B there might be two valid solutions (one per case). If there are two valid solutions, a simple comparison of the expected gross profits reveals the optimal order quantity.

The noncooperative and cooperative optima can be determined similarly to the noncooperative/cooperative optima in the base case model. For the complete mathematical derivation, see **Appendices V.A.5** and **V.A.6**.

The Nash equilibrium in case ME.1, where $h_3x_B\geq h_1x_A$:

(18) $$x_A^*=h_3F^{-1}\left(\frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}\right)$$

(19) $$x_B^*=h_2F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A^*$$

The Nash equilibrium in case ME.2, where $h_3 x_B < h_1 x_A$:

(20) $$x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - (h_4 - h_3) F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

(21) $$x_B^* = h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

In **Appendix V.A.5**, we show that up to three Nash equilibria can coexist within the two cases. The two Nash equilibria outlined above and an additional Nash equilibrium at $x_A^* = 0$ and $x_B^* = h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$ within case ME.1 but only if:

$$\left[\left(v_A - r + \frac{p(h_2 - h_1)}{h_3}\right) F(0) - \left(\frac{p(h_2 - h_1)}{h_3}\right)\left(\frac{r-c_B-p}{r-v_B-p}\right) + (r - c_A)\right] < 0$$

Because *B* pays a commission fee p to *A*, it could be optimal for *A* to not order at all. Given that it is possible that multiple Nash equilibria coexist in one parameter combination, the question arises whether there is always a pure-strategy Nash equilibrium, that is, whether one of the Nash equilibria dominates the other Nash equilibria. In our numerical examples, we consistently observed that there was a pure-strategy Nash equilibrium. For *B,* this is easy to prove. For *A*, however, we do not see any way to prove our anecdotal numerical evidence for every possible case.

Regarding the order quantities of the cooperative optimum we derive in case ME.1, where $h_3 x_B \geq h_1 x_A$:

(22)
$$x_A^*=\begin{cases} h_3F^{-1}\left(\dfrac{(r-c_A)-(r-c_B)\left(\dfrac{h_2-h_1}{h_3}\right)}{(r-v_A)-(r-v_B)\left(\dfrac{h_2-h_1}{h_3}\right)}\right) & \text{if } (r-v_B)(h_2-h_1)<h_3(r-v_A) \\ & \text{and } 0<\dfrac{(r-c_A)-(r-c_B)(h_2-h_1)/h_3}{(r-v_A)-(r-v_B)(h_2-h_1)/h_3}<1 \\ & \text{and } \dfrac{r-c_B}{r-v_B}\geq\dfrac{(r-c_A)-(r-c_B)(h_2-h_1)/h_3}{(r-v_A)-(r-v_B)(h_2-h_1)/h_3} \text{ (case ME.1)} \\ 0 & \text{else} \end{cases}$$

(23)
$$x_B^*=h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A^*$$

The order quantities of the cooperative optimum in case ME.2, where $h_3 x_B < h_1 x_A$, are:

(24) $$x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \left(\frac{h_4-h_3}{h_1}\right) x_B^*$$

(25) $$x_B^* = \begin{cases} h_1 F^{-1}\left(\dfrac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}\right) & \text{if } \mathrm{h}_1(r-v_B) > (h_4-h_3)(r-v_A) \\ & \text{and } 0 < \dfrac{(r-c_B)-(r-c_A)(h_4-h_3)/h_1}{(r-v_B)-(r-v_A)(h_4-h_3)/h_1} < 1 \\ & \text{and } \dfrac{r-c_A}{r-v_A} \geq \dfrac{(r-c_B)-(r-c_A)(h_4-h_3)/h_1}{(r-v_B)-(r-v_A)(h_4-h_3)/h_1} \text{ (case ME.2)} \\ 0 & \text{else} \end{cases}$$

V.4.3. Numerical example for the model extension

To demonstrate the model extension, we introduce a new example with more realistic values for the h_i-parameters. The new example compares to our base case model in the following way:

- Numerical example 1 (base case model):
  $h_1 = 0$, $h_3 = 1$, $h_2 = 1$, $h_4 = 1$
- Numerical example 3 (more realistic model):
  $h_1 = 0.2$, $h_3 = 0.75$, $h_2 = 0.4$, $h_4 = 0.90$

The top two subfigures in **Figure V.6.1** show the reaction functions of the new example. Compared to the base case model, the order quantity of *B* now has a more pronounced influence on the optimal order quantity of *A* because customers order from both the marketplace operator *A* and the third-party vendor *B* during the first demand phase. The kink in the reaction functions (see **Figure V.6.1**) is where the model switches between ME.2 (valid before the kink) and ME.1 (valid after the kink). Before the kink, in ME.2, the reaction function of *A* is not dependent on the commission fee p because *A* takes on more demand risk than *B*. Furthermore, it can be observed that before the kink, the best possible expected profit decreases for *A* and then increases again after the kink. In our base case model, such a decline would not be possible because more order quantity from *B* could not be negative for *A*. In our model extension, however, more order quantity from *B* means longer competition between *A* and *B*, which decreases the expected profit for *A*. The expected commission, on the other hand, increases the higher the order quantity from *B* is.

The bottom subfigure in **Figure V.6.1** depicts the Nash-EQ when $p = 3$. The equilibrium lies in the ME.1 case, and the red parts of the graph indicate only hypothetical expected profits because these order quantity combinations would violate the condition of the ME.1 case. Thus, in these regions, the profit would need to be calculated using the formulas from the ME.2 case.

Figure V.6.2 contains a sensitivity analysis of the commission fee p, which can be freely set by *A*. By "Critical ratio quantity B", we mean $F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$. We again see a kink at the crossover point between the ME.1 and the ME.2 case. The kinks are not an artifact of our approximate model formulation. They also exist in the exact model, although slightly less sharp and smoother. The explanation for the piecewise convexity/concavity of the curves is the following. The higher the commission fee p and the higher h_2 (probability that a customer orders from *B* when *A* is out of stock) and the higher the order quantity of *B*, the more attractive it is for *A* to cede some demand to *B* because *A* can expect sizeable commission payments from *B*. This is why the "Nash-EQ Quantity A" (see **Figure V.6.2**) first decreases until it increases again because *B* is ordering increasingly less. The slope of the curve changes at the kink because in ME.2, the commission fee p does no longer play a role in the reaction function of *A*. This transition is less sharp in the exact model when the demand quantity is low but nearly equally sharp when demand is high.

The bottom subfigure in **Figure V.6.2** shows the cooperative gross profit for numerical example 3. The gray/white line through the plot marks the crossover line between cases ME.1 and ME.2. In our example, *B* has better cost/salvage value parameters, but *A* has better demand probabilities ($h_2 < h_4$). This leads to a situation where it is optimal that both parties order some amount. The slope is somewhat steep, and one has to be rather precise to land in an area with a high cooperative gross profit. Our analytical model empowers the parties to do so. Depending on the parameter values, especially the h_i-parameters and the signs and sizes of the sums $h_2 - h_1$, $h_4 - h_3$, the model can exhibit very convoluted behavior. However, as we will discuss in the following section in greater detail, it will indeed probably often be the case that it is optimal that both parties *A* and *B* order some units.

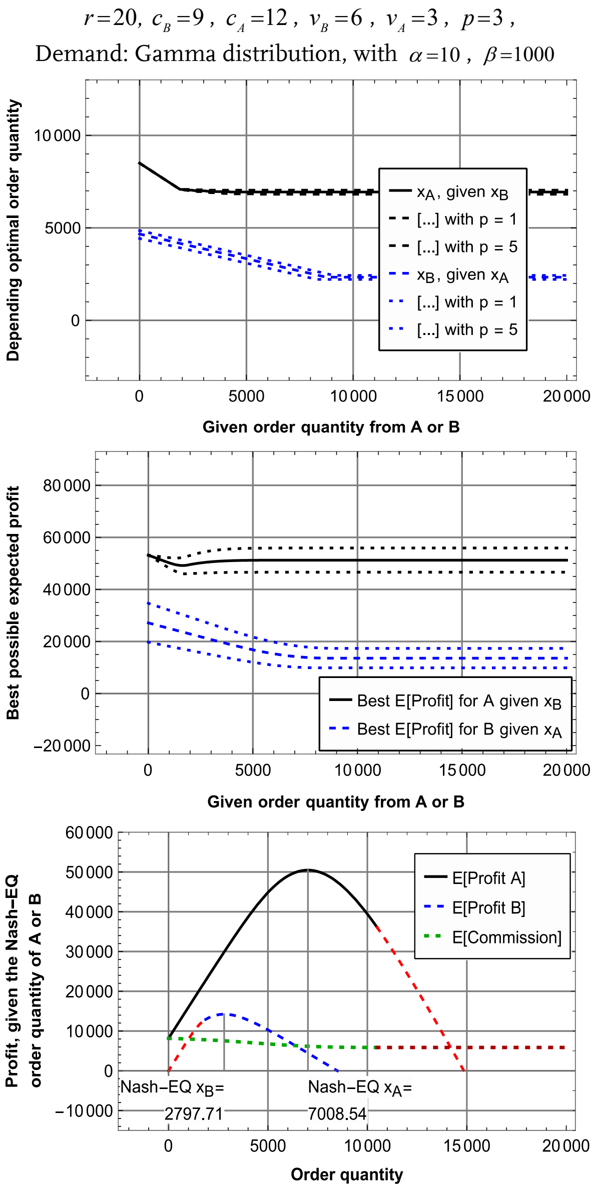


Figure V.6.1 Numerical example 3 (ME): reaction functions, Nash equilibria, optimum commission p, and cooperative optimum (part 1)

$r=20\,,\ c_B=9\,,\ c_A=12\,,\ v_B=6\,,\ v_A=3\,,\ p=3\,,$
Demand: Gamma distribution, with $\alpha=10\,,\ \beta=1000$

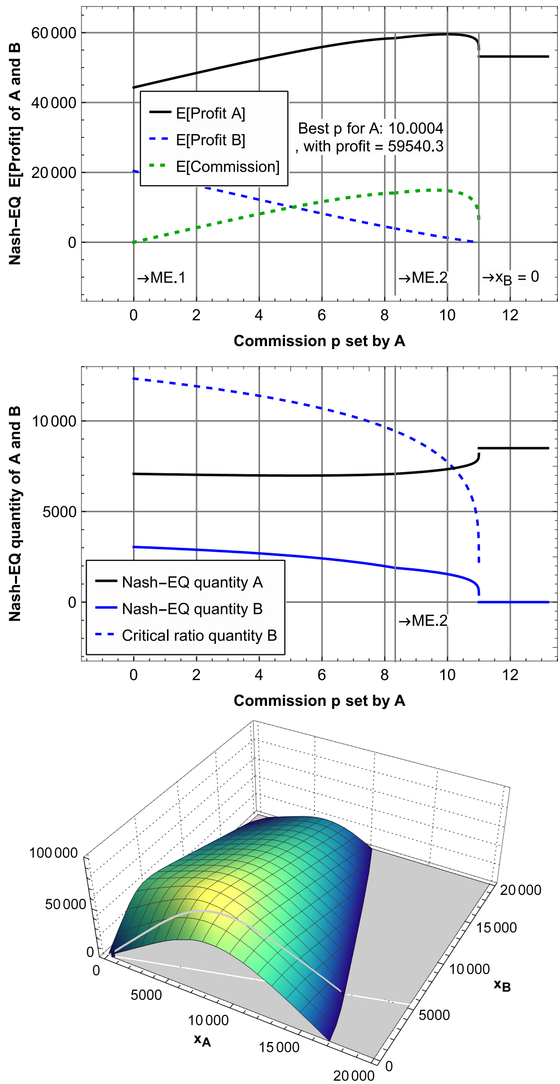


Figure V.6.2 Numerical example 3 (ME): reaction functions, Nash equilibria, optimum commission *p*, and cooperative optimum (part 2)

V.5. Numerical studies

Before we discuss managerial implications, we want to briefly present some additional numerical results across all our models. For **Table V.2**, we have calculated another numerical example 4, in which both *A* and *B* have the same cost/salvage revenue parameters. We have also split the α-service level into an α-service level for *A* and an α-service level for *B*. For example, it could be that during the demand period, the marketplace operator *A* runs out of stock, but the third-party vendor *B* does not. The α-service levels given in **Table V.2** are calculated with our approximate model. We also simulated the α-service levels, and the approximation is again excellent. Only in the cooperative case of numerical example 4 did we observe a small but significant difference. While our approximate model indicates an α-service level of 78.6% for both *A* and *B*, the simulation results indicate an α-service level of approximately 78% for both parties. The order quantities are right on the edge between cases ME.1 and ME.2, so this small difference between the approximate and exact numbers was to be expected.

The distinction between the two α-service levels (for *A* and *B*) is relevant for business practice, as there may be customers who want to exclusively order from the marketplace operator *A* or the third-party vendor *B*, for example, because they have had good experiences with the vendor and are otherwise risk averse and do not like to order from a vendor they do not know. If such a customer observes that his or her favorite vendor is often out of stock, he or she may develop a growing distaste for his or her favorite vendor or the marketplace platform as a whole. This would change the h_i-parameters in the long term. In future research, it would certainly be worthwhile to analyze the service levels, including the more accurate β-service level, in greater detail.

Given or expected values	**Numerical example 1**	**Numerical example 3**	**Numerical example 4**
Parameters	$r=20$, $c_B=9$, $v_B=6$, $p=3$, Demand: Gamma distribution, with $\alpha=10$, $\beta=1000$		
	$c_A=12$, $v_A=3$		$c_A=9$, $v_A=6$
	$h_1=0$, $h_2=1$, $h_3=1$, $h_4=1$	$h_1=0.2$, $h_2=0.4$, $h_3=0.75$, $h_4=0.9$	
Nash equilibrium	$x_A^{*Nash}=9023$ $x_B^{*Nash}=2644$	$x_A^{*Nash}=7009$ $x_B^{*Nash}=2798$	$x_A^{*Nash}=9352$ $x_B^{*Nash}=2333$
Nash-EQ gross profits	Total: 66493 A: 62209, B: 4284	Total: 64687 A: 50465, B: 14222	Total: 91352 A: 77781, B: 13571
α-service level	A: 41.6%, B: 72.7%	A: 45.8%, B: 72.7%	A: 78.6%, B: 72.7%
Cooperative optimum	$x_A^{*Coop}=0$ $x_B^{*Coop}=12336$	$x_A^{*Coop}=6575$ $x_B^{*Coop}=3181$	$x_A^{*Coop}=9252$ $x_B^{*Coop}=2467$
Coop. optimum gross profits	Total: 96175 A: 28539, B: 67636	Total: 65012 A: 50487, B: 14525	Total: 91366 A: 77839, B: 13527
Shapley values (payoffs)	A: 77050, B: 19125	A: 50628, B: 14383	A: 77788, B: 13578
α-service level	A: 0.0%, B: 78.57%	A: 38.2%, B: 78.6%	A: 78.6%, B: 78.6%
Assuming $x_B=0$	$x_A^*=9442$ Profit A: 59073 α-service level: A: 47.06%	$x_A^*=8498$ Profit A: 53166 α-service level: A: 47.06%	$x_A^*=11102$ Profit A: 86557 α-service level: A: 78.57%
Assuming $x_A=0$	$x_B^*=11667$ Profit A: 28052 Profit B: 67857 α-service level: B: 72.73%	$x_B^*=4667$ Profit A: 11221 Profit B: 27143 α-service level: B: 72.73%	
Assuming A and B order based on their normal critical ratio	$x_A^*=9442$ $x_B^*=11667$ Profit A: 63640 Profit B: –18253 α-service level: A: 47.1%, B: 99.8%	$x_A^*=8498$ $x_B^*=4667$ Profit A: 47903 Profit B: 9470 α-service level: A: 69.4%, B: 97.6%	$x_A^*=11102$ $x_B^*=4667$ Profit A: 76179 Profit B: 8270 α-service level: A: 92.6%, B: 99.1%

Table V.2 Numerical comparisons

We also present the calculation for the case where both parties would order according to their normal critical ratio, i.e., $x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$ and $x_B^* = h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$, and the results indeed show that it is important to account for the behavior of the other party. This is especially true for third-party vendors, who have to pay the commission and often have lower demand probabilities and thus must be careful when considering which quantity to order. The marketplace operator is often in a much more comfortable position.

Obviously, the higher the commission fee p, the less attractive intra-platform competition is for *B*. Less obvious is the effect of the sales price r. The commission p is usually pegged to the sales price (e.g., $0.15r = p$), but c_A, c_B and v_A, v_B are not necessarily correlated with the sales price r. Low-margin ($r \approx c$) and high-margin ($r >> c$) products exist, and expensive products can become scrap ($r >> v$). Recall our initial numerical example 1 (see **Table V.2**). In this example ($r = 20$, $c_B = 9$, $c_A = 12$, $v_B = 6$, $v_A = 3$, $p = 3 = 0.15r$), the Nash equilibrium has the following critical ratios: $CR_B = \frac{r-c_B-p}{r-v_B-p} = 0.727$ for *B* and $CR_A = \frac{r-c_A-p \cdot CR_B}{r-v_A-p} = 0.416$ for *A*. Compare this with a higher-margin product, where the sales price (and the commission fee) is doubled: $r = 40$, $c_B = 9$, $c_A = 12$, $v_B = 6$, $v_A = 3$, $p = 6 = 0.15r$. In this case, the critical ratio of *B* is $CR_B^{new} = 0.893$, and for *A* it is $CR_A^{new} = 0.730$. The difference $CR_B^{new} - CR_A^{new} = 0.163$ is less than $CR_B - CR_A = 0.311$. This means that *B* orders less. Moreover, the demand that *B* receives is also riskier because CR_A^{new} is higher than CR_A. Both effects can be observed in all low-margin vs. high-margin product comparisons. A high margin makes a product more attractive to all retailers, including *A*, and thus CR_A and

CR_B will increase and converge the higher the margin (given that c_A, c_B and/or v_A, v_B remain unchanged). However, the margin has also changed, for example for *B*, from $r^{old} - c_B - p^{old} = 8$ to $r^{new} - c_B - p^{new} = 25$ per unit sold. The negative demand reduction effect and the positive effect of the higher margin act against each other. In this concrete example, the profit is higher for *B* in the high-margin case ($x_A^* = 11700$, $x_B^* = 2348$, E[Profit *B*] = 4725 vs. 4284 in the low-margin case; see **Table V.2**). Nevertheless, the higher the margin is, the stronger the negative demand reduction effect for *B,* and at some point this negative effect will become stronger than the positive margin effect, and the expected profit will decrease for *B*. Thus, we can conclude based on our noncooperative model that while in most cases both *A* and *B* prefer high-margin products, *B* actually has to expect a lower profit for very high-margin products (assuming that $h_1 << h_3$, i.e., customers prefer *A* over *B*) when facing intra-platform competition.

V.6. Managerial implications

V.6.1. Coopetition between the third-party vendors and the marketplace operator

It is interesting and highly relevant for business practice that, at least in the examples that we tested, the commission fee has to be very high if *A* wants to have an expected profit that is at least as high as when *A* would close its marketplace to third-party vendors (i.e., $x_B = 0$). In our numerical example 3 (see **Figures V.6**), the marketplace operator would face reduced gross profits with typical fees found in practice (e.g., typically 8% to 20% at Amazon).

Of course, *A* might have more complex reasons to find the presence of third-party vendor offers on its marketplace platform desirable. Amazon, for example, openly speaks about its strategy to increase the third-party vendor revenue on its platform. It wants to make the platform more attractive for third-party vendors and in turn make the platform also more attractive to customers. Low platform fees lead to more product offerings and perhaps also lower prices, both of which are assumed to be attractive to the typical customer. Furthermore, some of the products that third-party vendors sell are maybe so exotic or proprietary (direct-to-consumer) that the marketplace operator cannot or has no interest in offering them itself. The fees paid for these products, however, are of course welcome additional revenue.

Another relevant economic concept in this context is risk-adjusted returns. Many companies want to maximize their risk-adjusted returns, for example, because their investors want to receive robust and steady cash flows out of the business. Some articles about risk-averse competitive newsvendor games already exist (e.g., Wu et al., 2014). Fee revenue can be considered less risky for the marketplace operator, as it takes on no inventory risk (overaging risk). Under this assumption, it is logical for the marketplace operator to want to increase the amount of commission revenue relative to the revenue it generates through its own product offerings. Interestingly, our results indicate that the highest commission revenue is generated with rather high fees (>20% of the product price). Thus, there seems to be a tradeoff between maximizing commission revenue in the short term with high fees and maximizing commission revenue in the long term with low fees and thus an attractive platform. This certainly warrants further investigation in future research.

In any case, to achieve more commission revenue, the marketplace operator and third-party vendors need to convince customers to order from the third-party vendors when the marketplace operator is out of stock (h_2 in our model). This could be achieved through marketing campaigns or through so-called institutional mechanisms and programs (Pavlou & Gefen, 2004). Amazon, for example, has the so-called “A-to-Z guarantee” and, more prominently, the “Prime” logo and the “Fulfillment by Amazon” (FBA) program. Amazon uses the Prime logo as a quality/service insurance logo and as an additional source of revenue. Many Amazon customers also have an Amazon Prime subscription; therefore, they receive free and fast delivery when buying an offer with the Prime logo. Furthermore, third-party vendor offers usually only receive the Prime logo if the third-party vendor uses the FBA service. It stands to reason that the demand probabilities (does a customer order from a third-party vendor or not; h_1 and h_2 in our model) increase if the third-party vendor offer has the Prime logo.

In our model, we assume that the third-party vendor's order/offer quantity is exclusively, or at least to a large extent, destined for *A's* marketplace. If the third-party vendor does not use FBA but instead ships its goods from its own warehouse to customers of *A's* marketplace as well as to customers of other marketplaces and its own online shop, a different demand distribution arises for the third-party vendor. This could mean less risk (because of demand pooling) but also less expected demand from the Amazon marketplace. Therefore, a direct comparison between a scenario with FBA and without FBA is not possible with our model alone. Nevertheless, our model represents one piece of the puzzle, and it would certainly be natural to investigate this tradeoff (between total expected demand and risk pooling) in further research. Amazon seems to have recognized this channel-commitment dilemma and has opened its fulfillment network in recent years. Amazon offers a "multichannel fulfillment" (MFC) service that allows third-party vendors to instruct Amazon to ship their goods to customers who bought them on websites other than the Amazon Marketplace. However, the fulfillment fees when using MFC (i.e., a third-party sale not on Amazon) are much higher than the fulfillment fees when using FBA (i.e., a third-party sale on Amazon) (Amazon, 2022a; Amazon, 2022b), and arguably, programs such as MFC have had a limited impact thus far. However, the results of our model provide a compelling explanation for why Amazon is pushing these services. MFC not only increases economies of scale for Amazon's logistic network but also makes storing inventory in Amazon warehouses more attractive to third-party vendors, which in turn makes the Amazon marketplace more attractive to customers (more third-party FBA offers with Prime service).

Our model reveals the difficult competitive position that third-party vendors have in a marketplace when the marketplace operator is also a retailer of the same product. However, not every third-party vendor and not every product is affected by this intra-platform competition in the same way. In our model, offering a product on the marketplace is profitable for a third-party vendor or not depending on the parameters $h_i, r, p, c_A, c_B, v_A, v_B$.

As mentioned in **Section V.3**, offering the same product as the marketplace operator is more attractive to a third-party vendor the higher the differences $c_A - c_B$ and/or $v_B - v_A$. It is an educated guess that this is more probable when the product is not a standard product (e.g., a normal pen) but belongs to some special product category (e.g., fashionable clothes) or is itself special and rarely demanded. In the case of standard products, it is, for example, likely that Amazon, as a high-volume customer, receives sizeable quantity discounts (implying that $c_A < c_B$). Moreover, the salvage value of standard products can be estimated rather accurately by liquidating businesses (to which large retailers like Amazon often sell their unsold goods). In the case of special products, however, liquidating businesses may have no expertise and would offer low prices (i.e., low v_A, v_B). A third-party vendor that specializes in these non-standard products on the other hand, may find more lucrative opportunities (implying that $v_B > v_A$) and may also have longstanding relationships with suppliers (implying potentially $c_B < c_A$). We have also shown in **Section V.5** that, while third-party vendors on a platform generally prefer products with high margins, they have to expect lower profits for very high-margin products when facing intra-platform competition because the critical ratios of *A* and *B* converge the higher the margin.

The h_i-parameters in our model are exogenously determined by non-price variables (e.g., the brand image; recall the discussion at the beginning of **Section V.4**). It is very difficult, if not impossible, for third-party vendors to build a powerful company brand on marketplaces such as Amazon. Thus, for many third-party vendors, the only real lever for distinguishing themselves are the offered products. If *A* offers the same product as *B*, this is of course not possible. However, *B* could offer a slightly different, substitutable product. Assume, for example, that both *A* and *B* offer calendars for a market average price. The calendars are very similar but have some distinguishing features (e.g., the quality of the binding), which could also lead to different reviews of the products by customers. For such standard products, even small differences in the reviews (e.g., 4.3/5 points vs. 4.5/5 points) can have considerable impacts on the h_i-parameters. Nevertheless, the company brand of the marketplace operator is probably still going to dominate many customer decisions. Most customers prefer the marketplace operator as a retailer when buying on a marketplace.

Imagine now a third-party vendor *B* that produces a product and wants to sell it on *A's* marketplace. The product is not easily copyable, and *B*, therefore, has the choice of selling it on the marketplace on its own (with a mediocre h_2, because many customers do not know/trust *B*) or to additionally sell the product to *A*, so that *A* can also offer the product on the marketplace (with a high h_3 and h_4, because customers know and trust *A*). A detailed analysis of this kind of decision is beyond the scope of this article. Nevertheless, it is certainly a very relevant and important topic for future research, and our noncooperative and cooperative models may be used as a basis for this.

V.6.2 Cooperation between the third-party vendors and the marketplace operator

Regarding our cooperative model, we remark that the value of cooperation is highest when *A* and *B* have different cost/salvage revenue values and when customers are willing to order from both vendors. From example 4, where both *A* and *B* have the same cost/salvage revenue parameters, it also becomes clear that in business practice, it is probably often optimal (in the cooperative case) that both *A* and *B* order some units. It is quite realistic that in practice both *A* and *B* often have similar, albeit not exactly the same, cost/salvage revenue parameters. Additionally, it is quite realistic that $h_1 + h_3 > h_4$ and $h_1 + h_3 > h_2$, that is, not all customers switch to the other vendor if their favorite vendor is out of stock.

Thus, if the cost/salvage revenue parameters are the same, then it is optimal in the cooperative case to maximize the demand probability (that is, to force the demand probabilities h_1 and h_3), which means that both parties should be out of stock at the same time. However, example 4 also demonstrates that the added value of cooperation is limited in such a case where both A and B have identical or very similar cost/salvage revenue parameters. The negative contract/transaction costs of cooperation could outweigh the positive effects.

Nevertheless, it is intriguing that cooperation, as described in our model, has comparatively few and low hurdles. In the practical example of the Amazon marketplace, it might be possible to integrate a contract mechanism into the FBA service to facilitate more cooperation between Amazon and third-party vendors. If a third-party vendor is considering using the FBA service, perhaps it could be offered a tool that suggests cooperation with Amazon under certain conditions. The third-party vendor enters its cost and salvage revenue parameters, and the tool calculates the order quantity that is required of the third-party vendor and fixes the payments between the marketplace operator and the third-party vendor (i.e., the payoffs). The third-party vendor can then decide whether it wants to take the deal or to act noncooperatively. This might be a very promising opportunity for introducing true value-adding cross-company supply chain management into corporate practice, and customers would also benefit because such a cooperation creates a win–win–win situation.

V.7. Summary, limitations and outlook

We have presented a model that describes the competition between a B2C e-commerce marketplace operator and a third-party vendor on the B2C e-commerce marketplace platform. Fundamentally, our model can be used to determine the optimal order quantities, accounting for the behavior of the other party without the need for expensive simulations. We have mathematically derived the Nash equilibria and have shown that it is economically important to consider the behavior of the other party when deciding the order quantity. We have shown that the commission fee, which is set by the marketplace operator and must be paid by the third-party vendor to the marketplace operator when selling a product on the marketplace, has a strong impact on the profits of both parties. Generally, the marketplace operator is in a more comfortable position, as it can at least partially influence the behavior of the third-party vendor through the amount of the commission fee. In addition, it can be assumed that many customers are willing to buy from the marketplace operator because they know the marketplace operator well and it may have a good reputation. The third-party vendors, on the other hand, are in a much weaker position. Thus, our results pertaining to the Nash equilibrium are especially important for third-party vendors, since even a moderate deviation from the Nash equilibrium can lead to a financial loss for a third-party vendor. Furthermore, going beyond the mere determination of optimal order quantities, our model can be used in both academia and practice to understand important features of the competitive situation described.

Our model is versatile and comes close to many realistic scenarios in practice. Nevertheless, the model is, of course, not equally accurate for all scenarios, and we have made some simplifying assumptions. We have built on and extended the research stream on competitive newsvendors that was started by Parlar (1988) and further developed by, among others, Lippmann and McCardle (1997), Mahajan and van Ryzin (2001), and Netessine and Rudi (2003). Within this research stream (and our model), it is commonly assumed that the selling price is exogenously given. Although this assumption is realistic in many cases (in our case, for example, because of the broader market pressure outside the marketplace platform), there are also cases where price competition between newsvendors exists. However, to model price competition, a price-demand function would be required. This would make the model substantially more complicated because parties would now optimize their selling price and order quantity simultaneously, and closed-form solutions, such as those in this article, are usually not obtainable (Zhao & Atkins, 2008; Transchel, 2017).

Another assumption in our model is that there is only one third-party vendor in addition to the marketplace operator. In future research, our model could be extended to $n \geq 3$ newsvendors, similar to Wang and Parlar (1994), Mahajan and van Ryzin (2001), and Netessine and Rudi (2003). Our approximate model (model extension) uses thresholds to determine when a party is expected to run out of stock. In principle, this procedure can also be applied to a situation with multiple third-party vendors, but the number of possible cases (who is out of stock before whom?) explodes relatively quickly (permutation complexity).

Furthermore, to calculate the optimal order quantities, the parties must estimate the demand distribution and know or estimate the cost and salvage revenue parameters of the other parties. This problem can only be circumvented to a limited extent, but our model is nevertheless useful because an inaccurate assumption is still usually better than complete ignorance of the competitor's behavior. In addition, the competitor's parameters will probably rarely differ substantially from a firm's own parameters. Thus, the vendors usually have quite a good idea about which parameters should be used. Nevertheless, the potential negative effects of incorrect assumptions and potential information-sharing mechanisms certainly deserve deeper analysis in future research.

Other topics for future research that we have touched upon include the effects of a Stackelberg-like leader-follower model, information sharing, or the effects of a high or low commission fee on the risk-adjusted returns of the parties. Moreover, a deeper analysis of the service levels, also taking into account the β-service level, is worthwhile. Future research could also integrate a minimum service level target into our model. Furthermore, it is important to better understand the tradeoff between channel commitment and expected greater demand, when a third-party seller uses FBA, and the risk-pooling through channel freedom, but expected lower demand, when a third-party seller uses its own storage and fulfillment. A related model arises when a direct-to-consumer third-party vendor has to decide whether it wants to sell a product on the marketplace on its own or whether it wants to offer the products to the marketplace operator for resale because many customers prefer the marketplace operator as a retailer.

V. References

Amazon. (2022a). *Pricing. Let's talk numbers*. Retrieved March 15, 2022, from https://sell.amazon.com/pricing#referral-fees

Amazon. (2022b). *Fulfillment fees for Multi-Channel Fulfillment orders*. Retrieved September 29, 2022, from https://sellercentral.amazon.com/help/hub/reference/external/201112650

Andersson, J., Jörnsten, K., Nonås, S. L., Sandal, L., & Ubøe, J. (2013). A maximum entropy approach to the newsvendor problem with partial information. *European Journal of Operational Research, 228*(1), 190–200. https://doi.org/10.1016/j.ejor.2013.01.031

Arrow, K., Harris, T., & Marshak, J. (1951). Optimal inventory policy. *Econometrica, 19*(3), 250–272. https://doi.org/10.2307/1906813

Brynjolfsson, E. & Smith, M. D. (2000). Frictionless commerce? A comparison of Internet and conventional retailers, *Management Science, 46*(4), 563–85. https://doi.org/10.1287/mnsc.46.4.563.12061

Cachon, G. P. (2003). Supply Chain Coordination with Contracts. *Handbooks in Operations Research and Management Science, 11*, 227–339. https://doi.org/10.1016/S0927-0507(03)11006-7

Cao, K., & He, P. (2016). The competition between B2C platform and third-party seller considering sales effort. *Kybernetes, 45*(7), 1084–1108. https://doi.org/10.1108/K-01-2016-0009

Cui, T. H., Raju, J. S., & Zhang, Z. J. (2007). Fairness and channel coordination. *Management Science, 53*(8), 1303–1314. https://doi.org/10.1287/mnsc.1060.0697

Gefen, D. (2000). E-commerce: the role of familiarity and trust. *Omega, 28*(6), 725–737. https://doi.org/10.1016/S0305-0483(00)00021-9

Güler, K., Körpeoğlu, E., & Şen, A. (2018). Newsvendor competition under asymmetric cost information, *European Journal of Operational Research, 271*(2), 561–576. https://doi.org/10.1016/j.ejor.2018.05.035

Jiang, H., Netessine, & S., Savin., S. (2011). Robust newsvendor competition under asymmetric information. *Operations Research, 59*(1), 254–261. https://doi.org/10.1287/opre.1100.0858

Khouja, M. (1999). The single-period (news-vendor) problem: literature review and suggestions for future research, *Omega, 27*(5), 537–553. https://doi.org/10.1016/S0305-0483(99)00017-1

Lariviere, M.A. (1999). Supply Chain Contracting and Coordination with Stochastic Demand. In S. Tayur, R. Ganeshan & M. Magazine (Eds.), *Quantitative Models for Supply Chain Management. International Series in Operations Research & Management Science* (Vol. 17, pp. 233–268). Springer. https://doi.org/10.1007/978-1-4615-4949-9_8

Li, L., & Liu, K. (2020). Coordination contract design for the newsvendor model. *European Journal of Operational Research, 283*(1), 380–389. https://doi.org/10.1016/j.ejor.2019.10.045

Li, M., Petruzzi, N. C., & Zhang, J. (2017). Overconfident competing newsvendors. *Management Science, 63*(8), 2637–2646. https://doi.org/10.1287/mnsc.2016.2469

Li, G., Tian, L., & Zheng, H. (2021). Information sharing in an online marketplace with co-opetitive sellers. *Production and Operations Management, 30*(10), 3713–3734. https://doi.org/10.1111/poms.13460

Lippman, S. A., & McCardle, K. F. (1997). The Competitive Newsboy. *Operations Research, 45*(1), 54–65. https://doi.org/10.1287/opre.45.1.54

Liu, W., Song, S., & Wu, C. (2013). Impact of loss aversion on the newsvendor game with product substitution. *International Journal of Production Economics, 141*(1), 352–359. https://doi.org/10.1016/j.ijpe.2012.08.017

Mahajan, S., & Van Ryzin, G. (2001). Inventory competition under dynamic consumer choice. *Operations Research, 49*(5), 646–657. https://doi.org/10.1287/opre.49.5.646.10603

Netessine, S., & Rudi, N. (2003). Centralized and Competitive Inventory Models with Demand Substitution. *Operations Research, 51*(2), 329–335. https://doi.org/10.1287/opre.51.2.329.12788

Netessine, S., & Shumsky, R. A. (2005). Revenue management games: Horizontal and vertical competition. *Management Science, 51*(5), 813–831. https://doi.org/10.1287/mnsc.1040.0356

Parlar, M. (1988). Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Research Logistics, 35*, 397–409. https://doi.org/10.1002/1520-6750(198806)35:3<397::AID-NAV3220350308>3.0.CO;2-Z

Pavlou, P. A., & Gefen, D. (2004). Building effective online marketplaces with institution-based trust. *Information Systems Research, 15*(1), 37–59. https://doi.org/10.1287/isre.1040.0015

Qin, Y., Wang, R., Vakharia, A. J., Chen, Y., & Seref, M. (2011). The newsvendor problem: Review and directions for future research. *European Journal of Operational Research, 213*(2), 361–374. https://doi.org/10.1016/j.ejor.2010.11.024

Ryan, J. K., Sun, D., & Zhao, X. (2012). Competition and Coordination in Online Marketplaces. *Production and Operations Management, 21*, 997–1014. https://doi.org/10.1111/j.1937-5956.2012.01332.x

Serin, Y. (2007). Competitive newsvendor problems with the same Nash and Stackelberg solutions. *Operations Research Letters, 35*(1), 83–94. https://doi.org/10.1016/j.orl.2006.01.002

Silbermayr, L. (2020). A review of non-cooperative newsvendor games with horizontal inventory interactions. *Omega, 92*, 102–148. https://doi.org/10.1016/j.omega.2019.102148

Spence, M. (1977). Nonprice competition. *The American Economic Review, 67*(1), 255–259. https://www.jstor.org/stable/1815912

Stigler, G. J. (1968). Price and non-price competition. *Journal of Political Economy, 76*(1), 149–154. https://doi.org/10.1086/259391

Transchel, S. (2017). Inventory management under price-based and stockout-based substitution. *European Journal of Operational Research, 262*(3), 996–1008. https://doi.org/10.1016/j.ejor.2017.03.075

Wang, Q., & Parlar, M. (1994). A three-person game theory model arising in stochastic inventory control theory. *European Journal of Operational Research, 76*(1), 83–97. https://doi.org/10.1016/0377-2217(94)90008-6

Wu, M., Zhu, S. X., & Teunter, R. H. (2014). A risk-averse competitive newsvendor problem under the CVaR criterion. *International Journal of Production Economics, 156*, 13–23. https://doi.org/10.1016/j.ijpe.2014.05.009

Zha, Y., Li, Q., Huang, T., & Yu, Y. (2023). Strategic Information Sharing of Online Platforms as Resellers or Marketplaces. *Marketing Science, 42*(4), 659–678. https://doi.org/10.1287/mksc.2022.1397

Zhao, X. & Atkins, D. R. (2008). Newsvendors Under Simultaneous Price and Inventory Competition. *Manufacturing & Service Operations Management, 10*(3), 539–546. https://doi.org/10.1287/msom.1070.0186

Zhao, Y., & Zhao, X. (2016). How a competing environment influences newsvendor ordering decisions. *International Journal of Production Research*, *54*(1), 204–214. https://doi.org/10.1080/00207543.2015.1034330

Zhu, F., & Liu, Q. (2018). Competing with complementors: An empirical look at Amazon.com. *Strategic Management Journal*, *39*(10), 2618–2642. https://doi.org/10.1002/smj.2932

Paper VI:
A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties

Christian Straubert

Abstract. Most existing location-inventory models use exact locations (coordinates) for warehouses and customers but use simple approximations for inventory costs. A different approach is presented in this paper. While the locations are loosely modeled using a continuous approximation approach, a reorder point, order quantity inventory model with exact costs is used. Exact expressions for the transport time distribution and backorder time distribution (for Poisson demand) are combined with an exogenously given warehouse fulfillment time and form the delivery lead time. Customers perceive a long delivery lead time negatively. This creates a dynamic between the number of stocking locations used (which affects the transport time and cost) and the inventory policy (which affects the backorder time and inventory costs) because both influence the delivery lead time. Furthermore, we are able to model a nonlinear customer perception of the delivery lead time. Based on the presented model, the general characteristics and mechanisms of such a location-inventory problem are investigated. Among other things, we show for many probability distributions that the average backorder time per backorder is convex and that our model is quasi-convex, which allows for a wide range of optimization methods.

Keywords: inventory, backordering, e-commerce, location-inventory problem, continuous approximation

Reference: Straubert, C. (2024). A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties. International Journal of Production Economics, 268, 1–25. doi.org/10.1016/j.ijpe.2023.109092

VI.1. Introduction

The model presented in this paper is motivated by recent trends in the B2C e-commerce market. B2C e-commerce retailers (e-tailers), especially the largest e-tailers, offer increasingly fast delivery lead times. To achieve such fast delivery lead times, a product is stored in multiple warehouses (stocking locations) and thus has moved closer to customers. This inventory decentralization reduces transport distances to customers, which should lower transport costs and shorten the delivery lead time. On the other hand, inventory decentralization increases inventory holding, replenishment, and stockout-related costs. **Figure VI.1** illustrates this relationship. Generally, in inventory management, more demand [per time unit] is better. Indeed, the square root law of inventories and the risk pooling effect can be regarded as some of the most famous and important effects in supply chain management (Maister, 1976; Eppen, 1979; Schwarz, 1981; Zinn et al., 1989). If demand is split between multiple stocking locations, then this increases the inventory replenishment, holding and stockout costs. However, such an isolated inventory (de-)centralization model ignores the delivery lead time. Fleischmann (2016, p. 903), for example, justifies the absence of the delivery lead time in his model with the industry-standard "Euro Logistics" system, which is coordinated/timed in such a way that a shipment usually takes 2 or 3 days, independent of the locations of the sender and receiver within Europe (Fleischmann, 2016, p. 900). However, this argument is less valid in a B2C e-commerce context today.

Nowadays, large e-tailers such as Amazon already operate dedicated warehouses for larger cities and metro areas. Moreover, e-tailers and logistics service providers are increasingly gearing their logistics network toward fast delivery lead times (Berman, 2019; Trebilcock, 2020). Many empirical studies have shown that B2C e-commerce customers value fast delivery (e.g., Gupta et al., 2004; Hsiao, 2009; Marino et al., 2018; Gawor & Hoberg, 2019). E-tailers have recognized this, and large e-tailers such as Amazon now offer next-day and often even same-day delivery (Berman, 2019). In addition to the typical B2C e-commerce business, our model also has relevance for the emerging field of quick commerce grocery delivery services. These services typically operate within metropolitan areas and strive to deliver groceries within one hour of lead time, often even faster (Nierynck, 2020). To achieve this, they operate multiple small warehouses spatially distributed over a metropolitan area.

The trade-off between inventory cost minimization and transport distance minimization is often called a location-inventory problem (LIP) and is a rather old concept (see, e.g., Magee, 1968, pp. 192–268), which, however, has received relatively little attention in the B2C e-commerce context thus far. Most LIP papers use a form of integer programming for their model formulation (Farahani et al., 2015). The usual formulation assumes a list of possible production/warehouse locations (coordinates) and a list of customer locations, and it must be decided which warehouse locations to use and which customers to supply from which location. These models usually focus on the "location" and "transport" parts of the location-inventory problem. On the other hand, most LIP papers have only a very rudimentary integration of inventory costs (e.g., no holding costs, no backordering/lost sales costs), and the papers that incorporate these costs often use simple approximations.

We take a different approach and approximate the "location" and "transportation" parts while solving the "inventory" part of the problem exactly. This approach of approximating customer locations also makes our model easy to understand. Geoffrion (1976) "[...] advocates the use of highly simplified analytic models to help explain the 'whys' behind the solutions of conventional mathematical programming models" (p. 81). The methods used by Geoffrion (1976) are now known as "continuous approximation" (Daganzo, 2005). We also use a continuous approximation (CA) approach for the demand side of our LIP model.

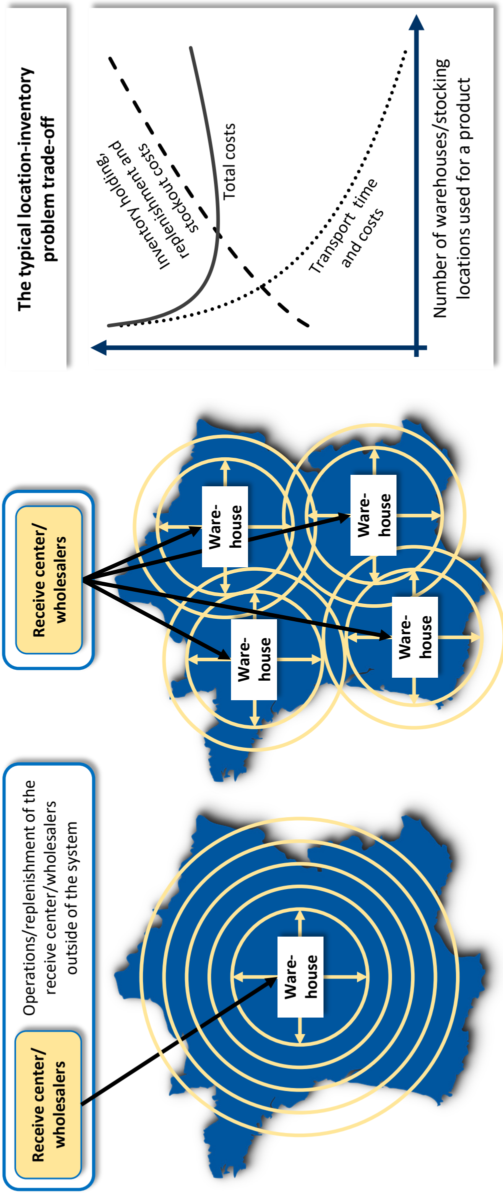


Figure VI.1 Location-inventory problem trade-off between inventory centralization and transport costs/times

Our motivation for using CA is due to the B2C e-commerce context of this paper. In a B2C e-commerce context, one generally observes a high number of customers per e-tailer. Indeed, the locations of demands are generally unknown before customers order from an e-tailer. This means that there is no definite list of customer coordinates and assigned demand quantities. Instead, demand is distributed rather evenly over an area, perhaps with some density peaks (cities) scattered over the plane. Modeling the locations and the transport distances between the locations as precisely as possible is therefore not important. Instead, it makes sense to focus on the inventory management side of the LIP because it is at the core of an e-tailer's value chain. This distinguishes our model from the usual mathematical programming formulation of the LIP.

The inventory management part of our LIP is a continuous review, reorder point, order quantity (s, Q)-model with backordering, Poisson demand and exact costs. That is, customers demand the product one unit at a time, and when the inventory position reaches s, a replenishment order over Q pieces is started (more details later in **Subsection VI.3.5**). If an ordered item is not on hand, a backorder occurs, and the delivery lead time is prolonged until the replenishment arrives at the warehouse. For this mechanism, we derive the exact backorder time distribution for our model, which enables us to incorporate nonlinear delivery lead time costs into our model. While most inventory models assume linear time costs (e.g., costs per time unit out of stock), it is more realistic to allow for nonlinear time costs. In a B2C e-commerce context, customers observe the delivery lead time, which is the time from ordering until receiving the parcel. A nonlinear perception of the delivery lead time from a customer's viewpoint appears logical, and Marino et al. (2018, p. 621), as well as Gawor and Hoberg (2019, p. 88), also found indications for this in their survey of B2C e-commerce customers. Customer perception of the delivery lead time (the utility curve) could, for example, be convex, which is a typical assumption found in economics. Put more simply, it is assumed that the difference in utility between a delivery lead time of three days vs. two days is, on average, smaller than the difference in utility between a lead time of one day vs. two days. However, it may be noted that the exact version of our model works with every form of the utility curve. We are not aware of any other literature that considers the effect of nonlinear delivery lead time costs within an (s, Q)-model.

Our approach allows for analytical insights into the cost structure (quasi-convexity), and therefore, it is possible to decompose the problem and to use fast algorithms (e.g., bisection search) to solve the subproblems. In fact, our approximate model has a semiclosed form (using only the cumulative Poisson distribution function) and can be easily solved using, for example, the Microsoft Excel spreadsheet program and its built-in Excel Solver. Even large problem instances with high demand levels (e.g., one million pieces demand during the replenishment lead time) can be optimized in seconds. This is in contrast to the typical mathematical programming methods used in other literature on LIP models, which can often only solve small problem instances (with few locations and/or low demand levels) and require long computation times to do so. At the same time, these models also use simplifications and heuristics (in their case for the inventory system) and therefore, like our model, should be viewed as approximate decision support models. Thus, our CA approach, with its focus on inventory management, is not only better suited to a B2C e-commerce context but is also easier to understand and solve (Hall, 1986).

The paper is structured as follows. We continue in **Section VI.2** with a closer look at the related literature. In **Section VI.3**, we introduce and explain the mathematical model. Subsequently, **Section VI.4** contains a sensitivity analysis and discussion of the model. Finally, in **Section VI.5**, we discuss warehouse outsourcing decisions both from an e-tailer's and an on-demand warehousing provider's point of view.

VI.2. Related literature and model background

Outside the B2C e-commerce context, the trade-off between inventory centralization and delivery lead time has already received more widespread attention and has become increasingly popular in recent years. Farahani et al. (2015) provide a recent review of LIP papers. Of the papers cited therein, Berman et al. (2007) stands out within the context of this paper. They developed a model that incorporates delivery lead time-sensitive demand and noted that their model has potential applications in B2C e-commerce. They considered simple inventory systems such as deterministic demand or Poisson demand with an alpha service level constraint and used heuristics for the optimal inventory policy parameters. Our model does not feature delivery lead time-sensitive demand; however, a longer delivery lead time does create penalty costs in our model. An e-tailer can offer its products for market average prices with market average delivery lead times, or it can charge below (above) market average prices and offer slower (faster) delivery lead times (Brynjolfsson & Smith, 2000; Gupta et al., 2004; Hsiao, 2009; Gawor & Hoberg, 2019). This price (revenue) mechanism is what we mean when we talk about delivery lead time penalty costs. Alternatively, as is the case in Berman et al. (2007), it is possible to offer a product for a market average price and at the same time provide a faster than average delivery lead time, which should raise demand for the offer. For this paper, we focus on the former case. However, models with lead time sensitive demand are certainly a worthwhile topic for future research.

Wang et al. (2007), Mahar et al. (2009) and Bretthauer et al. (2010) developed models explicitly for a B2C e-commerce context. Similar to our model, Wang et al. (2007) considered Poisson demand and included fixed order costs, inventory holding costs, and time length-dependent backorder costs. However, their model focuses on the effects of product returns and does not contain nonlinear delivery lead time costs. Instead of reducing the delivery lead time through more stocking locations, their model optimizes the number of return points. More return points in an area make returning a product more attractive to customers, which influences the optimal parameters of the inventory policy because some of the returned products can be resold. They presented a bi-level programming formulation of the problem and remarked that they were only able to solve small problem instances (≤ 36 pieces demand during the replenishment lead time in their numerical studies).

Mahar et al. (2009) and Bretthauer et al. (2010) considered a dual-channel model with both brick-and-mortar and e-commerce demand. The number and locations of brick-and-mortar stores and a central warehouse are given, and their model optimizes the decisions about what portion of the e-commerce demand should be fulfilled from a central warehouse and what portion should be fulfilled from which brick-and-mortar store. Their inventory model is of a periodic nature with sequential steps. Both models feature inventory holding and linear backorder time costs, and demand per period is assumed to be normal. Similar to our model, they also considered transport costs from the stocking locations to customers around the stocking locations. Different from our model, they indicate (Bretthauer et al. 2010, p. 129) that a high fraction of e-commerce demand may lead to a lower optimal number of stocking locations. In our case, 100% of demand is e-commerce demand (as we do not consider a dual-channel model), and more demand always leads to a higher optimal number of stocking locations. The models are solved using a branch-and-bound algorithm.

More recent papers about location-inventory problems within a B2C e-commerce context are Li et al. (2014), Liu et al. (2015), Deng et al. (2016), Yao et al. (2018), Moghadam et al. (2021), Correia and Melo (2022), and Kim et al. (2023). The first five papers listed above are all rather similar. They all integrate product returns in one form or another, and they also all use a typical location-inventory problem formulation with coordinates (or distance matrices) for the locations and heuristics for inventory costs. Moreover, since the integration of product returns makes the models very complicated, all of them use some form of metaheuristic (the ant colony or the genetic algorithm) for solving the model. These papers do not focus on the delivery lead time to customers. Kim et al. (2023), on the other hand, focused on delivery lead times. Their motivation is similar to ours. They also emphasize the current trend toward on-demand warehousing and the resulting flexibility in inventory stocking. Other than that, however, the articles differ quite significantly. Among other things, Kim et al. (2023) presented a newsvendor model, while we use a continuous review policy, and similar to Berman et al. (2007), they modeled lead time sensitive demand, while we use penalty costs. Correia and Melo (2022) also emphasize the flexibility that is made possible by the trend of on-demand warehousing. They used a classical LIP formulation but within a context of non-stationary demand that fluctuates significantly from period to period. On-demand warehousing makes it possible to decide from period to period to use or not use a stocking location.

Our continuous approximation approach for the demand side of our model enables us to solve large problem instances within seconds, even while using an exact inventory model. Reviews about the use of CA models in a logistics context can be found in Erlenkotter (1989), Langevin et al. (1996), and Ansari et al. (2018). Interestingly, CA is widely used in logistics research but has rarely been applied to LIP models. Farahani et al. (2015, p. 3780), found in their review of LIP papers only two papers that explicitly used a CA model.

Pujari et al. (2008) presented a model for optimizing integrated manufacturing-transportation decisions. Their CA approach is related to the classical economic order quantity model (for determining optimal production quantities) and its extensions. In their model, the number of storage locations is assumed to be fixed, while in our model, the number of storage locations is optimized. Tsao and Lu (2012), Tsao et al. (2012), Tsao (2016), Tsao et al. (2016), and Tsao (2019) presented models with a continuous approximation of the demand that is similar to our model. They introduced different extensions depending on the respective paper (e.g., volume discounts, trade credits, special warehouse technology). Differing from our model, they emphasized the multi-level nature of the studied fulfillment networks and used simple inventory policies with alpha service level constraints (or even no safety stock at all) and heuristically determined inventory holding costs. Pulido et al. (2015) also used a similar continuous approximation. Moreover, they modeled a scenario with a delivery lead time focus for timely home delivery in a mail-order context. However, they focused on approximating vehicle routing problems and considered only very rudimentary inventory holding costs (linear, no safety stock).

Of the few LIP papers with CA models, those by Erlenkotter (1989) and Rutten et al. (2001) are the most relevant to this work. Erlenkotter (1989) presented a so-called “market-area model” that incorporates economies of scale in facilities/process costs and economies of distance in transport costs. The market-area model calculates the optimal number of locations for a total market area. Rutten et al. (2001) extended the model from Erlenkotter (1989) by changing the facility cost function and introducing simple approximations of inventory holding and vehicle routing transport costs. We use the same continuous approximation approach as Erlenkotter (1989) and Rutten et al. (2001), but the rest of our model is different.

Both Erlenkotter (1989), Rutten et al. (2001), and many other LIP models implicitly assume that the considered company sells only one product. Li et al. (2021) and Li et al. (2023), for example, focused explicitly on (dis-)economies of scale but nevertheless assumed that the throughput of one single product determines the total (dis-)economies of scale in facility operating costs.

These models feature a fixed cost per location (e.g., Erlenkotter, 1989, p. 47), irrespective of the size of the location. This creates economies of scale because more throughput at a location distributes the fixed costs of this location over more orders/pieces. That is, in these models, the warehouse fulfillment cost per order (e.g., order picking and packing cost) is endogenous and dependent on how much of the singular product is fulfilled per time unit in the respective warehouse. If more stocking locations are used (for the singular product), the throughput per warehouse decreases, and the warehouse fulfillment cost per order increases.

An e-tailer usually offers many different products with similar fulfillment processes but different demand intensities, inventory holding costs, and so on. Thus, it is often not possible to aggregate the various products offered by an e-tailer into an average product in any meaningful way. Moreover, since the safety stock levels depend on the demand intensities of the individual products (perfect substitution excluded), one cannot simply aggregate demand into one cumulative product, even if all the products were similar. Therefore, we assume exogenously given warehouse parameters (warehouse fulfillment cost and time per order; $c_{\text{warehouse}}$ and $t_{\text{warehouse}}$ in our model, see also **Subsection VI.3.4**). This means that we do not model (dis-)economies of scale in warehouse operations. Instead, we assume that the warehouse sizes and therefore also the (dis-)economies of scale in warehouse operations are exogenously given. Thus, in our model, the number of stocking locations of a single product does not affect the size of the warehouses or their efficiency. Approximatively, this is very realistic and accurate in many cases. However, if all products of an e-tailer were to be considered at the same time, then the total system-wide effect of the individual stocking decisions could influence the efficiency of the fulfillment network.

In this context, a distinction must be made between a fulfillment network used exclusively by an e-tailer itself and a network also used by other e-tailers (e.g., on-demand warehousing). The latter would be less affected by the decisions of one e-tailer. Thus, our model developed in this paper is a very good fit when on-demand warehousing is used. If an e-tailer wants to use its own warehouses, then our product-specific model is well suited to play an important part in a broader network-wide (dis-)economies of scale perspective (see also our outlook in **Subsection VI.6.2**). For this paper, however, we focus on the optimal number of stocking locations for a specific product. This decision influences the inventory holding and replenishment costs, the backorder time distribution and the transport time distribution of a product. Backordering and longer transport times lead to a longer delivery lead time and thus higher penalty costs. Depending on the product, the optimal number of stocking locations can vary greatly.

As already mentioned, we also consider a nonlinear customer perception of the delivery lead time. Nonlinear backorder time costs, in general, have seemingly not been studied much in the literature. The only two papers we found that analyzed nonlinear backorder costs somewhat similar to our model formulation are Johansson and Olsson (2017) and Johansson and Olsson (2018), who used an $(S-1, S)$ base-stock model in a spare parts context. Berk and Toy (2023) also modeled their inventory system in detail and focused on the delivery lead time. The customers accept some backorder time, but demand is lost when the lead time is too long. They also used a $(S-1, S)$ base-stock model. We use the (s, Q) reorder point, order quantity model for our analysis. A few papers exist about the newsvendor problem with nonlinear costs (Porteus, 1990, p. 618; Parlar & Rempala, 1992; Gerchak & Wang, 1997; Ghosh et al., 2021). In contrast to the classical linear newsvendor model, the nonlinear newsvendor model does not have a simple solution in most cases. The (s, Q)-model is only marginally more complicated in comparison but more realistic in crucial aspects. In particular, the simple one-period newsvendor model does not consider backorders, which are, however, an important feature in our model. In future research, one could use a periodic review model and compare it to our continuous review (s, Q)-model. However, most of the key findings presented in this paper are likely to remain unchanged. The same should be true if lateral transshipments and product substitutability are included in the model.

In the case of lateral transshipments, customers could be served from another stocking location when the nearest one is out of stock (reactive transshipments), or the stocking locations could proactively transship inventory to each other so that stockouts are reduced. Paterson et al. (2011, p. 125), remark that for a retail setting, proactive transshipments are more suited than reactive transshipments. Reactive transshipments are often special, that is, expensive processes that can be worthwhile for example, in the case of important and expensive spare parts, but are often not economical for normal merchandise. Periodic proactive transshipments between stocking locations are easier to manage. However, the use of proactive lateral transshipments only means that the inventory system becomes more efficient (less costly). For our model, this would mean that the optimal number of stocking locations would increase.

In the case of product substitutability, some demand would not be backordered but would switch over to other similar products that are in stock, possibly with a penalty because the originally demanded product is not in stock (Nagarajan & Rajagopalan, 2008; Shin et al., 2015). Such a model extension would make the inventory system more costly (if the penalty is high) or less costly (if the penalty is low), which would lead to a lower or higher optimal number of stocking locations.

Finally, it is worth noting that LIP models with different customer classes exist (Escalona et al., 2015; Escalona et al., 2018; Puga et al., 2019). These models emphasize delivery lead times in the sense that some customers require faster lead times than others. Such models can be optimized by holding back part of the inventory for orders from high-priority customers. Such a critical level policy is difficult to implement in B2C e-commerce because the estimated delivery lead time is usually displayed on the normal product page. That is, customers do not need to be logged in to see the delivery lead time. Because of this, it is not always clear who is looking at an offer, and more fundamentally, it is often even difficult to tell which customer expects a fast delivery time and which customer does not. Nevertheless, it is certainly an interesting instrument to avoid inventory costs and still be able to offer fast delivery lead times to customers who value it.

In summary, our model stands out from the literature not only because it is anchored in a B2C e-commerce context but also because of its mathematical properties. The model is sufficiently general so that it may also be of interest in other scientific or practical fields of application.

VI.3. Model formulation

An overview of the parameters and decision variables used in our model can be found in **Tables VI.1**. **Figures VI.2** and **VI.3** may help the reader understand which parameter is used in which context.

VI.3.1. The decision variables

Our model has the following decision variables: y, s and Q, where y is the number of stocking locations for a specific product, s is the reorder point, and Q is the fixed reorder quantity for the product. The goal is to simultaneously find values for these variables that minimize the costs per order for the product. Note that in our model, the optimal values y^*, s^* and Q^* for one product do not depend on the optimal values of another product. This is of course not true in the real world. However, given that e-tailers (and many other types of businesses) usually sell many products, the influence of an inventory policy of one product on the optimal inventory policy of another product is often small.

VI.3.2. Market area demand

As an approximation, we assume that all market areas are circular. The total market area (region) has an area of A. If an e-tailer uses two stocking locations for a product, the market area per stocking location is again circular with an approximate area of $A/2$. Expected demand is evenly spread across the market areas with density d (demand per time unit per area unit). That is, two market areas of the same size also have the same expected demand intensity. Thus, dA/y represents the expected demand per stocking location for the product. Such an approximation of the real world creates inaccuracies. However, as discussed in Erlenkotter (1989), these inaccuracies can often be ignored in the grand scheme of things. Nevertheless, the approximation increasingly deviates from the real world as the number of market areas (i.e., stocking locations) increases. This is a limitation of our model.

Demand parameters:

$d \in \mathrm{R}^+$	Expected demand for the product per time unit per unit area (e.g., 0.012 pieces per year per km^2)
$A \in \mathrm{R}^+$	Total area of the considered region (e.g., 543,940 km^2 for France)

Decision variables:

$y = 1,2,\ldots$	Product-specific # of stocking locations (i.e., warehouses) used within the region
$s = 0,1,\ldots$ or $s \in \mathrm{Z}$	Reorder point: when the *inventory position* of the product in a warehouse reaches s, a replenishment order of size Q is triggered.
$Q = 1,2,3,\ldots$	Order quantity per replenishment order of the product

Warehouse fulfillment cost/time and transport cost/time parameters:

$t_{\text{transport}} \in \mathrm{R}^+$	Transport time scale parameter (product independent)
$0 < \beta_t \leq 1$	Transport time slope parameter, economies of distance (product independent)
$t_{\text{warehouse}} \in \mathrm{R}^+$	Constant fulfillment time per order (product independent)
$c_{\text{transport}} \in \mathrm{R}^+$	Transport cost scale parameter for the product (depending on weight and volume)
$0 < \beta_c \leq 1$	Transport cost slope parameter, economies of distance (product independent)
$c_{\text{warehouse}} \in \mathrm{R}^+$	Constant fulfillment cost per item (product independent)
$u \in \mathrm{R}^+$	Delivery time perception scale parameter for the product (delivery lead time penalty)
$\omega \in \mathrm{R}^+$	Delivery time perception slope parameter for the product

Table VI.1.1 Model parameters and decision variables (part 1)

Inventory cost and demand parameters:

$h \in \mathrm{R}^{+}$	Inventory holding costs for one piece of the product per time unit
$R \in \mathrm{R}^{+}$	Reorder costs per reorder/replenishment of the product
$f_{W\|W>0}(\tau_b)$	Backorder time distribution of backordered pieces of the product
$\overline{\tau_b}(y, s, Q)$	Average backorder time per backorder
$\overline{\tau_o}(y, s, Q)$	Average backorder time per order
$B(y, s, Q)$	Expected number of backorders of the product at any point in time
$P_{\mathrm{out}}(y, s, Q)$	Probability that there is no stock of the product on hand at any point in time
$\ell \in \mathrm{R}^{+}$	Replenishment lead time for the product
$F_{\mathrm{Poi}}(v, \mu)$	Cumulative distribution function ($Prob(X \leq v)$) of a Poisson distribution; mean = μ

Table VI.1.2 Model parameters and decision variables (part 2)

VI.3.3. Transport cost and time distribution

Identical to Erlenkotter (1989) and Rutten et al. (2001), we assume that economies of distance exist. The transport cost per distance traveled is given by $c_{\mathrm{transport}} r^{\beta_c}$, where $c_{\mathrm{transport}}$ is the product-dependent cost scale parameter (monetary units). Depending on the characteristics of a product (e.g., weight, volume), transporting it is more or less expensive. r is the distance (e.g., kilometers), and β_c is the transport cost slope parameter (economies of distance). We also define a similar formula for the transport time, in which the transport time per distance traveled is given by $t_{\mathrm{transport}} r^{\beta_t}$. We need to differentiate between transport cost and transport time because our model explicitly models the delivery lead time, which is then perceived by customers and valued monetarily.

Furthermore, we assume that the warehouse is always located at the center of the market area. This allows for an elegant simplification of the model (Erlenkotter, 1989, p. 51). Let $Rad(y) = \left(\frac{A}{y\pi}\right)^{0.5}$ be the market area radius of a stocking location. Then, $\int_{r=0}^{Rad(y)} (2\pi r d) c_{\text{transport}} r^{\beta_c}\, dr = \frac{2\pi}{2+\beta_c} c_{\text{transport}} d \left(\frac{A}{y\pi}\right)^{(1+0.5\beta_c)}$ equals the total expected transport costs when fulfilling all demand in a market area. Dividing by dA/y (expected demand) leads to $\frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c}$, that is, the expected transport cost per piece of the product sold.

VI.3.4. Warehouse fulfillment cost and time

For this paper, we focus on the product-specific case and therefore use a constant warehouse fulfillment cost per order $c_{\text{warehouse}} > 0$ and a constant warehouse fulfillment time per order $t_{\text{warehouse}} > 0$. These constants actually represent the average values of random variables. For example, the order picking and packing operations in an e-tailer's warehouse are usually M/M/m queue systems with a waiting and service time distribution. However, the warehouse fulfillment time distribution as a whole (including outbound logistics) combines several processes, and the mean value of the distribution depends, for example, on personnel scheduling, processes and technologies used within the warehouse. The actual values of y, s and Q for one specific product do not materially influence the warehouse fulfillment time distribution of a warehouse as a whole, which is why we assume for our model that the distribution is exogenously given. If an e-tailer outsources warehouse operations, which is one of the main areas of application for our model, then the warehouse fulfillment costs and times are exogenously given anyway.

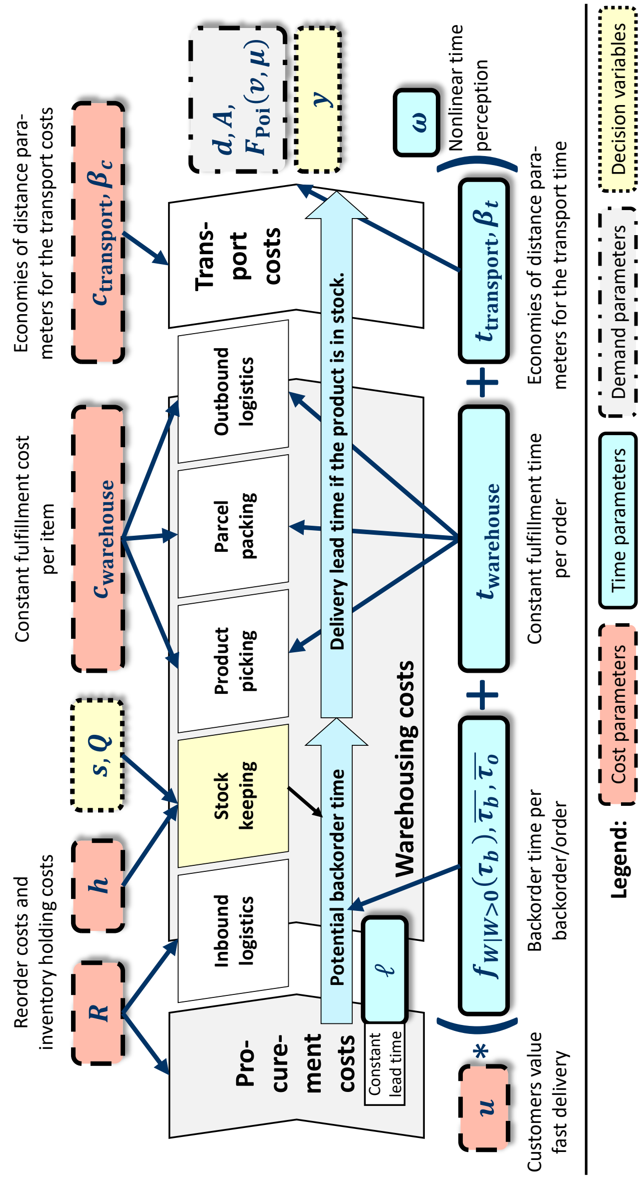


Figure VI.2 Stylized overview of the base case model parameters and decision variables

VI.3.5. Reorder and inventory holding costs and the backorder time distribution

For the inventory system (Hadley & Whitin, 1963, pp. 181–188), we use a

- continuous review,
- reorder point (s), fixed order quantity (Q) model,
- with a fixed replenishment lead time (ℓ),
- unit-sized Poisson distributed demand ($F_{\text{Poi}}(v, \mu)$),
- inventory holding cost, per piece per time unit in stock (h),
- reorder costs per replenishment shipment (R),
- and backordering.

Furthermore, our model also contains a special type of time length-dependent backorder cost per piece backordered not found in other literature. Because customers value a fast delivery lead time, the backorder time, which increases the delivery lead time, creates a penalty cost (see **Subsection VI.3.6**).

The replenishment transport costs are independent of the decision variables (i.e., y, s, Q). The replenishment from a producer, wholesaler or receive center to the customer-facing warehouses (y) is usually performed in shipments that contain several products at the same time. If an e-tailer uses third-party logistics service providers for these shipments, then the replenishment transport costs are even further detached from one specific product. Thus, the main replenishment costs in our model are the fixed reorder costs per replenishment shipment (R).

We decided to use the (s, Q)-model with Poisson demand because it fits quite well within the considered B2C e-commerce context where e-tailers receive many independent orders of small size. Moreover, despite being mathematically exact, the (s, Q)-model is relatively simple. The exact expected costs of the (s, Q)-model have a semiclosed form and can be calculated using standard spreadsheet programs such as Microsoft Excel. This makes the model easier to interpret and more suitable for practical applications.

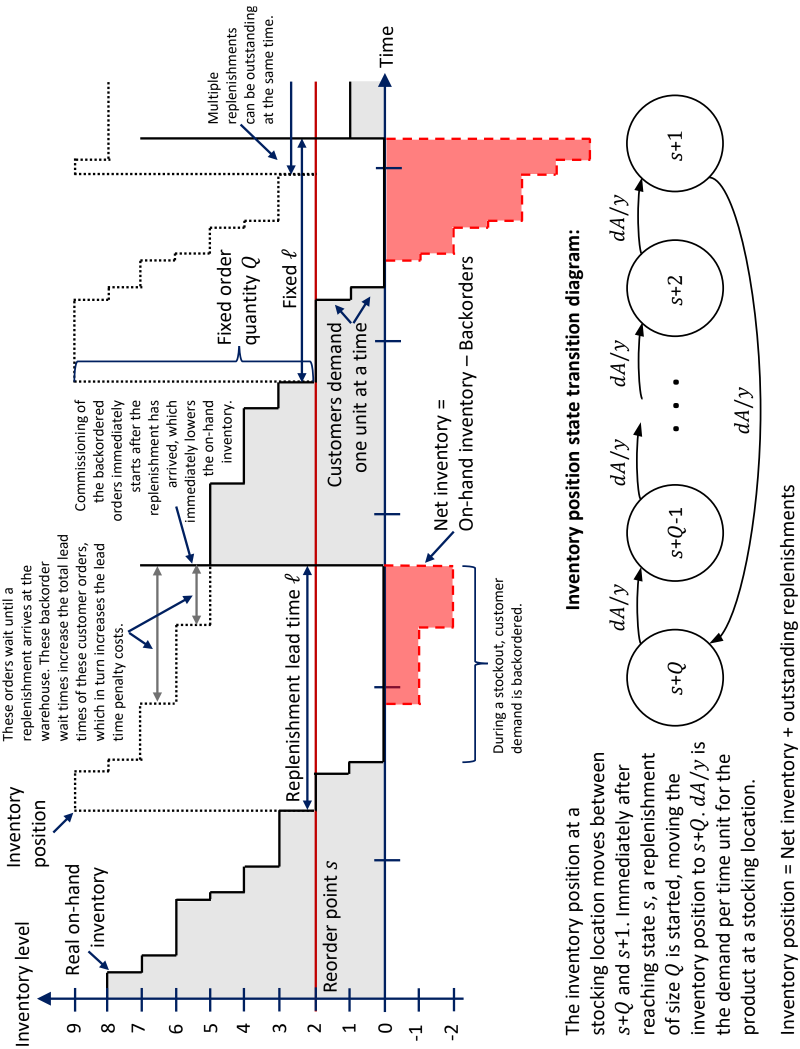


Figure VI.3 Inventory system behavior: sample path and inventory position state transition diagram

The above-described (s, Q)-model is a very prominent model, and we, therefore, would like to refer the reader to Hadley and Whitin (1963, pp. 181–188), for an in-depth explanation.

The expected proportionate reorder costs per piece sold are simply:

$$R/Q \tag{1}$$

The expected inventory holding costs per piece sold are given by:

$$h \cdot \frac{y}{dA} \cdot \left(0.5(Q+1) + s - \left(\frac{\ell dA}{y}\right) + B(y,s,Q)\right) \tag{2}$$

$B(y, s, Q)$ is the expected number of backorders at any point in time, with $\mu(y) = \ell dA/y$:

$$B(y,s,Q) = \frac{1}{Q}\begin{bmatrix} 0.5\big(\mu(y)\big)^2\big[1 - F_{\text{Poi}}\big(s-2,\mu(y)\big)\big] - \\ \big(\mu(y)\big)(s)\big[1 - F_{\text{Poi}}\big(s-1,\mu(y)\big)\big] + \\ 0.5(s+1)(s)\big[1 - F_{\text{Poi}}\big(s,\mu(y)\big)\big] - \\ 0.5\big(\mu(y)\big)^2\big[1 - F_{\text{Poi}}\big(s+Q-2,\mu(y)\big)\big] + \\ \big(\mu(y)\big)(s+Q)\big[1 - F_{\text{Poi}}\big(s+Q-1,\mu(y)\big)\big] - \\ 0.5(s+Q+1)(s+Q)\big[1 - F_{\text{Poi}}\big(s+Q,\mu(y)\big)\big] \end{bmatrix} \tag{3}$$

$P_{\text{out}}(y, s, Q)$ is the probability that there is no stock on hand at any point in time:

$$P_{\text{out}}(y,s,Q) = \frac{1}{Q}\begin{bmatrix} \big(\mu(y)\big)\big[1 - F_{\text{Poi}}\big(s-1,\mu(y)\big)\big] - \\ (s)\big[1 - F_{\text{Poi}}\big(s,\mu(y)\big)\big] - \\ \big(\mu(y)\big)\big[1 - F_{\text{Poi}}\big(s+Q-1,\mu(y)\big)\big] + \\ (s+Q)\big[1 - F_{\text{Poi}}\big(s+Q,\mu(y)\big)\big] \end{bmatrix} \tag{4}$$

Thus far, these formulas are identical to those used in the typical (s, Q)-model (Hadley & Whitin, 1963, pp. 181–188). The usual formulation of the (s, Q)-model minimizes the average cost per time unit. In such a model, it is possible to calculate the time length-dependent backorder costs by simply multiplying $B(y, s, Q)$ with a cost factor. However, this approach implicitly assumes linear time length-dependent lead time/backorder costs. In this paper, however, we model the more realistic case of a nonlinear customer perception (cost) of the delivery lead time. Therefore, in our model, we must calculate the time lengths and costs per piece sold.

Using Little's Law (Kruse, 1981, p. 204), the average backorder time per order is:

$$\overline{\tau_o}(y,s,Q) = \frac{B(y,s,Q)}{\frac{dA}{y}} \tag{5}$$

The average backorder time per order is composed of orders that were backordered and orders that were not backordered. When orders were backordered, the system was out of stock at that time ($P_{\text{out}}(y,s,Q)$). Thus, the average backorder time per backorder is:

$$\overline{\tau_b}(y,s,Q) \cdot P_{\text{out}}(y,s,Q) + \big(1 - P_{\text{out}}(y,s,Q)\big) \cdot 0 = \frac{B(y,s,Q)}{\frac{dA}{y}}$$

$$\Rightarrow \overline{\tau_b}(y,s,Q) \ = \frac{B(y,s,Q)}{\frac{dA}{y} \cdot P_{\text{out}}(y,s,Q)} \tag{6}$$

However, it is unclear whether the average backorder time per backorder is sufficient for our model. In the case of the warehouse fulfillment time distribution, we were satisfied with using the average of an exogenously given distribution. However, the backorder time distribution depends endogenously on all three decision variables of our model (s, Q, and y). Thus, it is prudent to check whether using the average backorder time is a sufficiently accurate approximation or whether it is necessary to use the exact backorder time distribution in our model.

Kruse (1981, pp. 203–204), provided a general formula for the waiting time distribution in an (s,Q)-model:

$$F_w(\tau_b) = \frac{1}{Q}\sum_{k=1}^{Q}\left[1 - H_{(s+k)}(\ell - \tau_b)\right]; 0 \leq \tau_b < \ell \tag{7}$$

where $H_{(j)}(\ell - \tau_b)$ is the j-fold convolution of $H(\ell - \tau_b)$. $H(x)$ is the cumulative distribution function of the time between two consecutive demands. Given that we assume Poisson distributed demand, the time between two consecutive demands follows the exponential distribution. A convolution of the exponential distribution creates an Erlang distribution. Therefore, the formula becomes (for $s \geq 0$, with $\mu(\tau_b) = (dA/y)(\ell - \tau_b)$):

$$F_w(\tau_b) = \frac{1}{Q}\sum_{k=1}^{Q}\left[\frac{\Gamma\big(s+k, \mu(\tau_b)\big)}{(s+k-1)!}\right] \tag{8}$$

where $\Gamma\big(s+k, \mu(\tau_b)\big)$ is the upper incomplete gamma function:

$$\Gamma\big(s+k, \mu(\tau_b)\big) = \int_{g=\mu(\tau_b)}^{\infty} g^{(s+k-1)} e^{-g} dg \tag{9}$$

We note that $1 - \Gamma\big(s+k, \mu(\tau_b)\big)/(s+k-1)!$ is equal to the cumulative distribution function of the Erlang distribution. The fraction $\Gamma\big(s+k, \mu(\tau_b)\big)/(\mathrm{s}+k-1)!$ is also known as the "regularized" upper incomplete gamma function. Because s and k are integers, the regularized upper incomplete gamma function is in our case also the cumulative distribution function of a Poisson distribution. Thus:

$$F_w(\tau_b) = \frac{1}{Q}\sum_{k=1}^{Q}\left[F_{\mathrm{Poi}}\big(s+k-1, \mu(\tau_b)\big)\right] \tag{10}$$

Note that $F_w(\tau_b)$ represents the distribution for the customer waiting time, including customers who do not wait at all. In $(1 - P_{\mathrm{out}}(y, s, Q))$ of the cases, customers do not wait, and their orders have a backorder time of $\tau_b = 0$ (thus $F_w(0) = (1 - P_{\mathrm{out}}(y, s, Q))$.

For the cumulative distribution function of the backorder time of actually backordered customer orders $F_{W|W>0}(\tau_b) = P(W \leq \tau_b | W > 0)$, we therefore need to subtract $(1 - P_{\mathrm{out}}(y, s, Q))$ from $F_w(\tau_b)$ and then rescale the distribution (dividing by $P_{\mathrm{out}}(y, s, Q)$) so that $0 \leq F_{W|W>0}(\tau_b) \leq 1$:

$$F_{W|W>0}(\tau_b) = \frac{\frac{1}{Q}\sum_{k=1}^{Q}\left[F_{\text{Poi}}\left(s+k-1, \mu(\tau_b)\right)\right] - \left(1 - P_{\text{out}}(y,s,Q)\right)}{P_{\text{out}}(y,s,Q)} \quad (11)$$

The derivative of $F_{W|W>0}(\tau_b)$ with respect to τ_b, that is, the probability density function $f_{W|W>0}(\tau_b)$, is:

$$f_{W|W>0}(\tau_b) = \frac{\frac{1}{Q}\frac{dA}{y}\left[F_{\text{Poi}}\left(s+Q-1, \mu(\tau_b)\right) - F_{\text{Poi}}\left(s-1, \mu(\tau_b)\right)\right]}{P_{\text{out}}(y,s,Q)} \quad (12)$$

With this distribution, we now check in the next subsection whether using simple averages in our model ($\overline{\tau_b}(y,s,Q)$, or even $\overline{\tau_o}(y,s,Q)$) is an acceptable approximation.

VI.3.6. Nonlinear customer perception of the delivery lead time

We assume a nonlinear customer perception (i.e., a nonlinear penalty/utility) of the delivery lead time. **Figure VI.4** illustrates the notion behind our model, which is based on the following general formula:

$$\text{Penalty} = u * (\text{delivery lead time})^{\omega} \quad (13)$$

where delivery lead time = backorder time + warehouse fulfillment time + transport time and $u > 0$ is the penalty scale parameter. Because the model has a cost per order perspective, the delivery lead time penalty can be interpreted as another cost (negative utility compared to the best possible lead time). A more practice-oriented interpretation is that an e-tailer may charge different delivery fees depending on the delivery time. $0 < \omega$ is the slope parameter that determines how nonlinear the customer perception is ($\omega = 1$ would mean a linear perception). For this paper, we focus on the range $0 < \omega \leq 1$ because it agrees with typical utility functions found in economics and with the survey results of Gawor and Hoberg (2019, p. 88). However, the following exact model also works for $\omega > 1$ or more complex functions such as S-shaped/logit-type functions. An S-shaped function can be realistic. However, the tails of an S-shaped function would only apply in the case of very short lead times (physically impossible) or very long lead times (see **Figure VI.4**).

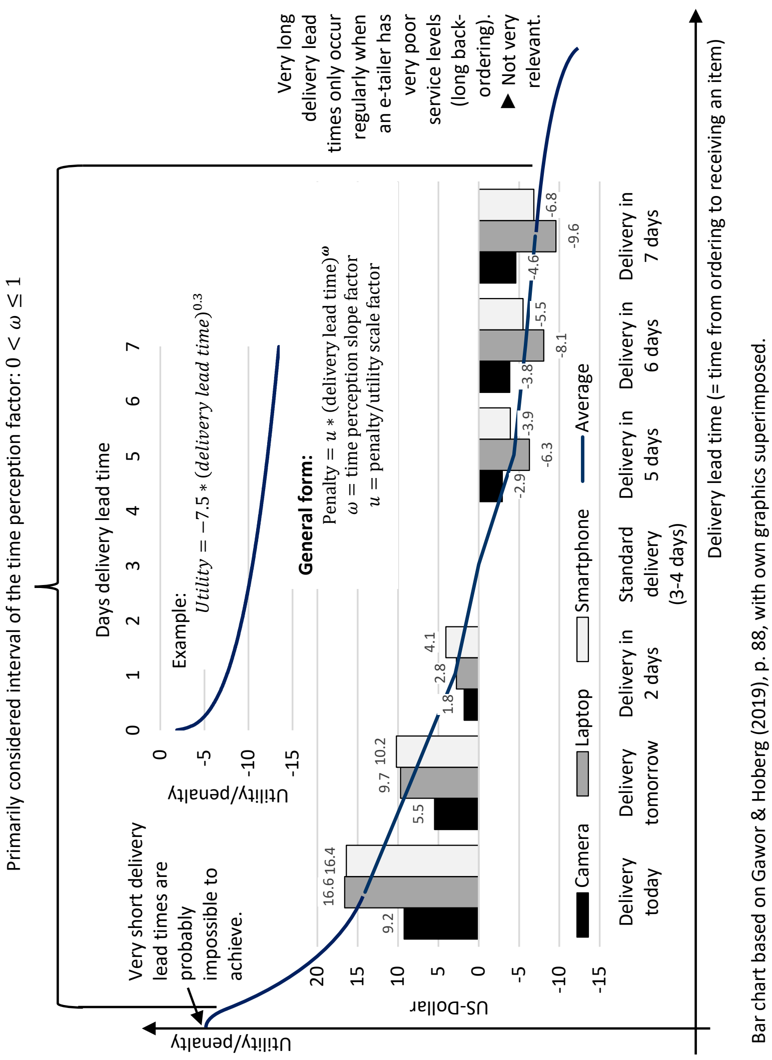


Figure VI.4 Stylized visualization of a nonlinear lead time perception

Normally, an e-tailer sets the reorder point $s \geq 0$, which means that a customer has to wait at most: maximum delivery lead time = replenishment lead time + warehouse fulfillment time + transport time. Thus, the lead time is usually not very long, and using a model with $0 < \omega \leq 1$ is not only easier to understand but also quite realistic in most cases.

By using the continuous approximation market area model from **Subsection VI.3.3** and the exact backorder time distribution derived in the previous **Subsection VI.3.5** (see also Equation 12), it is now possible to exactly calculate the delivery lead time penalty cost:

Exact lead time penalty cost =

$$\begin{gathered} u \cdot \left(\frac{y}{dA}\right)\left(1 - P_{\text{out}}(y,s,Q)\right) * \\ \int_{r=0}^{\left(\frac{A}{y\pi}\right)^{0.5}} (2\pi r d)\begin{pmatrix} t_{\text{warehouse}} + \\ t_{\text{transport}} r^{\beta_t} \end{pmatrix}^{\omega} dr + \\ u \cdot \left(\frac{y}{dA}\right) P_{\text{out}}(y,s,Q) * \\ \int_{\tau_b=0}^{\ell} \left[f_{W|W>0}(\tau_b)\right]\left[\int_{r=0}^{\left(\frac{A}{y\pi}\right)^{0.5}} (2\pi r d)\begin{pmatrix} \tau_b + t_{\text{warehouse}} + \\ t_{\text{transport}} r^{\beta_t} \end{pmatrix}^{\omega} dr\right] d\tau_b \end{gathered} \tag{14}$$

This formula contains two distributions: the probabilistic backorder time distribution and the deterministic transport time distribution based on the distances from the warehouse to the customers within the market area. These (nested) integrals make the formula somewhat slow and complex and therefore unattractive for use in business. We observed in our tests that, depending on the parameters, especially the demand rate, the calculation of the exact formula is about 0.5 to 1.5 orders of magnitude slower than the approximation that we will use later in our paper (e.g., instead of 0.3 seconds for the approximation, the calculation of the exact formula finished after 1.5–15 seconds). Thus, it is often certainly possible to use the exact formula, but a trade-off between accuracy and speed/cost in both the implementation and operation of software must always be considered, especially in business practice. Furthermore, our reported differences in computational speed require that the software implementation of the model uses a performant method for numerical integration of the nested integrals. Not every software program offers such possibilities out-of-the-box.

Therefore, we will now check two approximations that use only the cumulative distribution function of the Poisson distribution and do not use any numerical integration.

Approximation 1 of the lead time penalty cost =

$$u \cdot \left(1 - P_{\text{out}}(y,s,Q)\right)\left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{(0.5\beta_t)}\right)^{\omega} + \quad (15)$$

$$u \cdot P_{\text{out}}(y,s,Q)\left(\overline{\tau_b}(y,s,Q) + t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{(0.5\beta_t)}\right)^{\omega}$$

Approximation 2 of the lead time penalty cost =

$$u \cdot \left(\overline{\tau_o}(y,s,Q) + t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{(0.5\beta_t)}\right)^{\omega} \quad (16)$$

Instead of the exact representation of the delivery lead time, approximation 1 uses the average backorder time per backorder, and approximation 2 uses the average backorder time per order. In addition, both approximations use the average transport times from stocking locations to customers. In reality, some customers are closer to the nearest stocking location than others. The nonlinear delivery lead time penalty (ω) makes these expressions approximations. In approximation 2, every customer has the same delivery lead time but, for example, $\frac{10^{\omega}+100^{\omega}}{2} \neq \left[\frac{10+100}{2}\right]^{\omega}$. In this example, the right side of the equation represents the behavior of approximation 2. Generally, such an approximation becomes worse as the coefficient of variation of the delivery lead time distribution increases. We expect approximation 1 to perform significantly better than approximation 2 because all customers with zero backorder time are treated separately. This creates two sets of delivery lead times, each having a smaller coefficient of variation than the joint set.

An approximation is considered good if it performs well in challenging but realistic circumstances. We constructed scenarios 1.) and 2.) in **Ta-**

ble VI.2 to reflect this. For example, the value of Q should be high enough so that the coefficient of variation of the backorder time distribution is maximized, but not too high, because then $P_{\mathrm{out}}(y,s,Q)$ would be very low. We also set $s = 0$ so that the average backorder time is long. This means that the approximation gaps reported in **Table VI.2** are more or less as worse as it gets in the real world. From all the scenarios in **Table VI.2**, Scenario 5 is the most realistic. Note that the approximation quality worsens the larger $\omega > 1$ is (not reported in **Table VI.2**) because a $\omega > 1$ accentuates the variability in the time distributions. A perception slope of $0 < \omega \leq 1$, on the other hand, decreases the impact of the variability of the time distributions.

As expected, the approximation quality is generally only (very) good for approximation 1. Approximation 2 often performs well, but in more challenging parametrizations, it breaks down. Note that the quality of approximation 2 is particularly bad in scenario 2.) and 4.). The reason why $P_{\mathrm{out}}(y,s,Q) = 0.05$ is low in both scenarios is because $Q = 8000$ is very high and not because $s = 0$ is high, which is also the reason why approximation 2 performs poorly in these scenarios. The combination of $s = 0$ and $Q = 8000$ creates an inventory policy where 95% of the orders have zero backorder time and the remaining 5% have an, on average, long backorder time and a backorder time distribution with a relatively high coefficient of variation. Such a combination is especially challenging for approximation 2.

Finally, observe that the quality of approximation 1 is worst around $\omega = 0.5$ and that we set $\beta_t = 0.5$. In the case of very small ω and β_t, the time distributions barely matter because the distributions are heavily flattened. In the case of a large $\omega \to 1, \beta_t \to 1$, the approximate model converges into the exact model. Thus, models with values between these two extremes are more challenging to approximate. However, approximation 1 generally performs very well, and given that approximation 1 is not much more complex than approximation 2, it is reasonable to always use approximation 1.

Scenarios:	Variation of ω	Exact penalty	Approximation 1	Approximation 2
1.) $s = 0$, $Q = 400$ $P_{\text{out}}(y, s, Q) = 0.98$ $t_{\text{transport}} = 0$	$\omega = 0.1$	3.2532	3.3280 (2.30%)	3.3889 (4.17%)
	$\omega = 0.5$	4.1964	4.4247 (5.43%)	4.4695 (6.51%)
	$\omega = 0.9$	5.7865	5.8827 (1.66%)	5.8946 (1.87%)
2.) $s = 0$, $Q = 8000$ $P_{\text{out}}(y, s, Q) = 0.05$ $t_{\text{transport}} = 0$	$\omega = 0.1$	0.1651	0.1695 (2.63%)	2.5119 (1421%)
	$\omega = 0.5$	0.2108	0.2236 (6.07%)	1.0000 (374%)
	$\omega = 0.9$	0.2898	0.2951 (1.82%)	0.3981 (37.38%)
3.) No backorder time ($\tau_b \equiv 0$); $t_{\text{transport}} = 0.00625$	$\omega = 0.1$	2.5059	2.5119 (0.24%)	2.5119 (0.24%)
	$\omega = 0.5$	0.9938	1.0000 (0.62%)	1.0000 (0.62%)
	$\omega = 0.9$	0.3973	0.3981 (0.21%)	0.3981 (0.21%)
4.) $s = 0$, $Q = 8000$ $P_{\text{out}}(y, s, Q) = 0.05$ $t_{\text{transport}} = 0.00625$	$\omega = 0.1$	2.5474	2.5566 (0.36%)	2.6922 (5.68%)
	$\omega = 0.5$	1.1621	1.1791 (1.47%)	1.4142 (21.70%)
	$\omega = 0.9$	0.6808	0.6865 (0.83%)	0.7429 (9.11%)
5.) $s = 320$, $Q = 1600$ $P_{\text{out}}(y, s, Q) = 0.05$ $t_{\text{transport}} = 0.00625$	$\omega = 0.1$	2.5267	2.5344 (0.31%)	2.5603 (1.33%)
	$\omega = 0.5$	1.0541	1.0640 (0.94%)	1.1000 (4.36%)
	$\omega = 0.9$	0.4641	0.4660 (0.40%)	0.4726 (1.83%)
Other parameters (identical in all scenarios): $y = 1$, $A = 502655$, $d = 0.000198943$, $\beta_t = 0.5$, $\ell = 4$ days, $u = 3.1623$, $t_{\text{warehouse}} = 0$				

Table VI.2 Numerical comparisons between the exact model and the two approximations

VI.3.7. The complete model

The complete model using approximation 1 is:

$$
\begin{aligned}
&Min\ C(y,s,Q) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} + \\
&u \cdot \left(1 - P_{\text{out}}(y,s,Q)\right)\left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega} + \\
&u \cdot P_{\text{out}}(y,s,Q)\begin{pmatrix} \dfrac{B(y,s,Q)}{\left(\frac{dA}{y}\right)P_{\text{out}}(y,s,Q)} + \\ t_{\text{warehouse}} + \dfrac{2t_{\text{transport}}}{2+\beta_t}\left(\dfrac{A}{y\pi}\right)^{0.5\beta_t} \end{pmatrix}^{\omega} + \\
&\frac{R}{Q} + h \cdot \frac{y}{dA} \cdot \left(0.5(Q+1) + s - \left(\frac{\ell dA}{y}\right) + B(y,s,Q)\right)
\end{aligned} \tag{17}
$$

, with $y, Q \in \mathbb{Z}_{\geq 1}$, $s \in \mathbb{Z}$.

This model is valid for $0 < \omega \leq 1$. For $\omega \gg 1$ or other nonlinear curves (e.g., an S-shaped curve), the exact model should be used. To obtain the exact model, one merely needs to exchange the two lines starting with "$u \cdot$" with the exact lead time penalty cost expression from **Subsection VI.3.6** (Equation 14) and exchange $\frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c}$ with $\left(\frac{y}{dA}\right)\int_{r=0}^{\left(\frac{A}{y\pi}\right)^{0.5}} (2\pi r d)\left(c_{\text{transport}} r^{\beta_c}\right) dr$. For the remainder of this paper, we restrict ourselves to the above approximate model because most cases within a B2C e-commerce context can realistically be modeled with $0 < \omega \leq 1$. This approximate model is also valid for $s < 0$ (intentional backordering). Nevertheless, most of our numerical examples have optimal reorder points $s^* \geq 0$, and all core statements we make in relation to our examples remain the same, even when applying a restriction that the reorder point s must be $s \geq 0$. Such a restriction is often assumed in business practice, although negative s could very well be optimal, even in a practical setting. Caution is advised when ω is very low (near zero) because in such cases, even long delivery lead times (extreme negative values for s) generate very small penalty costs. In such cases, using the exact model and a different penalty curve would be more adequate.

VI.3.8. Parameter estimation

Before turning to the question of how to optimize the above model, it is worthwhile to touch upon how the various parameters of the model can be calculated or estimated. The parameters h, R and ℓ are fundamental to inventory management. The parameter ℓ can, for example, be set based on the average lead time of past replenishments or based on contractual service levels with wholesalers and/or manufacturers. h is often dependent on the purchasing price of the product, its depreciation, space requirements in the warehouse, and so on. R mainly depends on the complexity and time requirements of the processes involved in a replenishment delivery. The parameters $t_{\text{transport}}$, β_t, $c_{\text{transport}}$ and β_c are specific to location-inventory problems. The models in Erlenkotter (1989) and Rutten et al. (2001) have the same or similar parameters. These parameters can be set based on physical and processual economies of scale effects in combination with standard accounting. A long delivery trip between a stocking location and a customer is more likely to have (longer) passages over free- and highways than a short drive. Based on many simulated trips, it is possible to obtain average economies of distance parameters β_t and β_c. $t_{\text{transport}}$ is dependent on the allowed/economic speed, and $c_{\text{transport}}$ is dependent on the depreciation of the vehicles used for deliveries. $t_{\text{warehouse}}$ and $c_{\text{warehouse}}$ are important key performance indicators of warehouse operations and should be known by any large warehouse operator. In the case of on-demand warehousing, these parameters are usually part of the contract. The parameter A (the market area) is just a geographic fact and known, but the parameter d (demand intensity) has to be estimated/forecasted (Brown, 1959). Every (s, Q) inventory model is based on the assumption of a stationary demand distribution (i.e., the average demand per time unit stays the same over time). Thus, our model, which integrates an (s, Q)-model, is also only an adequate choice if the product has a more or less stationary demand distribution, which makes estimating/forecasting d easier. The lead time penalty scale factor u and the lead time penalty slope factor ω also have to be estimated. For individual products, this seems to be a difficult task. However, it is an educated guess that customer perceptions do not vary much within a product (sub)category. Gupta et al. (2004), Hsiao (2009), and Gawor and Hoberg (2019), for example, investigated books, wine and electronic devices using surveys. If surveys are considered too expensive, aggregation on a product (sub)category level also has the advantage that it should be easier to estimate u and ω on the basis of the different offers in the market.

VI.3.9. Optimization of the model

Given these parameters, we now want to optimize the model, that is, find optimal y^*, s^* and Q^*. The model can be decomposed into two quasi-convex subproblems:

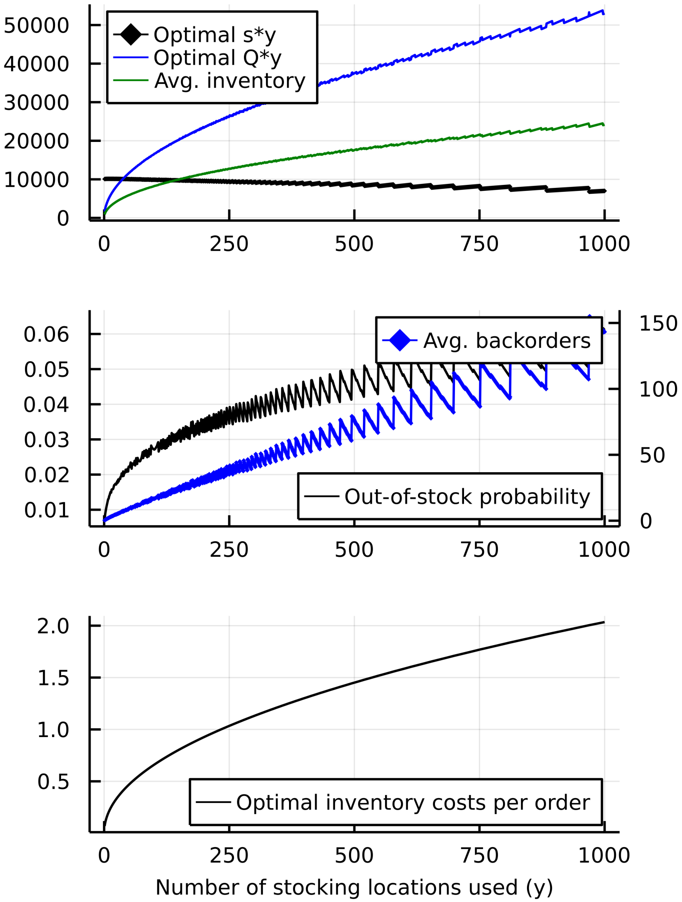
- Solving the model for optimal s^* and Q^*, given a fixed y.
- Solving the model for optimal y^*, given optimal $s^*(y)$ and $Q^*(y)$.

It is well known that the classical (s, Q)-model as presented in Hadley and Whitin (1963, pp. 181–188) is quasi-convex (Rubalski, 1972; Federgruen & Zheng, 1992). A quasi-convex function has exactly one minimum and is either nondecreasing, nonincreasing or first nonincreasing (until the minimum is reached) and then nondecreasing. Such a function can be solved for optimal parameters using a wide range of methods for nonlinear optimization. For example, the quasi-Newton BFGS algorithm or the conjugate gradient GRG algorithm often perform very well (Lasdon et al., 1978). Alternatively, a bisection search is also fast and guarantees an optimal solution in $\log_2$(upper bound for Q^* – lower bound for Q^*) times $\log_2$(upper bound for s^* – lower bound for s^*) steps (in the case of a (s, Q)-model).

Rubalski (1972) and Federgruen and Zheng (1992) presented a special algorithm for efficiently solving the classical (linear) (s, Q)-model. Federgruen and Zheng (1992, p. 812), hinted that their algorithm can be modified to solve general nonlinear holding, backlogging and/or stockout penalty costs. However, to the best of our knowledge, the algorithm does not work for our (s, Q)-model. We explain this in more detail in **Appendix VI.A.3** and show how the linear version of the algorithm can be used to calculate a lower bound for Q^* and an upper bound for s^*. The higher ω is, the worse is a high backorder time. That is, the weight of the backorder time is higher when $\omega = 1$ (linear case) compared to $0 < \omega < 1$ (nonlinear case). When the weight of the backorder time increases, it is optimal to increase s and to decrease Q. This is always the case in inventory management models such as the one above. If backorders would incur no penalty at all, then $s \to -\infty$, $Q \to \infty$ would be optimal because both the inventory holding and the reorder costs would be minimized. Therefore, when we optimize our model with $\omega = 1$ (linear case) instead of the (assumed) real parameter $0 < \omega < 1$, the value $s^*_{|\omega=1} > s^*$ is an upper bound for s^*, and the value $Q^*_{|\omega=1} < Q^*$ is a lower bound for Q^*.

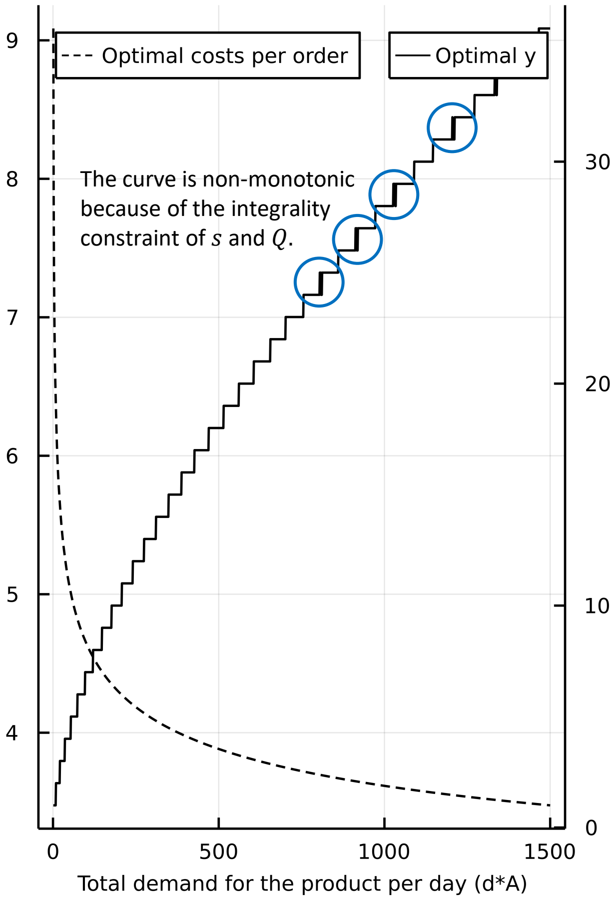
In **Appendix VI.A.1**, we prove that our (s, Q)-model (fixed y), with nonlinear costs that depend on the average backorder time per backorder, is quasi-convex. This means that our (s, Q)-model can be solved for optimality using one of the abovementioned general methods for nonlinear optimization.

A central part of our proof is the joint convexity of the average backorder time per backorder in s and Q. We prove this joint convexity for Poisson distributed demand and specify a general necessary and sufficient condition for probability distributions. While this necessary and sufficient condition is true for a wide range of probability distributions, it is slightly more restricting than the conditions for convexity of $B(y, s, Q)$ (Zipkin, 1986; Zhang, 1998). To our knowledge, no other publication has yet shown convexity of the average backorder time per backorder in the case of a (s, Q)-model with Poisson distributed demand, nor has a general sufficient condition been derived.



Inventory policy based only on inventory costs and backorder time: $dA = 5000$, $\ell = 2$, $h = 0.2$, $R = 50$, $u = 3$, $\omega = 0.5$

Figure VI.5.1 Discrete nature of the problem (part 1)



Inventory policy based only on inventory costs and backorder time: $dA = 5000$, $\ell = 2$, $h = 0.2$, $R = 50$, $u = 3$, $\omega = 0.5$

Figure additionally based on a LIP model: $A = 502655$, $c_{\text{transport}} = 0.03$, $\beta_c = 0.7$, $t_{\text{transport}} = 0.01845$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 0.247$, $c_{\text{warehouse}} = 0.247$

Figure VI.5.2 Discrete nature of the problem (part 2)

Turning to the second subproblem of finding an optimal y^*, we can observe the following. An increase in y has two effects. It decreases the replenishment lead time demand per stocking location (because demand splits among more stocking locations), which increases costs because the coefficient of demand variation increases, and it decreases the average transport time and the average transport costs from the stocking locations to customers, which decreases costs. We show in **Appendix VI.A.2** that these two opposite effects (visualized in **Figure VI.1**) make $C(y, s^*, Q^*)$ a quasi-convex function, and the optimal y^* can be found using the same solution methods as mentioned for the (s, Q)-model.

This subproblem of finding the optimal y^* is only guaranteed to be quasi-convex when the integrality constraints of s and Q are relaxed. The reason for this is that the average backorder time per backorder, as well as the inventory holding and replenishment costs, do not produce a smooth function of y given integers s^* and Q^* (see **Figures VI.5**). Note that the backorder time forms a sum with the transport time as part of the lead time penalty. When the backorder time (depending on y) jumps, then quasi-convexity is not guaranteed. Indeed, we encountered this in our numerical examples. However, it did not occur often, and when it occurred, the optimal y^* was in the vicinity, and the optimal (minimum) costs were within a fraction of a percent of the local optimum. Whether s and Q are relaxed to real numbers usually does not matter because if nonconvexity occurs in the integer case, then it usually only occurs at the flatter part of the cost curve. In **Appendices VI.A.2** and **VI.A.4**, we dive deeper into the integrality relaxation of y, s and Q.

VI.4. Sensitivity analysis

VI.4.1. General parameter variation

The following analyses and discussions are oriented around our motivating example of the B2C e-commerce market. However, the mathematical properties of our model and many results of our analyses have general validity.

Figure VI.6 shows the results of a sensitivity analysis with parameter variations of h, R, $c_{\text{transport}}$, $t_{\text{transport}}$, and u. All these parameters were quadrupled relative to the base case scenario. The reader may notice that a comparison between the case "$h \equiv h \cdot 4$" and the case "$R \equiv R \cdot 4$" reveals that an increase in the reorder costs (i.e., $R \equiv R \cdot 4$) is worse. This is always the case and can also be seen in the mathematical model. When optimizing its inventory policy, an e-tailer must choose an optimal combination of s and Q. When R increases, then it is optimal to attenuate the otherwise proportional increase in reorder costs. The only way to do so is to increase Q. Similarly, when h increases, it is optimal to attenuate the inventory holding cost increase. However, in this case, the e-tailer can use Q as well as s to do so. Basically, the e-tailer is less restricted when h increases compared to when R increases. This can also be seen in the two bottom subfigures within **Figures VI.7**. The curves are more erratic in the case of "$h \equiv h \cdot 4$" compared to "$R \equiv R \cdot 4$" because the e-tailer uses both s and Q to attenuate the effect of the increased inventory holding cost rate. Thus, if possible, e-tailers should avoid high reorder costs more than they should avoid high inventory holding costs.

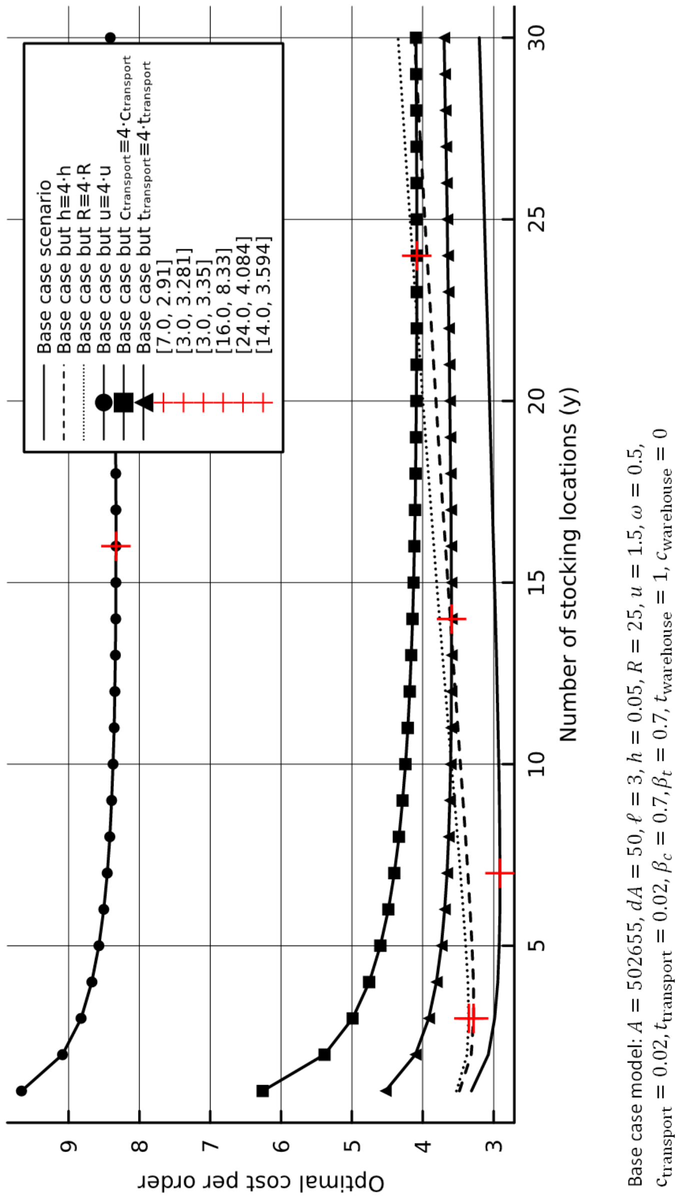
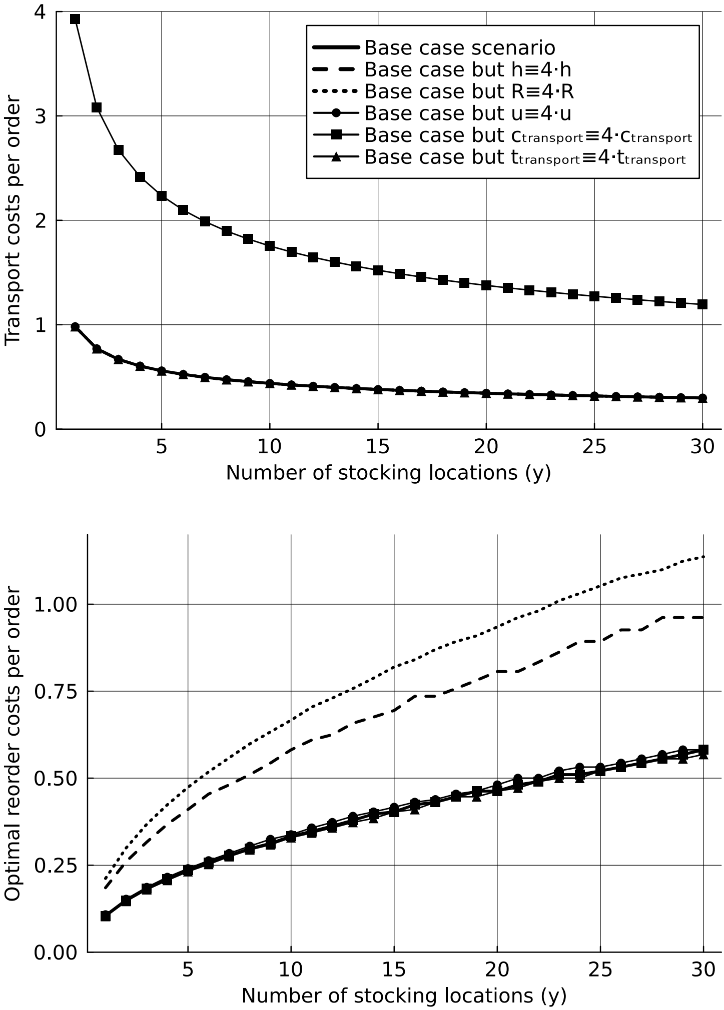
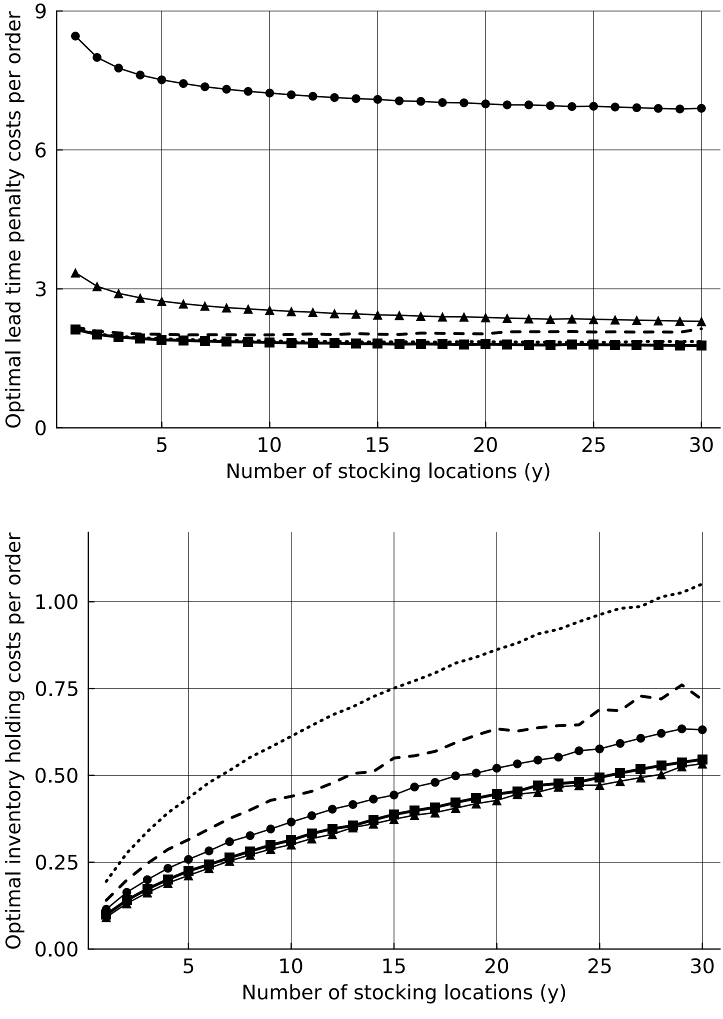


Figure VI.6 Sensitivity analysis 1: optimal costs per order depending on the number of stocking locations



Base case model: $A = 502655$, $dA = 50$, $\ell = 3$, $h = 0.05$, $R = 25$, $u = 1.5$, $\omega = 0.5$, $c_{\text{transport}} = 0.02$, $t_{\text{transport}} = 0.02$, $\beta_c = 0.7$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 1$, $c_{\text{warehouse}} = 0$

Figure VI.7.1 Sensitivity analysis 2: optimal costs per order depending on the number of stocking locations (part 1)



Base case model: $A = 502655$, $dA = 50$, $\ell = 3$, $h = 0.05$, $R = 25$, $u = 1.5$, $\omega = 0.5$, $c_{\text{transport}} = 0.02$, $t_{\text{transport}} = 0.02$, $\beta_c = 0.7$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 1$, $c_{\text{warehouse}} = 0$

Figure VI.7.2 Sensitivity analysis 2: optimal costs per order depending on the number of stocking locations (part 2)

A similar statement can be made when comparing a proportional increase in u with a proportional increase in $t_{\text{transport}}$. When studying the mathematical model, it becomes evident that a proportional increase in u is always worse if $\omega \leq 1$. For one, $t_{\text{transport}}$ is only one part of the complete delivery lead time, but more importantly, the delivery lead time is exponentiated by the delivery lead time perception factor ω. If we accept the educated assumption that $\omega \leq 1$ represents the most relevant part of the delivery lead time perception curve, then the following economic arguments can be made. The transport time scale parameter $t_{\text{transport}}$ is predominantly determined by the transport technology used and by legal regulations. The faster the transport vehicle (e.g., a car) can/is allowed to move, the smaller $t_{\text{transport}}$. An e-tailer therefore has only a very limited opportunity to influence the value of $t_{\text{transport}}$, and creating a lasting competitive advantage based on the value of $t_{\text{transport}}$ is difficult. u, on the other hand, is based on the experiences, expectations and wishes of customers. If a large e-tailer such as Amazon provides ever faster delivery lead times for free, then customer expectations toward the other e-tailers also rise. Smaller e-tailers have a hard time meeting these increased expectations because they simply lack the necessary economies of scale. Smaller e-tailers should therefore concentrate on products for which a fast delivery lead time is less important and/or try to outsource parts of the fulfillment process.

In **Figure VI.6**, it also stands out that the highest optimal number of stocking locations is calculated in the case of "$c_{\text{transport}} \equiv c_{\text{transport}} \cdot 4$" with $y^* = 24$ and not in the case of "$u \equiv u \cdot 4$" with $y^* = 16$. The effect of a higher c is rather direct because the transport costs only depend on the number of stocking locations and not on the inventory policy or the delivery lead time perception. In the term "$\frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c}$", the term "$1/y$" is only exponentiated by β_c. Compared to the term $\left(\ldots + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega}$, where the "$1/y$" term is exponentiated by β_t as well as by ω. Thus, if $\beta_c, \beta_t, \omega < 1$, then it is likely, although of course dependent on the specific values of β_c, β_t, ω, that the effect of more stocking locations is stronger with respect to the transport costs (dependent on $c_{\text{transport}}$) than with respect to the transport times (dependent on $t_{\text{transport}}$). Second, in our model, an e-tailer does not want to reduce the transport times but rather the delivery lead time penalty costs. The expected backorder time is the second endogenous part of the delivery lead time. Increasing the number of stocking locations (y) leads to shorter transport times but also to less demand risk pooling, and an e-tailer would need to increase s and/or Q and thus increase the inventory costs to keep the expected backorder time unchanged. Therefore, increasing y creates two opposing lead time effects, and an increase in u does not necessarily lead to a higher y^*.

Indeed, mathematically, the two opposing effects that form the delivery lead time mean that a higher u could also lead to a lower y^*. The optimal number of stocking locations $y^* \to \infty$ if $u \to 0$ (because, in this case, an infinitely long backorder time would not be penalized). Increasing u leads to a decrease in y^* until a point where the transport time becomes more important than the backorder time. From this point onward, an increasing u leads to an increasing y^*. It is an educated guess that most B2C e-commerce markets have rather high u, so that an increase in u usually leads to an increased y^*.

VI.4.2. The effects of a nonlinear delivery lead time perception

Figures VI.8, **VI.9** and **VI.10** show different scenarios with different values of $0 < \omega \leq 0$. The delivery lead time perception slope factor ω is a core element of our model. **Figure VI.8** represents a scenario in which an e-tailer has a fast delivery lead time relative to the market average. In the scenario of **Figure VI.9**, the e-tailer is as fast as the market average (if $y = 1$), and in **Figure VI.10**, the e-tailer is slower than the market average. The market average delivery lead time is the anchor point for customer perception.

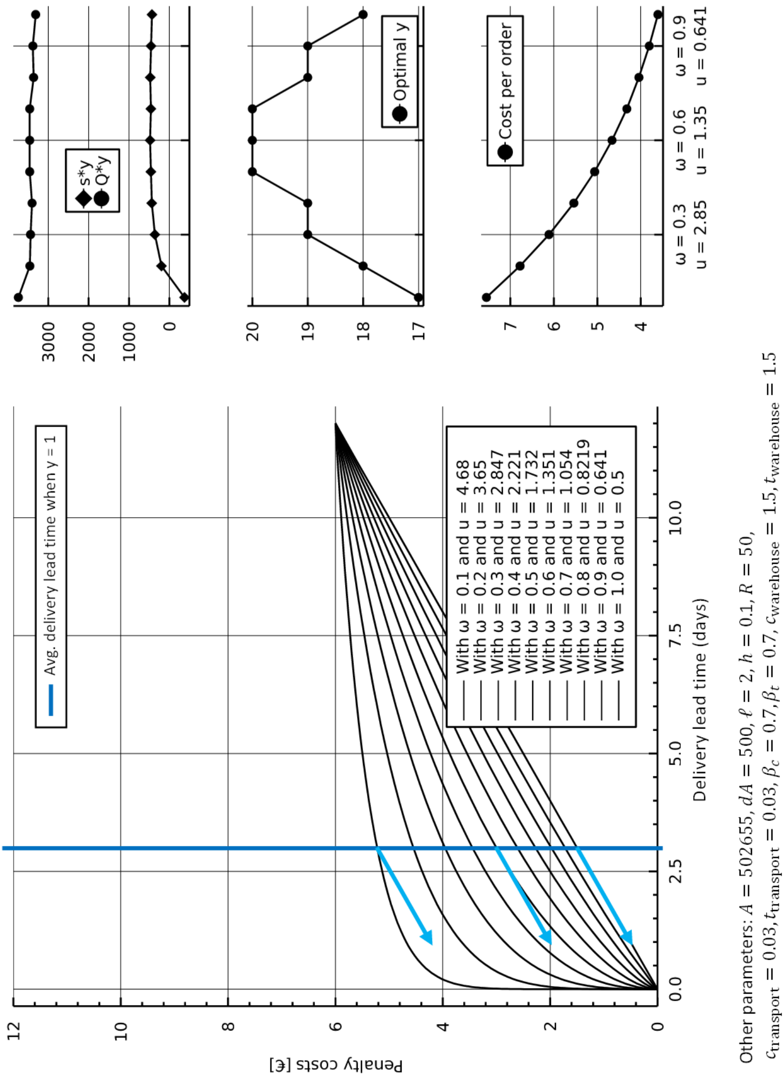


Figure VI.8 Sensitivity analysis 3.1: The company has a fast delivery lead time relative to the market

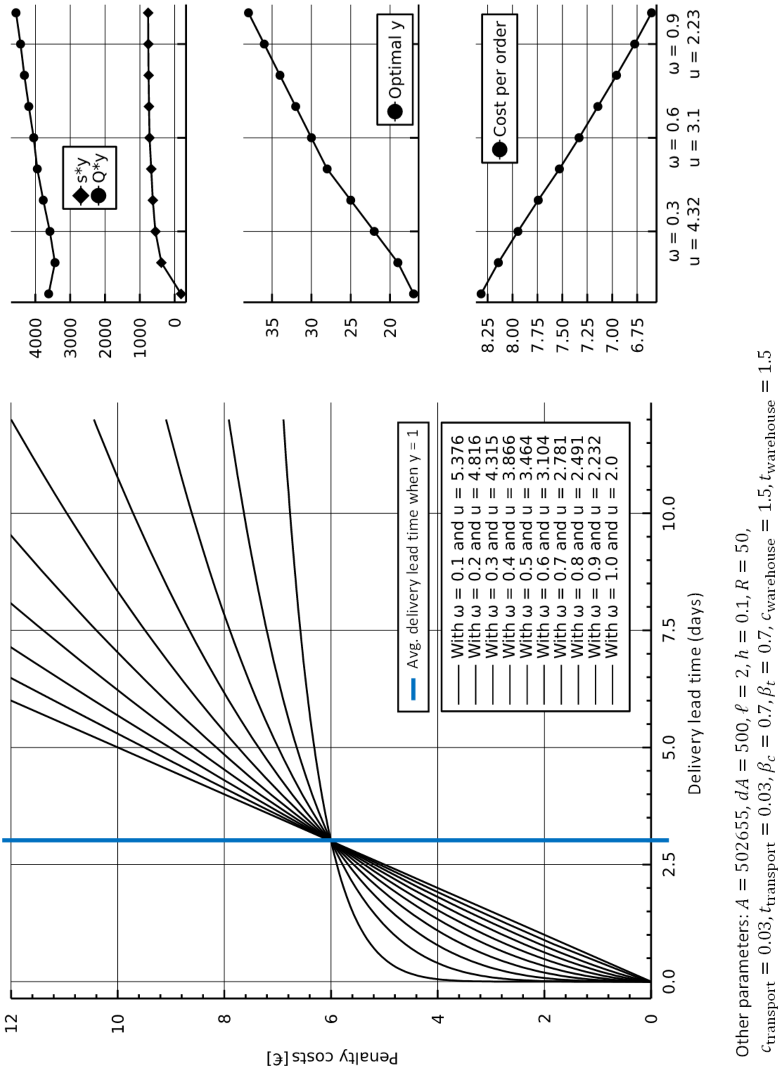


Figure VI.9 Sensitivity analysis 3.2: The company has a market average delivery lead time

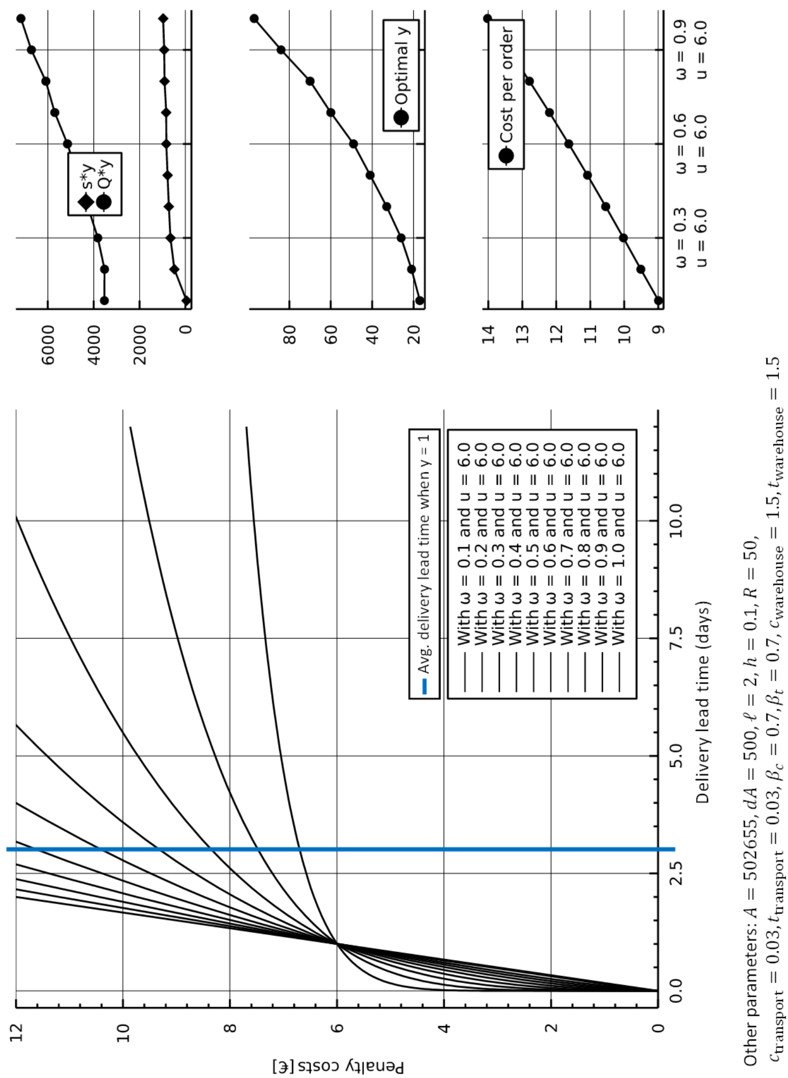


Figure VI.10 Sensitivity analysis 3.3: The company has a slow delivery lead time relative to the market

Regardless of what the value ω is in our examples, if an e-tailer provides a market average delivery lead time, then the penalty cost is always €6 on average across all customers. The delivery lead time perception ω affects how elastic the penalty cost curve is around this market average delivery lead time.

The three figures illustrate these differences in the elasticity (slope) and the absolute values of the penalty cost curves, depending on the relative competitive position. In **Figure VI.8**, for example, the curves in the case of a small ω and a high ω are flatter than the curves with a medium-high ω. Different y are optimal depending on the value of ω, and our examples additionally illustrate how y^* is also dependent on the competitive position of an e-tailer. The examples show that the slower the e-tailer is relative to the market average (measured with $y = 1$), the higher y^*. If, for example, the warehouse fulfillment time is high, then the e-tailer tries to compensate for this competitive disadvantage through more stocking locations, that is, a decrease in transport time. However, this compensation does not come without cost. The larger number of stocking locations leads to inventory decentralization and therefore more inventory costs. This can also be seen in the figures. The worse the competitive position is (**Figure VI.8** vs. **Fig. VI.9** vs. **Fig. VI.10**), the higher y^*, the higher s^* and Q^* and the higher the penalty costs. A fundamental competitive disadvantage, such as a long warehouse fulfillment time, cannot be adequately compensated for by optimizing the number of stocking locations.

This dilemma is aggravated by a nonlinear delivery lead time perception $\omega < 1$. If the warehouse fulfillment time ($t_{\text{warehouse}}$) stays the same, the effect of a decrease in the backorder ($\overline{\tau_b}$) and transport time $\left(\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{(0.5\beta_t)}\right)$ is weakened by the nonlinear perception of the delivery lead time, i.e., the sum of the three times: $\left(\overline{\tau_b} + t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{(0.5\beta_t)}\right)^{\omega}$. Thus, in such cases, outsourcing warehouse operations, with the goal of lowering warehouse fulfillment times, can be the most expedient course of action.

Finally, the examples also simply show how important it is to consider a potentially nonlinear delivery lead time perception when configuring/optimizing a (B2C e-commerce) fulfillment network. In **Figure VI.8**, for example, the cost per order is more than 100% higher for $\omega = 0.1$ than for the linear case $\omega = 1$. The optimal y^* in **Figure VI.10** is more than a third lower when $\omega = 0.7$ compared to the optimal y^* in the linear case. This is a large difference despite $\omega = 0.7$ being a rather moderate nonlinear perception.

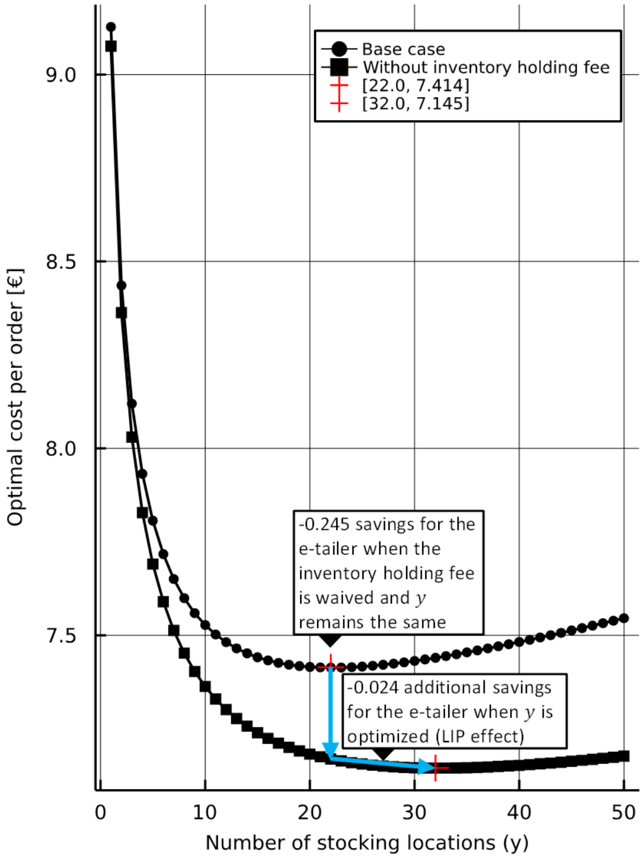
VI.5. Implications for warehouse outsourcing decisions

Some e-tailers (especially smaller ones) may want to use third-party warehouses to store their products. Roughly speaking, on-demand warehousing (ODW) means that an e-tailer purchases and owns a product but that the e-tailer does not store it in its own warehouse but in a warehouse of an ODW provider, which is often shared with other e-tailers (Ceschia et al., 2023). This creates economies of scale and risk pooling effects for warehouse operations. The ODW provider manages the inbound logistics of the products, inventory holding, picking, packing and outbound logistics. Such a service can be useful for e-tailers who want to offer fast delivery lead times and is thus particularly relevant to our model context (Unnu & Pazour, 2022; Lee et al., 2024).

The e-tailers receive services (e.g., inventory holding and fulfillment) from the ODW provider and pay money for these services. The following three fees are salient: an inbound logistics restocking fee per replenishment, an inventory holding fee per volume unit per time unit (warehouse space usage) and a one-time fulfillment fee per fulfilled (picked, packed, ...) product. At first glance, such a pricing scheme seems reasonable and intuitive since the fees match the typical inventory cost models. However, this can be questioned critically, especially in the context of an LIP.

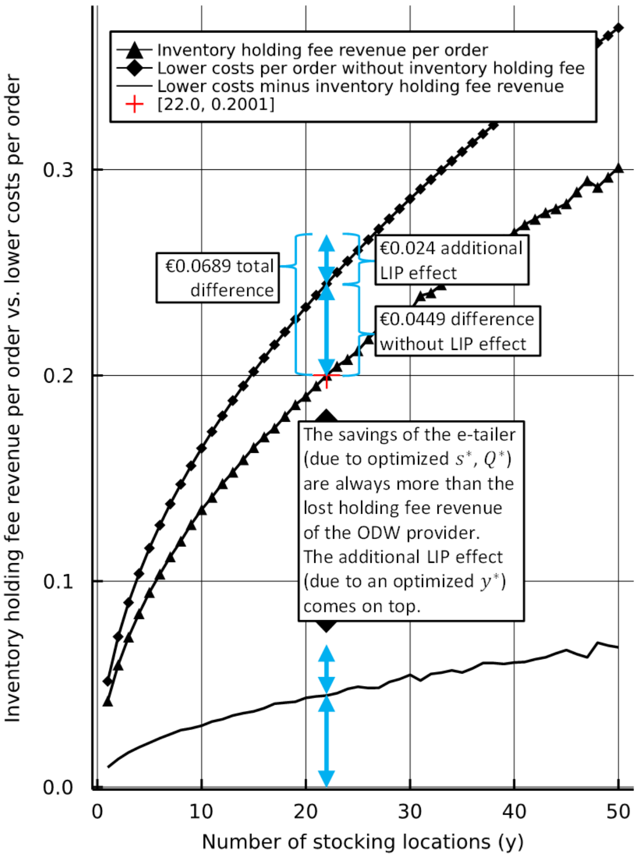
Figures VI.11 contain a comparison between an ODW provider that charges an inventory holding fee of $h_{\text{fee}} = 0.1$ [€ per piece of the product per day] and an ODW provider that does not. Note that in our example, the e-tailer incurs internal inventory holding costs of $h_{\text{internal}} = 0.1$ [€ per piece of the product per day] in any case, irrespective of h_{fee}, because bought inventory is tied-up capital until sold and thus generates opportunity costs. As one would expect, **Figure VI.11.1** shows that without the inventory holding fee (i.e., $h = h_{\text{internal}} = 0.1$), the costs per order are lower for the e-tailer (compared to the case of $h = h_{\text{internal}} + h_{\text{fee}} = 0.2$), and the optimal number of stocking locations is higher. The ODW provider, on the other hand, waives/loses the revenue from the inventory holding fee. However, the savings of the e-tailer are always more than the lost holding fee revenue of the ODW provider (see **Figure VI.11.2**). This means that the ODW provider can (without increasing the costs per order for the e-tailer) generate more revenue by waiving the inventory holding fee and instead increasing the fulfillment fee.

This is always the case because a higher fulfillment fee is not related to either y, s or Q and therefore does not change the optimal policy. The optimal cost curve is simply shifted upward. A higher h_{fee}, on the other hand, changes the optimal policy (y^*, s^*, Q^*) and thus leads to an overproportional cost increase for the e-tailer. Therefore, waiving or lowering the inventory holding fee is beneficial even if y is not optimized. The LIP effect, that is, lower costs through an optimized number of stocking locations y^*, comes on top and is especially beneficial because it is not associated with any additional inventory holding fee revenue decrease for the ODW provider.



Parameter variation: In the base case: $h = 0.2$. Without inventory holding fee: $h = 0.1$. **Reading example:** If the ODW provider does not charge an inventory holding fee, then the optimal number of stocking locations (from the e-tailers point of view) increases from $y^* = 22$ to $y^* = 32$ (left figure). The costs per order (for the e-tailer) decreases from €7.414 to €7.145 (= €0.269 savings per order).
Other parameters: $A = 502655$, $dA = 500$, $\ell = 2$, $R = 50$, $u = 2.78$, $\omega = 0.7$, $c_{\text{transport}} = 0.03$, $t_{\text{transport}} = 0.03$, $\beta_c = 0.7$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 1.5$, $c_{\text{warehouse}} = 1.5$

Figure VI.11.1 Inventory holding fee vs. fulfillment fee (part 1)



Parameter variation: In the base case: $h = 0.2$. Without inventory holding fee: $h = 0.1$. **Reading example:** Without an inventory holding fee, the ODW provider waives €0.2001 inventory holding fee revenue per order. But the ODW service provider could increase the fulfillment fee per order by €0.269 and thus increase the revenue per order by €0.0689. Indeed, even if y remains the same ($y = 22$ in this example), waiving the inventory holding fee is still always profitable.
Other parameters: $A = 502655$, $dA = 500$, $\ell = 2$, $R = 50$, $u = 2.78$, $\omega = 0.7$, $c_{\text{transport}} = 0.03, t_{\text{transport}} = 0.03$, $\beta_c = 0.7, \beta_t = 0.7$, $t_{\text{warehouse}} = 1.5, c_{\text{warehouse}} = 1.5$

Figure VI.11.2 Inventory holding fee vs. fulfillment fee (part 2)

The higher revenue that can be generated when h_{fee} is waived is of course only guaranteed to be profitable for the ODW provider if the warehouse space is not a bottleneck. A lower h for the e-tailer means that, optimally, the average inventory increases. If a warehouse is not fully utilized, then an ODW provider has nearly zero marginal costs if another item is stored in the warehouse, and lowering h_{fee} is likely to be a profitable decision. However, if the warehouse is well utilized, then the space is a scarce resource. In such a situation, an ODW provider should increase rather than decrease h_{fee}. Amazon, for example, offers the Fulfillment by Amazon service (where Amazon acts as an ODW provider for third-party sellers on the Amazon marketplace) and charges a higher h_{fee} for this service during the busy pre-Christmas months compared to the rest of the year when its warehouses are less utilized.

Similar arguments as for the inventory holding fee can also be made regarding a restocking fee. Indeed, as already explained in the previous section (sensitivity analysis), a high reorder cost ($R = R_{\text{internal}} + R_{\text{fee}}$) is even worse than a high inventory holding cost rate. However, not charging a restocking fee should be considered rather cautiously because inbound logistics costs are much more direct compared to inventory holding costs. Every restocking causes work (costs) directly associated with the restocking. In light of this, it seems very interesting that Amazon does not charge a restocking fee within its Fulfillment by Amazon service. Apparently, Amazon has also realized that a small R is important for e-tailers, and at the same time, they are confident that their inbound logistics processes are so efficient that many frequent replenishments from third-party sellers are not a big problem.

Finally, it must be noted that for a "fulfillment fee only" pricing scheme to be successful in general, an ODW provider should have at least a rough idea about the (partially) product-specific parameters (d, h_{internal}, R_{internal}, u, $c_{\text{transport}}$, ...) of the e-tailers' LIP problems. This could very well be inhibitively complex. Nevertheless, the trend toward faster deliveries in B2C e-commerce means that ODW providers are generally in a good position. In addition to warehouse outsourcing, they could also offer dropshipping services. One of the biggest problems when using multiple stocking locations is inventory decentralization. Warehouse outsourcing on its own can at best slightly mitigate the negative effects of inventory decentralization. Dropshipping (that is, virtual inventory centralization) is the most viable option to counteract inventory decentralization effects. Indeed, even large e-tailers may find dropshipping appealing if a demand-dependent dropshipping fee is charged. Much research remains to be done on both warehouse outsourcing and dropshipping considerations in the context of an LIP, some of which we touch upon in the following closing summary.

VI.6. Summary, limitations and outlook

VI.6.1. Summary

We have argued that location-inventory modeling is important within a B2C e-commerce context. The delivery lead time is an important service component, and it is likely that most customers perceive the delivery lead time nonlinearly. Our numerical results show that it is important to consider this when modeling an LIP. Existing LIP models, which predominantly use very simple approximations for the inventory part of the model, are not suited for such a nonlinear perception. Because the backorder time is part of the delivery lead time, we modeled the inventory part of the model exactly and instead approximated the location part of the LIP. Such an approach fits the B2C e-commerce context well because an e-tailer usually has many different customers. We derived the exact backorder time distribution for the inventory part of the model and the exact transport time distribution for the location part of the model. This resulted in an exact LIP model that is surprisingly easy to understand but at the same time detailed enough where it matters. The exact model can be used for any kind of nonlinear perception of the delivery lead time. If $0 < \omega < 1$ (concave penalty for longer lead times), which we assume to be representative of the most relevant part of the perception curve, then it is possible to utilize an approximative model that uses the mean backorder time and the mean transport time. This approximative model is so fast that it can be used within parameter variation simulations and is so simple that it can be calculated within a Microsoft Excel spreadsheet and solved with the built-in Excel solver. Indeed, we proved that the model is quasi-convex.

We presented a sensitivity analysis, which exemplified some general characteristics and mechanisms of the model. Selected results are as follows:

- A proportional decrease in the reorder costs (R) is better than a proportional decrease in the inventory holding cost rate (h). If $0 < \omega \leq 1$, a proportional decrease in the delivery lead time perception scale (u) is better than a proportional decrease in the transport time scale ($t_{\text{transport}}$).
- The inventory holding (h) and reorder costs (R), the delivery lead time perception scale u and the transport cost scale $c_{\text{transport}}$ are often the most important parameters within a B2C e-commerce context. $c_{\text{transport}}$ has a very direct influence on the absolute level and the slope of the total cost curve, and u is a powerful lever to create a lasting competitive advantage.
- Indeed, the relative speed of an e-tailer compared to the market average speed has pronounced effects on how the LIP model behaves. For example, a key takeaway is that an e-tailer should emphasize trying to avoid slow warehouse fulfillment times. Thus, (small) e-tailers may find outsourcing of warehouse operations very attractive.

Warehouse outsourcing is a natural consideration within the context of our model. We discussed some insights about this in **Section VI.5**.

- For example, it can be shown that if the warehouses of an ODW provider do not have a high utilization rate, then the ODW provider can generate more profit by waiving the inventory holding fee and instead increasing the fulfillment fee. This effect also exists within a normal inventory outsourcing model but is amplified by the location-inventory problem.
- While warehouse outsourcing is often beneficial, it can only slightly mitigate the effects of inventory decentralization. Drop-

shipping is a potential solution and should be researched in more detail in the future.

VI.6.2. Limitations and outlook

Mathematical models typically contain explicit or implicit simplifications. Our model is no exception. One of the most obvious simplifications in our model is the chosen continuous approximation approach. For strategic/tactical location-inventory decisions, this is an acceptable approximation. However, for some regions in the world, this model might not fit very well. Furthermore, we aggregated variables into mean values and used these mean values in our model. In **Subsection VI.3.6**, we discussed in detail the effects of using mean values for the backorder time, warehouse fulfillment time, and transport time. If $0 < \omega \leq 1$, these simplifications are rather unproblematic. However, the parameters u and ω themselves also vary from customer to customer, and u and ω represent some kind of average over all customer demand. However, given that both u and ω are bounded by ≥ 0, the coefficient of variation is typically not too high.

Moreover, while our model incorporates a somewhat sophisticated inventory management model, it also has some simplifications. For one, we consider only backordering and no lost sales. Lost sales can, for example, occur when a customer needs a product quickly and the delivery lead time is too long. Because the delivery lead time is a central part of our model, lost sales could also potentially play an important role in determining the optimal number of warehouses. It could be worthwhile to integrate delivery lead time-dependent demand ($d(\text{avg. lead time})$) into our model (similar to Berman et al., 2007). This would be a logical extension of our model because shorter delivery lead times could also shift some of the retail market share from brick-and-mortar retail toward B2C e-commerce.

Two additional simplifications we made in our inventory model were the assumption of unit-sized Poisson demand (a customer orders only one piece per product per order) and fixed deterministic replenishment lead times. Both assumptions do not match reality perfectly, but neither assumption is particularly problematic given the B2C e-commerce context of this paper. Although there is lumpy demand (customers ordering two or more pieces of one product) in B2C e-commerce, the overshoot past the reorder point is often relatively small. Nevertheless, future research could extend the model to lumpy compound Poisson demand and apply a reorder point, order up to (s, S)-policy. A variable replenishment lead time, on the other hand, is notoriously difficult to solve exactly (Hadley & Whitin, 1963, pp. 200–204). However, if we assume that the e-tailers use their own receive centers or local wholesalers for the replenishment of the regional warehouses, then it is realistic to assume that the replenishment lead time of the regional warehouses does not fluctuate much (e.g., 2–3 days in the Euro logistics system; Fleischmann, 2016, p. 900).

In any case, lumpy demand and a variable replenishment lead time basically increase only the variability of demand during the replenishment lead time and thus the variability of the backorder times (decreasing the fit of our approximate model slightly). This increases the inventory and stockout penalty costs and indirectly (through a higher stockout probability) the average backorder time. The impact on the model is predictable and in many cases not too strong. Future research could investigate the use of the well-known normal approximation of demand, which allows for an arbitrary coefficient of variation (Hadley & Whitin, 1963, pp. 191–195). Another popular simplification is to assume that the probability that the replenishment lead time demand is greater than $s + Q$ is negligible. Hadley and Whitin (1963, pp. 188–191), have shown how, in their model, this assumption makes it easier to obtain optimal s^* and Q^*. Unfortunately, for our model, such a simplification accomplishes little. The terms $P_{\mathrm{out}}(y, s, Q)$ and $B(y, s, Q)$ in $\left(\frac{B(y,s,Q)}{(dA/y)P_{\mathrm{out}}(y,s,Q)} + \cdots\right)^{\omega}$ would be slightly easier to calculate; however, because the whole term is exponentiated by ω, further simplifications are not possible.

We also set aside the possibility of transshipments. In practice, retailers often prefer tiered warehouse types with assigned regions, for example, central, regional, and local warehouses (a so-called arborescence network; Tsao et al., 2012, p. 219). Delivery outside of these assigned regions (e.g., transshipment) is often inefficient because the fulfillment network has streamlined standard processes. This also has implications for the presented LIP because in practice, the decision variable y is often not going to have the domain $y \in 1, 2, 3, \ldots$ but instead, maybe $y \in \{1, 5, 20, 100\}$. Holzapfel et al. (2018) and Fichtinger et al. (2019), for example, proposed segmenting products into different categories that are then stored either in a central warehouse (e.g., $y = 1$) or in regional warehouses (e.g., $y = 20$). Furthermore, we note that the assignment decisions (which products are stored in which warehouses) also influence the demand throughput of the warehouses. Erlenkotter (1989) and Rutten et al. (2001), for example, considered (dis-)economies of scale for warehouses based on the demand quantity fulfilled through a warehouse. This also has implications for warehouse outsourcing decisions because demand would be shifted from the warehouses of the e-tailers to the warehouses of the ODW providers. Combining both the notion of warehouse types and the notion of (dis-)economies of scale into one integrated multiproduct LIP creates a very complex model (both Erlenkotter (1989) and Rutten et al. (2001) implicitly assumed a one-product firm, and Holzapfel et al. (2018) and Fichtinger et al. (2019) do not consider (dis-)economies of scale). Such a complex model could nevertheless be worthwhile because many processes may be influenced by (dis-)economies of scale. These processes could include, for example, the warehouse fulfillment time and perhaps even the inventory holding cost rate h and the reorder costs R. The product-specific model that we presented in this paper can be solved relatively quickly. It is therefore well suited for integration into a larger model with (dis-)economies of scale in warehouse operations. Such a model could perhaps iteratively approach a system-wide (near) optimum by solving the product-specific models several times.

For this paper, however, we performed a more focused analysis of the LIP effects, especially emphasizing exact backorder time costs and a nonlinear delivery lead time perception.

VI. References

Ansari, S., Başdere, M., Li, X., Ouyang, Y., & Smilowitz, K. (2018). Advancements in continuous approximation models for logistics and transportation systems: 1996–2016. *Transportation Research Part B: Methodological, 107,* 229–252. https://doi.org/10.1016/j.trb.2017.09.019

Berk, E., & Toy, A. Ö. (2023). A serial inventory system with lead-time-dependent backordering: A reduced-state approximation. *IISE Transactions, 55*(3), 259–270. https://doi.org/10.1080/24725854.2022.2057625

Berman, O., Krass, D., & Menezes, M. B. C. (2007). Inventory-location models for remote and direct retailing with time-sensitive demand. *2007 IEEE International Conference on Industrial Engineering and Engineering Management,* 347–351. https://doi.org/10.1109/IEEM.2007.4419209

Berman, J. (2019). *Amazon preps for One-Day Free Shipping*. Retrieved July 14, 2022, from https://www.logisticsmgmt.com/article/amazon_preps_for_one_day_free_shipping.

Bretthauer, K. M., Mahar, S., & Venakataramanan, M. A. (2010). Inventory and distribution strategies for retail/e-tail organizations. *Computers & Industrial Engineering, 58*(1), 119–132. https://doi.org/10.1016/j.cie.2009.09.005

Brown, R. G. (1959). *Statistical forecasting for inventory control*. McGraw-Hill.

Brynjolfsson, E. & Smith, M. D. (2000). Frictionless commerce? A comparison of Internet and conventional retailers, *Management Science, 46*(4), 563–85. https://doi.org/10.1287/mnsc.46.4.563.12061

Ceschia, S., Gansterer, M., Mancini, S., & Meneghetti, A. (2023). The on-demand warehousing problem. *International Journal of Production Research, 61*(10), 3152–3170. https://doi.org/10.1080/00207543.2022.2078249

Correia, I., & Melo, T. (2022). Distribution network redesign under flexible conditions for short-term location planning. *Computers & Industrial Engineering, 174,* 108747. https://doi.org/10.1016/j.cie.2022.108747

Daganzo, C. F. (2005). *Logistics systems analysis* (4^{th} ed.). Springer.

Deng, S., Li, Y., Guo, H., & Liu, B. (2016). Solving a closed-loop location-inventory-routing problem with mixed quality defects returns in e-commerce by hybrid ant colony optimization algorithm. *Discrete Dynamics in Nature and Society, 2016*(1), 6467812. https://doi.org/10.1155/2016/6467812

Eppen, G. D. (1979). Note—effects of centralization on expected costs in a multi-location newsboy problem. *Management Science, 25*(5), 498–501. https://doi.org/10.1287/mnsc.25.5.498

Erlenkotter, D. (1989). The general optimal market area model. *Annals of Operations Research, 18*(1), 43–70. https://doi.org/10.1007/BF02097795

Escalona, P., Marianov, V., Ordónez, F., & Stegmaier, R. (2018). On the effect of inventory policies on distribution network design with several demand classes. *Transportation Research Part E: Logistics and Transportation Review, 111*, 229–240. https://doi.org/10.1016/j.tre.2017.10.019

Escalona, P., Ordóñez, F., & Marianov, V. (2015). Joint location-inventory problem with differentiated service levels using critical level policy. *Transportation Research Part E: Logistics and Transportation Review, 83*, 141–157. https://doi.org/10.1016/j.tre.2015.09.009

Farahani, R. Z., Bajgan, H. R., Fahimnia, B., & Kaviani, M. (2015). Location-inventory problem in supply chains: a modelling review. *International Journal of Production Research, 53*(12), 3769–3788. https://doi.org/10.1080/00207543.2014.988889

Federgruen, A., & Zheng, Y.-S. (1992). An efficient algorithm for computing an optimal (r, Q) policy in continuous review stochastic inventory systems. *Operations Research, 40*(4), 808–813. https://doi.org/10.1287/opre.40.4.808

Fichtinger, J., Chan, W.-C., & Yates, N. (2019). A joint network design and multi-echelon inventory optimisation approach for supply chain segmentation. *International Journal of Production Economics, 209*, 103–111. https://doi.org/10.1016/j.ijpe.2017.09.003

Fleischmann, B. (2016). The impact of the number of parallel warehouses on total inventory. *OR Spectrum, 38*(4), 899–920. https://doi.org/10.1007/s00291-016-0442-2

Gawor, T., & Hoberg, K. (2019). Customers' valuation of time and convenience in e-fulfillment. *International Journal of Physical Distribution & Logistics Management, 49*(1), 75–98. https://doi.org/10.1108/IJPDLM-09-2017-0275

Geoffrion, A. M. (1976). The purpose of mathematical programming is insight, not numbers. *Interfaces, 7*(1), 81–92. https://doi.org/10.1287/inte.7.1.81

Gerchak, Y., & Wang, S. (1997). Liquid asset allocation using "newsvendor" models with convex shortage costs. *Insurance: Mathematics and Economics, 20*(1), 17–21. https://doi.org/10.1016/S0167-6687(96)00018-2

Ghosh, S., Sahare, M., & Mukhoti, S. (2021). A new generalized newsvendor model with random demand and cost misspecification. In B. K. Sinha, S. B. Bagchi (Eds.) *Strategic Management, Decision Theory, and Decision Science* (pp. 211–245). Springer.

Gupta, A., Su, B.-c., & Walter, Z. (2004). An Empirical Study of Consumer Switching from Traditional to Electronic Channels: A Purchase-Decision Process Perspective. *International Journal of Electronic Commerce, 8*(3), 131–161. https://doi.org/10.1080/10864415.2004.11044302

Hadley, G., & Whitin, T. M. (1963). *Analysis of inventory systems.* Prentice-Hall.

Hall, R. W. (1986). Discrete models/continuous models. *Omega, 14*(3), 213–220. https://doi.org/10.1016/0305-0483(86)90040-X

Holzapfel, A., Kuhn, H., & Sternbeck, M. G. (2018). Product allocation to different types of distribution center in retail logistics networks. *European Journal of Operational Research, 264*(3), 948–966. https://doi.org/10.1016/j.ejor.2016.09.013

Hsiao, M.-H. (2009). Shopping mode choice: Physical store shopping versus e-shopping. *Transportation Research Part E: Logistics and Transportation Review, 45*(1), 86–95. https://doi.org/10.1016/j.tre.2008.06.002

Ilienko, A. (2013). Continuous counterparts of Poisson and binomial distributions and their properties. *Annales Univ. Sci. Budapest. Sect. Comp., 39*, 137–147. https://doi.org/10.48550/arXiv.1303.5990

Johansson, L., & Olsson, F. (2017). Quantifying sustainable control of inventory systems with non-linear backorder costs. *Annals of Operations Research, 259*(1), 217–239. https://doi.org/10.1007/s10479-017-2542-z

Johansson, L., & Olsson, F. (2018). Age-based inventory control in a multi-echelon system with emergency replenishments. *European Journal of Operational Research, 265*(3), 951–961. https://doi.org/10.1016/j.ejor.2017.08.057

Kim, N., Montreuil, B., Klibi, W., & Babai, M. Z. (2023). Network inventory deployment for responsive fulfillment. *International Journal of Production Economics, 255*, 108664. https://doi.org/10.1016/j.ijpe.2022.108664

Kruse, W. K. (1981). Waiting time in a continuous review (s, S) inventory system with constant lead times. *Operations Research, 29*(1), 202–207. https://doi.org/10.1287/opre.29.1.202

Langevin, A., Mbaraga, P., & Campbell, J. F. (1996). Continuous approximation models in freight distribution: An overview. *Transportation Research Part B: Methodological, 30*(3), 163–188. https://doi.org/10.1016/0191-2615(95)00035-6

Lasdon, L. S., Waren, A. D., Jain, A., & Ratner, M. (1978). Design and testing of a generalized reduced gradient code for nonlinear programming. *ACM Transactions on Mathematical Software (TOMS), 4*(1), 34–50. https://doi.org/10.1145/355769.355773

Lee, J., Ko, C., & Moon, I. (2024). E-commerce supply chain network design using on-demand warehousing system under uncertainty. International *Journal of Production Research, 62*(5), 1901–1927. https://doi.org/10.1080/00207543.2022.2128462

Li, Y., Lin, Y., & Shu, J. (2021). Location and two-echelon inventory network design with economies and diseconomies of scale in facility operating costs. *Computers & Operations Research, 133*, 105347. https://doi.org/10.1016/j.cor.2021.105347

Li, Y., Lu, M., & Liu, B. (2014). A two-stage algorithm for the closed-loop location-inventory problem model considering returns in e-commerce. *Mathematical Problems in Engineering, 2014*(1), 260869. https://doi.org/10.1155/2014/260869

Li, Y., Shu, J., Wang, L., & Wu, T. (2023). Joint location–inventory problem with economies and diseconomies of scale: A general model and an efficient column generation algorithm. *Computers & Industrial Engineering, 183,* 109537. https://doi.org/10.1016/j.cie.2023.109537

Liu, B., Chen, H., Li, Y., & Liu, X. (2015). A pseudo-parallel genetic algorithm integrating simulated annealing for stochastic location-inventory-routing problem with consideration of returns in e-commerce. *Discrete Dynamics in Nature and Society*, 2015(1), 586581. https://doi.org/10.1155/2015/586581

Magee, J. F. (1968). *Industrial Logistics: Analysis and Management of Physical Supply and Distribution Systems*. McGraw-Hill.

Mahar, S., Bretthauer, K. M., & Venkataramanan, M. A. (2009). The value of virtual pooling in dual sales channel supply chains. *European Journal of Operational Research, 192*(2), 561–575. https://doi.org/10.1016/j.ejor.2007.09.034

Maister, D. H. (1976). Centralisation of Inventories and the "Square Root Law". *International Journal of Physical Distribution*, *6*(3), 124–134. https://doi.org/10.1108/eb014366

Marino, G., Zotteri, G., & Montagna, F. (2018). Consumer sensitivity to delivery lead time: a furniture retail case. *International Journal of Physical Distribution & Logistics Management, 48*(6), 610–629. https://doi.org/10.1108/IJPDLM-01-2017-0030

Moghadam, S. S., Aghsami, A., & Rabbani, M. (2021). A hybrid NSGA-II algorithm for the closed-loop supply chain network design in e-commerce. *RAIRO-Operations Research, 55*(3), 1643–1674. https://doi.org/10.1051/ro/2021068

Nagarajan, M., & Rajagopalan, S. (2008). Inventory models for substitutable products: Optimal policies and heuristics. *Management Science*, *54*(8), 1453–1466. https://doi.org/10.1287/mnsc.1080.0871

Nierynck, R. (2020). *Quick commerce: pioneering the next generation of delivery*. Retrieved October 17, 2022, from https://www.deliveryhero.com/blog/quick-commerce/.

Parlar, M., & Rempala, R. (1992). A stochastic inventory problem with piecewise quadratic costs. *International Journal of Production Economics, 26*(1–3), 327–332. https://doi.org/10.1016/0925-5273(92)90083-J

Paterson, C., Kiesmüller, G., Teunter, R., & Glazebrook, K. (2011). Inventory models with lateral transshipments: A review. *European Journal of Operational Research, 210*(2), 125–136. https://doi.org/10.1016/j.ejor.2010.05.048

Porteus, E. L. (1990). Stochastic Inventory Theory. *Handbooks in Operations Research and Management Science,* 2, 605–652. https://doi.org/10.1016/S0927-0507(05)80176-8

Puga, M. S., Minner, S., & Tancrez, J. S. (2019). Two-stage supply chain design with safety stock placement decisions. *International Journal of Production Economics, 209,* 183–193. https://doi.org/10.1016/j.ijpe.2018.05.018

Pujari, N. A., Hale, T. S., & Huq, F. (2008). A continuous approximation procedure for determining inventory distribution schemas within supply chains. *European Journal of Operational Research, 186*(1), 405–422. https://doi.org/10.1016/j.ejor.2007.01.030

Pulido, R., Muñoz, J. C., & Gazmuri, P. (2015). A continuous approximation model for locating warehouses and designing physical and timely distribution strategies for home delivery. *EURO Journal on Transportation and Logistics, 4*(4), 399–419. https://doi.org/10.1007/s13676-014-0059-z

Rubalski. GB. (1972). Calculation of optimum parameters in an inventory control problem. *Engineering Cybernetics, 10*(2), 182–187.

Rutten, W. G. M. M., Van Laarhoven, P. J. M., & Vos, B. (2001). An extension of the GOMA model for determining the optimal number of depots. *IIE Transactions, 33*(11), 1031–1036. https://doi.org/10.1023/A:1010998620314

Schwarz, L. B. (1981). Physical distribution: the analysis of inventory and location. *AIIE Transactions, 13*(2), 138–150. https://doi.org/10.1080/05695558108974546

Shin, H., Park, S., Lee, E., & Benton, W. C. (2015). A classification of the literature on the planning of substitutable products. *European Journal of Operational Research, 246*(3), 686–699. https://doi.org/10.1016/j.ejor.2015.04.013

Trebilcock, B. (2020). *UPS takes a look at the future of e-commerce fulfillment*. Retrieved July 14, 2022, from https://www.logisticsmgmt.com/article/ups_takes_a_look_at_the_future_of_e_commerce_fulfillment.

Tsao, Y.-C. (2016). Designing a supply chain network for deteriorating inventory under preservation effort and trade credits. *International Journal of Production Research*, *54*(13), 3837–3851. https://doi.org/10.1080/00207543.2016.1157272

Tsao, Y.-C. (2019). A continuous approximation approach for supply network design considering radio frequency identification adoption. *RAIRO-Operations Research*, *53*(5), 1843–1860. https://doi.org/10.1051/ro/2018112

Tsao, Y.-C., & Lu, J. C. (2012). A supply chain network design considering transportation cost discounts. *Transportation Research Part E: Logistics and Transportation Review*, *48*(2), 401–414. https://doi.org/10.1016/j.tre.2011.10.004

Tsao, Y.-C., Mangotra, D., Lu, J.-C., & Dong, M. (2012). A continuous approximation approach for the integrated facility-inventory allocation problem. *European Journal of Operational Research*, *222*(2), 216–228. https://doi.org/10.1016/j.ejor.2012.04.033

Tsao, Y.-C., Zhang, Q., & Chen, T. H. (2016). Multi-item distribution network design problems under volume discount on transportation cost. *International Journal of Production Research*, *54*(2), 426–443. https://doi.org/10.1080/00207543.2015.1018453

Unnu, K., & Pazour, J. (2022). Evaluating on-demand warehousing via dynamic facility location models. *IISE Transactions*, *54*(10), 988–1003. https://doi.org/10.1080/24725854.2021.2008066

Wang, Z., Yao, D.-Q., & Huang, P. (2007). A new location-inventory policy with reverse logistics applied to B2C e-markets of China. *International Journal of Production Economics*, *107*(2), 350–363. https://doi.org/10.1016/j.ijpe.2006.09.012

Yao, L., Wei, H., & Wang, X. (2018). A Closed-Loop Location-Inventory Problem Considering Returns with Mixed Quality Defects in E-Commerce. *Proceedings of the 8th International Conference on Logistics, Informatics and Service Sciences (LISS), Canada*. https://doi.org/10.1109/LISS.2018.8593258

Zhang, H. (1998). A note on the convexity of service-level measures of the (r, q) system. *Management Science*, *44*(3), 431–432. https://doi.org/10.1287/mnsc.44.3.431

Zinn, W., Levy, M., & Bowersox, D. J. (1989). Measuring the Effect of Inventory Centralization/ Decentralization on Aggregate Safety Stock: The “Square Root Law” Revisited. *Journal of Business Logistics, 10*(1), 1–14.

Zipkin, P. (1986). Inventory service-level measures: convexity and approximation. *Management Science, 32*(8), 975–981. https://doi.org/10.1287/mnsc.32.8.975

Paper III Appendix

Appendix of the paper “Rundle in the Jungle! Why Do People Subscribe to Amazon Prime? Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program”

Reference: Straubert, C., Sucky, E., & Altewischer, D. (2024). Rundle in the Jungle! Why Do People Subscribe to Amazon Prime? Analyzing the Combination of Flat Rate and Bundle Pricing within a Loyalty Program. Proceedings of the 57th Hawaii International Conference on System Sciences, USA, 4393–4402.

III.A.1. Prime benefits

The following Prime benefits were found by us in March 2023. To find the Prime benefits, we had searched Amazon’s web pages.

Prime benefit	Long description as shown to the survey participants (short description in bold)	Short description used in the paper
“FREE Two-Day Shipping on eligible items to addresses in the contiguous US and other shipping benefits.”[6]	**Free shipping** when buying products from Amazon itself and eligible third-party sellers on Amazon.com.	Free shipping
“FREE Two-Day Shipping on eligible items to addresses in the contiguous US and other shipping benefits.”[6] “FREE Same-Day Delivery in eligible zip codes.”[6]	**Free fast shipping** (two-day shipping or faster; same-day delivery in some cases) when buying products from Amazon itself and eligible third-party sellers on Amazon.com.	Free fast shipping
“FREE Release-Date Delivery on eligible pre-order items delivered on their release date to ZIP codes within the continental US.”[6]	**Free special shipping benefits** such as release-date delivery, no-rush shipping with rewards and a weekly delivery day for the items you buy throughout the week.	Free special shipping benefits

[6] https://www.amazon.com/gp/help/customer/display.html?nodeId=G6LDPN7YJHYKH2J6

“FREE No-Rush Shipping. Select No-Rush Shipping and earn rewards for future purchases.”[6]		
“Amazon Day, where you can choose a weekly delivery day for the items you buy throughout the week.”[6]		
“Prime Try Before You Buy: Try before you buy from eligible items across women’s, men’s, kids’, and baby clothing, shoes, and accessories. You get seven days to try-on the items at home and we’ll only charge for the items that you decide to keep.”[6]	**Free 7-day trial period at home for selected items** from Amazon and eligible third-party sellers, and you will only pay for the items that you decide to keep.	Free 7-day trial period
“Prime Video offers unlimited streaming of movies and TV episodes for paid or free trial members in the US and Puerto Rico.”[6]	**Free on-demand video streaming (in general)** of movies, tv series, documentaries and more.	Free video streaming
“Award-winning Amazon Originals.”[7]	**Free on-demand streaming of exclusive movies/tv series** not available on other platforms.	Free Amazon exclusives
“Kids on Prime Video”[8]	**Free on-demand streaming of kids specific movies/tv series.**	Free kids specific content
“Sports on Prime Video”[9]	**Free live streaming of sport broadcasts**	Free live sport broadcasts

[7] https://www.amazon.com/amazonprime?ref_=nav_cs_primelink_nonmember
[8] https://www.amazon.com/gp/video/storefront/ref=atv_cat_leftpanel_kids?contentType=merch&contentId=kids
[9] https://www.amazon.com/gp/video/storefront/ref=atv_cat_sports?contentType=merch&contentId=pvsports

"Amazon Music for Prime Members provides Prime members with access to 100 million songs ad-free, the largest catalogue of ad-free top podcasts, and thousands of playlists and stations - all included with Prime at no additional cost for members in the US and Puerto Rico."[6]	**Free music/audio streaming** of shuffled music (not on-demand), podcasts and curated stations/playlists.	Free audio/music streaming, ...
"Prime Gaming, where you get free games, in-game content, and a free channel subscription on Twitch.tv every month."[6]	**Free games, game content and Twitch channel subscriptions.**	Free game(s) content, ...
"Prime Reading: Borrow books, magazines, and more from the Prime Reading catalog. Read them on your Fire tablet, Kindle e-reader, or the Kindle reading apps for iOS and Android."[6] "Amazon First Reads: Get early access to download a new book for free every month from the Amazon First Reads picks. You can also purchase hardcover titles at exclusive prices."[6]	**Free e-books, magazines, comics, and audiobooks.**	Free e-books, audiobooks, ...
"Amazon Photos: Get secure unlimited photo storage and enhanced search and organization features in Amazon Drive for you and the members of your Family Vault with Amazon Photos."[6]	**Free photo/video cloud storage.**	Free photo and video storage

"Whole Foods Market provides exclusive savings for Prime members, and 5% back for eligible Prime members with the Amazon Prime Rewards Visa Card. You can get Two-Hour Delivery in select cities (with more to come)."[6]		
"Amazon 4-star and Amazon Books stores: Prime members pay Amazon.com prices on all products in-store."[6]		
"Amazon Prime Rewards Visa Signature Card; eligible Prime members earn 5% back every day on all Amazon.com purchases, in addition to rewards everywhere else you shop."[6]	**Special discounts on physical and digital products** (e.g., Prime Day, early access to Lightning Deals, 5% off at Whole Foods with the Amazon Prime Rewards Visa Card, special deals when renting a movie to stream or a discount on an Amazon Music Unlimited subscription, ...).	Special discounts
"Eligible Prime members can get 5% back every day on Amazon.com purchases and access exclusive financing offers with an Amazon Prime Store Card."[6]		
"Amazon Fresh: Select regions get fast grocery delivery on a wide selection of groceries. Items include meat, seafood, produce, snacks, and household essentials. There are options for fast one-hour and two-hour delivery windows in select regions."[6]		

<table>
<tr><td>“Personal Shopper by Prime Try Before You Buy: Prime members in select regions can pay an additional monthly membership fee to receive a monthly styling service.”[6]</td><td rowspan="5">Special discounts on physical and digital products (e.g., Prime Day, early access to Lightning Deals, 5% off at Whole Foods with the Amazon Prime Rewards Visa Card, special deals when renting a movie to stream or a discount on an Amazon Music Unlimited subscription, ...).</td><td rowspan="5">Special discounts</td></tr>
<tr><td>“Deals and Discounts, Compliments of Amazon Family: Get up to 20% off diapers, baby food, and more through Subscribe & Save and 15% off eligible products from your baby registry.”[6]</td></tr>
<tr><td>“Prime Early Access: Get 30-minute early access to Lightning Deals on Amazon.com.”[6]</td></tr>
<tr><td>“Amazon Elements: Get access to Amazon Elements products, Amazon’s own line of everyday essentials.”[6]</td></tr>
<tr><td>“Prime members can get discounted Amazon Music Unlimited monthly plans and there are annual plans available exclusively to Prime members. For more information, go to Amazon Music Unlimited.”[6]</td></tr>
</table>

<table>
<tr><td>“With Amazon Channels, you can watch your favorite shows and movies from HBO, SHOWTIME, and STARZ channels. You don’t need a cable or additional apps and you can cancel anytime. Amazon Channels costs $4.99–$14.99/month for Prime members.”[6]</td><td rowspan="6">Special discounts on physical and digital products (e.g., Prime Day, early access to Lightning Deals, 5% off at Whole Foods with the Amazon Prime Rewards Visa Card, special deals when renting a movie to stream or a discount on an Amazon Music Unlimited subscription, ...).</td><td rowspan="6">Special discounts</td></tr>
<tr><td>“Grubhub + Membership: Redeem a free one-year Grubhub+ membership by visiting amazon.com/grubhub. Grubhub+ includes unlimited $0 delivery fees on orders over $12 as well as exclusive perks and rewards like free food and order discounts.”[6]</td></tr>
<tr><td>“Rx savings - Save on prescriptions at 60,000 pharmacies, including Walgreens, CVS, and Amazon Pharmacy.”[10]</td></tr>
<tr><td>“Prime Day - A once-a-year savings event”[10]</td></tr>
<tr><td>“Exclusive deals - A wide selection of savings”[10]</td></tr>
<tr><td>Discounted subscription Amazon Kids+[11]</td></tr>
</table>

[10] https://www.amazon.com/amazonprime?ref_=nav_cs_primelink_nonmember
[11] https://www.amazon.com/ftu/plans/ANNUAL?language=en_US

"Membership Sharing: Two adults living in the same household can create an Amazon Household to share certain Amazon Prime benefits."[6]	Not featured in our survey.	

III.A.2. Study design

First survey with n= 2062 (before quality checks, n=2200)

Only participants with a paid Prime subscription were invited to take the second survey.

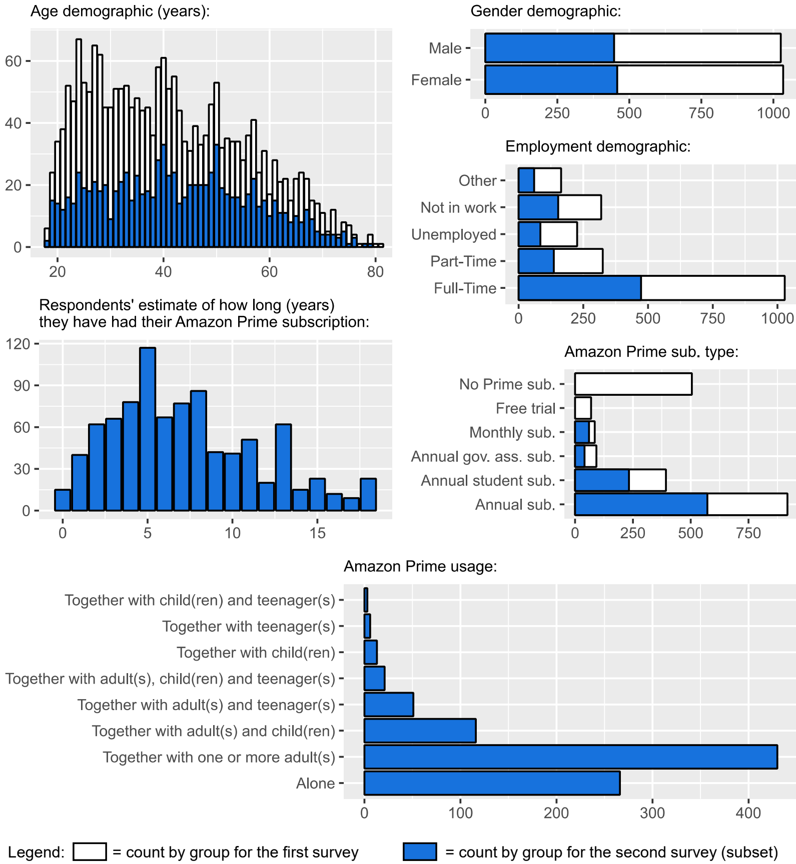
Second survey with n= 906 (before quality checks, n=1021)

Questions about:

- Online shopping frequency
- Percentage of online shopping on Amazon
- Prime membership
- Shopping basket sizes on Amazon
- US-Dollar spending on Amazon

Questions about:

- Best-worst scaling questions (repeated importance ranking of different Amazon Prime benefits)
- Likert scale questions about the importance and self-reported usage frequencies of the different benefits
- Satisfaction with the Prime subscription, …



III.A.3. Survey 1

1) Please estimate how often you order physical products on the internet **on average per month.** (one shopping basket with multiple items = one purchase)

Note: Decimal numbers are allowed.

[ ] times per month.

Prime customers	
Interval	**Frequency**
[0,1]	121
(1,2]	220
(2,3]	243
(3,4]	208
(4,5]	246
(5,6]	85
(6,7]	33
(7,8]	71
(8,9]	9
(9,10]	123
(10,11]	1
(11,12]	29
(12,13]	2
(13,14]	4
(14,15]	35
(15,16]	3
(16,17]	0
(17,18]	0
(18,19]	0
(19,20]	35
(20,30]	15
(30,40]	2
(40,50]	3

Customers without Prime	
Interval	**Frequency**
[0,1]	185
(1,2]	156
(2,3]	90
(3,4]	47
(4,5]	46
(5,6]	20
(6,7]	5
(7,8]	7
(8,9]	1
(9,10]	8
(10,11]	0
(11,12]	0
(12,13]	0
(13,14]	0
(14,15]	4
(15,16]	0
(16,17]	0
(17,18]	0
(18,19]	0
(19,20]	1
(20,30]	3
(30,40]	1
(40,50]	0

2) Do you have an Amazon Prime subscription?

Q2=1 Yes, a paid Prime annual subscription

Q2=2 Yes, a paid Prime monthly subscription

Q2=3 Yes, a paid Prime Students monthly subscription

Q2=4 Yes, a paid qualified Gov. Assistance monthly subscription

Q2=5 Yes, a free trial

Q2=6 No

Selection	Frequency
1	918
2	393
3	92
4	85
5	69
6	505

3) What percentage of your online purchases of physical products do you make through the Amazon website?

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

I do **not do** any of my online shopping for physical products on Amazon.

I do **all** my online shopping for physical products on Amazon.

Prime customers	
Interval	**Frequency**
[0,5]	7
(5,10]	20
(10,15]	38
(15,20]	45
(20,25]	43
(25,30]	65
(30,35]	57
(35,40]	73
(40,45]	33
(45,50]	110
(50,55]	47
(55,60]	101
(60,65]	50
(65,70]	169
(70,75]	100
(75,80]	169
(80,85]	92
(85,90]	159
(90,95]	64
(95,100]	46

Customers without Prime	
Interval	**Frequency**
[0,5]	40
(5,10]	32
(10,15]	38
(15,20]	28
(20,25]	33
(25,30]	35
(30,35]	17
(35,40]	39
(40,45]	9
(45,50]	55
(50,55]	11
(55,60]	31
(60,65]	14
(65,70]	42
(70,75]	21
(75,80]	56
(80,85]	6
(85,90]	35
(90,95]	12
(95,100]	20

4) When shopping on Amazon, how likely are you to order _____ item(s) per order (shopping basket size)?

Note: If you do not do any of your shopping on Amazon, please select „Never"

	Very likely	Likely	Neither likely nor unlikely	Unlikely	Very unlikely	Never
1 item per order	Q4_r1=1	Q4_r1=2	Q4_r1=3	Q4_r1=4	Q4_r1=5	Q4_r1=6
2 items per order	Q4_r2=1	Q4_r2=2	Q4_r2=3	Q4_r2=4	Q4_r2=5	Q4_r2=6
3 items per order	Q4_r3=1	Q4_r3=2	Q4_r3=3	Q4_r3=4	Q4_r3=5	Q4_r3=6
4-5 items per order	Q4_r4=1	Q4_r4=2	Q4_r4=3	Q4_r4=4	Q4_r4=5	Q4_r4=6
6-10 items per order	Q4_r5=1	Q4_r5=2	Q4_r5=3	Q4_r5=4	Q4_r5=5	Q4_r5=6

	Frequency table for Prime customers					
	1	**2**	**3**	**4**	**5**	**6**
Q4_r1	594	564	92	152	70	16
Q4_r2	357	892	123	86	19	11
Q4_r3	244	645	301	219	67	12
Q4_r4	100	319	261	451	283	74
Q4_r5	47	123	162	370	458	328

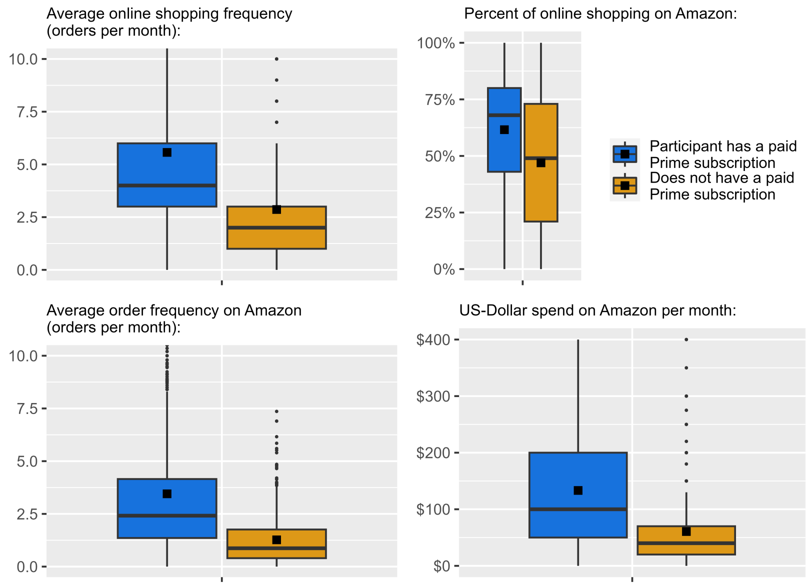
	Frequency table for customers without Prime					
	1	**2**	**3**	**4**	**5**	**6**
Q4_r1	157	166	49	97	69	36
Q4_r2	77	312	76	64	21	24
Q4_r3	69	252	107	74	43	29
Q4_r4	47	123	77	141	119	67
Q4_r5	10	36	47	126	179	176

5) On average, how much money (US-Dollar) do you spend on Amazon **per month**?

______ $ per month.

Prime customers	
Interval	**Frequency**
(0,20]	117
(20,40]	207
(40,60]	266
(60,80]	107
(80,100]	244
(100,120]	27
(120,140]	27
(140,160]	109
(160,180]	8
(180,200]	154
(200,220]	1
(220,240]	6
(240,260]	57
(260,280]	2
(280,300]	65
(300,320]	1
(320,340]	0
(340,360]	14
(360,380]	1
(380,400]	16
(400,420]	0
(420,440]	0
(440,460]	4
(460,480]	0
(480,500]	28
(500,750]	11
(750,1000]	12
(1000,1250]	0
(1250,1500]	4

Customers without Prime	
Interval	**Frequency**
(0,20]	155
(20,40]	160
(40,60]	111
(60,80]	29
(80,100]	54
(100,120]	6
(120,140]	3
(140,160]	19
(160,180]	1
(180,200]	11
(200,220]	1
(220,240]	0
(240,260]	5
(260,280]	1
(280,300]	8
(300,320]	0
(320,340]	0
(340,360]	1
(360,380]	0
(380,400]	3
(400,420]	0
(420,440]	0
(440,460]	1
(460,480]	0
(480,500]	4
(500,750]	0
(750,1000]	0
(1000,1250]	0
(1250,1500]	0


Average online shopping frequency
(orders per month):
10.0
7.5
5.0
2.5
0.0
Percent of online shopping on Amazon:
100%
75%
50%
25%
0%
Participant has a paid
Prime subscription
Does not have a paid
Prime subscription
Average order frequency on Amazon
(orders per month):
10.0
7.5
5.0
2.5
0.0
US-Dollar spend on Amazon per month:
$400
$300
$200
$100
$0

The average order frequency on Amazon was calculated by multiplying the answers to the questions 1) and 3). Following, a frequency table for this calculated variable:

Prime customers	
Interval	**Frequency**
[0,1]	280
(1,2]	341
(2,3]	281
(3,4]	190
(4,5]	115
(5,6]	73
(6,7]	42
(7,8]	47
(8,9]	32
(9,10]	22
(10,11]	7
(11,12]	11
(12,13]	3
(13,14]	10
(14,15]	5
(15,30]	27
(30,45]	2

Customers without Prime	
Interval	**Frequency**
[0,1]	337
(1,2]	131
(2,3]	59
(3,4]	28
(4,5]	9
(5,6]	4
(6,7]	3
(7,8]	1
(8,9]	0
(9,10]	0
(10,11]	0
(11,12]	1
(12,13]	0
(13,14]	1
(14,15]	0
(15,30]	0
(30,45]	0

III.A.4. Survey 2

Do you have an Amazon Prime subscription?

PrimeSelect=1 Yes, a paid Prime annual subscription

PrimeSelect=2 Yes, a paid Prime monthly subscription

PrimeSelect=3 Yes, a paid Prime Students monthly subscription

PrimeSelect=4 Yes, a paid qualified Gov. Assistance monthly subscription

PrimeSelect=5 Yes, a free trial

PrimeSelect=6 No

Selection	Frequency
1	572
2	233
3	41
4	60
5	0
6	0

Within the next 13 questions you will identify the Amazon Prime benefits that are most and least important to you. Basically, you are answering the question of why you have an Amazon Prime subscription. Within these questions, we use **abbreviated labels** for the various Amazon Prime benefits. So please take a moment to read the following detailed information about the Amazon Prime benefits we will be referring to in the following questions:

Free shipping when buying products from Amazon itself and eligible third-party sellers on Amazon.com.

Free fast shipping (two-day shipping or faster; same-day delivery in some cases) when buying products from Amazon itself and eligible third-party sellers on Amazon.com.

Free special shipping benefits such as release-date delivery, no-rush shipping with rewards and a weekly delivery day for the items you buy throughout the week.

Free 7-day trial period at home for selected items from Amazon and eligible third-party sellers, and you will only pay for the items that you decide to keep.

Free on-demand video streaming (in general) of movies, tv series, documentaries and more.

Free on-demand streaming of exclusive movies/tv series not available on other platforms.

Free on-demand streaming of kids specific movies/tv series.

Free live streaming of sport broadcasts.

Free music/audio streaming of shuffled music (not on-demand), podcasts and curated stations/playlists.

Free games, game content and Twitch channel subscriptions.

Free e-books, magazines, comics, and audiobooks.

Free photo/video cloud storage.

Special discounts on physical and digital products (e.g., Prime Day, early access to Lightning Deals, 5% off at Whole Foods with the Amazon Prime Rewards Visa Card, special deals when renting a movie to stream or a discount on an Amazon Music Unlimited subscription, ...).

But before we start with our questions, a comprehension test. **Based on the information above**, which services are part of the Amazon Prime subscription?

- [ ] Free audiobooks
- [ ] Free on-demand video streaming
- [ ] Free drone delivery
- [ ] Free personalized fashion advice
- [ ] Free games
- [ ] Free photo cloud storage
- [ ] Free CO2 neutral delivery
- [ ] Free e-books

Please answer the following questions from your individual perspective.

We will now start with our most-least scaling questions ...

Please think about how important different **benefits of the Amazon Prime subscription** are to you. Why do you have an Amazon Prime subscription? Considering just the following four benefits: Which of the four benefits is the **most important** to you, and which is the **least important**?

(1 of 13)

Most Important		Least Important
◯	Special discounts on physical and digital products	◯
◯	Free on-demand streaming of exclusive movies/tv	◯
◯	Free music/audio streaming	◯
◯	Free games, game content and Twitch channel subscriptions	◯

• • •

Currently, the following Amazon Prime benefit **is important** to me:

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
Free shipping	Q3_r1=1	Q3_r1=2	Q3_r1=3	Q3_r1=4	Q3_r1=5
Free fast shipping	Q3_r2=1	Q3_r2=2	Q3_r2=3	Q3_r2=4	Q3_r2=5
Free special shipping benefits	Q3_r3=1	Q3_r3=2	Q3_r3=3	Q3_r3=4	Q3_r3=5
Free 7-day trial period at home for selected items	Q3_r4=1	Q3_r4=2	Q3_r4=3	Q3_r4=4	Q3_r4=5
Select "Neutral" here, or you will fail this attention check.	Q3_r5=1	Q3_r5=2	Q3_r5=3	Q3_r5=4	Q3_r5=5
Free on-demand video streaming (in general)	Q3_r6=1	Q3_r6=2	Q3_r6=3	Q3_r6=4	Q3_r6=5
Free on-demand streaming of exclusive movies/tv	Q3_r7=1	Q3_r7=2	Q3_r7=3	Q3_r7=4	Q3_r7=5

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
Free on-demand streaming of kids specific movies/tv series	Q3_r8=1	Q3_r8=2	Q3_r8=3	Q3_r8=4	Q3_r8=5
Free live streaming of sport broadcasts	Q3_r9=1	Q3_r9=2	Q3_r9=3	Q3_r9=4	Q3_r9=5
Free music/audio streaming	Q3_r10=1	Q3_r10=2	Q3_r10=3	Q3_r10=4	Q3_r10=5
Free games, game content and Twitch channel subscriptions	Q3_r11=1	Q3_r11=2	Q3_r11=3	Q3_r11=4	Q3_r11=5
Free e-books, magazines, comics, and audiobooks	Q3_r12=1	Q3_r12=2	Q3_r12=3	Q3_r12=4	Q3_r12=5
Free photo/video cloud storage	Q3_r13=1	Q3_r13=2	Q3_r13=3	Q3_r13=4	Q3_r13=5
Special discounts on physical and digital products	Q3_r14=1	Q3_r14=2	Q3_r14=3	Q3_r14=4	Q3_r14=5

	Frequency table				
	1	**2**	**3**	**4**	**5**
Q3_r1	785	112	6	1	2
Q3_r2	729	150	21	5	1
Q3_r3	293	380	195	34	4
Q3_r4	41	157	307	282	119
Q3_r5	0	2	904	0	0
Q3_r6	303	429	104	61	9
Q3_r7	235	427	166	72	6
Q3_r8	38	148	217	276	227
Q3_r9	73	156	184	194	299
Q3_r10	77	252	293	215	69
Q3_r11	53	151	228	258	216
Q3_r12	50	268	299	200	89
Q3_r13	58	157	264	253	174
Q3_r14	156	389	249	87	25

Still looking at the same 13 benefits, which benefits are **actually important** to you?

	Important	Not important
Free shipping	Q4_r1=1	Q4_r1=2
Free fast shipping	Q4_r2=1	Q4_r2=2
Free special shipping benefits	Q4_r3=1	Q4_r3=2
Free 7-day trial period at home for selected items	Q4_r4=1	Q4_r4=2
Free on-demand video streaming (in general)	Q4_r5=1	Q4_r5=2
Free on-demand streaming of exclusive movies/tv	Q4_r6=1	Q4_r6=2
Free on-demand streaming of kids specific movies/tv series	Q4_r7=1	Q4_r7=2
	Important	Not important
Free live streaming of sport broadcasts	Q4_r8=1	Q4_r8=2
Select "Important" here, or you will fail this attention check.	Q4_r9=1	Q4_r9=2
Free music/audio streaming	Q4_r10=1	Q4_r10=2
Free games, game content and Twitch channel subscriptions	Q4_r11=1	Q4_r11=2
Free e-books, magazines, comics, and audiobooks	Q4_r12=1	Q4_r12=2
Free photo/video cloud storage	Q4_r13=1	Q4_r13=2
Special discounts on physical and digital products	Q4_r14=1	Q4_r14=2

	Frequency table	
	1	**2**
Q4_r1	901	5
Q4_r2	875	31
Q4_r3	664	242
Q4_r4	178	728
Q4_r5	719	187
Q4_r6	633	273
Q4_r7	173	733
Q4_r8	228	678
Q4_r9	903	3
Q4_r10	369	537
Q4_r11	209	697
Q4_r12	344	562
Q4_r13	229	677
Q4_r14	550	356

Which Amazon Prime benefits are you **currently using**?

	Very often	Often	Sometimes	Rarely	Very rarely or never
Free shipping	Q5_r1=1	Q5_r1=2	Q5_r1=3	Q5_r1=4	Q5_r1=5
Free fast shipping	Q5_r2=1	Q5_r2=2	Q5_r2=3	Q5_r2=4	Q5_r2=5
Free special shipping benefits	Q5_r3=1	Q5_r3=2	Q5_r3=3	Q5_r3=4	Q5_r3=5
Free 7-day trial period at home for selected items	Q5_r4=1	Q5_r4=2	Q5_r4=3	Q5_r4=4	Q5_r4=5
Free on-demand video streaming (in general)	Q5_r5=1	Q5_r5=2	Q5_r5=3	Q5_r5=4	Q5_r5=5
Free on-demand streaming of exclusive movies/tv	Q5_r6=1	Q5_r6=2	Q5_r6=3	Q5_r6=4	Q5_r6=5
Free on-demand streaming of kids specific movies/tv series	Q5_r7=1	Q5_r7=2	Q5_r7=3	Q5_r7=4	Q5_r7=5

	Very often	Often	Sometimes	Rarely	Very rarely or never
Free live streaming of sport broadcasts	Q5_r8=1	Q5_r8=2	Q5_r8=3	Q5_r8=4	Q5_r8=5
Free music/audio streaming	Q5_r9=1	Q5_r9=2	Q5_r9=3	Q5_r9=4	Q5_r9=5
Free games, game content and Twitch channel subscriptions	Q5_r10=1	Q5_r10=2	Q5_r10=3	Q5_r10=4	Q5_r10=5
Free e-books, magazines, comics, and audiobooks	Q5_r11=1	Q5_r11=2	Q5_r11=3	Q5_r11=4	Q5_r11=5
Free photo/video cloud storage	Q5_r12=1	Q5_r12=2	Q5_r12=3	Q5_r12=4	Q5_r12=5
Special discounts on physical and digital products	Q5_r13=1	Q5_r13=2	Q5_r13=3	Q5_r13=4	Q5_r13=5
Select "Often" here, or you will fail this attention check.	Q5_r14=1	Q5_r14=2	Q5_r14=3	Q5_r14=4	Q5_r14=5

	Frequency table				
	1	**2**	**3**	**4**	**5**
Q5_r1	724	149	30	3	0
Q5_r2	598	187	88	27	6
Q5_r3	215	167	293	143	88
Q5_r4	11	29	103	173	590
Q5_r5	213	233	275	109	76
Q5_r6	165	208	301	154	78
Q5_r7	28	47	118	223	490
Q5_r8	37	60	118	146	545
Q5_r9	59	93	194	194	366
Q5_r10	54	55	112	141	544
Q5_r11	26	76	229	234	341
Q5_r12	57	58	87	114	590
Q5_r13	77	164	322	165	178
Q5_r14	1	902	1	0	2

On average, how many hours **per week** do you spend watching **Amazon Prime Video**?

Note: 21 hours per week equals 3 hours per day. Keep in mind that you may have a different usage time on weekends compared to weekdays.

[ ] hours per week.

Interval	Frequency
[0,1]	194
(1,2]	123
(2,3]	73
(3,4]	80
(4,5]	96
(5,6]	66
(6,7]	30
(7,8]	27
(8,9]	4
(9,10]	66

Interval	Frequency
(10,11]	3
(11,12]	22
(12,13]	0
(13,14]	18
(14,15]	26
(15,16]	8
(16,17]	5
(17,18]	9
(18,19]	1
(19,20]	17

Interval	Frequency
(20,21]	6
(21,22]	0
(22,23]	0
(23,24]	4
(24,25]	3
(25,26]	0
(26,27]	1
(27,28]	4
(28,29]	0
(29,30]	9

Interval	Frequency
(30,40]	8
(40,50]	2
(50,60]	0
(60,70]	0
(70,80]	0
(80,90]	1
(90,100]	0
(100,110]	0
(110,120]	0
(120,130]	0

How **satisfied** are you with your Amazon Prime subscription?

	Very satisfied	Satisfied	Neutral	Dissatisfied	Very dissatisfied
Your satisfaction with your Amazon Prime subscription	Q7_r1=1	Q7_r1=2	Q7_r1=3	Q7_r1=4	Q7_r1=5

Selection	**Frequency**
1	318
2	484
3	76
4	23
5	5

What mainly applies to you? **I use my/our Amazon Prime subscription ...**

Q2=1 Alone

Q2=2 Together with one or more adult(s)

Q2=3 Together with adult(s) and child(ren)

Q2=4 Together with adult(s) and teenager(s)

Q2=5 Together with adult(s), child(ren) and teenager(s)

Q2=6 Together with child(ren)

Q2=7 Together with teenager(s)

Q2=8 Together with child(ren) and teenager(s)

Selection	**Frequency**
1	266
2	430
3	116
4	51
5	21
6	13
7	6
8	3

Are you planning to **cancel** your Amazon Prime subscription within the next year?

Q8=1 Yes

Q8=2 No

Selection	Frequency
1	55
2	851

Please estimate how long you have had your Amazon Prime subscription.

since 2023
since 2022
since 2021
since 2020
since 2019
since 2018
since 2017
since 2016
since 2015
since 2014
since 2013
since 2012
since 2011
since 2010
since 2009
since 2008
since 2007
since 2006
since 2005

Selection	Frequency
since 2023	15
since 2022	40
since 2021	62
since 2020	66
since 2019	78
since 2018	117
since 2017	67
since 2016	77
since 2015	86
since 2014	42
since 2013	41
since 2012	51
since 2011	20
since 2010	62
since 2009	15
since 2008	23
since 2007	12
since 2006	9
since 2005	23

On average, how much money (US-Dollar) do you personally have available **for shopping each month**?

Q1=1 Less than 150$

Q1=2 150$ to 299$

Q1=3 300$ to 449$

Q1=4 450$ to 599$

Q1=5 600$ to 799$

Q1=6 800$ to 999$

Q1=7 1 000$ to 1 499$

Q1=8 1 500$ to 1 999$

Q1=9 2 000$ to 2 999$

Q1=10 3 000$ to 3 999$

Q1=11 4 000$ and more

Selection	Frequency
1	221
2	259
3	149
4	100
5	38
6	33
7	58
8	15
9	20
10	4
11	9

III.A.4. Additional statistics

Count statistics for the BWS experiment:

Label	Times Shown - Best	Times Selected Best	Best Count Proportion	Times Shown - Worst	Times Selected Worst	Worst Count Proportion
Free shipping	3628.0	2610.0	0.719	3628.0	31.0	0.009
Free fast shipping	3628.0	2974.0	0.820	3628.0	48.0	0.013
Free special shipping benefits	3628.0	1287.0	0.355	3628.0	313.0	0.086
Free 7-day trial period at home for selected items	3628.0	238.0	0.066	3628.0	1523.0	0.420
Free on-demand video streaming (in general)	3628.0	1295.0	0.357	3628.0	182.0	0.050
Free on-demand streaming of exclusive movies/tv	3628.0	1066.0	0.294	3628.0	262.0	0.072
Free on-demand streaming of kids specific movies/tv series	3628.0	200.0	0.055	3628.0	1772.0	0.488
Free live streaming of sport broadcasts	3628.0	294.0	0.081	3628.0	1871.0	0.516
Free music/audio streaming	3628.0	356.0	0.098	3628.0	856.0	0.236
Free games, game content and Twitch channel subscriptions	3628.0	249.0	0.069	3628.0	1754.0	0.483
Free e-books, magazines, comics, and audiobooks	3628.0	270.0	0.074	3628.0	1088.0	0.300
Free photo/video cloud storage	3628.0	204.0	0.056	3628.0	1551.0	0.428
Special discounts on physical and digital products	3628.0	748.0	0.206	3628.0	540.0	0.149

Hierarchical Bayes results of the BWS experiment without anchoring:

Label	Average
Free shipping	19.47868
Free fast shipping	19.47830
Free special shipping benefits	13.42198
Free 7-day trial period at home for selected items	1.82947
Free on-demand video streaming (in general)	13.26041
Free on-demand streaming of exclusive movies/tv	11.21289
Free on-demand streaming of kids specific movies/tv series	1.51290
Free live streaming of sport broadcasts	2.62654
Free music/audio streaming	3.23441
Free games, game content and Twitch channel subscriptions	2.01936
Free e-books, magazines, comics, and audiobooks	2.32705
Free photo/video cloud storage	1.82498
Special discounts on physical and digital products	7.77301

Multiple regression: Satisfaction with the Prime subscription against the answers to the Likert scale questions about the importance of the 13 Prime benefits.

```
Call:
lm(formula = Q7_r1 ~ Q3_r1 + Q3_r2 + Q3_r3 + Q3_r4 + Q3_r6 +
    Q3_r7 + Q3_r8 + Q3_r9 + Q3_r10 + Q3_r11 + Q3_r12 + Q3_r13 +
    Q3_r14, data = RawDataCombined)

Residuals:
    Min      1Q  Median      3Q     Max
-1.6674 -0.6488  0.0900  0.2957  3.4004

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.771386   0.174493   4.421  1.1e-05 ***
Q3_r1        0.186795   0.063065   2.962  0.00314 **
Q3_r2        0.092083   0.051248   1.797  0.07271 .
Q3_r3       -0.021715   0.031694  -0.685  0.49342
Q3_r4        0.057882   0.024166   2.395  0.01682 *
Q3_r6        0.082153   0.038313   2.144  0.03228 *
Q3_r7        0.051066   0.037958   1.345  0.17886
Q3_r8        0.007920   0.021818   0.363  0.71670
Q3_r9        0.019335   0.018708   1.034  0.30164
Q3_r10       0.063099   0.025065   2.517  0.01200 *
Q3_r11      -0.009341   0.021135  -0.442  0.65862
Q3_r12      -0.004758   0.025060  -0.190  0.84944
Q3_r13      -0.004150   0.023023  -0.180  0.85701
Q3_r14       0.026688   0.027230   0.980  0.32731
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7199 on 892 degrees of freedom
Multiple R-squared:  0.06569, Adjusted R-squared:  0.05208
F-statistic: 4.825 on 13 and 892 DF,  p-value: 3.503e-08
```

Multiple regression: Satisfaction with the Prime subscription against the answers to the Likert scale questions about the usage of the 13 Prime benefits.

```
Call:
lm(formula = Q7_r1 ~ Q5_r1 + Q5_r2 + Q5_r3 + Q5_r4 + Q5_r5 +
    Q5_r6 + Q5_r7 + Q5_r8 + Q5_r9 + Q5_r10 + Q5_r11 + Q5_r12 +
    Q5_r13, data = RawDataCombined)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3932 -0.5925  0.0569  0.3359  3.3141

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.626389   0.181559   3.450 0.000587 ***
Q5_r1        0.241669   0.050491   4.786 1.99e-06 ***
Q5_r2        0.053638   0.032258   1.663 0.096712 .
Q5_r3       -0.019331   0.021534  -0.898 0.369570
Q5_r4        0.046763   0.027530   1.699 0.089742 .
Q5_r5        0.055486   0.032724   1.696 0.090321 .
Q5_r6        0.059413   0.033020   1.799 0.072307 .
Q5_r7       -0.003564   0.023844  -0.149 0.881226
Q5_r8       -0.012120   0.021529  -0.563 0.573599
Q5_r9        0.061699   0.020885   2.954 0.003217 **
Q5_r10       0.013844   0.019934   0.694 0.487557
Q5_r11       0.006762   0.023439   0.288 0.773045
Q5_r12      -0.001184   0.020432  -0.058 0.953788
Q5_r13       0.026436   0.021661   1.220 0.222623
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.702 on 892 degrees of freedom
Multiple R-squared:  0.1118,	Adjusted R-squared:  0.09885
F-statistic: 8.636 on 13 and 892 DF,  p-value: < 2.2e-16
```

Multiple regression: Satisfaction with the Prime subscription against the answers to the Likert scale questions both about the importance and the usage of the 13 Prime benefits.

```
Call:
lm(formula = Q7_r1 ~ Q5_r1 + Q5_r2 + Q5_r3 + Q5_r4 + Q5_r5 +
    Q5_r6 + Q5_r7 + Q5_r8 + Q5_r9 + Q5_r10 + Q5_r11 + Q5_r12 +
    Q5_r13 + Q3_r1 + Q3_r2 + Q3_r3 + Q3_r4 + Q3_r6 + Q3_r7 +
    Q3_r8 + Q3_r9 + Q3_r10 + Q3_r11 + Q3_r12 + Q3_r13 + Q3_r14,
    data = RawDataCombined)

Residuals:
    Min      1Q  Median      3Q     Max
-1.5190 -0.5644  0.0370  0.3301  3.2748

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.277293   0.214116   1.295   0.1956
Q5_r1        0.242944   0.055043   4.414 1.14e-05 ***
Q5_r2        0.018178   0.039341   0.462   0.6441
Q5_r3       -0.013972   0.025913  -0.539   0.5899
Q5_r4        0.015423   0.034335   0.449   0.6534
Q5_r5        0.038805   0.037790   1.027   0.3048
Q5_r6        0.056963   0.039223   1.452   0.1468
Q5_r7        0.011262   0.035076   0.321   0.7482
Q5_r8       -0.048250   0.035826  -1.347   0.1784
Q5_r9        0.052076   0.030212   1.724   0.0851 .
Q5_r10       0.059479   0.031998   1.859   0.0634 .
Q5_r11       0.010563   0.032405   0.326   0.7445
Q5_r12      -0.001534   0.028089  -0.055   0.9565
Q5_r13       0.018555   0.027758   0.668   0.5040
Q3_r1        0.090964   0.066224   1.374   0.1699
Q3_r2        0.081148   0.061252   1.325   0.1856
Q3_r3       -0.005421   0.036824  -0.147   0.8830
Q3_r4        0.059855   0.029336   2.040   0.0416 *
Q3_r6        0.044271   0.043916   1.008   0.3137
Q3_r7        0.002695   0.044105   0.061   0.9513
Q3_r8       -0.005767   0.031423  -0.184   0.8544
Q3_r9        0.047478   0.030169   1.574   0.1159
Q3_r10       0.014561   0.035489   0.410   0.6817
Q3_r11      -0.047916   0.032847  -1.459   0.1450
Q3_r12      -0.011284   0.033842  -0.333   0.7389
Q3_r13       0.008322   0.030990   0.269   0.7884
Q3_r14       0.009157   0.034467   0.266   0.7906
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7001 on 879 degrees of freedom
Multiple R-squared:  0.1294,	Adjusted R-squared:  0.1037
F-statistic: 5.025 on 26 and 879 DF,  p-value: 1.231e-14
```

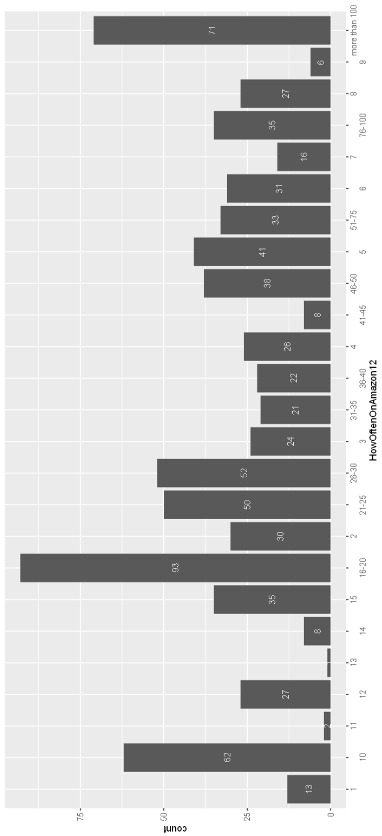
Paper IV Appendix

Appendix of the paper „Making Third-Party Sellers More Attractive – The Case of Amazon“

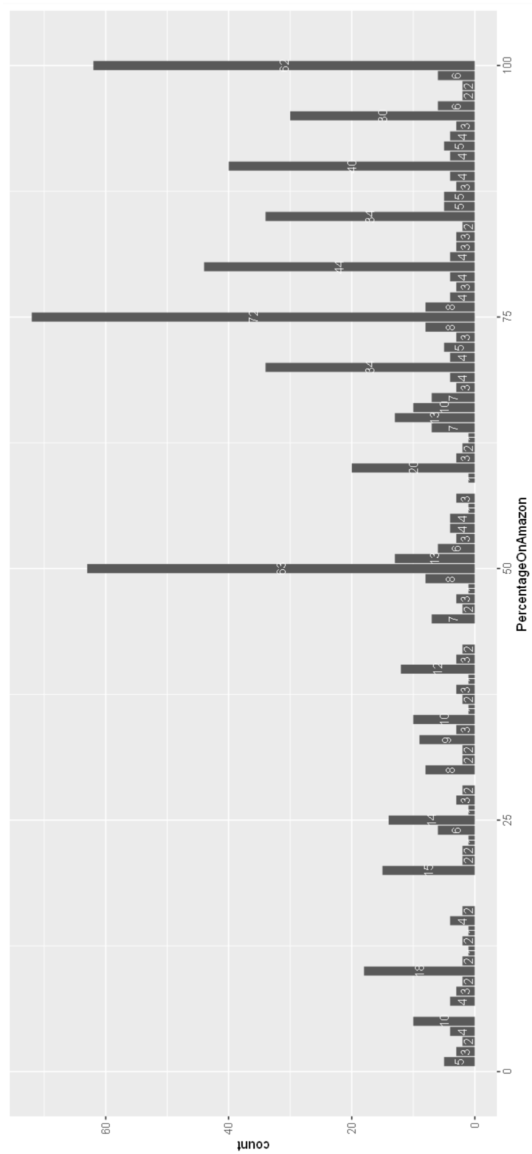
Reference: Straubert, C., Sucky, E., Felch, V., Karl, D., & Altewischer, D. (2023). Making Third-Party Sellers More Attractive—The Case of Amazon. Proceedings of the 56th Hawaii International Conference on System Sciences, USA, 3838–3847.

IV.A.1. Survey questions and descriptive results

Please estimate how many physical products you have ordered from the Amazon online store, i.e., Amazon.com, in the past 12 months (excluding fresh food/groceries)?

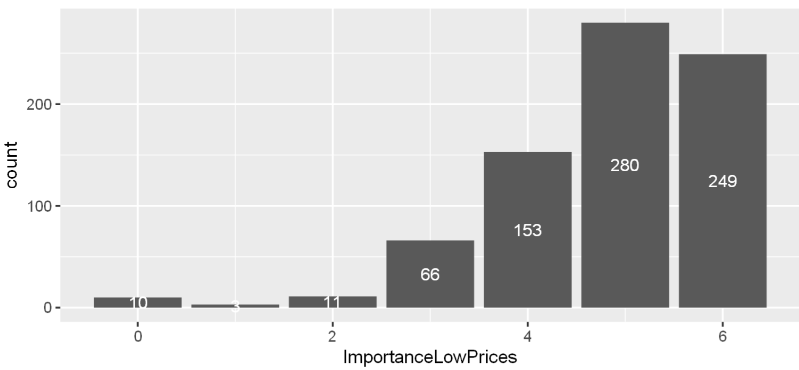


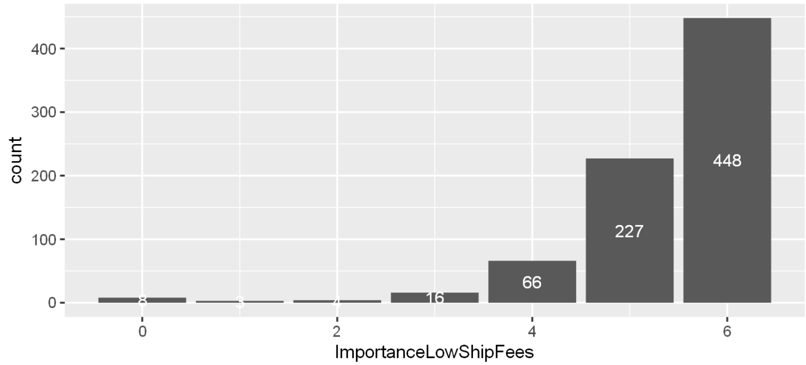
What **percentage** of your total online purchases **in the past 12 months** have you bought on Amazon.com (based on the items you purchased online)?

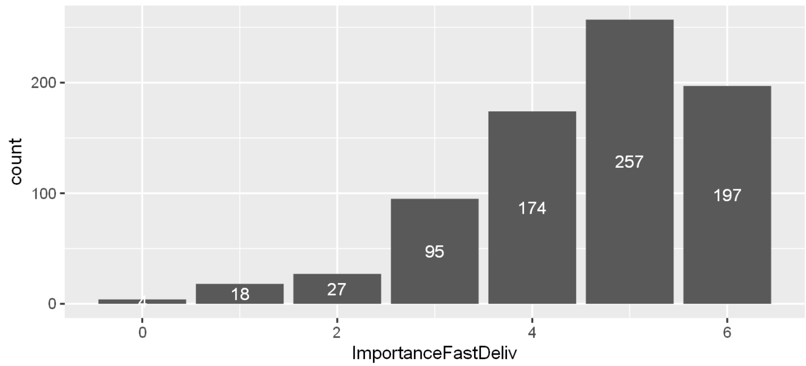


In general, how important are the following factors/aspects to you when shopping online?

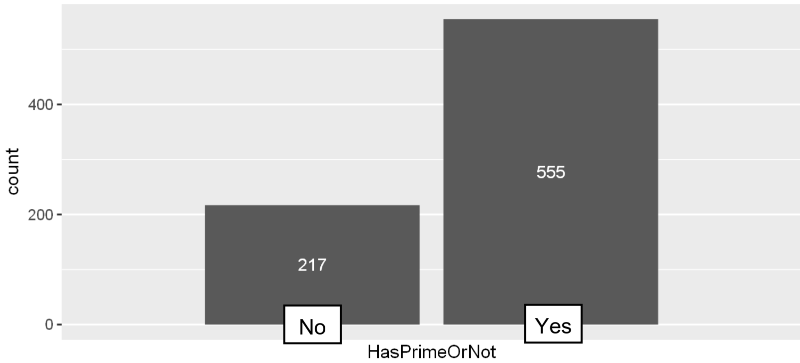
	Strongly disagree	Disagree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
Low item prices are important to me when shopping online.							
Low shipping fees are important to me when shopping online.							
Fast delivery is important to me when shopping online.							




count
ImportanceLowShipFees
8
3
4
16
66
227
448


count
ImportanceFastDeliv
4
18
27
95
174
257
197

Do you have an Amazon Prime subscription? (Yes/No)

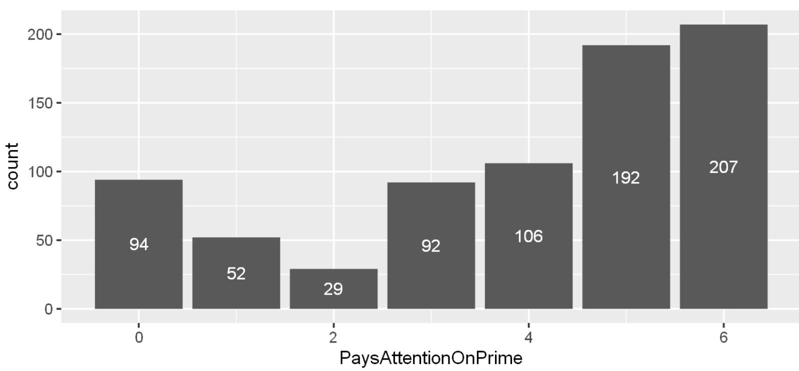


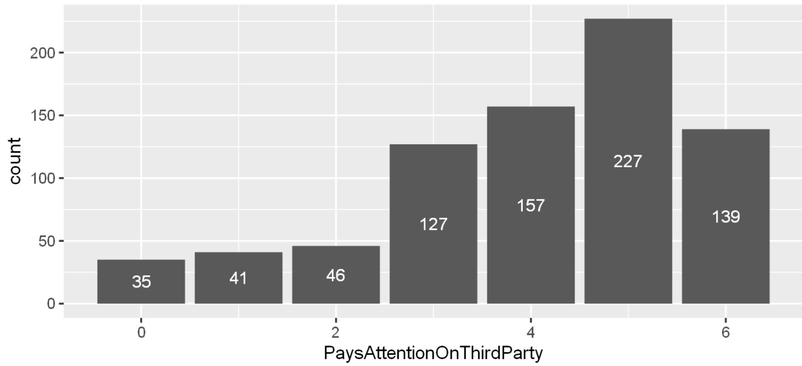
Additional Information

In addition to classic online stores (e.g., Nike.com, Apple.com, Wine.com, ...), there are online marketplaces, such as the Amazon website (Amazon.com) with the Amazon marketplace. On these marketplaces, various sellers can offer their products. On Amazon.com these non-Amazon sellers are called "third-party sellers".

When I shop on Amazon.com, I consciously pay attention to ...
(If you do not pay attention to the distinctions mentioned below, please select "Strongly Disagree".)

	Strongly disagree	Disagree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
.. whether an offer on Amazon.com has **Prime service** available or not.							
... whether an offer is from **Amazon itself** or from **a third-party seller**.							
... whether a **third-party seller offer/item** is **shipped** from the **third-party seller** or from **Amazon itself.**							

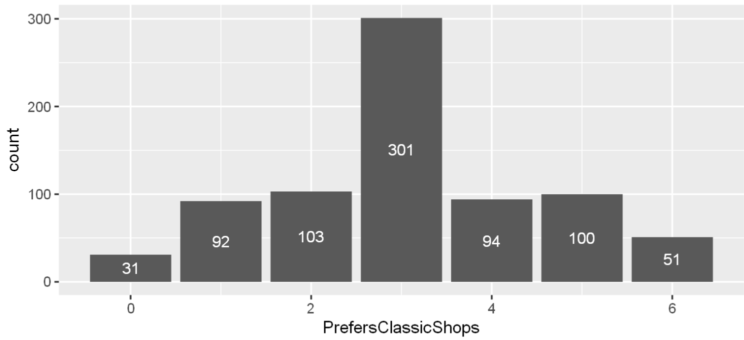


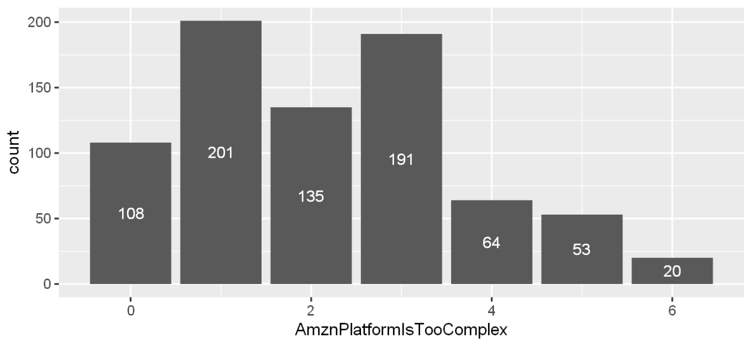

count
200
150
100
50
0
35
41
46
127
157
227
139
0
2
4
6
PaysAttentionOnThirdParty


count
200
150
100
50
0
38
51
45
132
142
221
143
0
2
4
6
PaysAttentionOnShipFromAmzn

How do you rate the following statements?

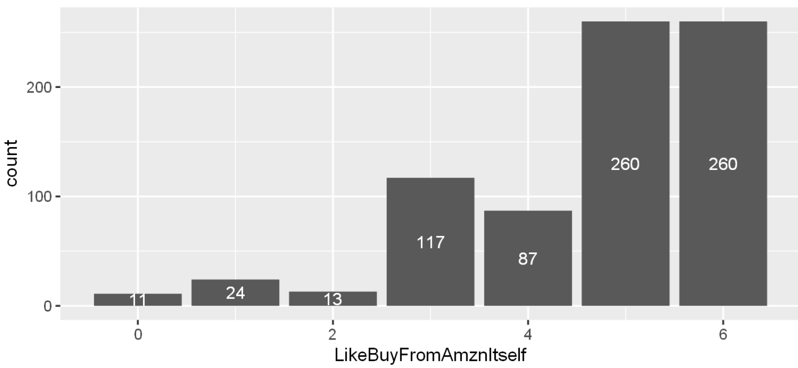
	Strongly disagree	Disa-gree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
I prefer classic online stores compared to online shopping marketplaces where many different sellers offer their goods.							
I find the Amazon marketplace (i.e., the Amazon.com store with the optional Prime subscription, its different seller types, and rules) too complex.							

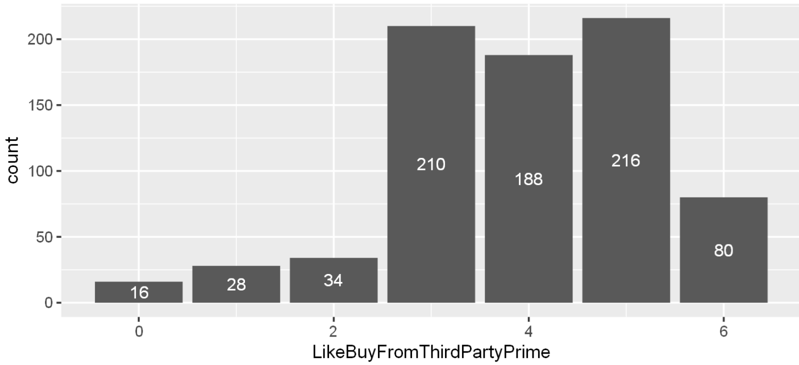


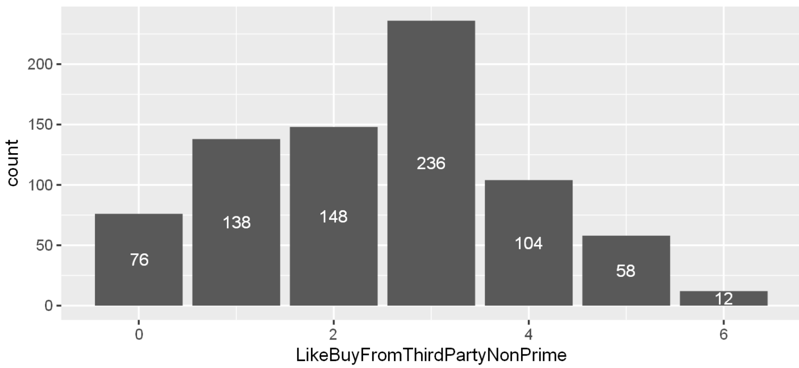


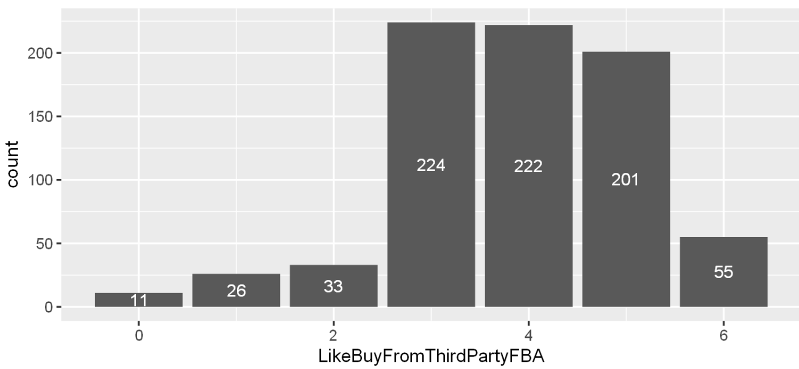
How much do you like buying on Amazon.com ...

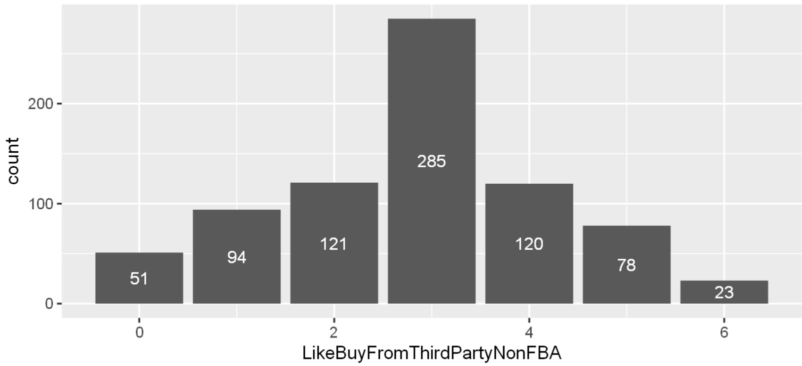
	Strongly dislike	Dislike	More or less dislike	Neutral	More or less like	Like	Strongly like
... from Amazon itself (**with** Prime service available)?							
... from a third-party seller **with** Prime service available?							
... from a third-party seller **without** Prime service available?							
... from a third-party seller **but** the product is shipped from Amazon itself?							
... from a third-party seller that **also** ships the product (i.e., the product is not shipped from Amazon).							




count
200
150
100
50
0
16
28
34
210
188
216
80
0
2
4
6
LikeBuyFromThirdPartyPrime


count
200
150
100
50
0
76
138
148
236
104
58
12
0
2
4
6
LikeBuyFromThirdPartyNonPrime


count
200
150
100
50
0
11
26
33
224
222
201
55
0
2
4
6
LikeBuyFromThirdPartyFBA


count
200
100
0
285
51
94
121
120
78
23
0
2
4
6
LikeBuyFromThirdPartyNonFBA

Scenario:

You plan to buy a specific product on Amazon.com. All offers for the product on the Amazon marketplace (Amazon.com) have the same product price and this price is acceptable to you.

Assume that the product is not sold by Amazon itself at all. Please assume that the other buying options listed below have normal product availability.

(The sum of the drop downs must equal 100%.)

(Percent drop-downs; 0%–100%; 5% increments)	... order on Amazon from a third-party seller **with** Prime and **shipping from Amazon itself**, is	... order on Amazon from a third-party seller **without** Prime and **shipping from the third-party**, is	... order from another online store, is	... buy from a local physical store, is	... simply do not buy the product at all, is	... choose another alternative (not listed), is
On average, what do you do? The probability that I will ...	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)

Still assume that the product is not sold by Amazon itself at all.

Now additionally assume that the product is also not sold by a third-party seller with Prime service and/or with shipping from Amazon itself. Please assume that the other buying options listed below have normal product availability.

(The sum of the drop downs must equal 100%.)

(Percent drop-downs; 0%–100%; 5% increments)	... order on Amazon from a third-party seller **without** Prime and **shipping from the third-party**, is	... order from another online store, is	... buy from a local physical store, is	... simply do not buy the product at all, is	... choose another alternative (not listed), is
On average, what do you do? The probability that I will ...	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)	(Percent drop-down)

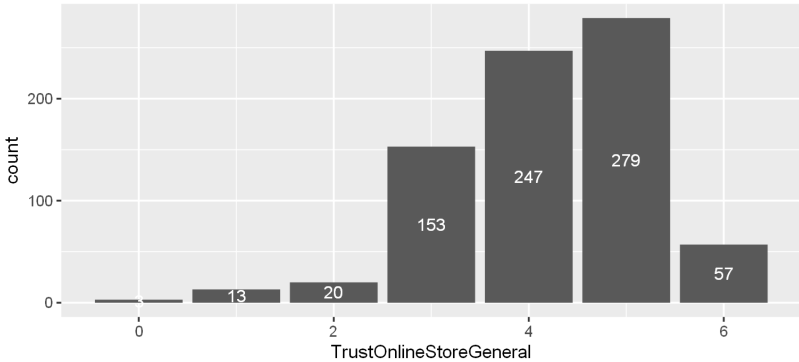
Additional Information

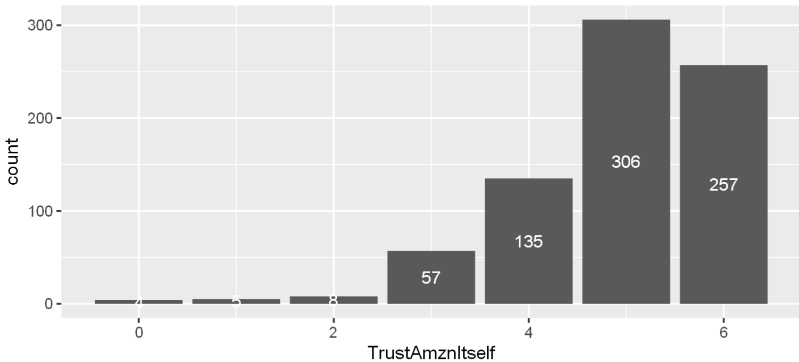
Please note for the following questions that when we talk about a "good experience" we mean that ideally everything goes according to plan, and if occasionally something does not go according to plan, that the service and troubleshooting are very good.

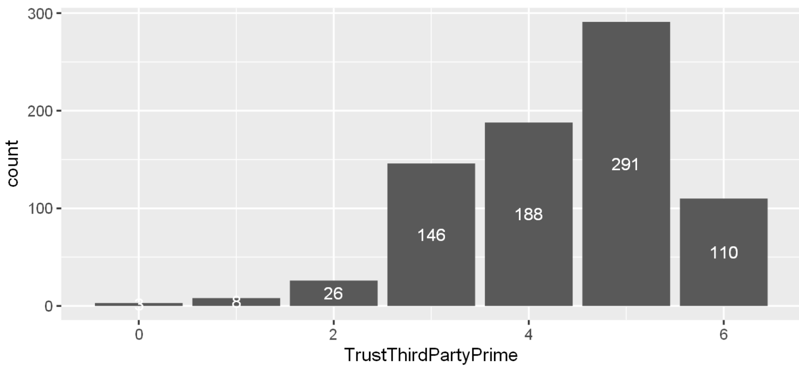
The concept "trust" is defined as "to have confidence in something, or to believe in someone". "Someone" in our context would be the companies/sellers and their personnel.

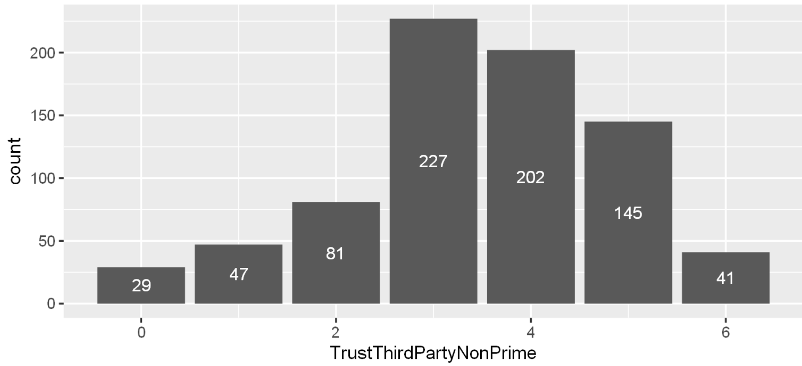
How do you rate the following statements? I trust that I will have a good experience when I buy something ...

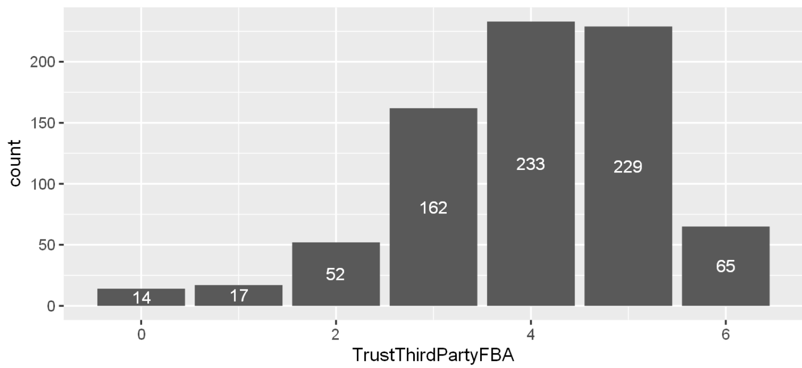
	Strongly disagree	Disagree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
... from an online shop in general.							
... on Amazon.com from **Amazon itself**.							
... on Amazon.com from a third-party seller **with** Prime service available.							
... on Amazon.com from a third-party seller **without** Prime service available.							
... on Amazon.com with shipping from Amazon itself **but** sold from a third-party seller.							
... on Amazon.com from a third-party seller that **also** ships the product (i.e., the product is not shipped from Amazon).							

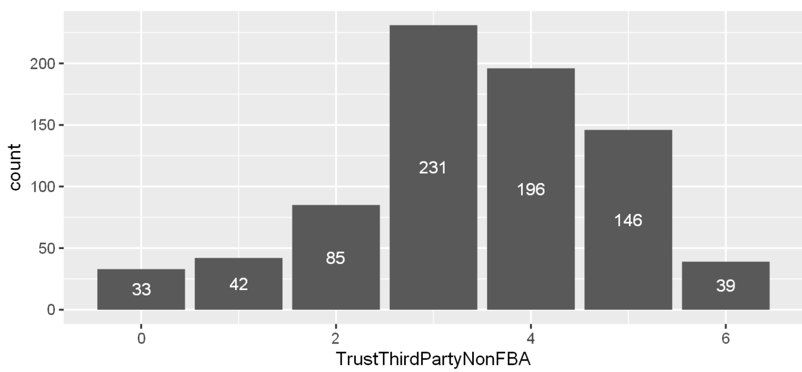

count
TrustOnlineStoreGeneral
0
100
200
0
2
4
6
13
20
153
247
279
57


count
TrustAmznItself
0
100
200
300
0
2
4
6
57
135
306
257


count
TrustThirdPartyPrime
0
100
200
300
0
2
4
6
26
146
188
291
110


200
150
100
50
0
count
29
47
81
227
202
145
41
0
2
4
6
TrustThirdPartyNonPrime


200
150
100
50
0
count
14
17
52
162
233
229
65
0
2
4
6
TrustThirdPartyFBA

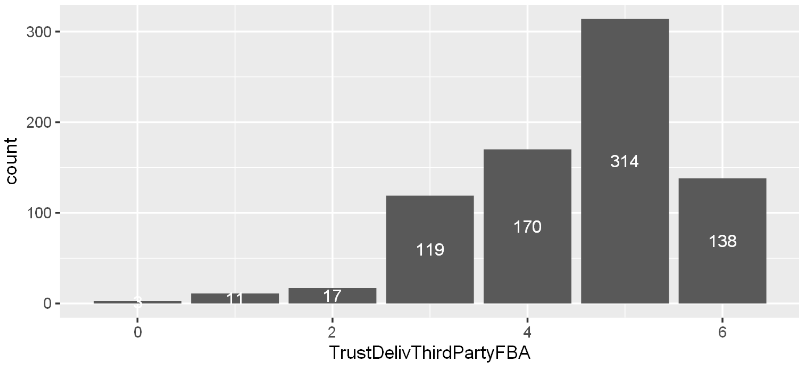

200
150
100
50
0
count
33
42
85
231
196
146
39
0
2
4
6
TrustThirdPartyNonFBA

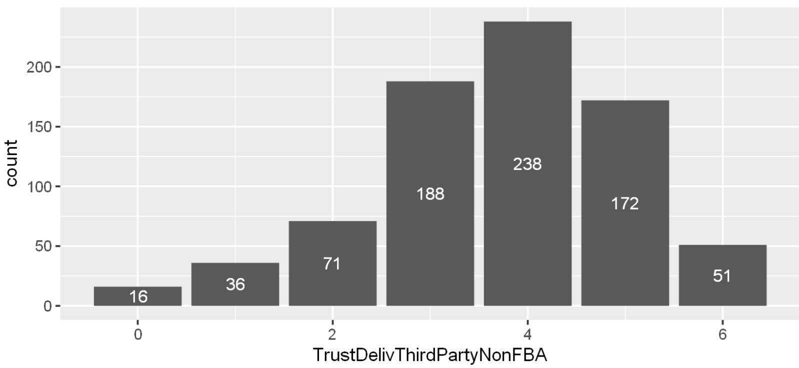
Additional Information

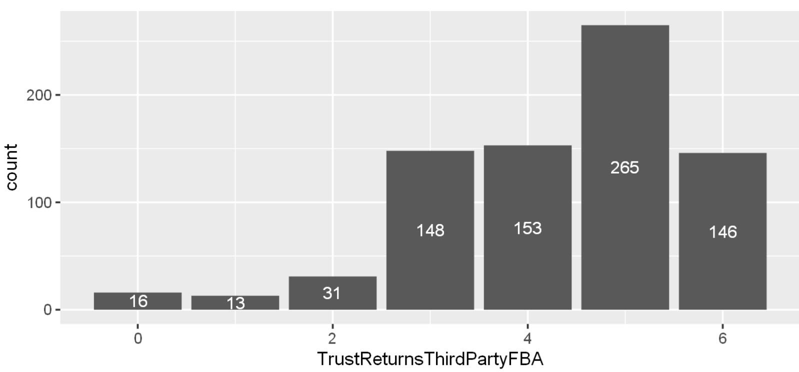
Please note for the following questions that we define the "delivery process" as the physical process between you finalizing your order and receiving the items (i.e., order picking, packing, transport, and last mile delivery) and that we define the "returns process" as the process between you deciding to return items and receiving the refund money (i.e., returns labeling, shipping, and money refunding).

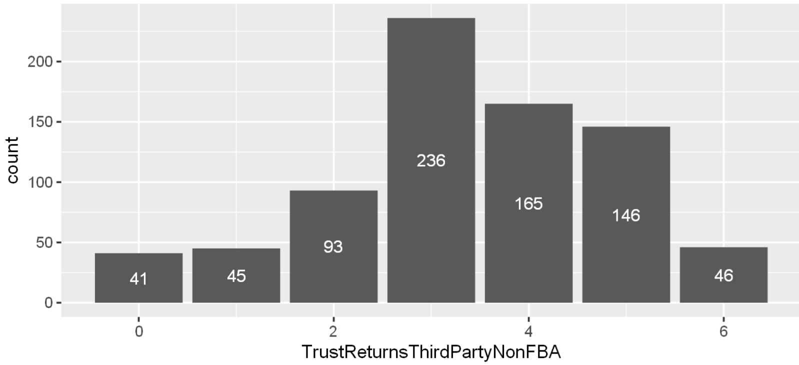
How do you rate the following statements? I trust that I will have a good experience with ...

	Strongly disagree	Disagree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
... the **delivery process** when I buy something on Amazon.com from a third-party seller **but** the product is shipped from Amazon itself.							
... the **delivery process** when I buy something on Amazon.com from a third-party seller that **also** ships the product (i.e., the product is not shipped from Amazon).							
... the **returns process** (if needed) when I buy something on Amazon.com from a third-party seller **but** the return is received and processed by Amazon itself.							
... the **returns process** (if needed) when I buy something on Amazon.com from a third-party seller that **also** receives and processes the return.							


count
TrustDelivThirdPartyFBA
0
100
200
300
0
2
4
6
11
17
119
170
314
138


count
TrustDelivThirdPartyNonFBA
0
50
100
150
200
0
2
4
6
16
36
71
188
238
172
51

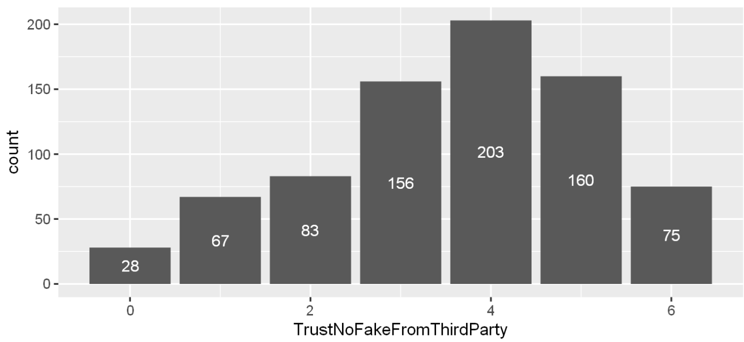

count
TrustReturnsThirdPartyFBA
0
100
200
0
2
4
6
16
13
31
148
153
265
146

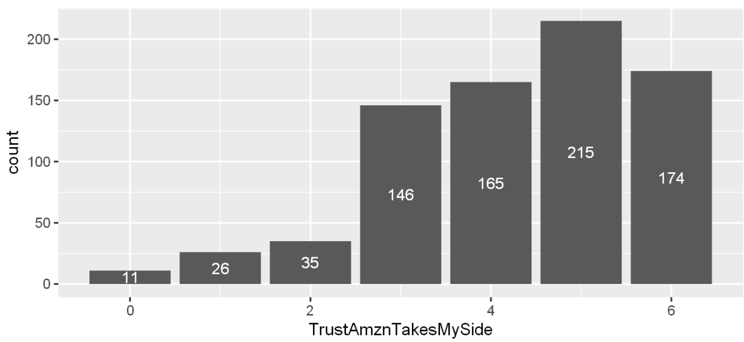


200
150
100
50
0
count
41
45
93
236
165
146
46
0
2
4
6
TrustReturnsThirdPartyNonFBA

Further questions about trust:

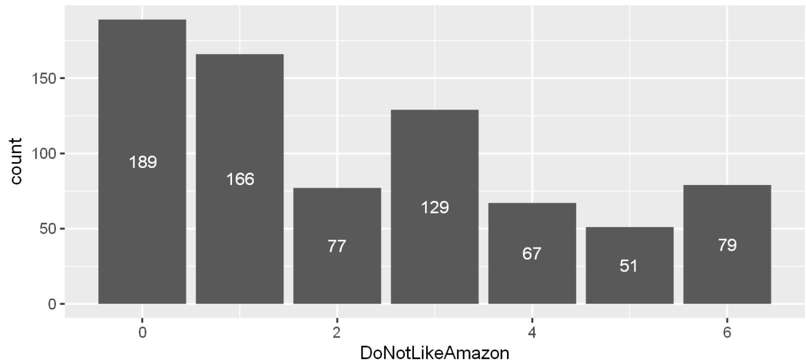
	Strongly disagree	Disa-gree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
I trust that I will **not** receive a fake/counterfeit product when I buy from a third-party seller on Amazon.com.							
I trust Amazon to take my side and to solve the problem if there is a dispute between me and a third-party seller active on the Amazon marketplace.							





How do you rate the following statements?

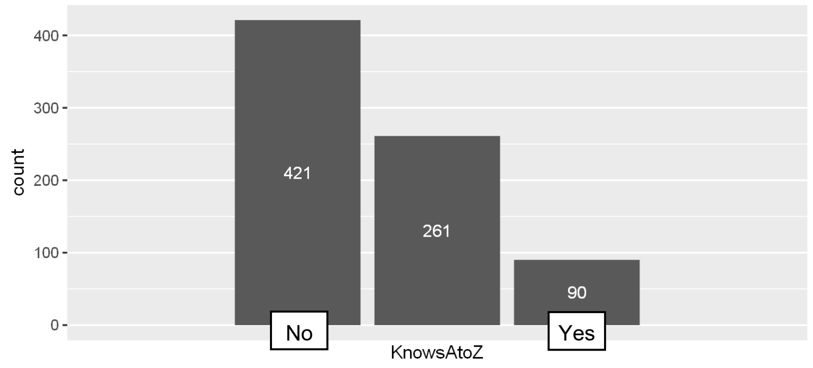
	Strongly disagree	Disagree	More or less disagree	Neutral	More or less agree	Agree	Strongly agree
I do not like Amazon as a company.							



Do you know what the “Amazon A-Z guarantee” is?

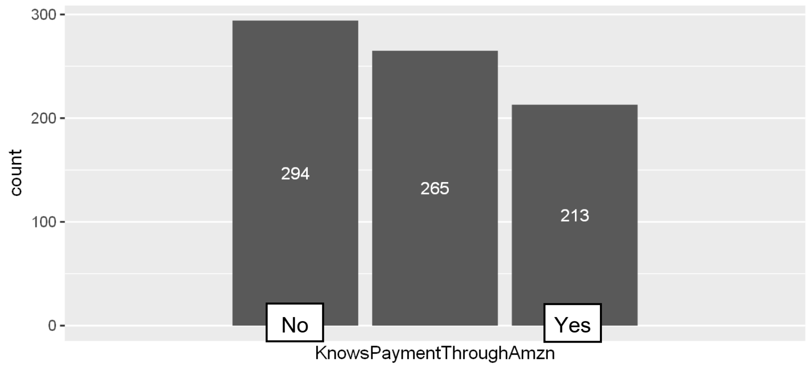
- Yes, I know the details.
- I have heard of it, but don’t know the details.
- No.

If you feel that your answer choice above does not adequately reflect your level of knowledge, please feel free to use this comment box to clarify.



Did you know (before you were asked this question) that all payments on Amazon.com go through Amazon and no third-party seller can see your payment details (e.g., your credit card number)?

○ Yes.
○ I had suspected this, but was not sure.
○ No.
If you feel that your answer choice above does not adequately reflect your level of knowledge, please feel free to use this comment box to clarify.

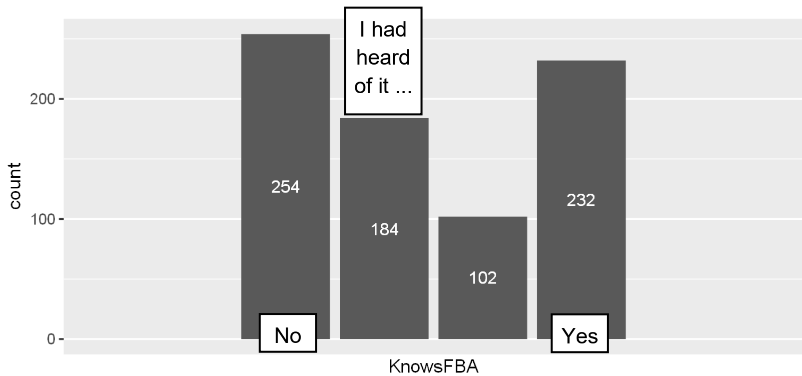


Additional Information

Amazon offers the third-party sellers on its marketplace a service called "Fulfillment by Amazon". Using this service, sellers can send products to Amazon fulfillment centers and when a customer makes a purchase, Amazon handles receiving, picking, packing, shipping, customer service, and returns for those orders. Basically, the third-party seller still owns the products but they are stored in an Amazon warehouse and Amazon is the logistics service provider.

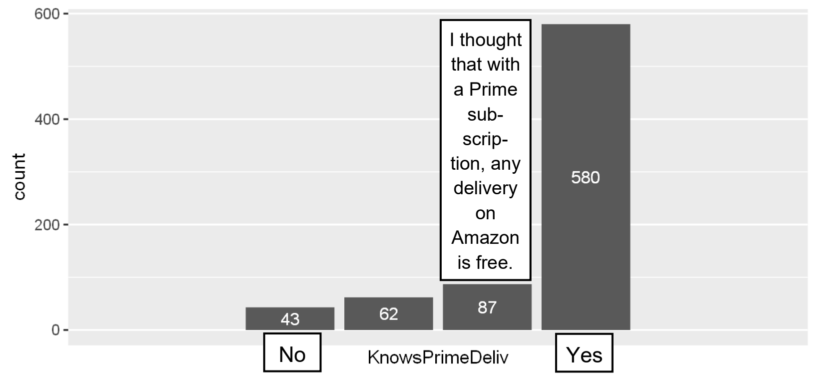
Did you know (before you were asked this question) that there is a service called "Fulfillment by Amazon" (FBA) and what this service includes?

○ Yes.
○ I knew that with FBA, Amazon ships the products from its warehouses, but I was not aware that the returns are also handled by Amazon.
○ I had heard of it, but didn't know any of the details.
○ No.
If you feel that your answer choice above does not adequately reflect your level of knowledge, please feel free to use this comment box to clarify.



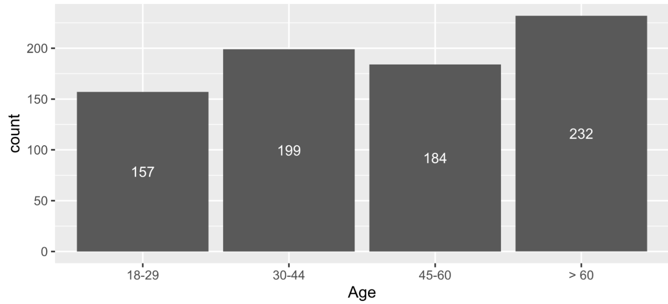
Did you know (before you were asked this question) that Amazon Prime subscribers do not pay any shipping fees for offers with the “Prime” logo, and that the free Prime delivery is often even faster than the regular delivery option?

○ Yes.
○ I thought that with a Prime subscription, any delivery on Amazon is free.
○ I knew about the free delivery for offers with the “Prime” logo, but not that the delivery is also faster than normal.
○ No.
If you feel that your answer choice above does not adequately reflect your level of knowledge, please feel free to use this comment box to clarify.

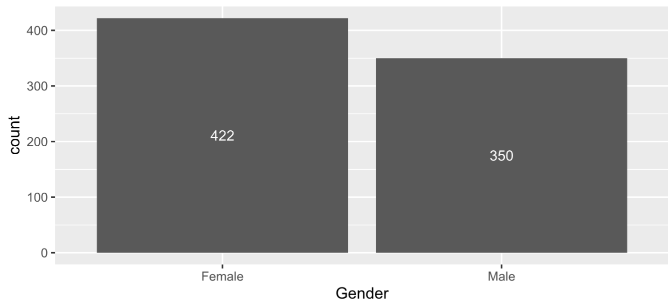


IV.A.2. Demographics

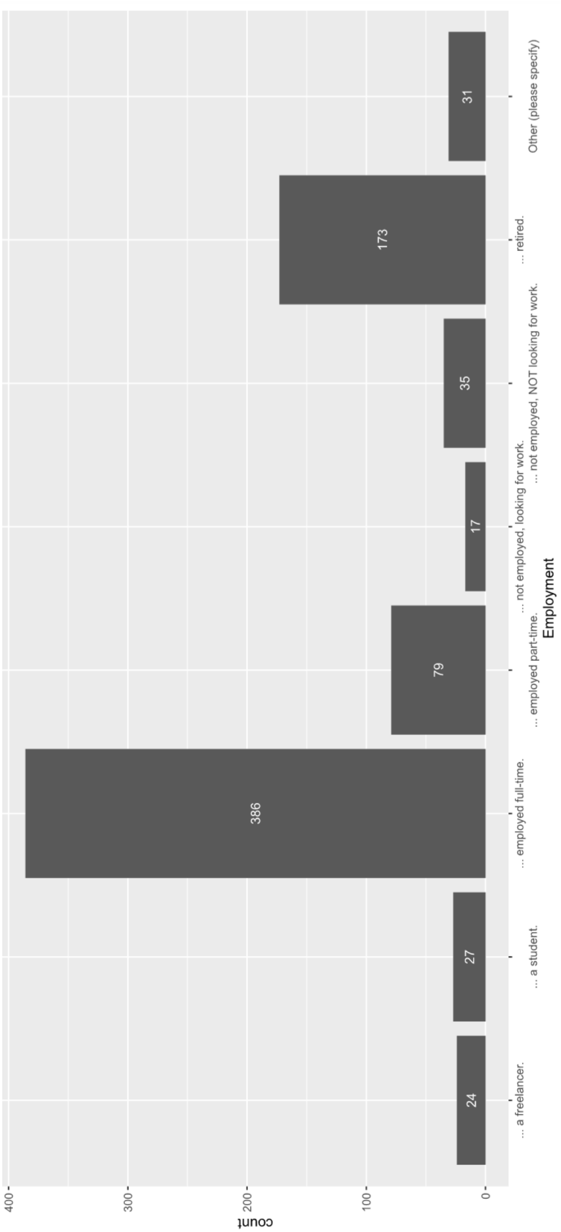
Age:



Gender:

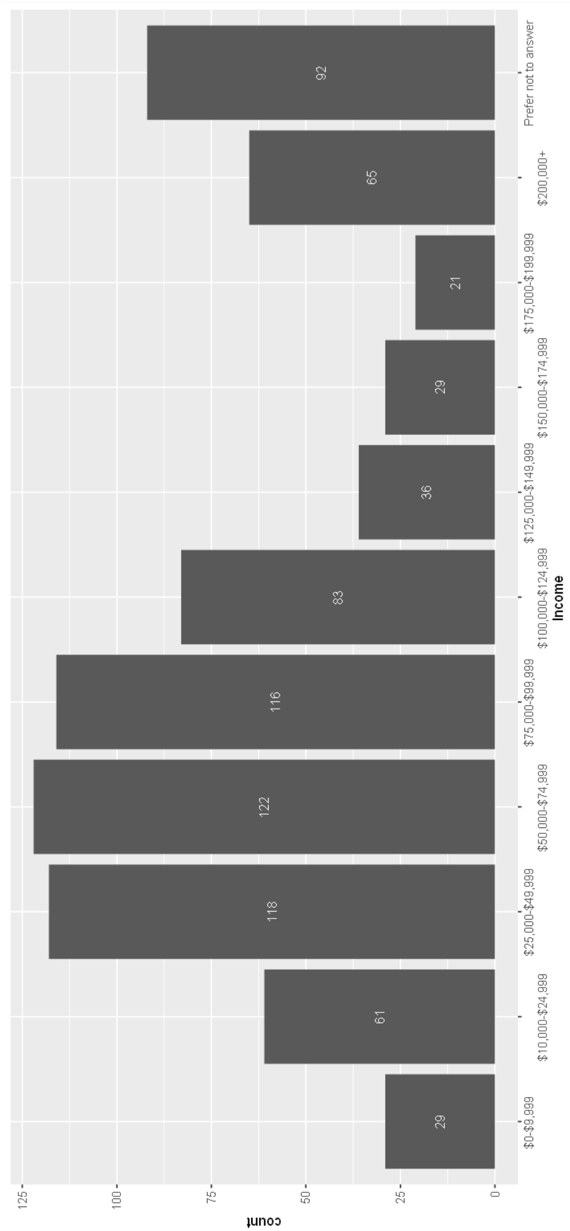


Employment:



count
0
100
200
300
400
Employment
... a freelancer.
... a student.
... employed full-time.
... employed part-time.
... not employed, looking for work.
... not employed, NOT looking for work.
... retired.
Other (please specify)
24
27
386
79
17
35
173
31

Income:


count
Income
$0-$9,999
$10,000-$24,999
$25,000-$49,999
$50,000-$74,999
$75,000-$99,999
$100,000-$124,999
$125,000-$149,999
$150,000-$174,999
$175,000-$199,999
$200,000+
Prefer not to answer
29
61
118
122
116
83
36
29
21
65
92
0
25
50
75
100
125

IV.A.3. SEM models

Mardia test for normality:

```
> mvn(RawDataSelect, mvnTest="mardia")
$multivariateNormality
             Test        Statistic               p value Result
1 Mardia Skewness 2360.23489676093 2.07507571253324e-322     NO
2 Mardia Kurtosis 45.8204782917009                     0     NO
3             MVN             <NA>                  <NA>     NO

$univariateNormality
               Test                      Variable Statistic   p value Normality
1  Anderson-Darling        TrustAmznItself          44.5192  <0.001      NO
2  Anderson-Darling     LikeBuyFromAmznItself       45.7945  <0.001      NO
3  Anderson-Darling     TrustThirdPartyPrime        31.2711  <0.001      NO
4  Anderson-Darling  LikeBuyFromThirdPartyPrime     23.7742  <0.001      NO
5  Anderson-Darling    TrustThirdPartyNonPrime      19.6215  <0.001      NO
6  Anderson-Darling LikeBuyFromThirdPartyNonPrime   17.4615  <0.001      NO
7  Anderson-Darling      TrustThirdPartyFBA         25.2938  <0.001      NO
8  Anderson-Darling   LikeBuyFromThirdPartyFBA      25.4494  <0.001      NO
9  Anderson-Darling     TrustThirdPartyNonFBA       19.7481  <0.001      NO
10 Anderson-Darling  LikeBuyFromThirdPartyNonFBA    20.2534  <0.001      NO
11 Anderson-Darling      PercentageOnAmazon         15.1788  <0.001      NO

$Descriptives
                                n      Mean   Std.Dev Median Min Max 25th 75th        Skew    Kurtosis
TrustAmznItself               772  4.927461  1.061004      5   0   6    4    6 -1.28061288  2.48071524
LikeBuyFromAmznItself         772  4.674870  1.399181      5   0   6    4    6 -1.19462785  1.06911905
TrustThirdPartyPrime          772  4.358808  1.139783      5   0   6    4    5 -0.61658706  0.26512511
LikeBuyFromThirdPartyPrime    772  3.935233  1.341139      4   0   6    3    5 -0.61019760  0.31714391
TrustThirdPartyNonPrime       772  3.457254  1.403899      4   0   6    3    4 -0.43505104 -0.07069014
LikeBuyFromThirdPartyNonPrime 772  2.487047  1.451718      3   0   6    1    3  0.08156284 -0.58150909
TrustThirdPartyFBA            772  3.981865  1.266011      4   0   6    3    5 -0.71290629  0.60051623
LikeBuyFromThirdPartyFBA      772  3.869171  1.233864      4   0   6    3    5 -0.54516002  0.47647633
TrustThirdPartyNonFBA         772  3.436528  1.410490      3   0   6    3    4 -0.44682590 -0.04496794
LikeBuyFromThirdPartyNonFBA   772  2.848446  1.430295      3   0   6    2    4 -0.07312763 -0.31679578
PercentageOnAmazon            772 63.946891 27.159347     70   1 100   50   85 -0.62648565 -0.57737336
```

Model 1 from the paper:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 81 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        37

                                                  Used       Total
  Number of observations                           758         772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                81.314      65.765
  Degrees of freedom                                26          26
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.276
  Shift parameter                                            2.045
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              2144.534    1708.139
  Degrees of freedom                                57          57
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.268

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.974       0.976
  Tucker-Lewis Index (TLI)                       0.942       0.947

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA


Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -13518.686  -13518.686
  Loglikelihood unrestricted model (H1)     -13478.029  -13478.029

  Akaike (AIC)                               27111.371   27111.371
  Bayesian (BIC)                             27282.707   27282.707
  Sample-size adjusted Bayesian (BIC)        27165.216   27165.216

Root Mean Square Error of Approximation:

  RMSEA                                          0.053       0.045
  90 Percent confidence interval - lower         0.040       0.032
  90 Percent confidence interval - upper         0.066       0.059
  P-value RMSEA <= 0.05                          0.333       0.714

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.023       0.023

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured


Regressions:
                                  Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  LikeBuyFromAmznItself ~
    TrustAmznItslf                   0.475    0.051    9.327    0.000    0.475    0.365
    ImportncFstDlv                   0.116    0.036    3.172    0.002    0.116    0.105
    DoNotLikeAmazn                  -0.166    0.024   -6.767    0.000   -0.166   -0.238
    HasPrimeOrNot                    0.904    0.107    8.462    0.000    0.904    0.294


  LikeBuyFromThirdPartyPrime ~
    TrstThrdPrtyPr                   0.455    0.040   11.287    0.000    0.455    0.393
    DoNotLikeAmazn                  -0.064    0.020   -3.160    0.002   -0.064   -0.097
```

```
    HasPrimeOrNot                 0.729    0.093    7.821    0.000    0.729    0.248
    AmznPltfrmIsTC               -0.083    0.026   -3.181    0.001   -0.083   -0.097
  LikeBuyFromThirdPartyNonPrime ~
    TrstThrdPrtyNP                0.556    0.032   17.596    0.000    0.556    0.542
    TrustAmznItslf               -0.168    0.046   -3.608    0.000   -0.168   -0.124
    HasPrimeOrNot                -0.386    0.101   -3.833    0.000   -0.386   -0.121
  S1FromAmzn3PwPrime ~
    LkByFrmAmznIts                3.838    0.828    4.637    0.000    3.838    0.167
    LkByFrmThrdPrP                2.790    1.041    2.680    0.007    2.790    0.116
    LkByFrmThrdPNP               -3.128    0.760   -4.118    0.000   -3.128   -0.141
    TrstThrdPrtyPr                4.261    1.060    4.021    0.000    4.261    0.153
    HasPrimeOrNot                19.481    2.580    7.549    0.000   19.481    0.275
    AmznPltfrmIsTC               -1.710    0.629   -2.718    0.007   -1.710   -0.083
  S1FromAmzn3PwoPrime ~
    LkByFrmThrdPNP                2.127    0.481    4.419    0.000    2.127    0.164
    TrstThrdPrtyNP                2.358    0.461    5.112    0.000    2.358    0.177
    HasPrimeOrNot               -13.163    1.797   -7.326    0.000  -13.163   -0.317
  PercentageOnAmazon ~
    LkByFrmAmznIts                2.677    0.901    2.972    0.003    2.677    0.137
    S1FrmAmzn3PwPr                0.191    0.038    5.064    0.000    0.191    0.225
    S1FrmAmzn3PwPr                0.239    0.056    4.297    0.000    0.239    0.165
    HasPrimeOrNot                 9.017    2.752    3.277    0.001    9.017    0.150
    DoNotLikeAmazn               -1.296    0.527   -2.461    0.014   -1.296   -0.096
    ImportncFstDlv                1.920    0.762    2.521    0.012    1.920    0.090
    AmznPltfrmIsTC               -1.314    0.593   -2.216    0.027   -1.314   -0.076

Covariances:
                               Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .LikeBuyFromAmznItself ~~
   .LkByFrmThrdPrP                0.420    0.058    7.262    0.000    0.420    0.384
   .LkByFrmThrdPNP                0.171    0.051    3.322    0.001    0.171    0.141
 .LikeBuyFromThirdPartyPrime ~~
   .LkByFrmThrdPNP                0.418    0.057    7.298    0.000    0.418    0.325
 .S1FromAmzn3PwPrime ~~
   .S1FrmAmzn3PwPr             -104.849   14.550   -7.206    0.000 -104.849   -0.246


Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .LkByFrmAmznIts    1.031    0.074   14.012    0.000    1.031    0.540
   .LkByFrmThrdPrP    1.157    0.068   17.103    0.000    1.157    0.665
   .LkByFrmThrdPNP    1.426    0.074   19.167    0.000    1.426    0.693
   .S1FrmAmzn3PwPr  675.116   32.981   20.470    0.000  675.116    0.667
   .S1FrmAmzn3PwPr  268.006   26.233   10.216    0.000  268.006    0.773
   .PercentgOnAmzn  566.571   28.099   20.163    0.000  566.571    0.779

R-Square:
                   Estimate
    LkByFrmAmznIts    0.460
    LkByFrmThrdPrP    0.335
    LkByFrmThrdPNP    0.307
    S1FrmAmzn3PwPr    0.333
    S1FrmAmzn3PwPr    0.227
    PercentgOnAmzn    0.221
```

Mediation analysis for model 1 from the paper – indirect and total effects:

```
Regressions:
                                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  LikeBuyFromAmznItself ~
    TrstAmI  (tLA)                    0.475    0.051    9.327    0.000    0.475    0.365
    ImprtFD  (dLA)                    0.116    0.036    3.172    0.002    0.116    0.105
    DNtLkAm (nALA)                   -0.166    0.024   -6.767    0.000   -0.166   -0.238
    HsPrmON (hpLA)                    0.904    0.107    8.462    0.000    0.904    0.294
  LikeBuyFromThirdPartyPrime ~
    TrstTPP  (tLp)                    0.455    0.040   11.287    0.000    0.455    0.393
    DNtLkAm (nALp)                   -0.064    0.020   -3.160    0.002   -0.064   -0.097
    HsPrmON (hpLp)                    0.729    0.093    7.821    0.000    0.729    0.248
    AmzPITC  (cLp)                   -0.083    0.026   -3.181    0.001   -0.083   -0.097
  LikeBuyFromThirdPartyNonPrime ~
    TrsTPNP (tLnp)                    0.556    0.032   17.596    0.000    0.556    0.542
    TrstAmI (tALn)                   -0.168    0.046   -3.608    0.000   -0.168   -0.124
    HsPrmON (hpLn)                   -0.386    0.101   -3.833    0.000   -0.386   -0.121
  S1FromAmzn3PwPrime ~
    LkByFAI (LAS1)                    3.838    0.828    4.637    0.000    3.838    0.167
    LkBFTPP (LpS1)                    2.790    1.041    2.680    0.007    2.790    0.116
    LBFTPNP (LnS1)                   -3.128    0.760   -4.118    0.000   -3.128   -0.141
    TrstTPP (tpS1)                    4.261    1.060    4.021    0.000    4.261    0.153
    HsPrmON (hpS1)                   19.481    2.580    7.549    0.000   19.481    0.275
    AmzPITC  (cS1)                   -1.710    0.629   -2.718    0.007   -1.710   -0.083
  S1FromAmzn3PwoPrime ~
    LBFTPNP (LnSo)                    2.127    0.481    4.419    0.000    2.127    0.164
    TrsTPNP  (tSo)                    2.358    0.461    5.112    0.000    2.358    0.177
    HsPrmON (hpSo)                  -13.163    1.797   -7.326    0.000  -13.163   -0.317
  PercentageOnAmazon ~
    LkByFAI  (LAP)                    2.677    0.901    2.972    0.003    2.677    0.137
    S1FA3PP  (S1P)                    0.191    0.038    5.064    0.000    0.191    0.225
    S1FA3PP  (SoP)                    0.239    0.056    4.297    0.000    0.239    0.165
    HsPrmON  (hpP)                    9.017    2.752    3.277    0.001    9.017    0.150
    DNtLkAm (nALP)                   -1.296    0.527   -2.461    0.014   -1.296   -0.096
    ImprtFD   (dP)                    1.920    0.762    2.521    0.012    1.920    0.090
    AmzPITC   (cP)                   -1.314    0.593   -2.216    0.027   -1.314   -0.076


inddP := dLA*LAP
totdP := inddP + dP

indcS1 := cLp*LpS1
totcS1 := cS1 + indcS1
indcS1P := totcS1*S1P
totcS1P := cP + indcS1P

indnLAp0 := nALp*LpS1
indnLAp1 := nALp*LpS1*S1P
indnLAP := nALA*LAP
totnLAP := nALP + indnLAp1 + indnLAP

indhpSo := hpLn*LnSo
tothpSo := hpSo + indhpSo

indhpS11 := hpLA*LAS1
indhpS12 := hpLp*LpS1
indhpS13 := hpLn*LnS1
tothpS1 := hpS1 + indhpS11 + indhpS12 + indhpS13

indhpSoP := tothpSo*SoP
indhpS1P := tothpS1*S1P
tothpP := hpP + indhpSoP + indhpS1P


indtASo := tALn*LnSo
indtAS11 := tALn*LnS1
indtAS12 := tLA*LAS1
tottAS1 := indtAS11 + indtAS12

indtASoP := indtASo*SoP
indtAS1P := tottAS1*S1P
indtALAP := tLA*LAP
tottAP := indtASoP + indtAS1P + indtALAP

indtpS1 := tLp*LpS1
tottpS1 := tpS1 + indtpS1
tottpP  := tottpS1*S1P

indtnS1 := tLnp*LnS1
indtnSo := tLnp*LnSo
tottpSo := tSo + indtnSo
tottnpP  := tottpSo*SoP + indtnS1*S1P
```

```
indLAP := LAS1*S1P
totLAP := LAP + indLAP

indLpP := LpS1*S1P

indLnpP1 := LnSo*SoP
indLnpP2 := LnS1*S1P
totLnpP := indLnpP1 + indLnpP2


Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    inddP             0.309    0.143    2.161    0.031    0.309    0.014
    totdP             2.230    0.757    2.947    0.003    2.230    0.104
    indcS1           -0.231    0.109   -2.117    0.034   -0.231   -0.011
    totcS1           -1.940    0.635   -3.057    0.002   -1.940   -0.095
    indcS1P          -0.370    0.142   -2.615    0.009   -0.370   -0.021
    totcS1P          -1.685    0.607   -2.775    0.006   -1.685   -0.097
    indnLAp0         -0.180    0.093   -1.925    0.054   -0.180   -0.011
    indnLAp1         -0.034    0.019   -1.787    0.074   -0.034   -0.003
    indnLAP          -0.443    0.168   -2.635    0.008   -0.443   -0.033
    totnLAP          -1.774    0.473   -3.747    0.000   -1.774   -0.131
    indhpSo          -0.822    0.294   -2.796    0.005   -0.822   -0.020
    tothpSo         -13.985    1.812   -7.720    0.000  -13.985   -0.337
    indhpS11          3.470    0.816    4.253    0.000    3.470    0.049
    indhpS12          2.033    0.793    2.564    0.010    2.033    0.029
    indhpS13          1.209    0.437    2.766    0.006    1.209    0.017
    tothpS1          26.193    2.335   11.216    0.000   26.193    0.370
    indhpSoP         -3.348    0.951   -3.520    0.000   -3.348   -0.056
    indhpS1P          4.996    1.060    4.714    0.000    4.996    0.083
    tothpP           10.665    2.571    4.149    0.000   10.665    0.177
    indtASo          -0.356    0.125   -2.857    0.004   -0.356   -0.020
    indtAS11          0.524    0.185    2.828    0.005    0.524    0.017
    indtAS12          1.825    0.435    4.197    0.000    1.825    0.061
    tottAS1           2.348    0.450    5.224    0.000    2.348    0.078
    indtASoP         -0.085    0.036   -2.395    0.017   -0.085   -0.003
    indtAS1P          0.448    0.123    3.631    0.000    0.448    0.018
    indtALAP          1.273    0.450    2.826    0.005    1.273    0.050
    tottAP            1.635    0.464    3.526    0.000    1.635    0.064
    indtpS1           1.270    0.484    2.625    0.009    1.270    0.045
    tottpS1           5.531    0.998    5.544    0.000    5.531    0.198
    tottpP            1.055    0.289    3.646    0.000    1.055    0.045
    indtnS1          -1.738    0.438   -3.966    0.000   -1.738   -0.076
    indtnSo           1.181    0.276    4.274    0.000    1.181    0.089
    tottpSo           3.539    0.431    8.220    0.000    3.539    0.266
    tottnpP           0.516    0.224    2.300    0.021    0.516    0.027
    indLAP            0.732    0.207    3.533    0.000    0.732    0.038
    totLAP            3.409    0.903    3.775    0.000    3.409    0.175
    indLpP            0.532    0.224    2.372    0.018    0.532    0.026
    indLnpP1          0.509    0.172    2.965    0.003    0.509    0.027
    indLnpP2         -0.597    0.197   -3.026    0.002   -0.597   -0.032
    totLnpP          -0.088    0.227   -0.386    0.699   -0.088   -0.005
```

Model 2 from the paper:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV", se="robust")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 32 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        48

  Number of observations                           772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                54.283      46.404
  Degrees of freedom                                24          24
  P-value (Chi-square)                           0.000       0.004
  Scaling correction factor                                  1.211
  Shift parameter                                            1.593
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              3509.838    1172.741
  Degrees of freedom                                63          63
  P-value                                        0.000       0.000
  Scaling correction factor                                  3.091

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.991       0.980
  Tucker-Lewis Index (TLI)                       0.977       0.947

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -10002.312  -10002.312
  Loglikelihood unrestricted model (H1)      -9975.171   -9975.171

  Akaike (AIC)                               20100.625   20100.625
  Bayesian (BIC)                             20323.776   20323.776
  Sample-size adjusted Bayesian (BIC)        20171.353   20171.353

Root Mean Square Error of Approximation:

  RMSEA                                          0.040       0.035
  90 Percent confidence interval - lower         0.026       0.019
  90 Percent confidence interval - upper         0.055       0.050
  P-value RMSEA <= 0.05                          0.856       0.954

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.032       0.032

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TrustAmznItself ~
    TrstOnlnStrGnr                  0.452    0.044   10.358    0.000    0.452    0.455
    HasPrimeOrNot                   0.509    0.079    6.443    0.000    0.509    0.216
    ImportncFstDlv                  0.092    0.028    3.306    0.001    0.092    0.109
  TrustAmznTakesMySide ~
    TrustAmznItslf                  0.550    0.052   10.538    0.000    0.550    0.412


  TrustNoFakeFromThirdParty ~
    TrstOnlnStrGnr                  0.141    0.050    2.844    0.004    0.141    0.097
    TrstAmznTksMyS                  0.516    0.039   13.338    0.000    0.516    0.471
  TrustDelivThirdPartyFBA ~
    TrstAmznTksMyS                  0.134    0.029    4.597    0.000    0.134    0.167
    TrustAmznItslf                  0.378    0.041    9.119    0.000    0.378    0.353
    TrstOnlnStrGnr                  0.106    0.045    2.356    0.018    0.106    0.099
```

```
  TrustDelivThirdPartyNonFBA ~
    TrstAmznTksMyS              0.254    0.036    7.122    0.000    0.254    0.271
    TrstOnlnStrGnr              0.292    0.046    6.327    0.000    0.292    0.236
  TrustReturnsThirdPartyFBA ~
    TrstAmznTksMyS              0.313    0.036    8.706    0.000    0.313    0.327
    TrustAmznItslf              0.356    0.042    8.414    0.000    0.356    0.279
  TrustReturnsThirdPartyNonFBA ~
    TrstAmznTksMyS              0.408    0.037   10.910    0.000    0.408    0.390
    TrstOnlnStrGnr              0.132    0.044    3.005    0.003    0.132    0.095
  TrustThirdPartyPrime ~
    TrstOnlnStrGnr              0.225    0.036    6.257    0.000    0.225    0.212
    HasPrimeOrNot               0.660    0.075    8.786    0.000    0.660    0.262
    TrstDlvThrPFBA              0.217    0.036    6.037    0.000    0.217    0.216
    TrstRtrnsTPFBA              0.141    0.030    4.696    0.000    0.141    0.168
    TrstNFkFrmThrP              0.075    0.021    3.617    0.000    0.075    0.102
  TrustThirdPartyNonPrime ~
    TrstOnlnStrGnr              0.137    0.040    3.401    0.001    0.137    0.107
    ImportncFstDlv             -0.090    0.029   -3.140    0.002   -0.090   -0.083
    HasPrimeOrNot              -0.219    0.076   -2.885    0.004   -0.219   -0.072
    TrstDlvThPNFBA              0.422    0.051    8.271    0.000    0.422    0.406
    TrstRtrnTPNFBA              0.168    0.043    3.905    0.000    0.168    0.181
    TrstNFkFrmThrP              0.086    0.034    2.577    0.010    0.086    0.098

Covariances:
                             Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .TrustAmznItself ~~
   .TrstThrdPrtyPr              0.357    0.044    8.170    0.000    0.357    0.470
   .TrstThrdPrtyNP              0.188    0.042    4.473    0.000    0.188    0.203
 .TrustThirdPartyPrime ~~
   .TrstThrdPrtyNP              0.263    0.040    6.658    0.000    0.263    0.300
 .TrustDelivThirdPartyFBA ~~
   .TrstDlvThPNFBA              0.556    0.058    9.666    0.000    0.556    0.467
   .TrstRtrnsTPFBA              0.506    0.061    8.301    0.000    0.506    0.444
   .TrstRtrnTPNFBA              0.352    0.059    5.922    0.000    0.352    0.267
 .TrustDelivThirdPartyNonFBA ~~
   .TrstRtrnTPNFBA              0.915    0.080   11.378    0.000    0.915    0.561
   .TrstRtrnsTPFBA              0.421    0.069    6.120    0.000    0.421    0.298
 .TrustReturnsThirdPartyFBA ~~
   .TrstRtrnTPNFBA              0.629    0.080    7.903    0.000    0.629    0.403
 .TrustNoFakeFromThirdParty ~~
   .TrstDlvThrPFBA              0.217    0.052    4.151    0.000    0.217    0.165
   .TrstDlvThPNFBA              0.575    0.066    8.683    0.000    0.575    0.352
   .TrstRtrnsTPFBA              0.157    0.067    2.339    0.019    0.157    0.100
   .TrstRtrnTPNFBA              0.618    0.075    8.211    0.000    0.618    0.343

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .TrustAmznItslf    0.804    0.061   13.279    0.000    0.804    0.720
   .TrstAmznTksMyS    1.657    0.100   16.648    0.000    1.657    0.831
   .TrstNFkFrmThrP    1.806    0.100   18.145    0.000    1.806    0.752
   .TrstDlvThrPFBA    0.962    0.071   13.614    0.000    0.962    0.750
   .TrstDlvThPNFBA    1.477    0.089   16.646    0.000    1.477    0.847
   .TrstRtrnsTPFBA    1.350    0.098   13.728    0.000    1.350    0.740
   .TrstRtrnTPNFBA    1.802    0.099   18.282    0.000    1.802    0.825
   .TrstThrdPrtyPr    0.718    0.046   15.735    0.000    0.718    0.559
   .TrstThrdPrtyNP    1.072    0.086   12.457    0.000    1.072    0.569

R-Square:
                   Estimate
    TrustAmznItslf    0.280
    TrstAmznTksMyS    0.169
    TrstNFkFrmThrP    0.248
    TrstDlvThrPFBA    0.250
    TrstDlvThPNFBA    0.153
    TrstRtrnsTPFBA    0.260
    TrstRtrnTPNFBA    0.175
    TrstThrdPrtyPr    0.441
    TrstThrdPrtyNP    0.431
```

Mediation analysis for model 2 from the paper – indirect and total effects:

```
Regressions:
                                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TrustAmznItself ~
    TrsOSG  (tGtA)                  0.452    0.044   10.358    0.000    0.452    0.455
    HsPrON  (hptA)                  0.509    0.079    6.443    0.000    0.509    0.216
    ImprFD   (dtA)                  0.092    0.028    3.306    0.001    0.092    0.109
  TrustAmznTakesMySide ~
    TrstAI  (tAtS)                  0.550    0.052   10.538    0.000    0.550    0.412
  TrustNoFakeFromThirdParty ~
    TrsOSG  (tGtF)                  0.141    0.050    2.844    0.004    0.141    0.097
    TrATMS  (tStF)                  0.516    0.039   13.338    0.000    0.516    0.471
  TrustDelivThirdPartyFBA ~
    TrATMS (tStdA)                  0.134    0.029    4.597    0.000    0.134    0.167
    TrstAI (tAtdA)                  0.378    0.041    9.119    0.000    0.378    0.353
    TrsOSG (tGtdA)                  0.106    0.045    2.356    0.018    0.106    0.099
  TrustDelivThirdPartyNonFBA ~
    TrATMS  (tStd)                  0.254    0.036    7.122    0.000    0.254    0.271
    TrsOSG  (tGtd)                  0.292    0.046    6.327    0.000    0.292    0.236
  TrustReturnsThirdPartyFBA ~
    TrATMS (tStrA)                  0.313    0.036    8.706    0.000    0.313    0.327
    TrstAI (tAtrA)                  0.356    0.042    8.414    0.000    0.356    0.279
  TrustReturnsThirdPartyNonFBA ~
    TrATMS  (tStr)                  0.408    0.037   10.910    0.000    0.408    0.390
    TrsOSG  (tGtr)                  0.132    0.044    3.005    0.003    0.132    0.095
  TrustThirdPartyPrime ~
    TrsOSG  (tGtp)                  0.225    0.036    6.257    0.000    0.225    0.212
    HsPrON  (hptp)                  0.660    0.075    8.786    0.000    0.660    0.262
    TDTPFB  (tdAt)                  0.217    0.036    6.037    0.000    0.217    0.216
    TRTPFB  (trAt)                  0.141    0.030    4.696    0.000    0.141    0.168
    TNFFTP  (tFtp)                  0.075    0.021    3.617    0.000    0.075    0.102
  TrustThirdPartyNonPrime ~
    TrsOSG  (tGtn)                  0.137    0.040    3.401    0.001    0.137    0.107
    ImprFD   (dtn)                 -0.090    0.029   -3.140    0.002   -0.090   -0.083
    HsPrON  (hptn)                 -0.219    0.076   -2.885    0.004   -0.219   -0.072
    TDTPNF  (tdnt)                  0.422    0.051    8.271    0.000    0.422    0.406
    TRTPNF  (trnt)                  0.168    0.043    3.905    0.000    0.168    0.181
    TNFFTP  (tFtn)                  0.086    0.034    2.577    0.010    0.086    0.098
```

```
inddtAtS := dtA*tAtS
inddtAtdA := dtA*tAtdA
inddtAtrA := dtA*tAtrA

inddtAtStdA := inddtAtS*tStdA
inddtAtStrA := inddtAtS*tStrA

inddtAtStF := inddtAtS*tStF

totdtdA := inddtAtStdA + inddtAtdA
totdtrA := inddtAtStrA + inddtAtrA

inddtAtStdn := inddtAtS*tStdn
inddtAtStrn := inddtAtS*tStrn

inddtp1 := tFtp*inddtAtStF
inddtp2 := tdAtp*inddtAtStdA
inddtp3 := trAtp*inddtAtStrA

totdtp := inddtp1 + inddtp2 + inddtp3

inddtn1 := tFtn*inddtAtStF
inddtn2 := tdntn*inddtAtStdn
inddtn3 := trntn*inddtAtStrn

totdtn := inddtn1 + inddtn2 + inddtn3 + dtn

indhptAtS := hptA*tAtS
indhptAtdA := hptA*tAtdA
indhptAtrA := hptA*tAtrA

indhptAtStdA := indhptAtS*tStdA
indhptAtStrA := indhptAtS*tStrA

indhptAtStF := indhptAtS*tStF
```

```
tothptdA := indhptAtStdA + indhptAtdA
tothptrA := indhptAtStrA + indhptAtrA

indhptAtStdn := indhptAtS*tStdn
indhptAtStrn := indhptAtS*tStrn

indhptp1 := tFtp*indhptAtStF
indhptp2 := tdAtp*indhptAtStdA
indhptp3 := trAtp*indhptAtStrA

tothptp := indhptp1 + indhptp2 + indhptp3 + hptp

indhptn1 := tFtn*indhptAtStF
indhptn2 := tdntn*indhptAtStdn
indhptn3 := trntn*indhptAtStrn

tothptn := indhptn1 + indhptn2 + indhptn3 + hptn

indtGtAtS := tGtA*tAtS
indtGtAtStdA := tStdA*indtGtAtS
indtGtAtStdn := tStdn*indtGtAtS
indtGtAtStrA := tStrA*indtGtAtS
indtGtAtStrn := tStrn*indtGtAtS
indtGtAtStF := tStF*indtGtAtS

indtGtAtdA := tGtA*tAtdA
indtGtAtrA := tGtA*tAtrA

tottGtF := indtGtAtStF + tGtF
tottGtdA := indtGtAtStdA + tGtdA + indtGtAtdA
tottGtrA := indtGtAtStrA + indtGtAtrA
tottGtdn := indtGtAtStdn + tGtdn
tottGtrn := indtGtAtStrn + tGtrn

indtGtFtp := tFtp*tottGtF
indtGtdAtp := tdAtp*tottGtdA
indtGtrAtp := trAtp*tottGtrA
tottGtp := indtGtFtp + indtGtdAtp + indtGtrAtp + tGtp

indtGtFtn := tFtn*tottGtF
indtGtdAtn := tdntn*tottGtdn
indtGtrAtn := trntn*tottGtrn
tottGtn := indtGtFtn + indtGtdAtn + indtGtrAtn + tGtn

indtAtdA := tAtS*tStdA
indtAtrA := tAtS*tStrA
indtAtdn := tAtS*tStdn
indtAtrn := tAtS*tStrn
indtAtF := tAtS*tStF
tottAtdA := indtAtdA + tAtdA
tottAtrA := indtAtrA + tAtrA

indtAtFtp := tFtp*indtAtF
indtAtdAtp := tdAtp*tottAtdA
indtAtrAtp := trAtp*tottAtrA
tottAtp := indtAtFtp + indtAtdAtp + indtAtrAtp

indtAtFtn := tFtn*indtAtF
indtAtdAtn := tdntn*indtAtdn
indtAtrAtn := trntn*indtAtrn
tottAtn := indtAtFtn + indtAtdAtn + indtAtrAtn

indtStp1 := tStdA*tdAtp
indtStp2 := tStrA*trAtp
indtStp3 := tStF*tFtp
tottStp := indtStp1 + indtStp2 + indtStp3

indtStn1 := tStdn*tdntn
indtStn2 := tStrn*trntn
indtStn3 := tStF*tFtn
tottStn := indtStn1 + indtStn2 + indtStn3
```

Defined Parameters:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
inddtAtS	0.050	0.015	3.277	0.001	0.050	0.045
inddtAtdA	0.035	0.011	3.280	0.001	0.035	0.039
inddtAtrA	0.033	0.010	3.143	0.002	0.033	0.031
inddtAtStdA	0.007	0.002	2.762	0.006	0.007	0.008
inddtAtStrA	0.016	0.005	3.050	0.002	0.016	0.015
inddtAtStF	0.026	0.008	3.192	0.001	0.026	0.021
totdtdA	0.041	0.012	3.365	0.001	0.041	0.046
totdtrA	0.048	0.015	3.266	0.001	0.048	0.045
inddtAtStdn	0.013	0.004	3.071	0.002	0.013	0.012
inddtAtStrn	0.021	0.007	3.144	0.002	0.021	0.018
inddtp1	0.002	0.001	2.351	0.019	0.002	0.002
inddtp2	0.001	0.001	2.406	0.016	0.001	0.002
inddtp3	0.002	0.001	2.564	0.010	0.002	0.002
totdtp	0.006	0.002	2.938	0.003	0.006	0.006
inddtn1	0.002	0.001	2.076	0.038	0.002	0.002
inddtn2	0.005	0.002	2.765	0.006	0.005	0.005
inddtn3	0.003	0.001	2.506	0.012	0.003	0.003
totdtn	-0.079	0.029	-2.710	0.007	-0.079	-0.073
indhptAtS	0.280	0.049	5.749	0.000	0.280	0.089
indhptAtdA	0.192	0.037	5.270	0.000	0.192	0.076
indhptAtrA	0.181	0.035	5.252	0.000	0.181	0.060
indhptAtStdA	0.037	0.011	3.543	0.000	0.037	0.015
indhptAtStrA	0.087	0.018	4.791	0.000	0.087	0.029
indhptAtStF	0.144	0.027	5.316	0.000	0.144	0.042
tothptdA	0.230	0.041	5.564	0.000	0.230	0.091
tothptrA	0.269	0.046	5.860	0.000	0.269	0.089
indhptAtStdn	0.071	0.016	4.352	0.000	0.071	0.024
indhptAtStrn	0.114	0.023	4.959	0.000	0.114	0.035
indhptp1	0.011	0.004	3.050	0.002	0.011	0.004
indhptp2	0.008	0.003	2.961	0.003	0.008	0.003
indhptp3	0.012	0.004	3.327	0.001	0.012	0.005
tothptp	0.692	0.076	9.059	0.000	0.692	0.274
indhptn1	0.012	0.005	2.386	0.017	0.012	0.004
indhptn2	0.030	0.008	3.871	0.000	0.030	0.010
indhptn3	0.019	0.006	3.014	0.003	0.019	0.006
tothptn	-0.158	0.080	-1.981	0.048	-0.158	-0.052
indtGtAtS	0.249	0.031	7.998	0.000	0.249	0.187
indtGtAtStdA	0.033	0.008	4.035	0.000	0.033	0.031
indtGtAtStdn	0.063	0.012	5.229	0.000	0.063	0.051
indtGtAtStrA	0.078	0.013	5.948	0.000	0.078	0.061
indtGtAtStrn	0.101	0.016	6.380	0.000	0.101	0.073
indtGtAtStF	0.128	0.019	6.864	0.000	0.128	0.088
indtGtAtdA	0.171	0.025	6.874	0.000	0.171	0.161
indtGtAtrA	0.161	0.024	6.574	0.000	0.161	0.127
tottGtF	0.269	0.051	5.250	0.000	0.269	0.185
tottGtdA	0.310	0.045	6.947	0.000	0.310	0.292
tottGtrA	0.239	0.030	8.086	0.000	0.239	0.188
tottGtdn	0.355	0.046	7.733	0.000	0.355	0.287
tottGtrn	0.234	0.046	5.081	0.000	0.234	0.168
indtGtFtp	0.020	0.007	3.032	0.002	0.020	0.019
indtGtdAtp	0.067	0.015	4.573	0.000	0.067	0.063
indtGtrAtp	0.034	0.008	4.042	0.000	0.034	0.032
tottGtp	0.346	0.041	8.399	0.000	0.346	0.326
indtGtFtn	0.023	0.011	2.175	0.030	0.023	0.018
indtGtdAtn	0.150	0.027	5.580	0.000	0.150	0.117
indtGtrAtn	0.039	0.012	3.192	0.001	0.039	0.031
tottGtn	0.350	0.049	7.166	0.000	0.350	0.272
indtAtdA	0.074	0.017	4.244	0.000	0.074	0.069
indtAtrA	0.172	0.025	6.820	0.000	0.172	0.135
indtAtdn	0.140	0.024	5.745	0.000	0.140	0.112
indtAtrn	0.225	0.031	7.311	0.000	0.225	0.161
indtAtF	0.284	0.034	8.324	0.000	0.284	0.194
tottAtdA	0.452	0.041	10.978	0.000	0.452	0.422
tottAtrA	0.528	0.044	11.947	0.000	0.528	0.413
indtAtFtp	0.021	0.006	3.379	0.001	0.021	0.020
indtAtdAtp	0.098	0.019	5.043	0.000	0.098	0.091
indtAtrAtp	0.074	0.017	4.379	0.000	0.074	0.069
tottAtp	0.194	0.023	8.284	0.000	0.194	0.181
indtAtFtn	0.025	0.010	2.506	0.012	0.025	0.019
indtAtdAtn	0.059	0.013	4.693	0.000	0.059	0.045
indtAtrAtn	0.038	0.011	3.385	0.001	0.038	0.029
tottAtn	0.121	0.019	6.461	0.000	0.121	0.093
indtStp1	0.029	0.008	3.494	0.000	0.029	0.036
indtStp2	0.044	0.011	4.011	0.000	0.044	0.055
indtStp3	0.039	0.011	3.451	0.001	0.039	0.048
tottStp	0.112	0.017	6.558	0.000	0.112	0.139
indtStn1	0.107	0.020	5.293	0.000	0.107	0.110
indtStn2	0.069	0.019	3.667	0.000	0.069	0.071
indtStn3	0.045	0.017	2.554	0.011	0.045	0.046
tottStn	0.221	0.026	8.449	0.000	0.221	0.227

Model 1 from the paper with insignificant but theoretically conceivable correlations:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 96 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        45

                                                  Used       Total
  Number of observations                           758         772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                72.881      57.965
  Degrees of freedom                                18          18
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.282
  Shift parameter                                            1.104
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              2144.534    1708.139
  Degrees of freedom                                57          57
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.268

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.974       0.976
  Tucker-Lewis Index (TLI)                       0.917       0.923

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -13514.469  -13514.469
  Loglikelihood unrestricted model (H1)     -13478.029  -13478.029

  Akaike (AIC)                               27118.939   27118.939
  Bayesian (BIC)                             27327.319   27327.319
  Sample-size adjusted Bayesian (BIC)        27184.426   27184.426

Root Mean Square Error of Approximation:

  RMSEA                                          0.063       0.054
  90 Percent confidence interval - lower         0.049       0.039
  90 Percent confidence interval - upper         0.079       0.070
  P-value RMSEA <= 0.05                          0.067       0.308

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.021       0.021

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                                Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  LikeBuyFromAmznItself ~
    TrustAmznItslf                 0.475    0.051    9.307    0.000    0.475    0.364
    ImportncFstDlv                 0.129    0.038    3.446    0.001    0.129    0.118
    DoNotLikeAmazn                -0.164    0.024   -6.717    0.000   -0.164   -0.236
    HasPrimeOrNot                  0.895    0.106    8.408    0.000    0.895    0.290


  LikeBuyFromThirdPartyPrime ~
    TrstThrdPrtyPr                 0.451    0.040   11.142    0.000    0.451    0.389
    ImportncFstDlv                 0.042    0.034    1.230    0.219    0.042    0.040
    DoNotLikeAmazn                -0.058    0.021   -2.783    0.005   -0.058   -0.088
```

```
    HasPrimeOrNot                 0.699    0.097    7.210    0.000    0.699    0.238
    AmznPltfrmIsTC               -0.098    0.027   -3.584    0.000   -0.098   -0.115
  LikeBuyFromThirdPartyNonPrime ~
    TrstThrdPrtyNP                0.548    0.032   17.038    0.000    0.548    0.536
    TrustAmznItslf               -0.166    0.048   -3.444    0.001   -0.166   -0.123
    ImportncFstDlv               -0.031    0.038   -0.813    0.416   -0.031   -0.027
    HasPrimeOrNot                -0.383    0.103   -3.701    0.000   -0.383   -0.120
    AmznPltfrmIsTC               -0.045    0.034   -1.312    0.190   -0.045   -0.048
  S1FromAmzn3PwPrime ~
    LkByFrmAmznIts                3.702    0.862    4.296    0.000    3.702    0.161
    LkByFrmThrdPrP                2.833    1.049    2.699    0.007    2.833    0.118
    LkByFrmThrdPNP               -3.115    0.763   -4.084    0.000   -3.115   -0.140
    TrstThrdPrtyPr                4.254    1.062    4.007    0.000    4.254    0.152
    ImportncFstDlv                0.429    0.823    0.521    0.602    0.429    0.017
    HasPrimeOrNot                19.254    2.620    7.348    0.000   19.254    0.271
    AmznPltfrmIsTC               -1.923    0.651   -2.956    0.003   -1.923   -0.094
  S1FromAmzn3PwoPrime ~
    LkByFrmAmznIts                0.179    0.596    0.300    0.764    0.179    0.013
    LkByFrmThrdPrP               -0.118    0.606   -0.195    0.846   -0.118   -0.008
    LkByFrmThrdPNP                2.154    0.504    4.277    0.000    2.154    0.166
    TrstThrdPrtyNP                2.370    0.472    5.018    0.000    2.370    0.178
    ImportncFstDlv               -0.404    0.580   -0.697    0.486   -0.404   -0.027
    HasPrimeOrNot               -12.802    2.025   -6.322    0.000  -12.802   -0.309
    AmznPltfrmIsTC                0.484    0.328    1.476    0.140    0.484    0.040
  PercentageOnAmazon ~
    LkByFrmAmznIts                2.677    0.900    2.975    0.003    2.677    0.137
    S1FrmAmzn3PwPr                0.191    0.038    5.066    0.000    0.191    0.225
    S1FrmAmzn3PwPr                0.239    0.056    4.279    0.000    0.239    0.165
    HasPrimeOrNot                 9.017    2.730    3.303    0.001    9.017    0.150
    DoNotLikeAmazn               -1.296    0.525   -2.467    0.014   -1.296   -0.096
    ImportncFstDlv                1.920    0.767    2.505    0.012    1.920    0.090
    AmznPltfrmIsTC               -1.314    0.594   -2.212    0.027   -1.314   -0.076

Covariances:
                                Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .LikeBuyFromAmznItself ~~
   .LkByFrmThrdPrP                 0.418    0.057    7.285    0.000    0.418    0.383
   .LkByFrmThrdPNP                 0.167    0.051    3.281    0.001    0.167    0.138
 .LikeBuyFromThirdPartyPrime ~~
   .LkByFrmThrdPNP                 0.417    0.057    7.331    0.000    0.417    0.326
 .S1FromAmzn3PwPrime ~~
   .S1FrmAmzn3PwPr              -104.414   14.540   -7.181    0.000 -104.414   -0.246

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .LkByFrmAmznIts    1.031    0.074   13.996    0.000    1.031    0.538
   .LkByFrmThrdPrP    1.153    0.067   17.152    0.000    1.153    0.661
   .LkByFrmThrdPNP    1.418    0.075   19.001    0.000    1.418    0.691
   .S1FrmAmzn3PwPr  674.791   33.034   20.427    0.000  674.791    0.666
   .S1FrmAmzn3PwPr  267.262   26.061   10.255    0.000  267.262    0.771
   .PercentgOnAmzn  566.571   28.099   20.163    0.000  566.571    0.779

R-Square:
                   Estimate
    LkByFrmAmznIts    0.462
    LkByFrmThrdPrP    0.339
    LkByFrmThrdPNP    0.309
    S1FrmAmzn3PwPr    0.334
    S1FrmAmzn3PwPr    0.229
    PercentgOnAmzn    0.221
```

Model 2 from the paper with insignificant but theoretically conceivable correlations:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV", se="robust")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 38 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        56

  Number of observations                           772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                38.753      31.498
  Degrees of freedom                                16          16
  P-value (Chi-square)                           0.001       0.012
  Scaling correction factor                                  1.268
  Shift parameter                                            0.937
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              3509.838    1172.741
  Degrees of freedom                                63          63
  P-value                                        0.000       0.000
  Scaling correction factor                                  3.091

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.993       0.986
  Tucker-Lewis Index (TLI)                       0.974       0.945

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -9994.547   -9994.547
  Loglikelihood unrestricted model (H1)      -9975.171   -9975.171

  Akaike (AIC)                               20101.094   20101.094
  Bayesian (BIC)                             20361.437   20361.437
  Sample-size adjusted Bayesian (BIC)        20183.611   20183.611

Root Mean Square Error of Approximation:

  RMSEA                                          0.043       0.035
  90 Percent confidence interval - lower         0.026       0.016
  90 Percent confidence interval - upper         0.060       0.054
  P-value RMSEA <= 0.05                          0.729       0.902

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.025       0.025

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                                Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TrustAmznItself ~
    TrstOnlnStrGnr                 0.450    0.044   10.304    0.000    0.450    0.453
    HasPrimeOrNot                  0.491    0.080    6.133    0.000    0.491    0.208
    ImportncFstDlv                 0.117    0.031    3.792    0.000    0.117    0.139
  TrustAmznTakesMySide ~
    TrstOnlnStrGnr                 0.059    0.053    1.106    0.269    0.059    0.044
    TrustAmznItslf                 0.525    0.060    8.792    0.000    0.525    0.394
    HasPrimeOrNot                 -0.021    0.110   -0.191    0.849   -0.021   -0.007

  TrustNoFakeFromThirdParty ~
    TrstOnlnStrGnr                 0.147    0.050    2.921    0.003    0.147    0.101
    TrstAmznTksMyS                 0.515    0.039   13.093    0.000    0.515    0.469
  TrustDelivThirdPartyFBA ~
```

```
    TrstAmznTksMyS              0.134    0.029    4.563    0.000    0.134    0.168
    TrustAmznItslf              0.379    0.042    9.063    0.000    0.379    0.355
    TrstOnlnStrGnr              0.122    0.049    2.472    0.013    0.122    0.115
    HasPrimeOrNot              -0.124    0.075   -1.644    0.100   -0.124   -0.049
  TrustDelivThirdPartyNonFBA ~
    TrstAmznTksMyS              0.257    0.036    7.118    0.000    0.257    0.274
    TrstOnlnStrGnr              0.305    0.050    6.103    0.000    0.305    0.245
    HasPrimeOrNot              -0.186    0.088   -2.123    0.034   -0.186   -0.063
  TrustReturnsThirdPartyFBA ~
    TrstAmznTksMyS              0.312    0.036    8.582    0.000    0.312    0.326
    TrustAmznItslf              0.319    0.045    7.135    0.000    0.319    0.250
    TrstOnlnStrGnr              0.064    0.049    1.294    0.196    0.064    0.050
    HasPrimeOrNot               0.047    0.095    0.501    0.616    0.047    0.016
  TrustReturnsThirdPartyNonFBA ~
    TrstAmznTksMyS              0.409    0.038   10.744    0.000    0.409    0.391
    TrstOnlnStrGnr              0.153    0.051    3.023    0.003    0.153    0.110
    HasPrimeOrNot              -0.160    0.098   -1.634    0.102   -0.160   -0.049
  TrustThirdPartyPrime ~
    TrstOnlnStrGnr              0.222    0.037    6.034    0.000    0.222    0.209
    ImportncFstDlv              0.050    0.024    2.070    0.038    0.050    0.056
    HasPrimeOrNot               0.625    0.077    8.111    0.000    0.625    0.248
    TrstDlvThrPFBA              0.219    0.036    6.156    0.000    0.219    0.219
    TrstRtrnsTPFBA              0.139    0.030    4.627    0.000    0.139    0.166
    TrstNFkFrmThrP              0.074    0.021    3.599    0.000    0.074    0.102
  TrustThirdPartyNonPrime ~
    TrstOnlnStrGnr              0.136    0.041    3.330    0.001    0.136    0.105
    ImportncFstDlv             -0.072    0.030   -2.386    0.017   -0.072   -0.066
    HasPrimeOrNot              -0.233    0.077   -3.018    0.003   -0.233   -0.076
    TrstDlvThPNFBA              0.423    0.051    8.274    0.000    0.423    0.406
    TrstRtrnTPNFBA              0.168    0.043    3.891    0.000    0.168    0.181
    TrstNFkFrmThrP              0.086    0.033    2.583    0.010    0.086    0.097

Covariances:
                             Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .TrustAmznItself ~~
   .TrstThrdPrtyPr              0.355    0.043    8.195    0.000    0.355    0.469
   .TrstThrdPrtyNP              0.188    0.042    4.470    0.000    0.188    0.202
 .TrustThirdPartyPrime ~~
   .TrstThrdPrtyNP              0.262    0.039    6.662    0.000    0.262    0.299
 .TrustDelivThirdPartyFBA ~~
   .TrstDlvThPNFBA              0.548    0.058    9.460    0.000    0.548    0.463
   .TrstRtrnsTPFBA              0.506    0.061    8.281    0.000    0.506    0.446
   .TrstRtrnTPNFBA              0.344    0.059    5.804    0.000    0.344    0.263
 .TrustDelivThirdPartyNonFBA ~~
   .TrstRtrnTPNFBA              0.903    0.080   11.235    0.000    0.903    0.558
   .TrstRtrnsTPFBA              0.423    0.069    6.131    0.000    0.423    0.301
 .TrustReturnsThirdPartyFBA ~~
   .TrstRtrnTPNFBA              0.631    0.080    7.920    0.000    0.631    0.406
 .TrustNoFakeFromThirdParty ~~
   .TrstDlvThrPFBA              0.210    0.052    4.022    0.000    0.210    0.160
   .TrstDlvThPNFBA              0.566    0.066    8.577    0.000    0.566    0.348
   .TrstRtrnsTPFBA              0.160    0.067    2.398    0.016    0.160    0.103
   .TrstRtrnTPNFBA              0.610    0.075    8.096    0.000    0.610    0.339

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .TrustAmznItslf    0.803    0.060   13.297    0.000    0.803    0.715
   .TrstAmznTksMyS    1.653    0.101   16.447    0.000    1.653    0.828
   .TrstNFkFrmThrP    1.806    0.100   18.135    0.000    1.806    0.749
   .TrstDlvThrPFBA    0.957    0.070   13.594    0.000    0.957    0.745
   .TrstDlvThPNFBA    1.464    0.089   16.427    0.000    1.464    0.833
   .TrstRtrnsTPFBA    1.348    0.098   13.748    0.000    1.348    0.739
   .TrstRtrnTPNFBA    1.791    0.099   18.095    0.000    1.791    0.817
   .TrstThrdPrtyPr    0.714    0.046   15.678    0.000    0.714    0.556
   .TrstThrdPrtyNP    1.072    0.086   12.422    0.000    1.072    0.564

R-Square:          Estimate                          R-Square:          Estimate
    TrustAmznItslf    0.285                              TrstRtrnsTPFBA    0.261
    TrstAmznTksMyS    0.172                              TrstRtrnTPNFBA    0.183
    TrstNFkFrmThrP    0.251                              TrstThrdPrtyPr    0.444
    TrstDlvThrPFBA    0.255                              TrstThrdPrtyNP    0.436
    TrstDlvThPNFBA    0.167
```

Model 1 from the paper but with a FBA focus instead of a Prime focus:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 84 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        37

                                                  Used       Total
  Number of observations                           758         772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                49.901      43.235
  Degrees of freedom                                26          26
  P-value (Chi-square)                           0.003       0.018
  Scaling correction factor                                  1.200
  Shift parameter                                            1.641
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              2019.717    1585.247
  Degrees of freedom                                57          57
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.288

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.988       0.989
  Tucker-Lewis Index (TLI)                       0.973       0.975

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -13492.392  -13492.392
  Loglikelihood unrestricted model (H1)     -13467.442  -13467.442

  Akaike (AIC)                               27058.785   27058.785
  Bayesian (BIC)                             27230.120   27230.120
  Sample-size adjusted Bayesian (BIC)        27112.630   27112.630

Root Mean Square Error of Approximation:

  RMSEA                                          0.035       0.030
  90 Percent confidence interval - lower         0.020       0.012
  90 Percent confidence interval - upper         0.049       0.045
  P-value RMSEA <= 0.05                          0.958       0.989

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.022       0.022

Parameter Estimates:

  Standard errors                             Robust.sem
  Information                                   Expected
  Information saturated (h1) model            Structured

Regressions:
                                Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  LikeBuyFromAmznItself ~
    TrustAmznItslf                 0.463    0.051    9.072    0.000    0.463    0.357
    ImportncFstDlv                 0.130    0.038    3.450    0.001    0.130    0.119
    DoNotLikeAmazn                -0.164    0.024   -6.764    0.000   -0.164   -0.238
    HasPrimeOrNot                  0.881    0.104    8.476    0.000    0.881    0.288
  LikeBuyFromThirdPartyFBA ~
    TrstThrdPrtFBA                 0.398    0.037   10.834    0.000    0.398    0.411
    DoNotLikeAmazn                -0.098    0.019   -5.001    0.000   -0.098   -0.158
  LikeBuyFromThirdPartyNonFBA ~
    TrstThrdPrNFBA                 0.636    0.031   20.287    0.000    0.636    0.628
    TrustAmznItslf                -0.170    0.040   -4.241    0.000   -0.170   -0.126
    AmznPltfrmIsTC                -0.059    0.030   -1.988    0.047   -0.059   -0.064
  S1FromAmzn3PwPrime ~
    LkByFrmAmznIts                 4.952    0.779    6.353    0.000    4.952    0.214
```

```
    LkByFrmThrPFBA              2.588    1.025    2.526    0.012    2.588    0.100
    LkByFrmThPNFBA             -2.211    0.806   -2.744    0.006   -2.211   -0.100
    TrstThrdPrtFBA              2.059    0.950    2.167    0.030    2.059    0.082
    HasPrimeOrNot              25.136    2.370   10.608    0.000   25.136    0.355
    AmznPltfrmIsTC             -2.338    0.649   -3.601    0.000   -2.338   -0.114
  S1FromAmzn3PwoPrime ~
    LkByFrmThrPFBA              1.090    0.516    2.110    0.035    1.090    0.072
    LkByFrmThPNFBA              1.570    0.499    3.147    0.002    1.570    0.121
    TrstThrdPrNFBA              2.432    0.479    5.081    0.000    2.432    0.184
    HasPrimeOrNot             -14.337    1.776   -8.071    0.000  -14.337   -0.345
    AmznPltfrmIsTC              0.776    0.337    2.304    0.021    0.776    0.065
  PercentageOnAmazon ~
    LkByFrmAmznIts              2.677    0.916    2.922    0.003    2.677    0.137
    S1FrmAmzn3PwPr              0.191    0.038    5.053    0.000    0.191    0.225
    S1FrmAmzn3PwPr              0.239    0.056    4.270    0.000    0.239    0.166
    HasPrimeOrNot               9.017    2.723    3.312    0.001    9.017    0.150
    DoNotLikeAmazn             -1.296    0.524   -2.474    0.013   -1.296   -0.096
    ImportncFstDlv              1.920    0.759    2.530    0.011    1.920    0.090
    AmznPltfrmIsTC             -1.314    0.592   -2.221    0.026   -1.314   -0.076

Covariances:
                              Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .LikeBuyFromAmznItself ~~
   .LkByFrmThrPFBA               0.331    0.057    5.855    0.000    0.331    0.298
   .LkByFrmThPNFBA               0.194    0.049    3.948    0.000    0.194    0.170
 .LikeBuyFromThirdPartyFBA ~~
   .LkByFrmThPNFBA               0.496    0.061    8.185    0.000    0.496    0.403
 .S1FromAmzn3PwPrime ~~
   .S1FrmAmzn3PwPr            -117.768   15.210   -7.743    0.000 -117.768   -0.273

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .LkByFrmAmznIts    1.032    0.074   13.983    0.000    1.032    0.545
   .LkByFrmThrPFBA    1.194    0.071   16.902    0.000    1.194    0.794
   .LkByFrmThPNFBA    1.272    0.084   15.210    0.000    1.272    0.619
   .S1FrmAmzn3PwPr  701.030   33.430   20.970    0.000  701.030    0.694
   .S1FrmAmzn3PwPr  265.681   24.878   10.679    0.000  265.681    0.765
   .PercentgOnAmzn  566.571   28.101   20.162    0.000  566.571    0.781

R-Square:
                   Estimate
    LkByFrmAmznIts    0.455
    LkByFrmThrPFBA    0.206
    LkByFrmThPNFBA    0.381
    S1FrmAmzn3PwPr    0.306
    S1FrmAmzn3PwPr    0.235
    PercentgOnAmzn    0.219
```

Model 2 from the paper but with a FBA focus instead of a Prime focus:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV", se="robust")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 28 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        46

  Number of observations                           772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                                90.691      74.363
  Degrees of freedom                                26          26
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.255
  Shift parameter                                            2.103
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              3678.514    1166.024
  Degrees of freedom                                63          63
  P-value                                        0.000       0.000
  Scaling correction factor                                  3.262

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.982       0.956
  Tucker-Lewis Index (TLI)                       0.957       0.894

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -10020.879  -10020.879
  Loglikelihood unrestricted model (H1)      -9975.534   -9975.534

  Akaike (AIC)                               20133.759   20133.759
  Bayesian (BIC)                             20347.612   20347.612
  Sample-size adjusted Bayesian (BIC)        20201.540   20201.540

Root Mean Square Error of Approximation:

  RMSEA                                          0.057       0.049
  90 Percent confidence interval - lower         0.044       0.036
  90 Percent confidence interval - upper         0.070       0.062
  P-value RMSEA <= 0.05                          0.177       0.522

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.046       0.046

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TrustAmznItself ~
    TrstOnlnStrGnr                  0.450    0.044   10.300    0.000    0.450    0.451
    HasPrimeOrNot                   0.496    0.076    6.505    0.000    0.496    0.210
    ImportncFstDlv                  0.125    0.030    4.139    0.000    0.125    0.149
  TrustAmznTakesMySide ~
    TrustAmznItslf                  0.550    0.052   10.657    0.000    0.550    0.414
  TrustNoFakeFromThirdParty ~
    TrstOnlnStrGnr                  0.141    0.050    2.845    0.004    0.141    0.097
    TrstAmznTksMyS                  0.516    0.039   13.364    0.000    0.516    0.471
  TrustDelivThirdPartyFBA ~
    TrstAmznTksMyS                  0.134    0.029    4.599    0.000    0.134    0.167
    TrustAmznItslf                  0.378    0.041    9.232    0.000    0.378    0.355
    TrstOnlnStrGnr                  0.106    0.045    2.363    0.018    0.106    0.099
  TrustDelivThirdPartyNonFBA ~
    TrstAmznTksMyS                  0.254    0.036    7.136    0.000    0.254    0.272
    TrstOnlnStrGnr                  0.292    0.046    6.328    0.000    0.292    0.236
```

```
  TrustReturnsThirdPartyFBA ~
    TrstAmznTksMyS                0.313    0.036    8.709    0.000    0.313    0.327
    TrustAmznItslf                0.356    0.042    8.492    0.000    0.356    0.280
  TrustReturnsThirdPartyNonFBA ~
    TrstAmznTksMyS                0.408    0.037   10.932    0.000    0.408    0.390
    TrstOnlnStrGnr                0.132    0.044    3.006    0.003    0.132    0.095
  TrustThirdPartyFBA ~
    TrstOnlnStrGnr                0.150    0.037    4.102    0.000    0.150    0.128
    TrstDlvThrPFBA                0.333    0.055    6.084    0.000    0.333    0.304
    TrstRtrnsTPFBA                0.129    0.039    3.320    0.001    0.129    0.140
    TrstNFkFrmThrP                0.188    0.027    7.053    0.000    0.188    0.235
  TrustThirdPartyNonFBA ~
    TrstOnlnStrGnr                0.159    0.038    4.220    0.000    0.159    0.126
    ImportncFstDlv               -0.062    0.023   -2.637    0.008   -0.062   -0.058
    TrstDlvThPNFBA                0.457    0.041   11.086    0.000    0.457    0.448
    TrstRtrnTPNFBA                0.199    0.033    5.954    0.000    0.199    0.218
    TrstNFkFrmThrP                0.124    0.030    4.102    0.000    0.124    0.142

Covariances:
                               Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .TrustAmznItself ~~
   .TrstThrdPrtFBA                0.185    0.044    4.209    0.000    0.185    0.214
   .TrstThrdPrNFBA                0.040    0.039    1.046    0.296    0.040    0.050
 .TrustThirdPartyFBA ~~
   .TrstThrdPrNFBA                0.403    0.048    8.341    0.000    0.403    0.462
 .TrustDelivThirdPartyFBA ~~
   .TrstDlvThPNFBA                0.556    0.058    9.666    0.000    0.556    0.467
   .TrstRtrnsTPFBA                0.506    0.061    8.301    0.000    0.506    0.444
   .TrstRtrnTPNFBA                0.352    0.059    5.922    0.000    0.352    0.267
 .TrustDelivThirdPartyNonFBA ~~
   .TrstRtrnTPNFBA                0.915    0.080   11.378    0.000    0.915    0.561
   .TrstRtrnsTPFBA                0.421    0.069    6.120    0.000    0.421    0.298
 .TrustReturnsThirdPartyFBA ~~
   .TrstRtrnTPNFBA                0.629    0.080    7.903    0.000    0.629    0.403
 .TrustNoFakeFromThirdParty ~~
   .TrstDlvThrPFBA                0.217    0.052    4.151    0.000    0.217    0.165
   .TrstDlvThPNFBA                0.575    0.066    8.683    0.000    0.575    0.352
   .TrstRtrnsTPFBA                0.157    0.067    2.339    0.019    0.157    0.100
   .TrstRtrnTPNFBA                0.618    0.075    8.211    0.000    0.618    0.343

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .TrustAmznItslf    0.804    0.060   13.330    0.000    0.804    0.711
   .TrstAmznTksMyS    1.657    0.100   16.648    0.000    1.657    0.829
   .TrstNFkFrmThrP    1.806    0.100   18.145    0.000    1.806    0.752
   .TrstDlvThrPFBA    0.962    0.071   13.614    0.000    0.962    0.749
   .TrstDlvThPNFBA    1.477    0.089   16.646    0.000    1.477    0.847
   .TrstRtrnsTPFBA    1.350    0.098   13.728    0.000    1.350    0.739
   .TrstRtrnTPNFBA    1.802    0.099   18.282    0.000    1.802    0.825
   .TrstThrdPrtFBA    0.928    0.068   13.573    0.000    0.928    0.599
   .TrstThrdPrNFBA    0.821    0.062   13.227    0.000    0.821    0.452

R-Square:
                   Estimate
    TrustAmznItslf    0.289
    TrstAmznTksMyS    0.171
    TrstNFkFrmThrP    0.248
    TrstDlvThrPFBA    0.251
    TrstDlvThPNFBA    0.153
    TrstRtrnsTPFBA    0.261
    TrstRtrnTPNFBA    0.175
    TrstThrdPrtFBA    0.401
    TrstThrdPrNFBA    0.548
```

Combination of model 1 and model 2 into one large model:

```
> fit<-sem(model = model, data=RawData, estimator="MLMV")
> summary(fit,fit.measures=T,standardized=T,rsq=T)
lavaan 0.6-11 ended normally after 106 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        89

                                                  Used       Total
  Number of observations                           758         772

Model Test User Model:
                                              Standard      Robust
  Test Statistic                               301.123     231.396
  Degrees of freedom                               106         106
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.423
  Shift parameter                                           19.739
       simple second-order correction

Model Test Baseline Model:

  Test statistic                              5945.059    2072.927
  Degrees of freedom                               180         180
  P-value                                        0.000       0.000
  Scaling correction factor                                  3.032

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.966       0.934
  Tucker-Lewis Index (TLI)                       0.943       0.888

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -23243.347  -23243.347
  Loglikelihood unrestricted model (H1)     -23092.786  -23092.786

  Akaike (AIC)                               46664.694   46664.694
  Bayesian (BIC)                             47076.825   47076.825
  Sample-size adjusted Bayesian (BIC)        46794.213   46794.213

Root Mean Square Error of Approximation:

  RMSEA                                          0.049       0.040
  90 Percent confidence interval - lower         0.043       0.033
  90 Percent confidence interval - upper         0.056       0.046
  P-value RMSEA <= 0.05                          0.560       0.994

  Robust RMSEA                                                  NA
  90 Percent confidence interval - lower                        NA
  90 Percent confidence interval - upper                        NA

Standardized Root Mean Square Residual:

  SRMR                                           0.045       0.045

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  TrustAmznItself ~
    TrstOnlnStrGnr                  0.431    0.043    9.923    0.000    0.431    0.435
    HasPrimeOrNot                   0.478    0.069    6.886    0.000    0.478    0.203
    DoNotLikeAmazn                 -0.174    0.017  -10.362    0.000   -0.174   -0.328
  TrustAmznTakesMySide ~
    TrustAmznItslf                  0.464    0.057    8.147    0.000    0.464    0.348
    DoNotLikeAmazn                 -0.108    0.028   -3.849    0.000   -0.108   -0.152

  TrustNoFakeFromThirdParty ~
    TrstOnlnStrGnr                  0.140    0.050    2.813    0.005    0.140    0.096
    TrstAmznTksMyS                  0.518    0.039   13.276    0.000    0.518    0.470
  TrustDelivThirdPartyFBA ~
    TrstAmznTksMyS                  0.131    0.029    4.453    0.000    0.131    0.163
    TrustAmznItslf                  0.378    0.042    9.002    0.000    0.378    0.353
    TrstOnlnStrGnr                  0.103    0.045    2.281    0.023    0.103    0.097
```

```
  TrustDelivThirdPartyNonFBA ~
    TrstAmznTksMyS        0.256    0.036    7.172    0.000    0.256    0.274
    TrstOnlnStrGnr        0.286    0.047    6.134    0.000    0.286    0.232
  TrustReturnsThirdPartyFBA ~
    TrstAmznTksMyS        0.314    0.036    8.654    0.000    0.314    0.329
    TrustAmznItslf        0.350    0.043    8.214    0.000    0.350    0.275
  TrustReturnsThirdPartyNonFBA ~
    TrstAmznTksMyS        0.410    0.038   10.861    0.000    0.410    0.391
    TrstOnlnStrGnr        0.126    0.044    2.873    0.004    0.126    0.092
  TrustThirdPartyPrime ~
    TrstOnlnStrGnr        0.219    0.036    6.090    0.000    0.219    0.207
    DoNotLikeAmazn       -0.054    0.016   -3.364    0.001   -0.054   -0.094
    HasPrimeOrNot         0.641    0.075    8.581    0.000    0.641    0.254
    TrstDlvThrPFBA        0.222    0.036    6.208    0.000    0.222    0.223
    TrstRtrnsTPFBA        0.133    0.030    4.384    0.000    0.133    0.158
    TrstNFkFrmThrP        0.075    0.021    3.632    0.000    0.075    0.104
  TrustThirdPartyNonPrime ~
    TrstOnlnStrGnr        0.148    0.040    3.734    0.000    0.148    0.116
    ImportncFstDlv       -0.083    0.029   -2.871    0.004   -0.083   -0.076
    HasPrimeOrNot        -0.221    0.077   -2.890    0.004   -0.221   -0.073
    TrstDlvThPNFBA        0.405    0.051    7.924    0.000    0.405    0.392
    TrstRtrnTPNFBA        0.183    0.043    4.226    0.000    0.183    0.198
    TrstNFkFrmThrP        0.099    0.034    2.950    0.003    0.099    0.113
  LikeBuyFromAmznItself ~
    TrustAmznItslf        0.484    0.054    9.033    0.000    0.484    0.371
    ImportncFstDlv        0.111    0.038    2.938    0.003    0.111    0.102
    DoNotLikeAmazn       -0.165    0.025   -6.629    0.000   -0.165   -0.237
    HasPrimeOrNot         0.903    0.111    8.152    0.000    0.903    0.294
  LikeBuyFromThirdPartyPrime ~
    TrstThrdPrtyPr        0.357    0.047    7.547    0.000    0.357    0.307
    TrstDlvThrPFBA        0.177    0.044    3.991    0.000    0.177    0.152
    DoNotLikeAmazn       -0.059    0.021   -2.864    0.004   -0.059   -0.089
    HasPrimeOrNot         0.789    0.095    8.291    0.000    0.789    0.269
    AmznPltfrmIsTC       -0.074    0.025   -2.972    0.003   -0.074   -0.087
  LikeBuyFromThirdPartyNonPrime ~
    TrstThrdPrtyNP        0.563    0.032   17.542    0.000    0.563    0.539
    TrustAmznItslf       -0.163    0.046   -3.558    0.000   -0.163   -0.121
    HasPrimeOrNot        -0.386    0.098   -3.917    0.000   -0.386   -0.122
  S1FromAmzn3PwPrime ~
    LkByFrmAmznIts        3.895    0.837    4.651    0.000    3.895    0.170
    LkByFrmThrdPrP        2.705    1.054    2.567    0.010    2.705    0.112
    LkByFrmThrdPNP       -3.092    0.744   -4.154    0.000   -3.092   -0.139
    TrstThrdPrtyPr        4.192    1.050    3.991    0.000    4.192    0.150
    HasPrimeOrNot        19.584    2.561    7.646    0.000   19.584    0.277
    AmznPltfrmIsTC       -1.679    0.597   -2.811    0.005   -1.679   -0.082
  S1FromAmzn3PwoPrime ~
    LkByFrmThrdPNP        2.019    0.478    4.227    0.000    2.019    0.156
    TrstThrdPrtyNP        1.285    0.599    2.145    0.032    1.285    0.095
    TrstDlvThPNFBA        1.877    0.543    3.459    0.001    1.877    0.134
    HasPrimeOrNot       -13.269    1.831   -7.248    0.000  -13.269   -0.323
  PercentageOnAmazon ~
    LkByFrmAmznIts        2.677    0.897    2.985    0.003    2.677    0.137
    S1FrmAmzn3PwPr        0.191    0.038    5.061    0.000    0.191    0.224
    S1FrmAmzn3PwPr        0.239    0.056    4.289    0.000    0.239    0.164
    HasPrimeOrNot         9.017    2.734    3.298    0.001    9.017    0.150
    DoNotLikeAmazn       -1.296    0.531   -2.440    0.015   -1.296   -0.096
    ImportncFstDlv        1.920    0.751    2.556    0.011    1.920    0.090
    AmznPltfrmIsTC       -1.314    0.589   -2.233    0.026   -1.314   -0.076

Covariances:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .TrustAmznItself ~~
   .TrstThrdPrtyPr      0.329    0.039    8.467    0.000    0.329    0.465
   .TrstThrdPrtyNP      0.174    0.039    4.485    0.000    0.174    0.203
 .TrustThirdPartyPrime ~~
   .TrstThrdPrtyNP      0.264    0.040    6.674    0.000    0.264    0.308
 .TrustDelivThirdPartyFBA ~~
   .TrstDlvThPNFBA      0.572    0.058    9.828    0.000    0.572    0.477
   .TrstRtrnsTPFBA      0.509    0.062    8.201    0.000    0.509    0.445
   .TrstRtrnTPNFBA      0.353    0.060    5.857    0.000    0.353    0.267

 .TrustDelivThirdPartyNonFBA ~~
   .TrstRtrnTPNFBA      0.930    0.081   11.453    0.000    0.930    0.569
   .TrstRtrnsTPFBA      0.435    0.070    6.253    0.000    0.435    0.309
 .TrustReturnsThirdPartyFBA ~~
   .TrstRtrnTPNFBA      0.628    0.081    7.780    0.000    0.628    0.403
 .TrustNoFakeFromThirdParty ~~
   .TrstDlvThrPFBA      0.222    0.053    4.187    0.000    0.222    0.167
   .TrstDlvThPNFBA      0.584    0.067    8.730    0.000    0.584    0.356
   .TrstRtrnsTPFBA      0.150    0.068    2.213    0.027    0.150    0.096
   .TrstRtrnTPNFBA      0.630    0.076    8.255    0.000    0.630    0.347
 .LikeBuyFromAmznItself ~~
   .LkByFrmThrdPrP      0.419    0.058    7.290    0.000    0.419    0.389
   .LkByFrmThrdPNP      0.171    0.051    3.323    0.001    0.171    0.141
 .LikeBuyFromThirdPartyPrime ~~
   .LkByFrmThrdPNP      0.414    0.057    7.217    0.000    0.414    0.327
 .S1FromAmzn3PwPrime ~~
```

```
   .S1FrmAmzn3PwPr              -104.475   14.499   -7.206    0.000 -104.475   -0.247

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .TrustAmznItslf    0.706    0.052   13.616    0.000    0.706    0.631
   .TrstAmznTksMyS    1.621    0.097   16.777    0.000    1.621    0.815
   .TrstNFkFrmThrP    1.823    0.101   18.033    0.000    1.823    0.755
   .TrstDlvThrPFBA    0.972    0.072   13.522    0.000    0.972    0.756
   .TrstDlvThPNFBA    1.479    0.089   16.575    0.000    1.479    0.850
   .TrstRtrnsTPFBA    1.345    0.099   13.524    0.000    1.345    0.743
   .TrstRtrnTPNFBA    1.803    0.100   18.028    0.000    1.803    0.826
   .TrstThrdPrtyPr    0.708    0.045   15.637    0.000    0.708    0.553
   .TrstThrdPrtyNP    1.039    0.081   12.905    0.000    1.039    0.558
   .LkByFrmAmznIts    1.031    0.074   13.943    0.000    1.031    0.542
   .LkByFrmThrdPrP    1.129    0.067   16.975    0.000    1.129    0.652
   .LkByFrmThrdPNP    1.425    0.075   19.113    0.000    1.425    0.702
   .S1FrmAmzn3PwPr  675.247   32.922   20.511    0.000  675.247    0.672
   .S1FrmAmzn3PwPr  264.170   25.627   10.308    0.000  264.170    0.779
   .PercentgOnAmzn  566.571   28.089   20.170    0.000  566.571    0.780

R-Square:
                   Estimate
    TrustAmznItslf    0.369
    TrstAmznTksMyS    0.185
    TrstNFkFrmThrP    0.245
    TrstDlvThrPFBA    0.244
    TrstDlvThPNFBA    0.150
    TrstRtrnsTPFBA    0.257
    TrstRtrnTPNFBA    0.174
    TrstThrdPrtyPr    0.447
    TrstThrdPrtyNP    0.442
    LkByFrmAmznIts    0.458
    LkByFrmThrdPrP    0.348
    LkByFrmThrdPNP    0.298
    S1FrmAmzn3PwPr    0.328
    S1FrmAmzn3PwPr    0.221
    PercentgOnAmzn    0.220
```

Paper V Appendix

Appendix of the paper „Inventory competition on electronic marketplaces – A competitive newsvendor problem with a unilateral sales commission fee“

Reference: Straubert, C., & Sucky, E. (2023). Inventory competition on electronic marketplaces–A competitive newsvendor problem with a unilateral sales commission fee. European Journal of Operational Research, 309(2), 656–670. doi.org/10.1016/j.ejor.2023.02.002

V.A.1. Appendix A

Determining the reaction functions of *A* and *B*.

Cost function for *A*:

$$
\begin{aligned}
Min\ E[K_A(x_A)] = c_{oA} \cdot & \int_{b=0}^{x_A} (x_A - b) f(b) db \\
& + c_{uA1} \int_{b=x_A}^{x_A+x_B} (b - x_A) f(b) db \\
& + \int_{b=x_A+x_B}^{\infty} \left[c_{uA1} \cdot x_B + c_{uA2} \cdot (b - x_A - x_B) \right] f(b) db
\end{aligned}
$$

First order derivatives (Leibniz integral rule). First term:

$$
\begin{aligned}
& \frac{\partial}{\partial x_A} \left[c_{oA} \cdot \int_{b=0}^{x_A} (x_A - b) f(b) db \right] \\
& = c_{oA} \int_{b=0}^{x_A} f(b) db + c_{oA}(x_A - x_A) f(x_A) \\
& = c_{oA} \cdot F(x_A)
\end{aligned}
$$

Second term:

$$\frac{\partial}{\partial x_A}\left[c_{uA1}\int_{b=x_A}^{x_A+x_B}(b-x_A)f(b)db\right]$$

$$=c_{uA1}\int_{b=x_A}^{x_A+x_B}(-1)f(b)db+$$

$$c_{uA1}\left((x_A+x_B)-x_A\right)f(x_A+x_B)-c_{uA1}\left(x_A-x_A\right)f(x_A)$$

$$=-c_{uA1}\left(F(x_A+x_B)-F(x_A)\right)+c_{uA1}\cdot x_B\cdot f(x_A+x_B)$$

Third term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=x_A+x_B}^{\infty}\left[c_{uA1}\cdot x_B+c_{uA2}\cdot(b-x_A-x_B)\right]f(b)db\right]$$

$$=c_{uA2}\int_{b=x_A+x_B}^{\infty}(-1)f(b)db-$$

$$\left[c_{uA1}\cdot x_B+c_{uA2}\cdot\left((x_A+x_B)-x_A-x_B\right)\right]f(x_A+x_B)$$

$$=-c_{uA2}\left(1-F(x_A+x_B)\right)-c_{uA1}\cdot x_B\cdot f(x_A+x_B)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[K_A(x_A)]}{\partial x_A}=$$
$$c_{oA}\cdot F(x_A)-c_{uA1}\left(F(x_A+x_B)-F(x_A)\right)-c_{uA2}\left(1-F(x_A+x_B)\right)$$

First order condition: $\frac{\partial E[K_A(x_A)]}{\partial x_A}=0$

$$c_{oA}\cdot F(x_A)-c_{uA1}\left(F(x_A+x_B)-F(x_A)\right)-c_{uA2}\left(1-F(x_A+x_B)\right)=0$$
$$\Leftrightarrow F(x_A^*)=\frac{c_{uA2}-\left(c_{uA2}-c_{uA1}\right)F(x_A^*+x_B)}{\left(c_{oA}+c_{uA1}\right)}$$

Given that $c_{uA2}-c_{uA1}=r-c_A-(r-c_A-p)=p$:

$$F(x_A^*)=\frac{c_{uA2}-p\cdot F(x_A^*+x_B)}{c_{oA}+c_{uA1}}=\frac{c_{uA2}-p\cdot F(x_A^*+x_B)}{c_{oA}+c_{uA2}-p}=$$
$$\frac{r-c_A-p\cdot F(x_A^*+x_B)}{r-v_A-p}$$

$$x_A^*=F^{-1}\left(\frac{c_{uA2}-p\cdot F(x_A^*+x_B)}{c_{oA}+c_{uA2}-p}\right)=F^{-1}\left(\frac{r-c_A-p\cdot F(x_A^*+x_B)}{r-v_A-p}\right)$$

Second order derivative:

$$\frac{\partial^2 E[K_A(x_A)]}{(\partial x_A)^2} = c_{oA} f(x_A) - c_{uA1} f\left(x_A + x_B\right) + c_{uA1} f\left(x_A\right) + c_{uA2} f\left(x_A + x_B\right)$$

Because of $c_{uA2} > c_{uA1}$, it is always true that:

$$\frac{\partial^2 E[K_A(x_A)]}{(\partial x_A)^2} > 0$$

Thus, the function is always convex and the second order condition for a minimum at

$x_A^* = F^{-1}\left(\frac{c_{uA2} - p \cdot F(x_A^* + x_B)}{c_{oA} + c_{uA2} - p}\right) = F^{-1}\left(\frac{r - c_A - p \cdot F(x_A^* + x_B)}{r - v_A - p}\right)$ is always fulfilled.

Cost function for B:

$$Min\ E[K_B(x_B)] = c_{oB} \cdot \int_{b=0}^{x_A} x_B f(b)db$$

$$+c_{oB} \int_{b=x_A}^{x_A+x_B} (x_A + x_B - b) f(b)db$$

$$+c_{uB} \int_{b=x_A+x_B}^{\infty} (b - x_A - x_B) f(b)db$$

First order derivatives (Leibniz integral rule). First term:

$$\frac{\partial}{\partial x_B}\left[c_{oB} \cdot \int_{b=0}^{x_A} x_B f(b)db \right] = c_{oB} \int_{b=0}^{x_A} f(b)db = c_{oB} \cdot F(x_A)$$

Second term:

$$\frac{\partial}{\partial x_B}\left[c_{oB} \int_{b=x_A}^{x_A+x_B} (x_A + x_B - b) f(b)db \right]$$

$$= c_{oB} \int_{b=x_A}^{x_A+x_B} f(b)db + c_{oB} \left(x_A + x_B - (x_A + x_B) \right) f(x_A + x_B)$$

$$= c_{oB} \left(F(x_A + x_B) - F(x_A) \right)$$

Third term:

$$\frac{\partial}{\partial x_B}\left[c_{uB} \int_{b=x_A+x_B}^{\infty} (b - x_A - x_B) f(b) db \right]$$

$$= c_{uB} \int_{b=x_A+x_B}^{\infty} (-1) f(b) db - \left[c_{uB} \cdot \left((x_A + x_B) - x_A - x_B \right) \right] f(x_A + x_B)$$

$$= -c_{uB} \cdot \left(1 - F(x_A + x_B) \right)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[K_B(x_B)]}{\partial x_B} = \left(c_{oB} + c_{uB} \right) \cdot F(x_A + x_B) - c_{uB}$$

First order condition: $\frac{\partial E[K_B(x_B)]}{\partial x_B} = 0$

$$\left(c_{oB} + c_{uB} \right) \cdot F(x_A + x_B) - c_{uB} = 0$$

$$\Leftrightarrow F(x_A + x_B^*) = \frac{c_{uB}}{c_{oB} + c_{uB}} = \frac{r - c_B - p}{r - v_B - p}$$

$$\Leftrightarrow x_B^* = F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) - x_A$$

Second order derivative:

$$\frac{\partial^2 E[K_B(x_B)]}{(\partial x_B)^2} = \left(c_{oB} + c_{uB}\right) \cdot f(x_A + x_B)$$

Because of $c_{uB} > 0$ and $c_{oB} > 0$, it is always true that:

$$\frac{\partial^2 E[K_B(x_B)]}{(\partial x_B)^2} > 0$$

Thus, the function is always convex and the second order condition for a minimum at $x_B^* = F^{-1}\left(\frac{r - c_B - p}{r - v_B - p}\right) - x_A$ is always fulfilled.

V.A.2. Appendix B

Determining the Nash equilibrium for our base case model.

Reaction function of *A*: $x_A^* = \begin{cases} F^{-1}\left(\dfrac{r-c_A-p\cdot F(x_A^*+x_B)}{r-v_A-p}\right) & \text{if } x_B>0 \\ F^{-1}\left(\dfrac{r-c_A}{r-v_A}\right) & \text{else} \end{cases}$

Reaction function of *B*: $x_B^* = \max\left\{0,\ F^{-1}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)-x_A\right\}$

The following assumes that $x_B>0$.

Substituting the reaction function of *B* for x_B in the reaction function of *A*:

$$x_A^* = F^{-1}\left(\frac{r-c_A-p\cdot F\left(x_A^*+F^{-1}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)-x_A^*\right)}{r-v_A-p}\right)$$

$$\Leftrightarrow x_A^* = F^{-1}\left(\frac{r-c_A-p\cdot\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$$

And it follows for *B*:

$$x_B^* = F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-F^{-1}\left(\frac{r-c_A-p\cdot\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$$

Note, that $F^{-1}\left(\frac{r-c_A}{r-v_A}\right) > F^{-1}\left(\frac{r-c_A-p\cdot\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$ is always true.

Thus, if $x_B^*=0$ would be optimal for *B*, this result ($x_B^*=0$) would also be stable. Therefore, the Nash equilibrium is either

$$x_A^*=F^{-1}\left(\frac{r-c_A-p\cdot\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$$ and

$$x_B^*=F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-F^{-1}\left(\frac{r-c_A-p\cdot\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}\right)$$ or

$F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$ and $x_B^*=0$.

V.A.3. Appendix C

Please refer to **Appendix V.A.6** (case ME.1) for the mathematical derivation in the general case. The results in **Appendix V.A.6** (case ME.1) can be easily adapted to our base case model using the following values for the *h*-paramaters are used:

$h_1 = 0$ Probability that a customer orders from *B* while both *A* and *B* still have stock.

$h_2 = 1$ Probability that a customer orders from *B* when *A* is out of stock.

$h_3 = 1$ Probability that a customer orders from *A* while both *A* and *B* still have stock.

$h_4 = 1$ Probability that a customer orders from *A* when *B* is out of stock.

In the following, we want to expand on the different possible cases encountered when determining the cooperative optimum:

$$x_A^* = \begin{cases} F^{-1}\left(\dfrac{c_B - c_A}{v_B - v_A}\right) & \text{if } v_B > v_A \text{ (S.O.C.) and } 0 < \dfrac{c_B - c_A}{v_B - v_A} \le \dfrac{r - c_B}{r - v_B} \text{ (case 1)} \\ F^{-1}\left(\dfrac{r - c_A}{r - v_A}\right) & \text{if } v_B > v_A \text{ (S.O.C.) and } \dfrac{c_B - c_A}{v_B - v_A} > \dfrac{r - c_B}{r - v_B} \text{ (case 2)} \\ 0 & \text{if } v_B > v_A \text{ (S.O.C.) and } \dfrac{c_B - c_A}{v_B - v_A} \le 0 \text{ (case 2)} \\ F^{-1}\left(\dfrac{r - c_A}{r - v_A}\right) & \text{if } v_B < v_A \text{ and } \dfrac{c_B - c_A}{v_B - v_A} \le 0 \text{ (case 3)} \\ 0 & \text{if } v_B < v_A \text{ and } \dfrac{c_B - c_A}{v_B - v_A} > \dfrac{r - c_B}{r - v_B} \text{ (case 3)} \\ 0 \text{ or } F^{-1}\left(\dfrac{r - c_A}{r - v_A}\right) & \text{if } v_B < v_A \text{ and } 0 < \dfrac{c_B - c_A}{v_B - v_A} \le \dfrac{r - c_B}{r - v_B} \text{ (case 4)} \end{cases}$$

Note that the following system of inequalities cannot be true:
$\frac{r - c_B}{r - v_B} > \frac{c_B - c_A}{v_B - v_A} > \frac{r - c_A}{r - v_A}$. Therefore, the condition $\frac{c_B - c_A}{v_B - v_A} > \frac{r - c_B}{r - v_B}$, also automatically ensures $\frac{c_B - c_A}{v_B - v_A} > \frac{r - c_A}{r - v_A}$.

$$x_B^* = \begin{cases} F^{-1}\left(\dfrac{r - c_B}{r - v_B}\right) - x_A^* & \text{if } 0 \le F^{-1}\left(\dfrac{r - c_B}{r - v_B}\right) - x_A^* \\ 0 & \text{else} \end{cases}$$

Generally, one can understand the solution space the following way. There is always one point where both the gradients with respect to x_A and x_B are zero. This point is either a maximum or a saddle point.

- Case 1: The point is a maximum and the maximum lies within the domain $x_A > 0 \wedge x_B > 0$. Then, the maximum also indicates the optimal order quantities for *A* and *B*.

- Case 2: The point is a maximum but does not lie within the domain $x_A > 0 \wedge x_B > 0$. Then, only *A* **or** *B* should order units (Going from the hilltop straight to the edge of the domain by increasing x_A or x_B and then ascending along the edge of the domain to the highest point on the edge).
- Case 3: The point is a saddle point outside of the domain. Also in this case, only *A* **or** *B* should order units (Ascending the convex surface towards the first edge of the domain, crossing the edge and ascending further to the second edge of the domain).
- Case 4: The point is a saddle point within the domain. Also in this case, only *A* **or** *B* should order units (Ascending the convex surface towards the edges). In this case it is unclear, whether the ascend towards $x_A = 0$ or towards $x_B = 0$, ends up with a higher total gross profit. Both solutions need to be compared.

V.A.4. Appendix D

Determining the reaction functions for *A* and *B* for the cases ME.1 and ME.2.

Case ME.1, where $h_3 x_B \geq h_1 x_A$.

1. The expected gross profit for *A*, is given by:

$$
\begin{aligned}
Max\ E[\pi_A(x_A)] = & \int_{b=0}^{\frac{x_A}{h_3}} \left[rh_3 b - c_A x_A + v_A(x_A - h_3 b) + ph_1 b \right] f(b)db \\
& + \int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)} \left[\begin{array}{l} rx_A - c_A x_A + \\ p\left(\dfrac{h_1 x_A}{h_3}\right) + ph_2\left(b - \dfrac{x_A}{h_3}\right) \end{array} \right] f(b)db \\
& + \int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)}^{\infty} \left[rx_A - c_A x_A + px_B \right] f(b)db
\end{aligned}
$$

First order derivatives (Leibniz integral rule). First term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=0}^{\frac{x_A}{h_3}}\left[rh_3b - c_Ax_A + v_A(x_A - h_3b) + ph_1b\right]f(b)db\right]$$

$$= \int_{b=0}^{\frac{x_A}{h_3}}\left[v_A - c_A\right]f(b)db$$

$$+\frac{1}{h_3}\left[rh_3\left(\frac{x_A}{h_3}\right) - c_Ax_A + v_A\left(x_A - h_3\left(\frac{x_A}{h_3}\right)\right) + ph_1\left(\frac{x_A}{h_3}\right)\right]f\left(\frac{x_A}{h_3}\right)$$

$$= \left(v_A - c_A\right)F\left(\frac{x_A}{h_3}\right)$$

$$+\left[\frac{rx_A}{h_3} - \frac{c_Ax_A}{h_3} + \frac{ph_1}{h_3}\left(\frac{x_A}{h_3}\right)\right]f\left(\frac{x_A}{h_3}\right)$$

Second term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[rx_A-c_Ax_A+p\left(\frac{h_1x_A}{h_3}\right)+ph_2\left(b-\frac{x_A}{h_3}\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[r-c_A+\frac{p(h_1-h_2)}{h_3}\right]f(b)db$$

$$+\left(\frac{1}{h_3}-\frac{h_1}{h_2h_3}\right)\left[\begin{array}{l} rx_A-c_Ax_A+p\left(\frac{h_1x_A}{h_3}\right)+ \\ ph_2\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)-\frac{x_A}{h_3}\right)\end{array}\right]f\left(\begin{array}{l}\frac{x_A}{h_3}+ \\ \frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\end{array}\right)$$

$$-\frac{1}{h_3}\left[rx_A-c_Ax_A+p\left(\frac{h_1x_A}{h_3}\right)+ph_2\left(\frac{x_A}{h_3}-\frac{x_A}{h_3}\right)\right]f\left(\frac{x_A}{h_3}\right)$$

$$=\left(r-c_A+\frac{p(h_1-h_2)}{h_3}\right)\left(F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-F\left(\frac{x_A}{h_3}\right)\right)$$

$$+\left[\frac{rx_A}{h_3}-\frac{c_Ax_A}{h_3}+\frac{px_B}{h_3}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$-\left[\frac{h_1rx_A}{h_2h_3}-\frac{h_1c_Ax_A}{h_2h_3}+\frac{h_1px_B}{h_2h_3}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$-\left[\frac{rx_A}{h_3}-\frac{c_Ax_A}{h_3}+\frac{p}{h_3}\left(\frac{h_1x_A}{h_3}\right)\right]f\left(\frac{x_A}{h_3}\right)$$

Third term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}^{\infty}\left[rx_A-c_Ax_A+px_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}^{\infty}\left[r-c_A\right]f(b)db$$

$$-\left(\frac{1}{h_3}-\frac{h_1}{h_2h_3}\right)\left[rx_A-c_Ax_A+px_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$=(r-c_A)\left(1-F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right)$$

$$-\left[\frac{rx_A}{h_3}-\frac{c_Ax_A}{h_3}+\frac{px_B}{h_3}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$+\left[\frac{h_1rx_A}{h_2h_3}-\frac{h_1c_Ax_A}{h_2h_3}+\frac{h_1px_B}{h_2h_3}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[\pi_A(x_A)]}{\partial x_A}=$$

$$\left(v_A-r-\frac{p(h_1-h_2)}{h_3}\right)F\left(\frac{x_A}{h_3}\right)+$$

$$\left(\frac{p(h_1-h_2)}{h_3}\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)+$$

$$(r-c_A)$$

First order condition: $\frac{\partial E[\pi_A(x_A)]}{\partial x_A}=0$

$$\left(v_A-r-\frac{p(h_1-h_2)}{h_3}\right)F\left(\frac{x_A}{h_3}\right)+$$

$$\left(\frac{p(h_1-h_2)}{h_3}\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)+$$

$$(r-c_A)=0$$

$$\Leftrightarrow F\left(\frac{x_A^*}{h_3}\right)=\frac{(r-c_A)+\left(\frac{p(h_1-h_2)}{h_3}\right)F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A^*}{h_3}\right)\right)}{\left(r-v_A+\frac{p(h_1-h_2)}{h_3}\right)}$$

$$\Leftrightarrow x_A^*=h_3F^{-1}\left(\frac{r-c_A-\frac{p(h_2-h_1)}{h_3}F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A^*}{h_3}\right)\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}\right)$$

Second order derivative:

$$\frac{\partial^2E[\pi_A(x_A)]}{(\partial x_A)^2}=$$

$$\left(v_A-r+\frac{p(h_2-h_1)}{h_3}\right)f\left(\frac{x_A}{h_3}\right)-$$

$$\left(\frac{p(h_2-h_1)}{h_3}\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

Generally, the second order condition (S.O.C.) for a maximum at the position determined in the first order condition, $\frac{\partial^2E[\pi_A(x_A^*)]}{(\partial x_A^*)^2}<0$, is not fulfilled for every possible parameter combination.

If $\left(v_A - r + \frac{p(h_2 - h_1)}{h_3}\right) > 0$, then the complete inequality for the second order condition must be checked:

$$\left(v_A - r + \frac{p(h_2 - h_1)}{h_3}\right) f\left(\frac{x_A^*}{h_3}\right) - \left(\frac{p(h_2 - h_1)}{h_3}\right) f\left(\frac{x_A^*}{h_3} + \frac{1}{h_2}\left(x_B - \frac{h_1 x_A^*}{h_3}\right)\right) < 0$$

However, for many realistic parameter combinations one can say for sure, that the S.O.C. is fulfilled. Assume for example the realistic case of $h_2 > h_1$. It follows that $\left(\frac{p(h_2 - h_1)}{h_3}\right)$ is positive. Furthermore assume the realistic case of $p < r - c_A$. And given that $r - c_A < r - v_A$, it follows, that the first term above, $\left(v_A - r + \frac{p(h_2 - h_1)}{h_3}\right) f\left(\frac{x_A^*}{h_3}\right)$, is negative, if $p\frac{(h_2 - h_1)}{h_3} \leq p$.

That is, if $\frac{(h_2 - h_1)}{h_3} \leq 1$, then the S.O.C. is fulfilled, because if $h_2 > h_1$, then the second term above, $-\left(\frac{p(h_2 - h_1)}{h_3}\right) f\left(\frac{x_A^*}{h_3} + \frac{1}{h_2}\left(x_B - \frac{h_1 x_A^*}{h_3}\right)\right)$, is always negative.

Case ME.1, where $h_3 x_B \geq h_1 x_A$.

2. The expected gross profit for *B*, is given by:

$$Max\ E[\pi_B(x_B)] =$$

$$\int_{b=0}^{\frac{x_A}{h_3}} \left[(r-p)h_1 b - c_B x_B + v_B(x_B - h_1 b)\right] f(b)db$$

$$+ \int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)} \begin{bmatrix} (r-p)\left(\dfrac{h_1 x_A}{h_3} + h_2\left(b - \dfrac{x_A}{h_3}\right)\right) - \\ c_B x_B + v_B\left(x_B - \dfrac{h_1 x_A}{h_3} - h_2\left(b - \dfrac{x_A}{h_3}\right)\right) \end{bmatrix} f(b)db$$

$$+ \int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty} \left[(r-p)x_B - c_B x_B\right] f(b)db$$

First order derivatives (Leibniz integral rule). First term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=0}^{\frac{x_A}{h_3}} \left[(r-p)h_1 b - c_B x_B + v_B(x_B - h_1 b)\right] f(b)db\right]$$

$$= \int_{b=0}^{\frac{x_A}{h_3}} \left[v_B - c_B\right] f(b)db = \left(v_B - c_B\right) F\left(\frac{x_A}{h_3}\right)$$

Second term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[(r-p)\left(\frac{h_1x_A}{h_3}+h_2\left(b-\frac{x_A}{h_3}\right)\right)-c_Bx_B+v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(b-\frac{x_A}{h_3}\right)\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[v_B-c_B\right]f(b)db+\frac{1}{h_2}\left[\begin{array}{l}(r-p)\left(\frac{h_1x_A}{h_3}+h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)-\\ c_Bx_B+\\ v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)\end{array}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$=\left(v_B-c_B\right)\left(F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-F\left(\frac{x_A}{h_3}\right)\right)+\left[\frac{rx_B}{h_2}-\frac{px_B}{h_2}-\frac{c_Bx_B}{h_2}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

Third term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[(r-p)x_B-c_B x_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[r-p-c_B\right]f(b)db-$$

$$\frac{1}{h_2}\left[(r-p)x_B-c_B x_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

$$=(r-p-c_B)\left(1-F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)\right)-$$

$$\left[\frac{rx_B}{h_2}-\frac{px_B}{h_2}-\frac{c_B x_B}{h_2}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[\pi_B(x_B)]}{\partial x_B}=(r-p-c_B)+\left(v_B-r+p\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

First order condition: $\frac{\partial E[\pi_B(x_B)]}{\partial x_B}=0$

$$(r-p-c_B)+\left(v_B-r+p\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)=0$$

$$\Leftrightarrow F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B^*-\frac{h_1x_A}{h_3}\right)\right)=\frac{r-c_B-p}{r-v_B-p}$$

$$\Leftrightarrow x_B^*=h_2F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A$$

x_B^* is of course constrained by zero, and therefore:

$$x_B^*=max\left\{0,\ h_2F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A\right\}$$

Second order derivative:

$$\frac{\partial^2E[\pi_B(x_B)]}{(\partial x_B)^2}=\left(v_B-r+p\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

Because of $p<r-c_B$ and $r-c_B<r-v_B$ it must be that: $r>p+v_B$. Therefore, the term is always negative and the second order condition (S.O.C.) for a maximum at the position determined in the first order condition, $\frac{\partial^2E[\pi_B(x_B^*)]}{(\partial x_B^*)^2}<0$, is always fulfilled.

Case ME.2, where $h_3 x_B < h_1 x_A$.

3. The expected gross profit for *A*, is given by:

$$Max\ E[\pi_A(x_A)] =$$

$$\int_{b=0}^{\frac{x_B}{h_1}} \left[rh_3 b - c_A x_A + v_A(x_A - h_3 b) + ph_1 b \right] f(b)db$$

$$+ \int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)} \left[\begin{array}{l} r\left(h_4\left(b - \dfrac{x_B}{h_1} \right) + h_3 \dfrac{x_B}{h_1} \right) - \\ c_A x_A + \\ v_A\left(x_A - h_3 \dfrac{x_B}{h_1} - h_4\left(b - \dfrac{x_B}{h_1} \right) \right) + \\ p x_B \end{array} \right] f(b)db$$

$$+ \int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)}^{\infty} \left[r x_A - c_A x_A + p x_B \right] f(b)db$$

First order derivatives (Leibniz integral rule). First term:

$$\frac{\partial}{\partial x_A} \left[\int_{b=0}^{\frac{x_B}{h_1}} \left[rh_3 b - c_A x_A + v_A(x_A - h_3 b) + ph_1 b \right] f(b)db \right]$$

$$= \int_{b=0}^{\frac{x_B}{h_1}} \left[v_A - c_A \right] f(b)db = \left(v_A - c_A \right) F\left(\frac{x_B}{h_1} \right)$$

Second term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[r\left(h_4\left(b-\frac{x_B}{h_1}\right)+h_3\frac{x_B}{h_1}\right)-c_A x_A+v_A\left(x_A-h_3\frac{x_B}{h_1}-h_4\left(b-\frac{x_B}{h_1}\right)\right)+px_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[v_A-c_A\right]f(b)db+\frac{1}{h_4}\left[\begin{array}{l} r\left(h_4\left(\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)-\frac{x_B}{h_1}\right)+h_3\frac{x_B}{h_1}\right)-c_A x_A+ \\ v_A\left(x_A-h_3\frac{x_B}{h_1}-h_4\left(\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)-\frac{x_B}{h_1}\right)\right)+px_B \end{array}\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

$$=(v_A-c_A)\left(F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)-F\left(\frac{x_B}{h_1}\right)\right)+\left[\frac{rx_A}{h_4}-\frac{c_A x_A}{h_4}+\frac{px_B}{h_4}\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

Third term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)}^{\infty}\left[rx_A-c_Ax_A+px_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)}^{\infty}\left[r-c_A\right]f(b)db$$

$$-\frac{1}{h_4}\left[rx_A-c_Ax_A+px_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$=(r-c_A)\left(1-F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right)$$

$$-\left[\frac{rx_A}{h_4}-\frac{c_Ax_A}{h_4}+\frac{px_B}{h_4}\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[\pi_A(x_A)]}{\partial x_A}=(r-c_A)-(r-v_A)F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

First order condition: $\dfrac{\partial E[\pi_A(x_A)]}{\partial x_A}=0$

$$(r-c_A)-(r-v_A)F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)=0$$

$$\Leftrightarrow F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A^*-\frac{h_3x_B}{h_1}\right)\right)=\frac{r-c_A}{r-v_A}$$

$$\Leftrightarrow x_A^*=h_4F^{-1}\left(\frac{r-c_A}{r-v_A}\right)-\left(\frac{h_4-h_3}{h_1}\right)x_B$$

x_A^* is of course constrained by zero, and therefore:

$$x_A^*=max\left\{0,\ h_4F^{-1}\left(\frac{r-c_A}{r-v_A}\right)-\left(\frac{h_4-h_3}{h_1}\right)x_B\right\}$$

Second order derivative:

$$\frac{\partial^2 E[\pi_A(x_A)]}{(\partial x_A)^2}=(v_A-r)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

Because $r>v_A$, the second order condition (S.O.C.) for a maximum at the position determined in the first order condition, $\dfrac{\partial^2 E[\pi_A(x_A^*)]}{(\partial x_A^*)^2}<0$, is always fulfilled.

Case ME.2, where $h_3 x_B < h_1 x_A$ **.**

4. The expected gross profit for *B*, is given by:

$$Max\ E[\pi_B(x_B)] = \int_{b=0}^{\frac{x_B}{h_1}} \left[(r-p)h_1 b - c_B x_B + v_B(x_B - h_1 b)\right] f(b)db$$

$$+ \int_{b=\frac{x_B}{h_1}}^{\infty} \left[(r-p)x_B - c_B x_B\right] f(b)db$$

First order derivatives (Leibniz integral rule). First term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=0}^{\frac{x_B}{h_1}} \left[(r-p)h_1 b - c_B x_B + v_B(x_B - h_1 b)\right] f(b)db\right]$$

$$= \int_{b=0}^{\frac{x_B}{h_1}} \left[v_B - c_B\right] f(b)db + \frac{1}{h_1}\left[\begin{array}{l}(r-p)h_1\left(\dfrac{x_B}{h_1}\right) - c_B x_B + \\ v_B\left(x_B - h_1\left(\dfrac{x_B}{h_1}\right)\right)\end{array}\right] f\left(\frac{x_B}{h_1}\right)$$

$$= \left(v_B - c_B\right)F\left(\frac{x_B}{h_1}\right) + \left[\frac{rx_B}{h_1} - \frac{px_B}{h_1} - \frac{c_B x_B}{h_1}\right] f\left(\frac{x_B}{h_1}\right)$$

Second term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_B}{h_1}}^{\infty}\left[(r-p)x_B-c_Bx_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}}^{\infty}\left[r-p-c_B\right]f(b)db-\frac{1}{h_1}\left[rx_B-px_B-c_Bx_B\right]f\left(\frac{x_B}{h_1}\right)$$

$$=(r-p-c_B)\left(1-F\left(\frac{x_B}{h_1}\right)\right)-\left[\frac{rx_B}{h_1}-\frac{px_B}{h_1}-\frac{c_Bx_B}{h_1}\right]f\left(\frac{x_B}{h_1}\right)$$

All terms together, after canceling and reordering:

$$\frac{\partial E[\pi_B(x_B)]}{\partial x_B}=(r-p-c_B)+\left(v_B-r+p\right)F\left(\frac{x_B}{h_1}\right)$$

First order condition: $\dfrac{\partial E[\pi_B(x_B)]}{\partial x_B}=0$

$$(r-p-c_B)+(v_B-r+p)F\left(\frac{x_B}{h_1}\right)=0$$

$$\Leftrightarrow F\left(\frac{x_B^*}{h_1}\right)=\frac{r-c_B-p}{r-v_B-p}$$

$$\Leftrightarrow x_B^*=h_1F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

Second order derivative:

$$\frac{\partial^2 E[\pi_B(x_B)]}{(\partial x_B)^2}=(v_B-r+p)\,f\left(\frac{x_B}{h_1}\right)$$

Because of $p<r-c_B$ and $r-c_B<r-v_B$ it must be that: $r>p+v_B$. Therefore, the term is always negative and the second order condition (S.O.C.) for a maximum at the position determined in the first order condition, $\dfrac{\partial^2 E[\pi_B(x_B^*)]}{(\partial x_B^*)^2}<0$, is always fulfilled.

V.A.5. Appendix E

1. The Nash equilibrium for the case ME.1, where $h_3 x_B \geq h_1 x_A$ **:**

Reaction function of *A*:

$$x_A^* = h_3 F^{-1}\left(\frac{r - c_A - \frac{p(h_2 - h_1)}{h_3} F\left(\frac{x_A^*}{h_3} + \frac{1}{h_2}\left(x_B - \frac{h_1 x_A^*}{h_3}\right)\right)}{r - v_A - \frac{p(h_2 - h_1)}{h_3}}\right)$$

Reaction function of *B*: $x_B^* = h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p}\right) - \left(\frac{h_2 - h_1}{h_3}\right) x_A$

Substituting the reaction function of *B* for x_B in the reaction function of *A*:

$$x_A^* = h_3 F^{-1}\left(\frac{r - c_A - \frac{p(h_2 - h_1)}{h_3} F\left(\frac{x_A^*}{h_3} + \frac{1}{h_2}\left(\begin{array}{c} h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p}\right) - \\ \left(\frac{h_2 - h_1}{h_3}\right) x_A - \frac{h_1 x_A^*}{h_3} \end{array}\right)\right)}{r - v_A - \frac{p(h_2 - h_1)}{h_3}}\right)$$

$$\Leftrightarrow x_A^* = h_3 F^{-1}\left(\frac{r - c_A - \frac{p(h_2 - h_1)}{h_3}\left(\frac{r - c_B - p}{r - v_B - p}\right)}{r - v_A - \frac{p(h_2 - h_1)}{h_3}}\right)$$

And it follows for B:

$$x_B^* = h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) - \left(h_2-h_1\right) F^{-1}\left(\frac{r-c_A-\frac{p(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}\right)$$

However, this is only a Nash equilibrium if the condition of case ME.1, $h_3 x_B \geq h_1 x_A$, is met. That is, if:

$$h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) - \left(h_2-h_1\right) F^{-1}\left(\frac{r-c_A-\frac{p(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}\right) \geq$$

$$\frac{h_1}{h_3} h_3 F^{-1}\left(\frac{r-c_A-\frac{p(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}\right)$$

$$\Leftrightarrow h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) \geq h_2 F^{-1}\left(\frac{r-c_A-\frac{p(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}\right)$$

$$\Leftrightarrow \frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-\frac{p(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-\frac{p(h_2-h_1)}{h_3}}$$

Furthermore, there can exist another Nash equilibrium at $x_A^* = 0$. More information about this Nash equilibrium can be found at the end of this **Appendix V.A.5**.

2. The Nash equilibrium for the case ME.2, where $h_3 x_B < h_1 x_A$:

Reaction function of *A*: $x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \left(\frac{h_4 - h_3}{h_1}\right) x_B$

Reaction function of *B*: $x_B^* = h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$

Note, that the reaction function of *B* is independent of the order quantity of *A*. Substituting the reaction function of *B* for x_B in the reaction function of *A*:

$$x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \left(h_4 - h_3\right) F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

However, this is only a Nash equilibrium if the condition of case ME.2, $h_3 x_B < h_1 x_A$, is met. That is, if:

$$h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) < \frac{h_1}{h_3} h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \frac{h_1}{h_3}\left(h_4 - h_3\right) F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

$$\Leftrightarrow h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) <$$

$$\frac{h_1 h_4}{h_3} F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \frac{h_1 h_4}{h_3} F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) + h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$$

$$\Leftrightarrow \frac{h_1 h_4}{h_3} F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) < \frac{h_1 h_4}{h_3} F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$$

$$\Leftrightarrow \frac{r-c_B-p}{r-v_B-p} < \frac{r-c_A}{r-v_A}$$

3. The derivation of the Nash equilibrium at $x_A^* = 0$, and a discussion about which Nash equilibria can co-exist within a parameter combination.

For better readability, we call the two above derived Nash equilibria "standard-form Nash equilibria":

The ME.1 case condition $h_3 x_B \geq h_1 x_A$, given

$$x_A^* = h_3 F^{-1}\left(\frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}\right) \text{and}$$

$$x_B^* = h_2 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)+(h_1-h_2)F^{-1}\left(\frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}\right),$$

is only fulfilled, if $\frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}$

The ME.2 case condition $h_3 x_B < h_1 x_A$, given

$$x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right)+(h_3-h_4)F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right) \text{ and } x_B^* = h_1 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right),$$

is only fulfilled, if $\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p}$

Therefore, if both conditions are fulfilled at the same time, it must hold true that:

$$\frac{r-c_A}{r-v_A}>\frac{r-c_B-p}{r-v_B-p}\geq\frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$$

If neither condition is fulfilled, it must hold true that:

$$\frac{r-c_A}{r-v_A}\leq\frac{r-c_B-p}{r-v_B-p}<\frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$$

3.1. When is it optimal that A orders nothing at all ($x_A^* = 0$)?

Because B pays a commission fee p to A, it could be optimal for A to order nothing at all. If A orders nothing at all, only the ME.1 case $h_3 x_B \geq h_1 x_A$ can be true. The first derivative of A's gross profit function, given $h_3 x_B \geq h_1 x_A$, is:

$$\frac{\partial E[\pi_A(x_A)]}{\partial x_A} = \left(v_A - r + \frac{p(h_2 - h_1)}{h_3} \right) F\left(\frac{x_A}{h_3} \right) -$$
$$\left(\frac{p(h_2 - h_1)}{h_3} \right) F\left(\frac{x_A}{h_3} + \frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3} \right) \right) +$$
$$(r - c_A)$$

First, consider the case where $h_2 = 1$ and $h_4 = 1$. In this case, the first order derivative reduces to:

$$(v_A - r + p) F\left(\frac{x_A}{h_3} \right) - (p) F\left(\frac{x_A}{h_3} + \left(x_B - \frac{h_1 x_A}{h_3} \right) \right) + (r - c_A)$$

Now, consider that for a Nash equilibrium it must be, that B orders:

$$x_B^* = F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) - x_A^*$$

Substituting $x_A^* = 0$ and $x_B^* = F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) - x_A^*$ into the first order derivative yields:

$$(v_A - r + p) F(0) - (p)\left(\frac{r - c_B - p}{r - v_B - p} \right) + (r - c_A)$$

The first order derivative at this position must be negative in order for $x_A = 0$ to be optimal. Thus, the condition is:

$$\left(v_A - r + p\right)F\left(0\right) - \left(p\right)\left(\frac{r - c_B - p}{r - v_B - p}\right) + (r - c_A) < 0$$

$$\Leftrightarrow r - c_A - p\left(\frac{r - c_B - p}{r - v_B - p}\right) < -\left(v_A - r + p\right)F\left(0\right)$$

$$\Leftrightarrow \frac{r - c_A - p\left(\frac{r - c_B - p}{r - v_B - p}\right)}{r - v_A - p} < F\left(0\right)$$

Thus, if $x_A = 0$ is optimal, this is also indicated by the critical ratio $\frac{r - c_A - p\left(\frac{r - c_B - p}{r - v_B - p}\right)}{r - v_A - p}$. Therefore, in the case where $h_2 = 1$ and $h_4 = 1$, the standard-form Nash equilibrium is the only possible Nash equilibrium but may take a form where $x_A^* = 0$.

Now consider our model extension, with $h_2 \neq 1$ and/or $h_4 \neq 1$. Again, the first order derivative at $x_A = 0$ and $x_B^* = h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p}\right) - \frac{(h_2 - h_1)}{h_3} x_A$ must be negative in order for $x_A = 0$ to be optimal.

$$\left(v_A - r + \frac{p(h_2 - h_1)}{h_3}\right)F\left(0\right) - \left(\frac{p(h_2 - h_1)}{h_3}\right)\left(\frac{r - c_B - p}{r - v_B - p}\right) + (r - c_A) < 0$$

$$\Leftrightarrow \begin{cases} \dfrac{r - c_A - \left(\frac{p(h_2 - h_1)}{h_3}\right)\left(\frac{r - c_B - p}{r - v_B - p}\right)}{\left(r - v_A - \frac{p(h_2 - h_1)}{h_3}\right)} < F\left(0\right) & \text{if} \left(r - v_A - \dfrac{p(h_2 - h_1)}{h_3}\right) > 0 \\ \dfrac{r - c_A - \left(\frac{p(h_2 - h_1)}{h_3}\right)\left(\frac{r - c_B - p}{r - v_B - p}\right)}{\left(r - v_A - \frac{p(h_2 - h_1)}{h_3}\right)} > F\left(0\right) & \text{if} \left(r - v_A - \dfrac{p(h_2 - h_1)}{h_3}\right) < 0 \end{cases}$$

If $h_1 \geq h_2$, the term $\left(r - v_A - \frac{p(h_2 - h_1)}{h_3} \right) > 0$ is always positive. Therefore, the standard-form Nash equilibrium is again the only possible Nash equilibrium but may take a form where $x_A^* = 0$.

If $h_1 < h_2$, however, the term $\left(r - v_A - \frac{p(h_2 - h_1)}{h_3} \right)$ can be negative. In this case, two Nash equilibria that both fulfill $h_3 x_B \geq h_1 x_A$, can potentially co-exist. One Nash equilibrium at

$$x_A^* = 0$$

$$x_B^* = F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right)$$

and another standard-form Nash equilibrium with

$$x_A^* = h_3 F^{-1}\left(\frac{r - c_A - p\frac{(h_2 - h_1)}{h_3}\left(\frac{r - c_B - p}{r - v_B - p} \right)}{r - v_A - p\frac{(h_2 - h_1)}{h_3}} \right)$$

$$x_B^* = h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p} \right) + (h_1 - h_2) F^{-1}\left(\frac{r - c_A - p\frac{(h_2 - h_1)}{h_3}\left(\frac{r - c_B - p}{r - v_B - p} \right)}{r - v_A - p\frac{(h_2 - h_1)}{h_3}} \right)$$

3.2. Is it possible that no (standard-form) Nash equilibrium could exist?

If neither condition for the standard-form Nash equilibria are fulfilled, it must hold true that:

$$\frac{r-c_A}{r-v_A} \leq \frac{r-c_B-p}{r-v_B-p} < \frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$$

First, consider the case where $h_2=1$ **and** $h_4=1$. The critical ratio in the ME.1 case $h_3 x_B \geq h_1 x_A$ reduces to: $\dfrac{r-c_A-p\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}$. This ratio, given that $\dfrac{r-c_A}{r-v_A} \leq \dfrac{r-c_B-p}{r-v_B-p}$ (from the system of inequalities above), is always $\dfrac{r-c_A}{r-v_A} \geq \dfrac{r-c_A-p\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}$, thus contradicting the system of inequalities $\dfrac{r-c_A}{r-v_A} \leq \dfrac{r-c_B-p}{r-v_B-p} < \dfrac{r-c_A-p\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}$. Therefore, a minimum of one standard-form Nash equilibrium must exist.

Now consider our model extension, with $h_2 \neq 1$ and/or $h_4 \neq 1$ and with $h_1 \geq h_2$. Then this system of inequalities also cannot be true. If $h_1 \geq h_2$,

then $\dfrac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$ is a mixture of the ratios $\dfrac{r-c_A}{r-v_A}$ and

$\dfrac{r-c_B-p}{r-v_B-p}$.

It must therefore be always true that

$$\frac{r-c_A}{r-v_A}<\frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}<\frac{r-c_B-p}{r-v_B-p}$$

, which contradicts the system of inequalities. Thus, also in this case a minimum of one standard-form Nash equilibrium must exist.

Now consider our model extension, with $h_2 \neq 1$ and/or $h_4 \neq 1$ but with $h_1 < h_2$. The only difference between $\dfrac{r-c_A}{r-v_A}$ and

$\dfrac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$ is the addition of the commission p. We

know that the addition of a commission p (given that $h_1 < h_2$ and $\dfrac{r-c_A}{r-v_A}\leq\dfrac{r-c_B-p}{r-v_B-p}$) can never raise the order quantity of *A* above the critical

ratio $\dfrac{r-c_A}{r-v_A}$. *A* orders according to $\dfrac{r-c_A}{r-v_A}$ either if there is no commission or if the commission is so high that *B* orders nothing at all.

Thus, the only reason why the system of inequalities

$$\frac{r-c_A}{r-v_A} \leq \frac{r-c_B-p}{r-v_B-p} < \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}$$

might be true, is that the effect of the commission $p\frac{(h_2-h_1)}{h_3}$ is so strong that the critical ratio has crossed the boundaries of the critical ratio. The critical ratio is only defined on the domain $0 \leq \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}} \leq 1$. Arithmetically, however, the critical ratio does not honor this boundary. Starting from $p=0$ and $\frac{r-c_A}{r-v_A} \leq \frac{r-c_B-p}{r-v_B-p}$: With an increasing p, the critical ratio decreases up to a certain point after which it increases back to $\frac{r-c_A}{r-v_A}$. If the slope of the decrease is very steep, the critical ratio will hit zero but arithmetically continues to exist. If both terms $-p\frac{(h_2-h_1)}{h_3} < -p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right) << 0$ are big negative numbers, then the critical ratio fraction is positive and close to one, possibly above $\frac{r-c_B-p}{r-v_B-p}$, fulfilling the condition for the case where neither condition $h_3 x_B \geq h_1 x_A$ nor $h_3 x_B < h_1 x_A$ is fulfilled.

However, because the boundary of the critical ratio was crossed, the valid critical ratio would be zero and the optimal order quantity of A would therefore be $x_A^* = 0$.

The Nash equilibrium $x_A^* = 0$, $x_B^* = h_2 F^{-1}\left(\frac{r - c_B - p}{r - v_B - p}\right)$ would be the only Nash equilibrium in this case. Note, that this Nash equilibrium is also indicated by (see above):

$$\frac{r - c_A - \left(\frac{p(h_2 - h_1)}{h_3}\right)\left(\frac{r - c_B - p}{r - v_B - p}\right)}{\left(r - v_A - \frac{p(h_2 - h_1)}{h_3}\right)} > F(0) \quad \text{if} \left(r - v_A - \frac{p(h_2 - h_1)}{h_3}\right) < 0$$

3.3. Can the Nash equilibria (the Nash-EQ with $x_A^* = 0$, one standard-form EQ for the case $h_3 x_B \geq h_1 x_A$, and one standard-form EQ for the case $h_3 x_B < h_1 x_A$) co-exist in one parameter combination?

If both conditions are fulfilled at the same time, it must hold true that:

$$\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}$$

First, consider the case where $h_2 = 1$ and $h_4 = 1$. Then the system of inequalities reduces to:

$$\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p}$$
$$\Leftrightarrow (r-v_B-p)(r-c_A) > (r-v_A)(r-c_B-p)$$

and

$$\frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-p\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p}$$
$$\Leftrightarrow (r-v_A-p)(r-c_B-p) \geq (r-v_B-p)\left(r-c_A-p\left(\frac{r-c_B-p}{r-v_B-p}\right)\right)$$
$$\Leftrightarrow (r-v_A-p)(r-c_B-p) \geq (r-v_B-p)(r-c_A) - p(r-c_B-p)$$
$$\Leftrightarrow (r-v_A)(r-c_B-p) \geq (r-v_B-p)(r-c_A)$$

These inequalities contradict each other. Therefore, in this case, only exactly one Nash equilibrium exists.

Now consider our model extension, with $h_2 \neq 1$ and/or $h_4 \neq 1$ and with $h_1 \geq h_2$. Again, the system of inequalities cannot be true. The same reason as outlined under point 3.2. is also valid in this case. If $h_1 \geq h_2$, then $\dfrac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}$ is a mixture of the ratios $\dfrac{r-c_A}{r-v_A}$ and $\dfrac{r-c_B-p}{r-v_B-p}$.

It must therefore be always true that

$$\frac{r-c_A}{r-v_A} > \frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}} > \frac{r-c_B-p}{r-v_B-p}$$

, which contradicts the system of inequalities. Thus, also in this case exactly one Nash equilibrium exists.

Now, consider the model extension, with $h_2 \neq 1$ and/or $h_4 \neq 1$ but with $h_1 < h_2$. Then the condition for two standard-form Nash equilibria at the same time,

$$\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}},$$

is indeed very well possible, even within the bounds

$$0 \le \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}} \le 1 .$$

Starting from $p=0$ and $\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p}$: With an increasing p, the critical ratio $\frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}$ increases up to a certain point, then becomes negative, and then positive again but will never cross $\frac{r-c_A}{r-v_A}$ or $\frac{r-c_B-p}{r-v_B-p}$.

Arithmetically, the ratio would never be between

$$\frac{r-c_A}{r-v_A} \ge \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}} \ge \frac{r-c_B-p}{r-v_B-p} .$$

If $\frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}} \ge \frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p}$, then the condition of case ME.1 $h_3 x_B \ge h_1 x_A$ is violated. If *A* wants to stay within the bounds of $h_3 x_B \ge h_1 x_A$, *A* can order a maximum of $x_A = h_3 F^{-1}\left(\frac{r-c_B-p}{r-v_B-p}\right)$. Also note, that in this case $x_A^* = 0$ is not a Nash equilibrium. Thus, a standard-form Nash equilibrium (the only one) must exist within case ME.2, with $h_3 x_B < h_1 x_A$.

If $\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p} \geq \frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}} \geq 0$, then the standard-form Nash equilibrium and the Nash equilibrium at $x_A^*=0$ co-exist within case ME.1 ($h_3 x_B \geq h_1 x_A$) and additionally another standard-form Nash equilibrium exists within case ME.2 ($h_3 x_B < h_1 x_A$).

A special case arises when $\frac{r-c_A-p\frac{(h_2-h_1)}{h_3}\left(\frac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\frac{(h_2-h_1)}{h_3}}$ becomes negative. Then it is not immediately clear, whether a Nash equilibrium exists at $x_A=0$ or not. The negative critical ratio would, at first glance, suggest an optimum at $x_A=0$. However, the formula from above says, that a Nash equilibrium at $x_A=0$ exists, only if

$$\begin{cases} \frac{r-c_A-\left(\frac{p(h_2-h_1)}{h_3}\right)\left(\frac{r-c_B-p}{r-v_B-p}\right)}{\left(r-v_A-\frac{p(h_2-h_1)}{h_3}\right)} < F(0) & \text{if} \left(r-v_A-\frac{p(h_2-h_1)}{h_3}\right) > 0 \\ \frac{r-c_A-\left(\frac{p(h_2-h_1)}{h_3}\right)\left(\frac{r-c_B-p}{r-v_B-p}\right)}{\left(r-v_A-\frac{p(h_2-h_1)}{h_3}\right)} > F(0) & \text{if} \left(r-v_A-\frac{p(h_2-h_1)}{h_3}\right) < 0 \end{cases}$$

It can be shown that $\left(r-v_A-\frac{p(h_2-h_1)}{h_3}\right)$ cannot be positive if $r-c_A-\left(\frac{p(h_2-h_1)}{h_3}\right)\left(\frac{r-c_B-p}{r-v_B-p}\right)$ is negative. Because $\frac{r-c_A}{r-v_A} > \frac{r-c_B-p}{r-v_B-p}$ and

$h_1 < h_2$, we can substitute $\frac{r-c_A}{r-v_A} \Leftrightarrow \frac{r-c_B-p}{r-v_B-p}$, without losing generality. Thus, the following system of inequalities must be true:

$$r-v_A-\frac{p(h_2-h_1)}{h_3}>0 \Leftrightarrow r-v_A>\frac{p(h_2-h_1)}{h_3} \text{ and}$$

$$r-c_A-\left(\frac{p(h_2-h_1)}{h_3}\right)\left(\frac{r-c_A}{r-v_A}\right)<0$$

Now, we can substitute $r-v_A \Leftrightarrow \frac{p(h_2-h_1)}{h_3}$, without losing generality. Thus, the following inequality must be true, $r-c_A-(r-v_A)\left(\frac{r-c_A}{r-v_A}\right)<0 \Leftrightarrow (r-c_A)-(r-c_A)<0$, which is impossible.

This means, that when $\dfrac{r-c_A-p\dfrac{(h_2-h_1)}{h_3}\left(\dfrac{r-c_B-p}{r-v_B-p}\right)}{r-v_A-p\dfrac{(h_2-h_1)}{h_3}}<0$, the conditions are not fulfilled and $x_A=0$ is not a Nash equilibrium.

Given that it is possible that multiple Nash equilibria co-exist in one parameter combination, the question arises whether there is always a pure-strategy Nash equilibrium or not, that is, whether one of the Nash equilibrium dominates the other Nash equilibria. In our numerical examples, we always observed that there was a pure-strategy Nash equilibrium. For *B* this is easy to prove because *B* always prefers it (given that $h_1 < h_2$), when *A* orders less. For *A* we do not see any way to proof our anecdotal numerical evidence for every possible case.

V.A.6. Appendix F

1. The cooperative optimum for the case ME.1, where $h_3 x_B \geq h_1 x_A$.

The expected total gross profit (to be maximized) is given by:

$$E[\pi(x_A, x_B)] = \int_{b=0}^{\frac{x_A}{h_3}} \begin{bmatrix} r(h_3 + h_1)b - \\ c_A x_A - c_B x_B + \\ v_A(x_A - h_3 b) + v_B(x_B - h_1 b) \end{bmatrix} f(b)db$$

$$+ \int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)} \begin{bmatrix} r\left(x_A + \dfrac{h_1 x_A}{h_3} + h_2\left(b - \dfrac{x_A}{h_3}\right)\right) - \\ c_A x_A - c_B x_B + \\ v_B\left(x_B - \dfrac{h_1 x_A}{h_3} - h_2\left(b - \dfrac{x_A}{h_3}\right)\right) \end{bmatrix} f(b)db$$

$$+ \int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)}^{\infty} \left[r(x_A + x_B) - c_A x_A - c_B x_B\right] f(b)db$$

Partial first order derivatives with respect to x_A (Leibniz integral rule).

First term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=0}^{\frac{x_A}{h_3}}\begin{bmatrix} r(h_3+h_1)b- \\ c_Ax_A-c_Bx_B+ \\ v_A(x_A-h_3b)+v_B(x_B-h_1b) \end{bmatrix} f(b)db\right]$$

$$=\int_{b=0}^{\frac{x_A}{h_3}}\left[v_A-c_A\right]f(b)db+\frac{1}{h_3}\begin{bmatrix} r(h_3+h_1)\frac{x_A}{h_3}- \\ c_Ax_A-c_Bx_B+ \\ v_A(x_A-h_3\frac{x_A}{h_3})+v_B(x_B-h_1\frac{x_A}{h_3}) \end{bmatrix} f\left(\frac{x_A}{h_3}\right)$$

$$=\left(v_A-c_A\right)F\left(\frac{x_A}{h_3}\right)+\frac{1}{h_3}\begin{bmatrix} r(x_A+\frac{h_1x_A}{h_3})- \\ c_Ax_A-c_Bx_B+ \\ v_B(x_B-\frac{h_1x_A}{h_3}) \end{bmatrix} f\left(\frac{x_A}{h_3}\right)$$

Second term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[r\left(x_A+\frac{h_1x_A}{h_3}+h_2\left(b-\frac{x_A}{h_3}\right)\right)-c_Ax_A-c_Bx_B+v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(b-\frac{x_A}{h_3}\right)\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[r+\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-c_A-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right]f(b)db+$$

$$\left(\frac{1}{h_3}-\frac{h_1}{h_2h_3}\right)\left[\begin{array}{l} r\left(x_A+\frac{h_1x_A}{h_3}+h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)- \\ c_Ax_A-c_Bx_B+ \\ v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)\end{array}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-$$

$$\frac{1}{h_3}\left[r\left(x_A+\frac{h_1x_A}{h_3}+h_2\left(\frac{x_A}{h_3}-\frac{x_A}{h_3}\right)\right)-c_Ax_A-c_Bx_B+v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(\frac{x_A}{h_3}-\frac{x_A}{h_3}\right)\right)\right]f\left(\frac{x_A}{h_3}\right)$$

$$=\left(r+\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-c_A-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right)\left(F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-F\left(\frac{x_A}{h_3}\right)\right)+$$

$$\left(\frac{1}{h_3}-\frac{h_1}{h_2h_3}\right)\left[r\left(x_A+x_B\right)-c_Ax_A-c_Bx_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{1}{h_3}\left[r\left(x_A+\frac{h_1x_A}{h_3}\right)-c_Ax_A-c_Bx_B+v_B\left(x_B-\frac{h_1x_A}{h_3}\right)\right]f\left(\frac{x_A}{h_3}\right)$$

Third term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[r-c_A\right]f(b)db$$

$$-\left(\frac{1}{h_3}-\frac{h_1}{h_2 h_3}\right)\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

$$=\left(r-c_A\right)\left(1-F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)\right)$$

$$-\left(\frac{1}{h_3}-\frac{h_1}{h_2 h_3}\right)\left[r\left(x_A+x_B\right)-c_A x_A-c_B x_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

All three terms together, after canceling and reordering:

$$\frac{\partial E[\pi(x_A,x_B)]}{\partial x_A}=\left(r-c_A\right)$$

$$-\left(r-v_A+\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right)F\left(\frac{x_A}{h_3}\right)$$

$$+\left(\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

First order condition: $\frac{\partial E[\pi(x_A,x_B)]}{\partial x_A}=0$

$$F\left(\frac{x_A^*}{h_3}\right)=\frac{\left(r-c_A\right)+\left(\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right)F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A^*}{h_3}\right)\right)}{\left(r-v_A+\frac{rh_1}{h_3}-\frac{rh_2}{h_3}-\frac{v_Bh_1}{h_3}+\frac{v_Bh_2}{h_3}\right)}$$

$$\Leftrightarrow x_A^*=h_3F^{-1}\left(\frac{(r-c_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A^*}{h_3}\right)\right)}{(r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

Partial first order derivatives with respect to x_B (Leibniz integral rule).

First term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=0}^{\frac{x_A}{h_3}}\begin{bmatrix} r(h_3+h_1)b-c_Ax_A-c_Bx_B+ \\ v_A(x_A-h_3b)+v_B(x_B-h_1b) \end{bmatrix} f(b)db\right]$$

$$=\int_{b=0}^{\frac{x_A}{h_3}}\left[v_B-c_B\right]f(b)db$$

$$=\left(v_B-c_B\right)F\left(\frac{x_A}{h_3}\right)$$

Second term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[r\left(x_A+\frac{h_1x_A}{h_3}+h_2\left(b-\frac{x_A}{h_3}\right)\right)-c_Ax_A-c_Bx_B+v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(b-\frac{x_A}{h_3}\right)\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}}^{\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)}\left[v_B-c_B\right]f(b)db+\left(\frac{1}{h_2}\right)\left[\begin{array}{l} r\left(x_A+\frac{h_1x_A}{h_3}+h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)- \\ c_Ax_A-c_Bx_B+ \\ v_B\left(x_B-\frac{h_1x_A}{h_3}-h_2\left(\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-\frac{x_A}{h_3}\right)\right)\end{array}\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$=\left(v_B-c_B\right)\left(F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)-F\left(\frac{x_A}{h_3}\right)\right)+\left(\frac{1}{h_2}\right)\left[r\left(x_A+x_B\right)-c_Ax_A-c_Bx_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

Third term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)}^{\infty}\left[r-c_B\right]f(b)db$$

$$-\left(\frac{1}{h_2}\right)\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

$$=\left(r-c_B\right)\left(1-F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)\right)$$

$$-\left(\frac{1}{h_2}\right)\left[r\left(x_A+x_B\right)-c_A x_A-c_B x_B\right]f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

All three terms together, after canceling and reordering:

$$\frac{\partial E[\pi(x_A,x_B)]}{\partial x_B}=\left(r-c_B\right)-\left(r-v_B\right)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

First order condition: $\frac{\partial E[\pi(x_A,x_B)]}{\partial x_B}=0$

$$(r-c_B)-(r-v_B)F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)=0$$

$$\Leftrightarrow F\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)=\frac{(r-c_B)}{(r-v_B)}$$

$$\Leftrightarrow x_B^*=h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A$$

Substituting the results for *A* and *B* respectively.

Optimum for *A*:

$$x_A^*=h_3F^{-1}\left(\frac{(r-c_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A^*}{h_3}\right)\right)}{(r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

Optimum for *B*:

$$x_B^*=h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A$$

It follows:

$$x_A^*=h_3F^{-1}\left(\frac{(r-c_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)F\left(\frac{x_A^*}{h_3}+\frac{1}{h_2}\left(\left(h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(\frac{h_2-h_1}{h_3}\right)x_A\right)-\frac{h_1x_A^*}{h_3}\right)\right)}{(r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

$$\Leftrightarrow x_A^*=h_3F^{-1}\left(\frac{(r-c_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\left(\frac{r-c_B}{r-v_B}\right)}{(r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

$$\Leftrightarrow x_A^*=h_3F^{-1}\left(\frac{(r-c_A)-(r-c_B)\left(\frac{h_2-h_1}{h_3}\right)}{(r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

And:

$$x_B^{*}=h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(h_2-h_1\right)F^{-1}\left(\frac{\left(r-c_A\right)-\left(r-c_B\right)\left(\frac{h_2-h_1}{h_3}\right)}{\left(r-v_A\right)-\left(r-v_B\right)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

However, this is only a valid cooperative optimum if the condition of case ME.1, $h_3x_B\geq h_1x_A$, is met. That is, if:

$$h_3\left(h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)-\left(h_2-h_1\right)F^{-1}\left(\frac{\left(r-c_A\right)-\left(r-c_B\right)\left(\frac{h_2-h_1}{h_3}\right)}{\left(r-v_A\right)-\left(r-v_B\right)\left(\frac{h_2-h_1}{h_3}\right)}\right)\right)\geq$$

$$h_1\left(h_3F^{-1}\left(\frac{\left(r-c_A\right)-\left(r-c_B\right)\left(\frac{h_2-h_1}{h_3}\right)}{\left(r-v_A\right)-\left(r-v_B\right)\left(\frac{h_2-h_1}{h_3}\right)}\right)\right)$$

$$\Leftrightarrow h_2F^{-1}\left(\frac{r-c_B}{r-v_B}\right)\geq h_2F^{-1}\left(\frac{\left(r-c_A\right)-\left(r-c_B\right)\left(\frac{h_2-h_1}{h_3}\right)}{\left(r-v_A\right)-\left(r-v_B\right)\left(\frac{h_2-h_1}{h_3}\right)}\right)$$

$$\Leftrightarrow\frac{r-c_B}{r-v_B}\geq\frac{\left(r-c_A\right)-\left(r-c_B\right)\left(\frac{h_2-h_1}{h_3}\right)}{\left(r-v_A\right)-\left(r-v_B\right)\left(\frac{h_2-h_1}{h_3}\right)}$$

Second order condition (Hessian matrix).

The function has two variables (x_A and x_B). For functions with more than one variable, the second order condition must be checked with the Hessian matrix.

For the 2x2 Hessian matrix four second derivatives must be calculated:

1. Case: First derivative w.r.t. x_A and second derivative also w.r.t. x_A.

2. Case: First derivative w.r.t. x_A and second derivative w.r.t. x_B.

3. Case: First derivative w.r.t. x_B and second derivative w.r.t. x_A.

4. Case: First derivative w.r.t. x_B and second derivative also w.r.t. x_B.

1. Case:

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{(\partial x_A)^2}=\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)-$$
$$\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)$$

2. Case:

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{\partial x_A \partial x_B}=\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

3. Case:

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{(\partial x_B)^2}=-\left(r-v_B\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

4. Case:

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{\partial x_B \partial x_A}=-\left(r-v_B\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1 x_A}{h_3}\right)\right)$$

These cases form the following Hessian matrix:

$$H_{E[\pi]}=\begin{pmatrix} \left[\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right) -\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)\right] & \left[\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right] \\ \left[-(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right] & \left[-(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right] \end{pmatrix}$$

The second order condition check is done via the principal minors of the Hessian matrix. The determinant of the principal minor of the first degree (odd degree) must be negative and the determinant of the principal minor of the second degree (even degree) must be positive, in order for a maximum to exist:

1. Principal minor:

$$Det1 = -\left(r - v_B\right) f\left(\frac{x_A}{h_3} + \frac{1}{h_2}\left(x_B - \frac{h_1 x_A}{h_3}\right)\right)$$

Because per definition $r > v_B$, the determinant of the first principal minor is always negative.

2. Principal minor:

$$Det2=$$

$$\left(\begin{array}{l}\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\\-\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)\end{array}\right)\left(-(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right)$$

$$-\left(\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right)\left(-(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right)$$

$$\Leftrightarrow\frac{Det2}{\left(-(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)\right)}=\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$-\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)-\left(-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)$$

$$\Leftrightarrow\frac{Det2}{(r-v_B)f\left(\frac{x_A}{h_3}+\frac{1}{h_2}\left(x_B-\frac{h_1x_A}{h_3}\right)\right)}=\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)$$

Thus, if $\left((r-v_A)-(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)\right)f\left(\frac{x_A}{h_3}\right)$ is positive, the Hessian matrix is negative-definite and the above derived point x_A^*, x_B^* is indeed a maximum (i.e., the cooperative optimum). Therefore, the following inequality must be fulfilled:

$$r-v_A>(r-v_B)\left(\frac{h_2-h_1}{h_3}\right)$$

If the determinant of the second principal minor would be negative (and given that the determinant of the first principal minor is always negative in our case), the derived point x_A^*, x_B^* would be a saddle point and it would be optimal that either, *A* or *B* orders everything. Of course, within case ME.1 ($h_3 x_B \geq h_1 x_A$), only $x_A^*=0$ would be a valid solution. But the other ME.2 case ($h_3 x_B < h_1 x_A$) must be checked for a potentially better cooperative optimum.

2. The cooperative optimum for the case ME.2, where $h_3 x_B < h_1 x_A$.

The expected total gross profit (to be maximized) is given by:

$$E[\pi(x_A,x_B)] =$$

$$\int_{b=0}^{\frac{x_B}{h_1}} \begin{bmatrix} r(h_3+h_1)b- \\ c_A x_A - c_B x_B + \\ v_A(x_A - h_3 b) + v_B(x_B - h_1 b) \end{bmatrix} f(b)db$$

$$+ \int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)} \begin{bmatrix} r\left(x_B + \frac{h_3 x_B}{h_1} + h_4\left(b - \frac{x_B}{h_1}\right)\right) - \\ c_A x_A - c_B x_B + \\ v_A\left(x_A - \frac{h_3 x_B}{h_1} - h_4\left(b - \frac{x_B}{h_1}\right)\right) \end{bmatrix} f(b)db$$

$$+ \int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)}^{\infty} \left[r(x_A + x_B) - c_A x_A - c_B x_B\right] f(b)db$$

Partial first order derivatives with respect to x_A (Leibniz integral rule).

First term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=0}^{\frac{x_B}{h_1}} \begin{bmatrix} r(h_3+h_1)b - c_A x_A - c_B x_B + \\ v_A(x_A - h_3 b) + v_B(x_B - h_1 b) \end{bmatrix} f(b)db\right]$$

$$= \int_{b=0}^{\frac{x_B}{h_1}} \left[v_A - c_A\right] f(b)db = \left(v_A - c_A\right) F\left(\frac{x_B}{h_1}\right)$$

Second term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[r\left(x_B+\frac{h_3 x_B}{h_1}+h_4\left(b-\frac{x_B}{h_1}\right)\right)-c_A x_A-c_B x_B+v_A\left(x_A-\frac{h_3 x_B}{h_1}-h_4\left(b-\frac{x_B}{h_1}\right)\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[v_A-c_A\right]f(b)db+\left(\frac{1}{h_4}\right)\begin{bmatrix} r\left(x_B+\frac{h_3 x_B}{h_1}+h_4\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)-\frac{x_B}{h_1}\right)\right)- \\ c_A x_A-c_B x_B+ \\ v_A\left(x_A-\frac{h_3 x_B}{h_1}-h_4\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)-\frac{x_B}{h_1}\right)\right)\end{bmatrix}f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

$$=\left(v_A-c_A\right)\left(F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)-F\left(\frac{x_B}{h_1}\right)\right)+\left(\frac{1}{h_4}\right)\left[r\left(x_B+x_A\right)-c_A x_A-c_B x_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

Third term:

$$\frac{\partial}{\partial x_A}\left[\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}^{\infty}\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}^{\infty}\left[r-c_A\right]f(b)db$$

$$-\left(\frac{1}{h_4}\right)\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

$$=\left(r-c_A\right)\left(1-F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)\right)$$

$$-\left(\frac{1}{h_4}\right)\left[r\left(x_A+x_B\right)-c_A x_A-c_B x_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

All three terms together, after canceling and reordering:

$$\frac{\partial E[\pi(x_A,x_B)]}{\partial x_A}=\left(r-c_A\right)-\left(r-v_A\right)F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

First order condition: $\dfrac{\partial E[\pi(x_A, x_B)]}{\partial x_A} = 0$

$$\left(r - c_A\right) - \left(r - v_A\right) F\left(\frac{x_B}{h_1} + \frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)\right) = 0$$

$$\Leftrightarrow F\left(\frac{x_B}{h_1} + \frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)\right) = \frac{\left(r - c_A\right)}{\left(r - v_A\right)}$$

$$\Leftrightarrow x_A^* = h_4 F^{-1}\left(\frac{r - c_A}{r - v_A}\right) - \frac{h_4 - h_3}{h_1} x_B$$

Partial first order derivatives with respect to x_B (Leibniz integral rule).

First term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=0}^{\frac{x_B}{h_1}}\begin{bmatrix} r(h_3+h_1)b-c_Ax_A-c_Bx_B+ \\ v_A(x_A-h_3b)+v_B(x_B-h_1b) \end{bmatrix} f(b)db\right]$$

$$=\int_{b=0}^{\frac{x_B}{h_1}}\left[v_B-c_B\right]f(b)db$$

$$+\frac{1}{h_1}\begin{pmatrix} r(h_3+h_1)\frac{x_B}{h_1}- \\ c_Ax_A-c_Bx_B+ \\ v_A\left(x_A-h_3\frac{x_B}{h_1}\right)+v_B\left(x_B-h_1\frac{x_B}{h_1}\right) \end{pmatrix} f\left(\frac{x_B}{h_1}\right)$$

$$=\left(v_B-c_B\right)F\left(\frac{x_B}{h_1}\right)$$

$$+\frac{1}{h_1}\left(r(h_3+h_1)\frac{x_B}{h_1}-c_Ax_A-c_Bx_B+v_A\left(x_A-h_3\frac{x_B}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)$$

Second term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[r\left(x_B+\frac{h_3 x_B}{h_1}+h_4\left(b-\frac{x_B}{h_1}\right)\right)-c_A x_A-c_B x_B+v_A\left(x_A-\frac{h_3 x_B}{h_1}-h_4\left(b-\frac{x_B}{h_1}\right)\right)\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}}^{\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}\left[r+r\frac{h_3}{h_1}-r\frac{h_4}{h_1}-c_B-v_A\frac{h_3}{h_1}+v_A\frac{h_4}{h_1}\right]f(b)db+$$

$$\left(\frac{1}{h_1}-\frac{h_3}{h_4 h_1}\right)\left[\begin{array}{l} r\left(x_B+\frac{h_3 x_B}{h_1}+h_4\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)-\frac{x_B}{h_1}\right)\right)- \\ c_A x_A-c_B x_B+ \\ v_A\left(x_A-\frac{h_3 x_B}{h_1}-h_4\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)-\frac{x_B}{h_1}\right)\right) \end{array}\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)-$$

$$\left(\frac{1}{h_1}\right)\left[r\left(x_B+\frac{h_3 x_B}{h_1}+h_4\left(\frac{x_B}{h_1}-\frac{x_B}{h_1}\right)\right)-c_A x_A-c_B x_B+v_A\left(x_A-\frac{h_3 x_B}{h_1}-h_4\left(\frac{x_B}{h_1}-\frac{x_B}{h_1}\right)\right)\right]f\left(\frac{x_B}{h_1}\right)$$

$$=\left(r+\frac{rh_3}{h_1}-\frac{rh_4}{h_1}-c_B-\frac{v_Ah_3}{h_1}+\frac{v_Ah_4}{h_1}\right)\left(\begin{array}{l}F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)- \\ F\left(\frac{x_B}{h_1}\right)\end{array}\right)$$

$$+\left(\frac{1}{h_1}-\frac{h_3}{h_4h_1}\right)\left[r\left(x_B+x_A\right)-c_Ax_A-c_Bx_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$-\left(\frac{1}{h_1}\right)\left[r\left(x_B+\frac{h_3x_B}{h_1}\right)-c_Ax_A-c_Bx_B+v_A\left(x_A-\frac{h_3x_B}{h_1}\right)\right]f\left(\frac{x_B}{h_1}\right)$$

Third term:

$$\frac{\partial}{\partial x_B}\left[\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}^{\infty}\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f(b)db\right]$$

$$=\int_{b=\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)}^{\infty}\left[r-c_B\right]f(b)db$$

$$-\left(\frac{1}{h_1}-\frac{h_3}{h_4 h_1}\right)\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

$$=\left(r-c_B\right)\left(1-F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)\right)$$

$$-\left(\frac{1}{h_1}-\frac{h_3}{h_4 h_1}\right)\left[r(x_A+x_B)-c_A x_A-c_B x_B\right]f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

All three terms together, after canceling and reordering:

$$\frac{\partial E[\pi(x_A,x_B)]}{\partial x_B}=\left(r-c_B\right)$$

$$-\left(r-v_B+\frac{rh_3}{h_1}-\frac{rh_4}{h_1}-\frac{v_A h_3}{h_1}+\frac{v_A h_4}{h_1}\right)F\left(\frac{x_B}{h_1}\right)$$

$$+\left(\frac{rh_3}{h_1}-\frac{rh_4}{h_1}-\frac{v_A h_3}{h_1}+\frac{v_A h_4}{h_1}\right)F\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B}{h_1}\right)\right)$$

First order condition: $\frac{\partial E[\pi(x_A,x_B)]}{\partial x_B}=0$

$$F\left(\frac{x_B^*}{h_1}\right)=\frac{\left(r-c_B\right)+\left(\frac{rh_3}{h_1}-\frac{rh_4}{h_1}-\frac{v_Ah_3}{h_1}+\frac{v_Ah_4}{h_1}\right)F\left(\frac{x_B^*}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B^*}{h_1}\right)\right)}{\left(r-v_B+\frac{rh_3}{h_1}-\frac{rh_4}{h_1}-\frac{v_Ah_3}{h_1}+\frac{v_Ah_4}{h_1}\right)}$$

$$\Leftrightarrow x_B^*=h_1F^{-1}\left(\frac{(r-c_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)F\left(\frac{x_B^*}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B^*}{h_1}\right)\right)}{(r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)}\right)$$

Substituting the results for *A* and *B* respectively.

Optimum for *A*:

$$x_A^* = h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right) - \left(\frac{h_4-h_3}{h_1}\right) x_B$$

Optimum for *B*:

$$x_B^* = h_1 F^{-1}\left(\frac{(r-c_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right) F\left(\frac{x_B^*}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3 x_B^*}{h_1}\right)\right)}{(r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)}\right)$$

It follows:

$$x_B^* = h_1 F^{-1}\left(\frac{(r-c_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right) F\left(\frac{x_B^*}{h_1}+\frac{1}{h_4}\left(\left(h_4 F^{-1}\left(\frac{r-c_A}{r-v_A}\right)-\left(\frac{h_4-h_3}{h_1}\right)x_B\right)-\frac{h_3 x_B^*}{h_1}\right)\right)}{(r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)} \right)$$

$$\Leftrightarrow x_B^* = h_1 F^{-1}\left(\frac{(r-c_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\left(\frac{r-c_A}{r-v_A}\right)}{(r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)} \right)$$

$$\Leftrightarrow x_B^* = h_1 F^{-1}\left(\frac{(r-c_B)-(r-c_A)\left(\frac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)} \right)$$

And:

$$x_A^*=h_4F^{-1}\left(\frac{r-c_A}{r-v_A}\right)-\left(h_4-h_3\right)F^{-1}\left(\frac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}\right)$$

However, this is only a valid cooperative optimum if the condition of case ME.2, $h_3x_B<h_1x_A$, is met. That is, if:

$$h_3\left(h_1F^{-1}\left(\frac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}\right)\right)<$$

$$h_1\left(h_4F^{-1}\left(\frac{r-c_A}{r-v_A}\right)-\left(h_4-h_3\right)F^{-1}\left(\frac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}\right)\right)$$

$$\Leftrightarrow h_4F^{-1}\left(\frac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}\right)<h_4F^{-1}\left(\frac{r-c_A}{r-v_A}\right)$$

$$\Leftrightarrow\frac{(r-c_B)-(r-c_A)\left(\dfrac{h_4-h_3}{h_1}\right)}{(r-v_B)-(r-v_A)\left(\dfrac{h_4-h_3}{h_1}\right)}<\frac{r-c_A}{r-v_A}$$

Second order condition (Hessian matrix).

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{(\partial x_A)^2}=-\left(r-v_A\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{\partial x_A\partial x_B}=-\left(r-v_A\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{(\partial x_B)^2}=\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)-$$
$$\left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)$$

$$\frac{\partial^2 E[\pi(x_A,x_B)]}{\partial x_B\partial x_A}=\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$H_{E[\pi]}=\begin{pmatrix} \left[-(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right] & \left[-(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right] \\ \left[\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right] & \left[\begin{matrix}\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)- \\ \left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)\end{matrix}\right] \end{pmatrix}$$

1. Principal minor:

$$Det1 = -(r - v_A) f\left(\frac{x_B}{h_1} + \frac{1}{h_4}\left(x_A - \frac{h_3 x_B}{h_1}\right)\right)$$

Because per definition $r > v_A$, the determinant of the first principal minor is always negative.

2. Principal minor:

$$Det2=$$

$$\left(-(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right)\left(\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)-\left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)\right)$$

$$-\left(\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right)\left(-(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right)$$

$$\Leftrightarrow\frac{Det2}{\left(-(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)\right)}=\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)-$$

$$\left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)-\left(-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)$$

$$\Leftrightarrow\frac{Det2}{(r-v_A)f\left(\frac{x_B}{h_1}+\frac{1}{h_4}\left(x_A-\frac{h_3x_B}{h_1}\right)\right)}=\left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)$$

Thus, if $\left((r-v_B)-(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)\right)f\left(\frac{x_B}{h_1}\right)$ is positive, the Hessian matrix is negative-definite and the above derived point x_A^*, x_B^* is indeed a maximum (i.e., the cooperative optimum). Therefore, the following inequality must be fulfilled:

$$r-v_B>(r-v_A)\left(\frac{h_4-h_3}{h_1}\right)$$

If the determinant of the second principal minor would be negative (and given that the determinant of the first principal minor is always negative in our case), the derived point x_A^*, x_B^* would be a saddle point and it would be optimal that either, *A* or *B* orders everything. Of course, within case ME.2 ($h_3 x_B < h_1 x_A$), only $x_B^* = 0$ would be a valid solution. But the other ME.1 case ($h_3 x_B \geq h_1 x_A$) must be checked for a potentially better cooperative optimum.

Paper VI Appendix

Appendix of the paper „A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties“

Reference: Straubert, C. (2024). A continuous approximation location-inventory model with exact inventory costs and nonlinear delivery lead time penalties. International Journal of Production Economics, 268, 1–25. doi.org/10.1016/j.ijpe.2023.109092

VI.A.1. Appendix A: Proof of the joint quasi-convexity of the approximate location-inventory model with respect to the reorder point s and the order quantity Q given a fixed number of stocking locations y.

Proof part 1: Unless otherwise indicated, the following statements and results refer to cases where cumulative demand is a nondecreasing stochastic process with a stationary probability distribution and a continuous sample path (e.g., unit-sized Poisson demand; Zipkin, 1986, p. 975). We want to prove that the following model is jointly quasi-convex in s and Q:

$$
\begin{aligned}
Min\ C(s,Q) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} + \\
u \cdot \left(1 - P_{\text{out}}(s,Q)\right)\left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega} + \\
u \cdot P_{\text{out}}(s,Q)\left(\begin{array}{c} \dfrac{B(s,Q)}{\left(\frac{dA}{y}\right)P_{\text{out}}(s,Q)} + \\ t_{\text{warehouse}} + \dfrac{2t_{\text{transport}}}{2+\beta_t}\left(\dfrac{A}{y\pi}\right)^{0.5\beta_t} \end{array}\right)^{\omega} + \\
\frac{R}{Q} + h \cdot \frac{y}{dA} \cdot \left(0.5(Q+1) + s - \left(\frac{\ell dA}{y}\right) + B(s,Q)\right)
\end{aligned}
\tag{A.1}
$$

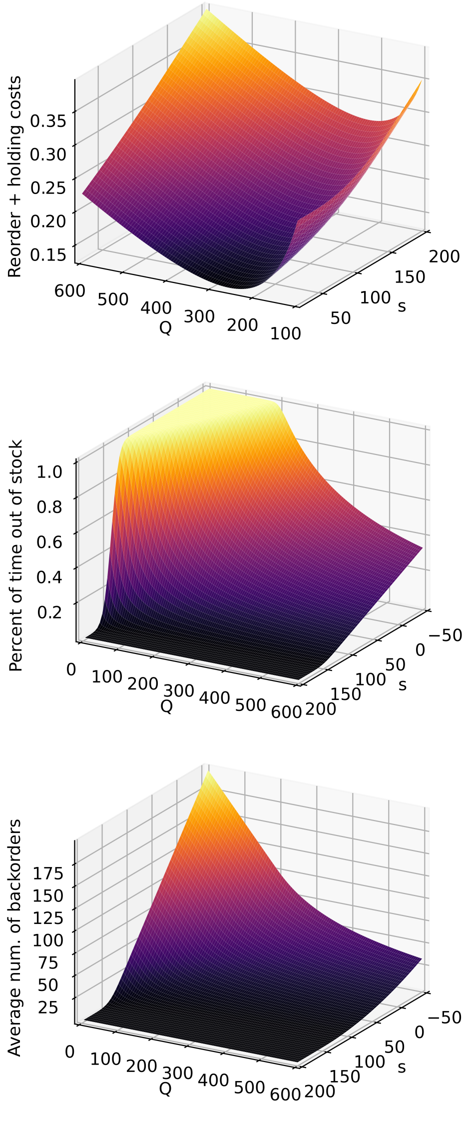
, with $y, Q \in \mathrm{Z}_{\geq 1}, s \in \mathrm{Z}$.

Without loss of generality, we set $y = 1$, $\lambda = dA$, and simplify the constants:

$$Min\ C(s,Q) = \text{const}_1 + \text{const}_2 +$$
$$P_{\text{out}}(s,Q)\left(u\cdot\left(\frac{1}{\lambda}\cdot\frac{B(s,Q)}{P_{\text{out}}(s,Q)} + \text{const}_3\right)^{\omega} - \text{const}_2\right) + \tag{A.2}$$
$$\frac{R}{Q} + h\cdot\frac{1}{\lambda}\cdot\big(0.5(Q+1) + s - (\ell\lambda) + B(s,Q)\big)$$

, with $Q \in \mathrm{Z}_{\geq 1}$, $s \in \mathrm{Z}$.

The term "R/Q" decreases and is convex in Q. Zipkin (1986) and Zhang (1998) showed that $B(s,Q)$ decreases and is jointly convex in s and Q; therefore, the same is true for "$h/\lambda \cdot B(s,Q)$". The term "$h/\lambda \cdot (0.5(Q+1) + s - (\ell\lambda))$" increases linearly in s and Q. Thus, the term "$R/Q + h/\lambda \cdot \big(0.5(Q+1) + s - (\ell\lambda) + B(s,Q)\big)$" is jointly convex in s and Q, which is also noted in Zipkin (1986) and Zhang (1998). $0 < P_{\text{out}}(s,Q) \leq 1$ is guaranteed to be either joint convex or joint quasi-convex. That is, starting from arbitrary s and Q, $P_{\text{out}}(s,Q)$ will, when s and Q increase, first decrease either concave or convex before ultimately always limiting with a convex decrease toward zero (see Zipkin, 1986; Zhang, 1998). Thus, $P_{\text{out}}(s,Q)$ can exhibit an s-shape, which is not convex but quasi-convex. **Figure VI.A.1** illustrates this.



Model parameters: $A = 502655$, $dA = 50$, $\ell = 3$, $h = 0.05$,
$R = 25$, $u = 1.5$, $\omega = 0.5$, $c_{\text{transport}} = 0.02$, $t_{\text{transport}} = 0.02$,
$\beta_c = 0.7$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 1$, $c_{\text{warehouse}} = 0$

Figure VI.A.1 Convexity and quasi-convexity of several model components

Proof part 2: We will now show that the term "$u \cdot \left(\frac{1}{\lambda} \cdot \frac{B(s,Q)}{P_{\text{out}}(s,Q)} + \text{const}_3\right)^{\omega} - \text{const}_2$" is jointly convex and decreasing in s and Q. Because u, λ, ω, const_2 and const_3 are constants, it suffices to show that the ratio $B(s,Q)/P_{\text{out}}(s,Q)$ is a jointly convex function decreasing in s and Q.

We start by observing that $B(s,Q)$ and $P_{\text{out}}(s,Q)$ can, in discrete form, be written as (Hadley & Whitin, 1963, p. 184):

$$P_{\text{out}}(s,Q) = \frac{1}{Q}\left(\sum_{x=s}^{\infty}[1 - P(x)] - \sum_{x=s+Q}^{\infty}[1 - P(x)]\right) \tag{A.3}$$

$$B(s,Q) = \frac{1}{Q}\left(\begin{array}{l} \displaystyle\sum_{x=s}^{\infty}(x - s)[1 - P(x)] - \\ \displaystyle\sum_{x=s+Q}^{\infty}(x - s - Q)[1 - P(x)] \end{array}\right) \tag{A.4}$$

where $P(x) = Prob(X \leq x)$ is the cumulative distribution function of the discrete replenishment lead time demand with mean $\ell\lambda$ and the probability mass function $p(x) = P(x + 1) - P(x)$.

Thus:

$$\frac{B(s,Q)}{P_{\text{out}}(s,Q)} = \frac{\frac{1}{Q}\left(\sum_{x=s}^{\infty}(x - s)[1 - P(x)] - \sum_{x=s+Q}^{\infty}(x - s - Q)[1 - P(x)]\right)}{\frac{1}{Q}\left(\sum_{x=s}^{\infty}[1 - P(x)] - \sum_{x=s+Q}^{\infty}[1 - P(x)]\right)} \tag{A.5}$$

For joint convexity of the average backorder time per backorder, we need to prove the following:

$$\frac{B(s,Q+1)}{P_{\text{out}}(s,Q+1)} \leq \frac{B(s,Q)}{P_{\text{out}}(s,Q)} \tag{A.6}$$

and that:

$$\frac{B(s+1,Q)}{P_{\text{out}}(s+1,Q)} \leq \frac{B(s,Q)}{P_{\text{out}}(s,Q)} \tag{A.7}$$

We will now first restrict ourselves to the cases where $s \geq 0$ and $Q > 0$ and explore what happens when the relative decrease of $[1-P(x)]$ in x is constant. We define the function $a(x)$ of the relative decrease in $[1-P(x)]$ as:

$$a(x) = \frac{[1-P(x+1)]}{[1-P(x)]} \tag{A.8}$$

When the relative decrease in $[1-P(x)]$ is constant, then $a(x+1) = a(x)$ is a constant $0 < a < 1$ independent of x. In such a case, $[1-P(x+k)] = a^k[1-P(x)]$. Accordingly, the fraction $B(s,Q)/P_{\text{out}}(s,Q)$ changes to:

$$\frac{B\big(s,Q|a(x+1)=a(x)\big)}{P_{\text{out}}\big(s,Q|a(x+1)=a(x)\big)} = \frac{\begin{pmatrix} \sum_{x=s}^{\infty} a^{(x-s)}(x-s)[1-P(s)] - \\ \sum_{x=s+Q}^{\infty} a^{(x-s)}(x-s-Q)[1-P(s)] \end{pmatrix}}{\begin{pmatrix} \sum_{x=s}^{\infty} a^{(x-s)}[1-P(s)] - \\ \sum_{x=s+Q}^{\infty} a^{(x-s)}[1-P(s)] \end{pmatrix}} \tag{A.9}$$

These sums simply converge to:

$$\frac{B\big(s,Q|a(x+1)=a(x)\big)}{P_{\text{out}}\big(s,Q|a(x+1)=a(x)\big)} = \frac{\frac{a[1-P(s)]}{(a-1)^2} - \frac{a^{(Q+1)}[1-P(s)]}{(a-1)^2}}{\frac{[1-P(s)]}{1-a} - \frac{a^Q[1-P(s)]}{1-a}} = \frac{a}{1-a} \tag{A.10}$$

That is, the ratio $B(s,Q)/P_{\text{out}}(s,Q)$, and therefore also the average backorder time per backorder, would always be $a/(1-a)$ irrespective of s and Q. The only distributions with a constant $a(x+1) = a(x) = a$ are the two memoryless distributions, the exponential distribution and its discrete counterpart, the geometric distribution. All other distributions have varying $a(x)$.

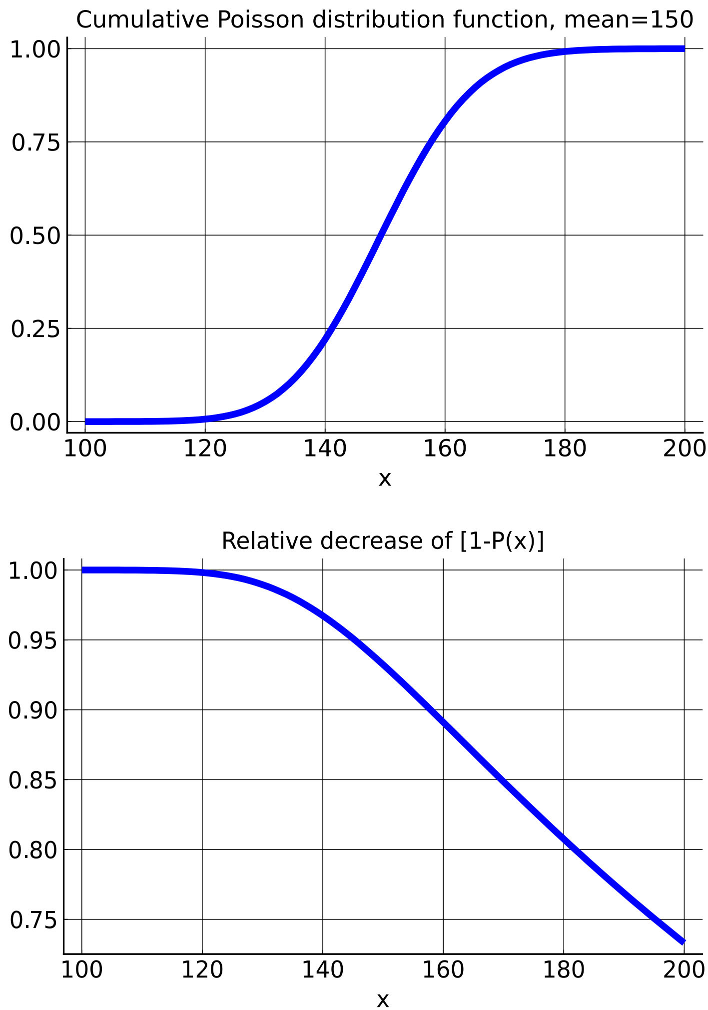


Figure VI.A.2 A Poisson distribution always has an accelerating relative decrease in $[1 - P(x)]$

For our proof, we are particularly interested in distributions that have an accelerating relative decrease of $[1 - P(x)]$, that is, where $a(x+1) < a(x)$, for example, $a(0) = 0.23$, $a(1) = 0.21$, $a(2) = 0.15$, $a(3) = 0.12$ and so on. **Figure VI.A.2** exemplifies this.

Recall that $B(s, Q)$ is always convex and decreasing in s and Q. Therefore, if $a(x+1) = a(x)$, both $B(s, Q)$ and $P_{\text{out}}(s, Q)$ are convex and decreasing in s and Q, and both functions have the same slope. If $a(x+1) < a(x)$, this is not the case. As proven by Zipkin (1986) and Zhang (1998), $P_{\text{out}}(s, Q)$ can exhibit an s-shape. For the concave part of $P_{\text{out}}(s, Q)$ (if it exists), it is trivial to see that Equations A.6 and A.7 are true.

However, for the convex part of $P_{\text{out}}(s, Q)$, these inequalities are not trivial. Without further investigation, it could be the case that the relative slope of $P_{\text{out}}(s, Q)$ is steeper than the relative slope of $B(s, Q)$, in which case the average backorder time per backorder would increase and not decrease when s and Q increase.

Fortunately, it is not very difficult to see that if $a(x+1) < a(x)$, then the relative slope of $B(s, Q)$ is indeed steeper than the relative slope of $P_{\text{out}}(s, Q)$, even if both functions are convex. Note that $\sum_{x=v}^{\infty}(x - v)[1 - P(x)]$ is a weighted sum. The weight $(x - v)$ increases linearly with x, regardless of whether $a(x+1) = a(x)$ or $a(x+1) < a(x)$. However, if $a(x+1) < a(x)$, then the relative decrease of $[1 - P(x)]$ is accelerating the higher x, and $[1 - P(x)]$ with a high x are underproportionally important compared to the case where $a(x+1) = a(x)$. Thus, the higher the weight $(x - v)$ is, the more underproportionally important it is. Furthermore, it is evident that both $\sum_{x=s+Q}^{\infty}[1 - P(x)]$ and $\sum_{x=s+Q}^{\infty}(x - v)[1 - P(x)]$ are always convexly decreasing in s and Q. Thus, it follows in the case of $a(x+1) < a(x)$:

$$\begin{gathered}\frac{\sum_{x=v+2}^{\infty}(x - v - 2)[1 - P(x)]}{\sum_{x=v+1}^{\infty}(x - v - 1)[1 - P(x)]} < \\ \frac{\sum_{x=v+1}^{\infty}(x - v - 1)[1 - P(x)]}{\sum_{x=v}^{\infty}(x - v)[1 - P(x)]} < 1\end{gathered} \tag{A.11}$$

$$\frac{\sum_{x=v+2}^{\infty}[1-P(x)]}{\sum_{x=v+1}^{\infty}[1-P(x)]} < \frac{\sum_{x=v+1}^{\infty}[1-P(x)]}{\sum_{x=v}^{\infty}[1-P(x)]} < 1 \tag{A.12}$$

and

$$\frac{\sum_{x=v+k}^{\infty}(x-v-k)[1-P(x)]}{\sum_{x=v}^{\infty}(x-v)[1-P(x)]} < \frac{\sum_{x=v+k}^{\infty}[1-P(x)]}{\sum_{x=v}^{\infty}[1-P(x)]} \tag{A.13}$$

That is, the relative slope of $\sum_{x=v}^{\infty}(x-v)[1-P(x)]$ is steeper than the relative slope of $\sum_{x=v}^{\infty}[1-P(x)]$.

Therefore, in the case of an increasing Q, $\sum_{x=s+Q}^{\infty}(x-s-Q)[1-P(x)]$ decreases relatively faster than $\sum_{x=s+Q}^{\infty}[1-P(x)]$. Thus, $B(s,Q)$ converges/decreases relatively faster toward $(1/Q)(\sum_{x=s}^{\infty}(x-s)[1-P(x)])$ (from Equation A.4) than $P_{\text{out}}(s,Q)$ converges toward $(1/Q)(\sum_{x=s}^{\infty}[1-P(x)])$ (from Equation A.3), which is the same as stating that the relative slope of $B(s,Q)$ is steeper than the relative slope of $P_{\text{out}}(s,Q)$. Accordingly, Inequality A.6 (condition for the convexity of the backorder time per backorder in the case of increasing Q) is indeed true for every demand distribution where $a(x+1) < a(x)$.

In the case of an increasing s, $\sum_{x=s}^{\infty}(x-s)[1-P(x)] - \sum_{x=s+Q}^{\infty}(x-s-Q)[1-P(x)]$ (from the $B(s,Q)$ Equation A.4) converges faster to $\sum_{x=s}^{\infty}(x-s)[1-P(x)]$ than $\sum_{x=s}^{\infty}[1-P(x)] - \sum_{x=s+Q}^{\infty}[1-P(x)]$ (from the $P_{\text{out}}(s,Q)$ Equation A.3) converges to $\sum_{x=s}^{\infty}[1-P(x)]$.

And $\sum_{x=s}^{\infty}(x-s)[1-P(x)]$ converges/decreases relatively faster to zero than $\sum_{x=s}^{\infty}[1-P(x)]$. Accordingly, Inequality A.7 (condition for the convexity of the backorder time per backorder in the case of increasing s) is indeed true for every demand distribution where $a(x+1) < a(x)$.

Convexity follows in both cases from the fact that the function is a ratio of two decreasing convex (or one convex and one quasi-convex) functions. Joint convexity is evident by the fact that the above results hold for every s and $Q > 0$.

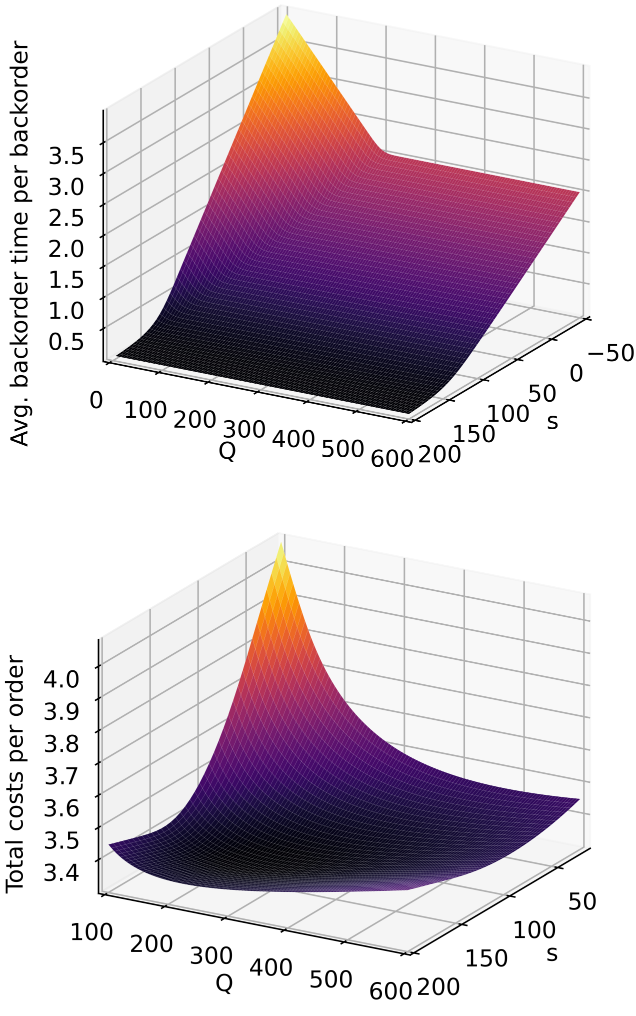
Proof part 3: Given the above, we now have to prove that the Poisson distribution always exhibits an accelerating relative decrease of $[1 - P(x)]$, that is, that $a(x+1) < a(x)$ for every possible Poisson distribution. First, note that for the Poisson distribution:

$$p(x+1) = p(x)\left(\frac{\lambda}{x+1}\right) \tag{A.14}$$

$\lambda/(x+1)$ is a convex function decreasing in x. This means that the ratio $p(x+1)/p(x)$ is a convex function decreasing in x. Conversely, in the case of a decelerating relative decrease in $[1 - P(x)]$ (that is, $a(x+1) > a(x)$), $p(x+1)/p(x)$ would be a function increasing in x and not decreasing. Therefore, the necessary and sufficient condition $a(x+1) < a(x)$ is fulfilled for every possible Poisson distribution.

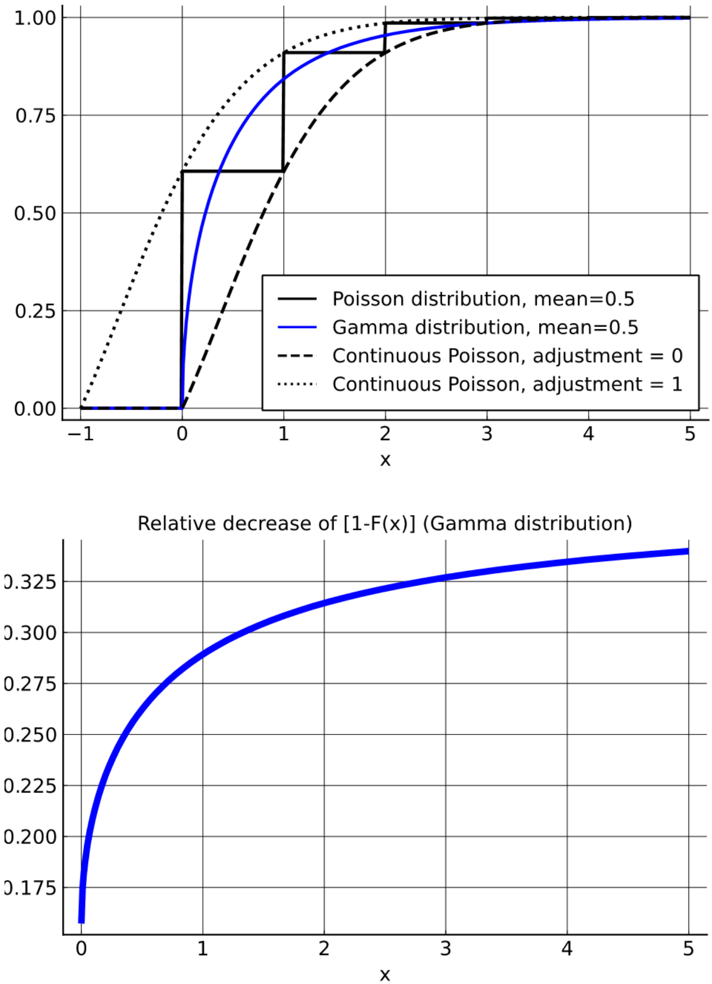
Proof part 4: We have shown that "$u \cdot \left(\frac{1}{\lambda} \cdot \frac{B(s,Q)}{P_{\text{out}}(s,Q)} + \text{const}_3\right)^{\omega} - \text{const}_2$" is jointly convex and decreasing in s and Q if demand follows a Poisson distribution. Multiplying a quasi-convex function decreasing in s and Q (i.e., $P_{\text{out}}(s,Q)$) with a convex function that also decreases in s and Q (i.e., $u \cdot \left(\frac{1}{\lambda} \cdot \frac{B(s,Q)}{P_{\text{out}}(s,Q)} + \text{const}_3\right)^{\omega} - \text{const}_2$) always creates a function that is quasi-convex and decreasing in s and Q. It follows that "$P_{\text{out}}(s,Q)\left(u \cdot \left(\frac{1}{\lambda} \cdot \frac{B(s,Q)}{P_{\text{out}}(s,Q)} + \text{const}_3\right)^{\omega} - \text{const}_2\right)$" is a joint quasi-convex function decreasing in s and Q. Thus, our model does not differ in its basic convexity from a normal inventory management model with stockout-related penalty costs (see **Figure VI.A.3**).

Because of the quasi-convexity of the model, many different algorithms for minimizing the cost function with respect to s and Q are conceivable (see **Subsection VI.3.9**).



Model parameters: $A = 502655$, $dA = 50$, $\ell = 3$, $h = 0.05$, $R = 25$, $u = 1.5$, $\omega = 0.5$, $c_{\text{transport}} = 0.02$, $t_{\text{transport}} = 0.02$, $\beta_c = 0.7$, $\beta_t = 0.7$, $t_{\text{warehouse}} = 1$, $c_{\text{warehouse}} = 0$

Figure VI.A.3 Convexity of the backorder time per backorder and quasi-convexity of the total costs

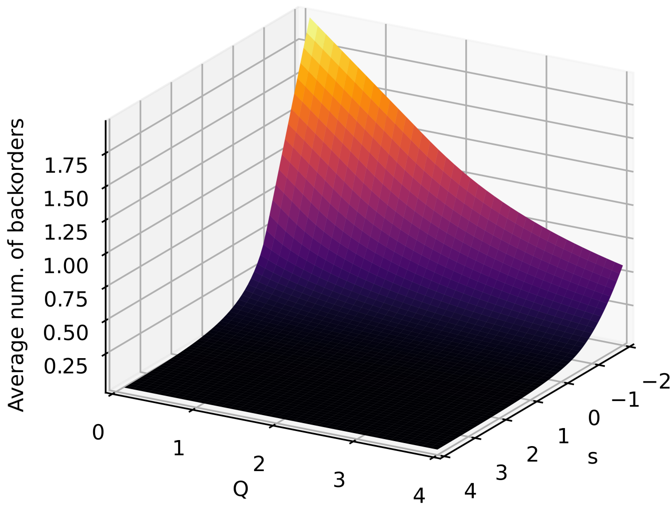


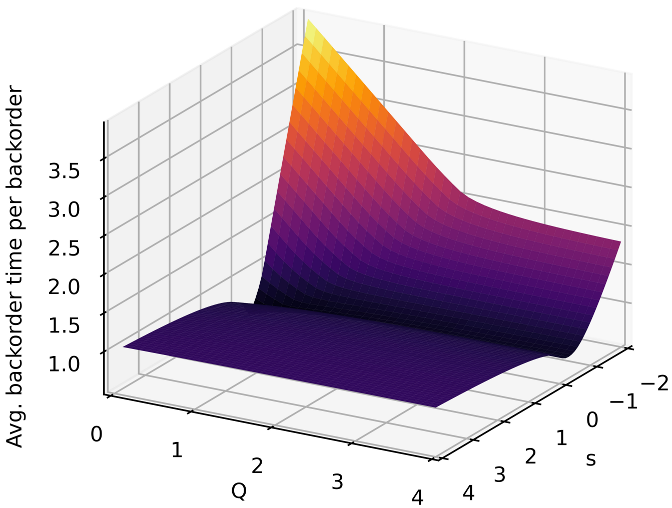
For this figure, the gamma distribution is defined as:

$F_{Gamma}(x, \ell\lambda) = \frac{\gamma(\ell\lambda, x)}{\Gamma(x,0)}$, with $\gamma(\ell\lambda, x) = \int_{g=0}^{x} g^{(\ell\lambda-1)} e^{-g} dg$

All four graphs use a gamma distribution with $\ell\lambda = 0.5$.

Figure VI.A.4.1 Example of a probability distribution with a decelerating relative decrease in $[1 - F(x)]$ (part 1)





For this figure, the gamma distribution is defined as:

$F_{Gamma}(x, \ell\lambda) = \frac{\gamma(\ell\lambda,x)}{\Gamma(x,0)}$, with $\gamma(\ell\lambda, x) = \int_{g=0}^{x} g^{(\ell\lambda-1)}e^{-g}dg$

All four graphs use a gamma distribution with $\ell\lambda = 0.5$.

Figure VI.A.4.2 Example of a probability distribution with a decelerating relative decrease in $[1 - F(x)]$ (part 2)

Further explanations:

- Until now, we have assumed $s \geq 0$ and $Q > 0$. $Q > 0$ is always the case because negative order quantities do not exist. It is trivial that the lower $s < 0$ is, the higher the average backorder time per backorder. In fact, the ratio $B(s,Q)/P_{\text{out}}(s,Q)$ converges to a linear increase as s decreases because $s < 0$ can be understood as an additional wait time with an average length of $|s|/\lambda$. This does not change the convexity of $B(s,Q)/P_{\text{out}}(s,Q)$.

- While an increasing s decreases the average backorder time per backorder toward zero (given that $a(x+1) < a(x)$), this is not the case for an increasing Q. A high Q only reduces the probability that a replenishment, with its arrival, does not eliminate all current backorders. Even if this probability were to be zero, there would still be a stockout probability determined by s. This stockout probability in combination with the replenishment lead time would then define the average backorder time per backorder:

$$\overline{\tau_b}(s, Q \to \infty) = \frac{1}{\lambda} \cdot \frac{B(s, Q \to \infty)}{P_{\text{out}}(s, Q \to \infty)} = \frac{1}{\lambda} \cdot \frac{\sum_{x=s}^{\infty}(x-s)[1-P(x)]}{\sum_{x=s}^{\infty}[1-P(x)]} \tag{A.15}$$

- The proof of convexity of the average backorder time per backorder ($\overline{\tau_b}(s,Q)$) can also be applied to the continuous domain, but only if one chooses the correct continuous counterpart of the Poisson distribution (Ilienko, 2013), that is, something between $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda) = \Gamma(x,\ell\lambda)/\Gamma(x,0)$ and $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda) = \Gamma(x+1,\ell\lambda)/\Gamma(x+1,0)$, with $\Gamma(k,n) = \int_{g=n}^{\infty} g^{(k-1)} e^{-g} dg$. The definitions of $P_{\text{out}}(s,Q)$ and $B(s,Q)$ would change to:

$$P_{\text{out}}(s,Q) = \frac{1}{Q}\left(\int_{x=s}^{\infty} \left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx - \int_{x=s+Q}^{\infty} \left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx \right) \tag{A.16}$$

$$B(s,Q) = \frac{1}{Q}\left(\begin{array}{c} \int\limits_{x=s}^{\infty} (x-s)\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx \\ - \int\limits_{x=s+Q}^{\infty} (x-s-Q)\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx \end{array} \right) \tag{A.17}$$

where $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda) = Prob(X \le x)$ is the cumulative distribution function of the replenishment lead time demand with mean $\ell\lambda$ (note that $\ell\lambda = \ell dA/y$ in our main text).

- We note that the conditions for the joint convexity of $B(s,Q)/P_{\text{out}}(s,Q)$ are more restrictive than the conditions for the joint convexity of $B(s,Q)$. Not all distributions fulfill the necessary and sufficient condition $a(x+1) < a(x)$. Take, for example, the gamma distribution $F_{Gamma}(x,\ell\lambda) = \gamma(\ell\lambda,x)/\Gamma(x,0)$, with $\gamma(\ell\lambda,x) = \int_{g=0}^{x} g^{(\ell\lambda-1)}e^{-g}dg$. This distribution is very similar to $\Gamma(x,\ell\lambda)/\Gamma(x,0)$ but has $a(x+1) < a(x)$ when $\ell\lambda > 1$, $a(x+1) = a(x)$ when $\ell\lambda = 1$ and $a(x+1) > a(x)$ when $\ell\lambda < 1$. In contrast, $B(s,Q)$ is guaranteed to be convex (see **Figures VI.A.4**).

VI.A.2. **Appendix B:** Proof of the quasi-convexity of the approximate location-inventory model with respect to the number of stocking locations **y**, given optimized continuous parameters $\boldsymbol{s}^*$ and $\boldsymbol{Q}^*$.

Consider our model with optimized continuous parameters s^* and Q^*:

$$\begin{aligned}
Min\ C(y) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} + \\
u \cdot \left(1 - P_{\text{out}}(y, s^*, Q^*)\right) \cdot \left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega} + \\
u \cdot P_{\text{out}}(y, s^*, Q^*) \cdot \left(\begin{array}{c} \dfrac{B(y, s^*, Q^*)}{\left(\frac{dA}{y}\right) P_{\text{out}}(y, s^*, Q^*)} + \\ t_{\text{warehouse}} + \dfrac{2t_{\text{transport}}}{2+\beta_t}\left(\dfrac{A}{y\pi}\right)^{0.5\beta_t} \end{array}\right)^{\omega} + \\
\frac{R}{Q^*} + h \cdot \frac{y}{dA} \cdot \left(0.5(Q^*+1) + s^* - \left(\frac{\ell dA}{y}\right) + B(y, s^*, Q^*)\right)
\end{aligned} \tag{B.1}$$

, with $y, Q^* \epsilon \mathrm{R}^+$, $s^* \epsilon \mathrm{R}$.

This model has a location and an inventory component. The location component includes all costs associated with the travel distances from stocking locations to customers. In particular, $\frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c}$ and $\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}$ are both strictly convex decreasing in y. The inventory component includes all costs associated with inventory holding and replenishment. It is a well-known fact that inventory management systems, as modeled above, exhibit two economies of scale effects (Maister, 1976; Eppen, 1979; Schwarz, 1981; Zinn et al., 1989). One effect relates to the cycle stock and can be derived from the economic order quantity model. This effect is often known as the square root law of the cycle stock. The other effect relates to the safety stock and can be derived from the additive property of variance. This effect is often known as the square root law of the safety stock, or more generally, risk pooling. Both effects are strictly concave increasing in y. For example, in their purest form, these effects mean that the total optimized inventory-related costs increase by a factor of $\sqrt{y}$ when y stocking locations are used compared to when one stocking location is used.

For our model, we define the sum of the optimized inventory holding, replenishment and stockout-related delivery lead time penalty costs as follows:

$$IM_{Cost}(y) = \frac{R}{Q^*} + h \cdot \frac{y}{dA} \cdot \left(\begin{array}{c} 0.5(Q^* + 1) + s^* - \\ \left(\frac{\ell dA}{y}\right) + B(y, s^*, Q^*) \end{array} \right) +$$

$$u \cdot P_{\text{out}}(y, s^*, Q^*) \left(\begin{array}{c} \frac{B(y, s^*, Q^*)}{\left(\frac{dA}{y}\right) P_{\text{out}}(y, s^*, Q^*)} + \\ t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t} \end{array} \right)^{\omega} - \tag{B.2}$$

$$u \cdot P_{\text{out}}(y, s^*, Q^*) \left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t} \right)^{\omega}$$

Both the location component and the inventory component of our model interact within the delivery lead time penalty terms. Because $t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t} > 0$, an increase in the average backorder time per backorder is less costly the lower $\omega < 1$. This affects the values of the optimized parameters s^* and Q^*. Higher $B(y, s, Q)$ and $P_{\text{out}}(y, s, Q)$ are optimal because a high average backorder time per backorder is not as detrimental. However, this does not change the fundamental fact that the sum of the optimized inventory holding, replenishment and stockout-related delivery lead time penalty costs (i.e., $IM_{Cost}(y)$) increases strictly concave in y. Only the slope of the concave function $IM_{Cost}(y)$ is affected. More specifically, because $\omega < 1$ and $t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t} > 0$, the steeper the slope of $t_{\text{warehouse}} +$

$\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}$ is, the steeper the slope of $IM_{Cost}(y)$. Because $t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}$ is steeper, the lower y, it follows that:

$$\frac{IM_{Cost}(y+1)}{IM_{Cost}(y)} > \frac{IM_{Cost}(y+2)}{IM_{Cost}(y+1)} \tag{B.3}$$

The total costs per order in our model can therefore be thought of as a sum of a constant, two strictly convex decreasing functions of the kind $1/y$, and a strictly concave, increasing function of the kind $y^{b(y)}$ (i.e., $IM_{Cost}(y)$), with $b(y)$ being a function dependent on y, $1 > b(y) > 0$, $b(y+1) < b(y)$ (see **Figure VI.B.1**).

$$Min\ C(y) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} + u \cdot \left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega} + IM_{Cost}(y) \tag{B.4}$$

Note that although $b(y+1) < b(y)$, it is also evident that $y^{b(y)} \rightarrow \infty$ when $y \rightarrow \infty$. When the average lead time demand converges toward zero, the average optimized inventory holding, replenishment and stockout-related delivery lead time penalty costs per order ($IM_{Cost}(y)$) converge toward infinity. Thus:

$$IM_{Cost}(2y) - IM_{Cost}(y) \leq IM_{Cost}(4y) - IM_{Cost}(2y) \tag{B.5}$$

At the same time:

$$-\left(\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{2y\pi}\right)^{0.5\beta_t} - \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right) > -\left(\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{4y\pi}\right)^{0.5\beta_t} - \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{2y\pi}\right)^{0.5\beta_t}\right) \tag{B.6}$$

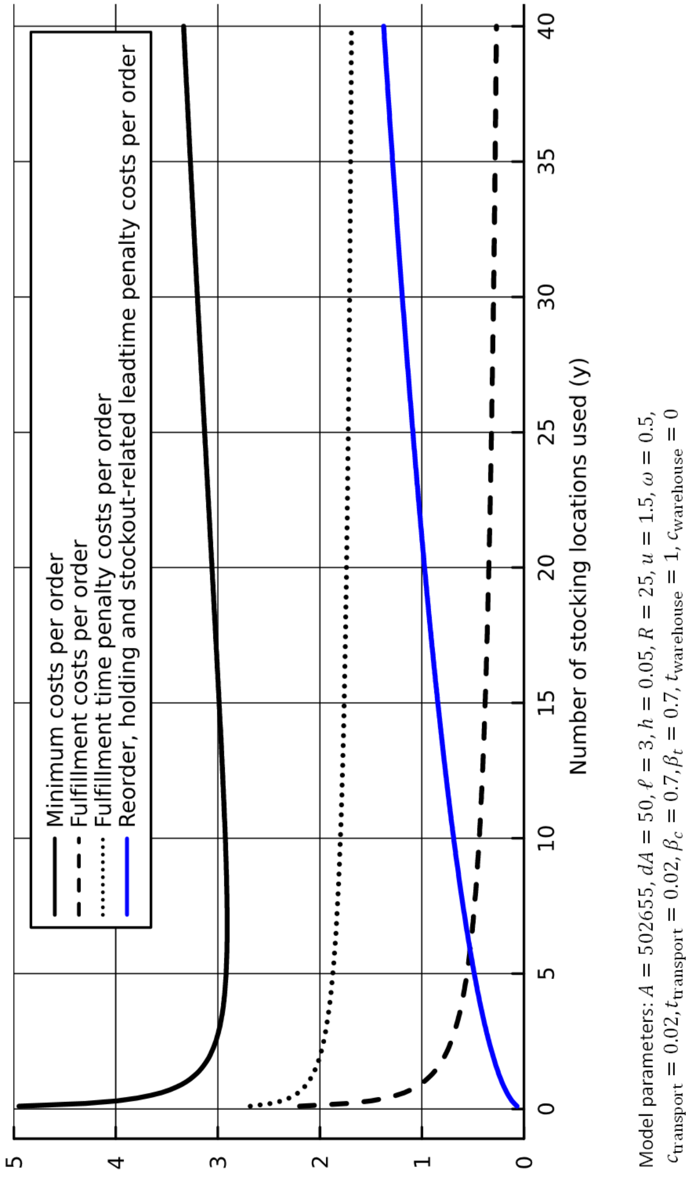
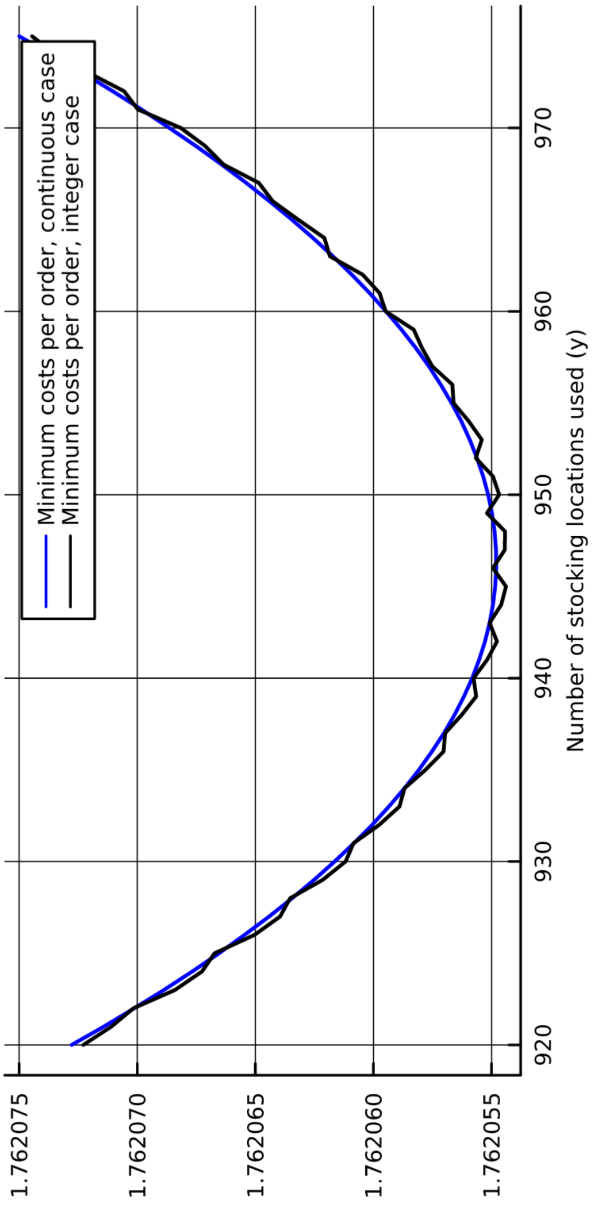


Figure VI.B.1 The curve of the optimal costs per order is a sum of two convex and one concave function



Model parameters: A = 502655, dA = 50, ℓ = 3, h = 0.05, R = 25, u = 1.5, ω = 0.5, $c_{\mathrm{transport}}$ = 0.02, $t_{\mathrm{transport}}$ = 0.02, β_c = 0.7, β_t = 0.7, $t_{\mathrm{warehouse}}$ = 1, $c_{\mathrm{warehouse}}$ = 0

Figure VI.B.2 Comparison between the continuous case and the integer case

Thus, the cost-decreasing effect of shorter transport distances/times may be stronger when y is low but inevitably becomes (and stays) smaller than the cost-increasing effect of the smaller replenishment lead time demand. Therefore, $Min\,C(y,s,Q)$ always has one unique minimum. Depending on the concrete parameters, the minimum may be at $y=1$, or the cost function first decreases convexly when y increases until the minimum is reached. Then, it increases convexly and ultimately changes to a concave increasing function. The higher y is, the more prominent the influence of the inventory management component, which is why the function will always converge into a concave function. Thus, our model is not convex in y but quasi-convex in y. As noted in the main text, quasi-convexity is only guaranteed if s and Q are continuous parameters. However, an integrality constraint on s and Q usually only creates problems if the demand rate per stocking location is relatively low (because then an integer change $+/-1$ of s and Q can substantially change $P_{\text{out}}(y,s,Q)$ and $B(y,s,Q)$) and when, at the same time, the slope of $\frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c}$ and $\frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}$ is relatively flat. For realistic parameter values, such a combination only occurs if both the demand rate (summed over all stocking locations) and y are very high. This has the positive side effect that any breaks in the quasi-convexity of the cost function $C(y,s^*,Q^*)$ usually exist only in a very flat region around the optimum $C(y^*,s^*,Q^*)$ (see **Figure VI.B.2**).

VI.A.3. **Appendix C:** A lower bound for $\boldsymbol{Q}^*$ and an upper bound for $\boldsymbol{s}^*$ using the linear approximation of the model.

Rubalski (1972) and Federgruen and Zheng (1992) presented an algorithm for efficiently solving a linear inventory management model with costs per unit backordered (u_1, event-based) and/or costs per unit of backordered demand, per unit of time (u_2, time-based). In the following, we lean on the notation used in Federgruen and Zheng (1992). Note, however, that the formulas in Federgruen and Zheng (1992) use costs per time unit, and we use (in line with the rest of our article) costs per order.

The algorithm is based on the fact that $C(s, Q)$ can be written as:

$$C(s,Q) = \left[K + \sum_{x=s+1}^{s+Q} G(x)\right] / Q \tag{C.1}$$

For example (with $\lambda = dA$ in our case), the following cost function

$$\begin{aligned} C(s,Q) = \frac{R}{Q} + h \cdot \frac{1}{\lambda} \cdot \big(0.5(Q+1) + s - (\ell\lambda) + B(s,Q)\big) \\ + u_1 \cdot P_{\text{out}}(s,Q) + u_2 \cdot \left(\frac{1}{\lambda}\right) B(s,Q) \end{aligned} \tag{C.2}$$

can be expressed as:

$$C(s,Q) = \left[R + \sum_{x=s+1}^{s+Q} G(x)\right] / Q \tag{C.3}$$

$$\begin{aligned} G(x) = \left(\frac{h}{\lambda} + \frac{u_2}{\lambda}\right) \sum_{j=0}^{x-1} F_{Poi}(j, \ell\lambda) \\ + \left(\frac{u_2}{\lambda}\right)(\ell\lambda - x) + u_1(1 - F_{Poi}(x-1, \ell\lambda)) \end{aligned} \tag{C.4}$$

where $F_{\text{Poi}}()$ is the cumulative distribution function of a Poisson distribution with mean $\mu = \ell\lambda$ (note that $\ell\lambda = \ell dA/y$ in our main text).

Federgruen and Zheng (1992, p. 812), hint that the algorithm can be modified to handle “general nonlinear holding, backlogging or stockout penalty cost functions”. However, to the best of our knowledge, this statement needs to be amended by the caveat that this is only possible when $G(x)$ is still only dependent on x and not on s or Q (a condition that is mentioned in Federgruen and Zheng (1992, p. 810), in regard to the linear case).

In our case, this seems to be impossible. We have:

$$\begin{gathered}
Min\ C(s,Q) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} + \\
u \cdot \left(1 - P_{\text{out}}(s,Q)\right)\left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right)^{\omega} + \\
u \cdot P_{\text{out}}(s,Q)\begin{pmatrix} \dfrac{B(s,Q)}{(\lambda)P_{\text{out}}(s,Q)} + \\ t_{\text{warehouse}} + \dfrac{2t_{\text{transport}}}{2+\beta_t}\left(\dfrac{A}{y\pi}\right)^{0.5\beta_t} \end{pmatrix}^{\omega} + \\
\frac{R}{Q} + h \cdot \frac{1}{\lambda} \cdot \left(0.5(Q+1) + s - (\ell\lambda) + B(s,Q)\right)
\end{gathered} \tag{C.5}$$

To the best of our knowledge, because of the term $P_{\text{out}}(s,Q)\left(\frac{B(s,Q)}{(\lambda)P_{\text{out}}(s,Q)} + \text{constant}\right)^{\omega}$ with $0 < \omega < 1$, there is no way to convert our $C(s,Q)$ into a general formula of the kind $\left[K + \sum_{x=s+1}^{s+Q} G(x)\right]/Q$.

However, we can use the linear approximation of the model to calculate a lower bound for Q^* and an upper bound for s^* (see **Subsection VI.3.9**). Such bounds can be useful to constrain a search procedure (e.g., a bisection search) to relevant areas. In the case of $\omega = 1$, our model simplifies to

$$Min\ C(s,Q) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} +$$
$$u \cdot \left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right) + \left(\frac{u}{\lambda}\right) \cdot B(s,Q) + \tag{C.6}$$
$$\frac{R}{Q} + h \cdot \frac{1}{\lambda} \cdot \big(0.5(Q+1) + s - (\ell\lambda) + B(s,Q)\big)$$

The linear version of our model can be efficiently solved by the algorithm in Rubalski (1972) and Federgruen and Zheng (1992). After similar transformations as the one above:

$$Min\ C(s,Q) = c_{\text{warehouse}} + \frac{2c_{\text{transport}}}{2+\beta_c}\left(\frac{A}{y\pi}\right)^{0.5\beta_c} +$$
$$u \cdot \left(t_{\text{warehouse}} + \frac{2t_{\text{transport}}}{2+\beta_t}\left(\frac{A}{y\pi}\right)^{0.5\beta_t}\right) + \left[R + \sum_{x=s+1}^{s+Q} G(x)\right]/Q \tag{C.7}$$

$$G(x) = \left[(h+u)\sum_{j=0}^{x-1} F_{\text{Poi}}(j,\ell\lambda) + u(\ell\lambda - x)\right]/\lambda \tag{C.8}$$

$$\Delta G(x) = G(x+1) - G(x) = [(h+u)F_{\text{Poi}}(x,\ell\lambda) - u]/\lambda \tag{C.9}$$

In **Appendix VI.A.5**, we give a concrete computer code example of how linear approximation bounds (obtained with the algorithm of Federgruen and Zheng 1992) can be used in combination with a bisection search algorithm to solve our model. Whether it makes sense to calculate a lower bound for Q^* and an upper bound for s^* using the linear approximation always depends on the concrete case, in particular, how efficient the implementation of the algorithms is in practice and which algorithm is used for solving the nonlinear problem. We saw practically no time difference between using and not using the linear approximation bounds when solving the nonlinear problem with a bisection search algorithm.

VI.A.4. **Appendix D:** The case of continuous $\boldsymbol{y}$, $\boldsymbol{s}$ and $\boldsymbol{Q}$.

Calculating the continuous model is slower than calculating the integer model. Since breaks in the quasi-convexity of the integer model cost function with respect to y usually occur, if at all, only in very flat regions around the optimum, we advise against solving the model with continuous y, s and Q. However, if the reader would like to solve the continuous model nonetheless, we can report the following observations.

The reason why calculating the continuous model is slow is due to the calculation of the service level measures $P_{\text{out}}(y,s,Q)$ and $B(y,s,Q)$. In the continuous case:

$$P_{\text{out}}(y,s,Q) = \frac{1}{Q}\left(\begin{array}{c}\displaystyle\int_{x=s}^{\infty}\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx \\ \displaystyle -\int_{x=s+Q}^{\infty}\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx\end{array}\right) \tag{D.1}$$

$$B(y,s,Q) = \frac{1}{Q}\left(\begin{array}{c}\displaystyle\int_{x=s}^{\infty}(x-s)\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx \\ \displaystyle -\int_{x=s+Q}^{\infty}(x-s-Q)\left[1 - F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right] dx\end{array}\right) \tag{D.2}$$

where $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda) = Prob(X \leq x)$ is the cumulative distribution function of the replenishment lead time demand with mean $\ell\lambda$ (note that $\ell\lambda = \ell dA/y$ in our main text).

The numerical integration of these integrals costs considerable time. Recall that in the integer case (see **Subsection VI.3.5**), both $P_{\text{out}}(y,s,Q)$ and $B(y,s,Q)$ can be calculated without using any infinite integrals or sums.

In our tests, we observed that numerical integration by the deterministic iterative adaptive algorithm “CUHRE” suits the above integrals well

(Berntsen et al., 1991; Hahn, 2005). Note that for this algorithm to work, one needs to rescale the integrals:

$$\int_{x=v}^{\infty}\left[1-F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right]= \int_{x=0}^{1}\left[1-F_{\text{Poi}}^{\text{Cont.}}\left(v+\frac{x}{1-x},\ell\lambda\right)\right]\left[\frac{1}{(1-x)^2}\right] \tag{D.3}$$

$$\int_{x=v}^{\infty}(x-v)\left[1-F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right]= \int_{x=0}^{1}\left[\frac{x}{1-x}\right]\left[1-F_{\text{Poi}}^{\text{Cont.}}\left(v+\frac{x}{1-x},\ell\lambda\right)\right]\left[\frac{1}{(1-x)^2}\right] \tag{D.4}$$

As noted by Zipkin (1986), inaccuracies arise when one tries to model a discrete system with continuous variables. A usual adjustment to improve the accuracy of the approximation of $B(y,s,Q)$ is a shift of the lower integral bound by +0.5 (see, for example, Zipkin, 1986, p. 976; Hadley & Whitin, 1963, p. 142). That is, instead of Equation D.4, one would use:

$$\int_{x=v+0.5}^{\infty}(x-v-0.5)\left[1-F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)\right]= \int_{x=0}^{1}\left[\frac{x}{1-x}\right]\left[1-F_{\text{Poi}}^{\text{Cont.}}\left(v+0.5+\frac{x}{1-x},\ell\lambda\right)\right]\left[\frac{1}{(1-x)^2}\right] \tag{D.5}$$

Regarding the continuous cumulative distribution function $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)$, something between $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)=\Gamma(x,\ell\lambda)/\Gamma(x,0)$ and $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)=\Gamma(x+1,\ell\lambda)/\Gamma(x+1,0)$, with $\Gamma(k,n)=\int_{g=n}^{\infty}g^{(k-1)}e^{-g}dg$, yields the most accurate approximation. We found that $F_{\text{Poi}}^{\text{Cont.}}(x,\ell\lambda)=\Gamma(x+0.5,\ell\lambda)/\Gamma(x+0.5,0)$, in combination with the abovementioned adjustment of $B(y,s,Q)$ provides an excellent approximation if the average replenishment lead time demand is greater than one ($\ell\lambda>1$).

Regarding the optimization of the continuous quasi-convex model, that is, the search for y^*, s^* and Q^*, we found gradient descent algorithms to be well suited (e.g., a box-constrained BFGS algorithm or the GRG algorithm; Lasdon et al., 1978).

Additional references not cited in our paper:

Berntsen, J., Espelid, T. O., & Genz, A. (1991). An adaptive algorithm for the approximate calculation of multiple integrals. *ACM Transactions on Mathematical Software (TOMS)*, *17*(4), 437–451. https://doi.org/10.1145/210232.210233

Hahn, T. (2005). Cuba—a library for multidimensional numerical integration. *Computer Physics Communications, 168*(2), 78–95. https://doi.org/10.1016/j.cpc.2005.01.010

VI.A.5. **Appendix E:** Example code for determining $\boldsymbol{y}^*$, $\boldsymbol{s}^*$ and $\boldsymbol{Q}^*$ via bisection search algorithms.

The following code is written in the "Julia" programming language. The following packages are needed, which can be installed via "`using Pkg; Pkg.add("Name of package")`":

```
using Distributions

using Cuba

using Optim
```

First, we define the functions for the linear-case algorithm from Federgruen & Zheng (1992):

```
function DeltaLinearG(u, h, x, MeanBaseDemandPerWarehouse, MeanLeadtimeDemand)

    Delta = ( (h + u)*cdf( Poisson(MeanLeadtimeDemand), x ) - u )/
            MeanBaseDemandPerWarehouse
#Ideally the CDF value would be calculated using the Poisson probability ratio
(mean of distribution / (x+1)). However, problems arise when the PDF value is
smaller than the machine precision. One can either use a big number library
(which is slow) or check the value until it is big enough to be manipulated by
the Poisson probability ratio. For readability, we refrain from doing so in
this code.

    return Delta

end



function LinearGFZ(u, h, x, MeanBaseDemandPerWarehouse, MeanLeadtimeDemand)

    Sum = 0
    for i in range(0, x)
        Sum = Sum + (x-i)*pdf( Poisson(MeanLeadtimeDemand), i )
    end
#Ideally the PDF value would be calculated using the Poisson probability ratio
(mean of distribution / (x+1)). For more information refer to the code of our
DeltaLinearG() function.

    HoldingCosts = h*Sum/MeanBaseDemandPerWarehouse

    BackorderPenaltyCosts = (u/MeanBaseDemandPerWarehouse)*
                            (Sum + MeanLeadtimeDemand - x)

    GCosts = HoldingCosts + BackorderPenaltyCosts

    return GCosts

end
```

```
function OptimizeLinearFZ(y, LT, A, d, u, bt, ttransport, twarehouse, bc,
ctransport, cwarehouse, R, h)

    TimeConstant = twarehouse + (((2*ttransport)/(2+bt))*(a/(y*pi))^(0.5*bt))
    CostConstant = cwarehouse + (((2*ctransport)/(2+bc))*(a/(y*pi))^(0.5*bc))

    MeanBaseDemandPerWarehouse = (d*A)/y
    MeanLeadtimeDemand = MeanBaseDemandPerWarehouse * LT

    x = 0
    Glower = 0
    Ghigher = 0
    G = LinearGFZ(u, h, x, MeanBaseDemandPerWarehouse, MeanLeadtimeDemand)
    Delta = DeltaLinearG(u, h, x, MeanBaseDemandPerWarehouse,
                         MeanLeadtimeDemand)

    while (Delta < 0)
        Glower = G
        G = G + Delta
        x = x + 1
        Delta = DeltaLinearG(u, h, x, MeanBaseDemandPerWarehouse,
                             MeanLeadtimeDemand)
        Ghigher = G + Delta
    end

    Q = 1
    S = G + R      #R=K in Federgruen & Zheng (1992)
    C = S/Q
    s = x-1                 #s=r in Federgruen & Zheng (1992)
    FZ = x+1                #FZ=R in Federgruen & Zheng (1992)
```

```
    LoopCondition = true
    while (LoopCondition == true)
        if (Glower <= Ghigher)
            if (C <= Glower)
                LoopCondition = false
            else
                S = S + Glower
                Q = Q + 1
                C = S/Q
                s = s - 1
                Delta = DeltaLinearG(u, h, s, MeanBaseDemandPerWarehouse,
                                     MeanLeadtimeDemand)
                Glower = Glower - Delta
            End
        elseif (C <= Ghigher)
           LoopCondition = false
           else
            S = S + Ghigher
            Q = Q + 1
            C = S/Q
            Delta = DeltaLinearG(u, h, FZ, MeanBaseDemandPerWarehouse,
                                     MeanLeadtimeDemand)
            Ghigher = Ghigher + Delta
            FZ = FZ + 1
        end
    end

    Opt_s = s
    Opt_Q = Q
    Opt_Objective = C + u*TimeConstant + CostConstant
    return Opt_s, Opt_Q, Opt_Objective

end
```

Now we define functions that return the cost per order, given y, s and Q. Note that these functions have some precision error corrections. These allow us to use the code for problem instances with very large mean replenishment lead time demands. In these instances, and with the following corrections, the function is not exact anymore but is an excellent approximation. In case of a moderate mean replenishment lead time demand, the following functions are exact. Alternatively, one can use a higher bit CPU architecture or a (slow) big number library.

```
function HWalpha(value, MeanLeadtimeDemand)

    HWalphaValue = MeanLeadtimeDemand *
                   ( 1 - cdf( Poisson(MeanLeadtimeDemand), (value-1) ) ) -
                   (value) *
                   ( 1 - cdf( Poisson(MeanLeadtimeDemand), (value) ) )

    return HWalphaValue
end


function HWbeta(value, MeanLeadtimeDemand)

    HWbetaValue = ((MeanLeadtimeDemand^2)/2) *
                  ( 1 - cdf( Poisson(MeanLeadtimeDemand), (value-2) ) ) -
                  MeanLeadtimeDemand * value *
                  ( 1 - cdf( Poisson(MeanLeadtimeDemand), (value-1) ) ) +
                  ( (value+1)*value/2 ) *
                  ( 1 - cdf( Poisson(MeanLeadtimeDemand), value ) )

    return HWbetaValue
end
```

```
function TotalCostsPerOrder(y, LT, A, d, v, u, bt, ttransport, twarehouse, bc,
ctransport, cwarehouse, R, h, s, Q)

    #Stockout probability:
    P_out = (1/Q)*( HWalpha(s, ( (LT*d*A)/(y) )) -
                    HWalpha(s+Q, ((LT*d*A)/(y) )) )

    if P_out < 0.000000001  #For correcting precision errors

        P_out = 0

    end

    #Mean number of backorders at any time:
    BQs = (1/Q)*( HWbeta(s, ( (LT*d*A)/(y) )) -
                  HWbeta(s+Q, ( (LT*d*A)/(y) )) )

    if BQs < 0.000000001    #For correcting precision errors

        BQs = 0

    end

    #Mean backorder time per backorder:

    if (P_out == 0)         #Because of possible precision errors (see above).

        BackordertimePerBackorder = 0

    else

        BackordertimePerBackorder = BQs / ( ( (d*A)/(y) ) * P_out )

    end

    WarehouseCostsPerOrder = cwarehouse
    TransportCostsPerOrder = ((2*ctransport)/(2+bc))*((A)/(y*pi))^(0.5*bc)

    DeliveryTimeCostsPerNormalOrder = u*((twarehouse +
                         (((2*ttransport)/(2+bt))*(a/(y*pi))^(0.5*bt)))^v)

    DeliveryTimeCostsPerBackorderedOrder = u*(( BackordertimePerBackorder +
            twarehouse + (((2*ttransport)/(2 + bt))* (a/(y*pi))^(0.5*bt)) )^v)

    DeliveryTimeCostsPerOrder = (1 - P_out)*DeliveryTimeCostsPerNormalOrder +
                           P_out*DeliveryTimeCostsPerBackorderedOrder

    ReorderCostsPerOrder = R / Q

    InventoryHoldingCostsPerOrder = h * (y/(d*A)) * (0.5*(Q+1) + s -
                                ( (LT*d*A)/y ) + BQs )

    CostsPerOrder = WarehouseCostsPerOrder + TransportCostsPerOrder +
                    DeliveryTimeCostsPerOrder + ReorderCostsPerOrder +
                    InventoryHoldingCostsPerOrder

    return CostsPerOrder
end
```

In case of continuous s and Q, the above functions become:

```
function ContinuosPoissonCDF(value, MeanLeadtimeDemand)

    Shift = 0.5

    if( value < -Shift )
        CDFvalue = 0
    else
          CDFvalue = gamma_inc((value + Shift), MeanLeadtimeDemand)[2]
          #gamma_inc()[2] = gamma(MeanLeadtimeDemand, (value + Shift))/
          #                 gamma(value + Shift)
    end

    return CDFvalue
end



function HWalpha(value, MeanLeadtimeDemand)

    result, err = cuhre((x,f) -> f[1] = ( 1 - ContinuosPoissonCDF( value +
                      x[1]/(1-x[1]), MeanLeadtimeDemand) ) *
                      ( 1/( (1-x[1])^2 ) ),
                      atol = 1e-12)

    HWalphaValue = result[1]
    return HWalphaValue
end



function HWbeta(value, MeanLeadtimeDemand)

    ContAdjustment = 0.5
    result, err = cuhre((x,f) -> f[1] = (x[1]/(1-x[1])) *
                      ( 1 - ContinuosPoissonCDF( value + ContAdjustment +
                      x[1]/(1-x[1]), MeanLeadtimeDemand) ) *
                      ( 1/( (1-x[1])^2 ) ),
                      atol = 1e-12)

    HWbetaValue = result[1]
    return HWbetaValue
end
```

The following functions implement a bisection algorithm for finding optimal integer y^*, s^* and Q^*:

```
function sBisectionSearch(y, LT, A, d, v, u, bt, ttransport, twarehouse, bc,
ctransport, cwarehouse, R, h, LowerBounds, UpperBounds, Q)

        s = 0
        Direction = 0
        stest = LowerBounds
        sStepSize = floor((UpperBounds - LowerBounds)/2)

        CostsPerOrder = TotalCostsPerOrder(y, LT, A, d, v, u, bt, ttransport,
                                           twarehouse, bc, ctransport,
                                           cwarehouse, R, h, stest, Q)

        LoopCondition = true
        while (LoopCondition == true)

            if(Direction == 0)
                stest = stest + sStepSize
            else
                stest = stest - sStepSize
            end

            TestCostsPerOrder = TotalCostsPerOrder(y, LT, A, d, v, u, bt,
                                ttransport, twarehouse, bc, ctransport,
                                cwarehouse, R, h, stest, Q)

            if(TestCostsPerOrder >= CostsPerOrder)
                sStepSize = floor(sStepSize/2)
                if (Direction == 0)
                    Direction = 1
                else
                    Direction = 0
                end
            end

            if (sStepSize >= 1)
                CostsPerOrder = TestCostsPerOrder
            else
                if (Direction == 0)
                    s = stest + 1
                else
                    s = stest - 1
                end
                LoopCondition = false
            end

        end

        return CostsPerOrder, s
end
```

```
function QBisectionSearch(y, LT, A, d, v, u, bt, ttransport, twarehouse, bc,
ctransport, cwarehouse, R, h)
    Q = 1
    Bounds = OptimizeLinearFZ(y, LT, A, d, u, bt, ttransport, twarehouse, bc,
                              ctransport, cwarehouse, R, h)
    UpperBounds = Bounds[1]
    LowerBoundQ = Bounds[2]
    UpperBoundQ = 100000000 #The upper bound of Q (the maximum order
#quantity/volume) is determined by limitations of the real world
    LowerBounds = -UpperBoundQ
    Direction = 0
    Qtest = LowerBoundQ
    QStepSize = floor((UpperBoundQ - LowerBoundQ)/2)
    Results = sBisectionSearch(y, LT, A, d, v, u, bt, ttransport, twarehouse,
                               bc, ctransport, cwarehouse, R, h, LowerBounds,
                               UpperBounds, Qtest)
    CostsPerOrder = Results[1]
    s = Results[2]
    LoopCondition = true
    while (LoopCondition == true)
        if (Direction == 0)
            Qtest = Qtest + QStepSize
        else
            if(Qtest - QStepSize < 1)
                Qtest = 1
                QStepSize = floor(QStepSize/2)
            else
                Qtest = Qtest - QStepSize
            end
        end
        Results = sBisectionSearch(y, LT, A, d, v, u, bt, ttransport,
                                   twarehouse, bc, ctransport, cwarehouse,
                                   R, h, LowerBounds, UpperBounds, Qtest)
        TestCostsPerOrder = Results[1]
        if (TestCostsPerOrder >= CostsPerOrder)
            QStepSize = floor(QStepSize/2)
            if (Direction == 0)
                Direction = 1
            else
                Direction = 0
            end
        end

        if (QStepSize >= 1)
            CostsPerOrder = TestCostsPerOrder
            s = Results[2]
        else
            if (Direction == 0)
                Q = Qtest + 1
            else
                Q = Qtest - 1
            end
            LoopCondition = false
        end
    end
    return CostsPerOrder, s, Q
end
```

```
function yBisectionSearch(LT, A, d, v, u, bt, ttransport, twarehouse, bc,
ctransport, cwarehouse, R, h)

    y = 1
    LowerBoundy = 1
    UpperBoundy = 100000000 #The upper bound of y (the maximum number of
#stocking locations) is determined by limitations of the real world
    Direction = 0
    ytest = 1
    yStepSize = floor((UpperBoundy - LowerBoundy)/2)
    Results = QBisectionSearch(ytest, LT, A, d, v, u, bt, ttransport,
                              twarehouse, bc, ctransport, cwarehouse, R, h)
    CostsPerOrder = Results[1]
    s = Results[2]
    Q = Results[3]
    LoopCondition = true
    while (LoopCondition == true)
        if (Direction == 0)
            ytest = ytest + yStepSize
        else
            if(ytest - yStepSize < 1)
                ytest = 1
                yStepSize = floor(yStepSize/2)
            else
                ytest = ytest - yStepSize
            end
        end

        Results = QBisectionSearch(ytest, LT, A, d, v, u, bt, ttransport,
                                  twarehouse, bc, ctransport, cwarehouse,
                                  R, h)
        TestCostsPerOrder = Results[1]
        if (TestCostsPerOrder >= CostsPerOrder)
            yStepSize = floor(yStepSize/2)
            if (Direction == 0)
                Direction = 1
            else
                Direction = 0
            end
        end

        if (yStepSize >= 1)
            CostsPerOrder = TestCostsPerOrder
            s = Results[2]
            Q = Results[3]
        else
            if (Direction == 0)
                y = ytest + 1
            else
                y = ytest - 1
            end
            LoopCondition = false
        end
    end

    return CostsPerOrder, y, s, Q
end
```

Note, that for certain parametrizations the integer model is not guaranteed to be quasi-convex in y. A possible implementation for finding optimal continuous y^*, s^* and Q^* using the BFGS algorithm is:

```
function Optimize(LT, A, d, v, u, bt, ttransport, twarehouse, bc, ctransport,
cwarehouse, R, h)

    f(x) = TotalCostsPerOrder(x[3], LT, A, d, v, u, bt, ttransport,
                              twarehouse, bc, ctransport, cwarehouse, R,
                              h, x[1], x[2])
    lower = [-Inf, 0.00000000001, 0.00000000001]
    upper = [Inf, Inf, Inf]
    initial_x = [1.0, 1.0, 1.0]
    inner_optimizer = BFGS()
    results = optimize(f, lower, upper, initial_x, Fminbox(inner_optimizer),
                 Optim.Options(x_tol = 1e-12, f_tol = 1e-12, g_tol = 1e-12))

    if (Optim.converged(results) == false)
        println( "Algorithm did not converge: ", Optim.converged(results) )
    end

    Opt_s = Optim.minimizer(results)[1]
    Opt_Q = Optim.minimizer(results)[2]
    Opt_y = Optim.minimizer(results)[3]
    Opt_Objective = Optim.minimum(results)
    return Opt_y, Opt_s, Opt_Q, Opt_Objective

end
```

After installing the programming language “Julia”, one could create a new file called “functions.jl” and copy paste the above functions (either the ones for the discrete case or the ones for the continuous case) into this file. In another new file “Main.jl”, one could use the following code to execute the functions:

```
include("functions.jl")
LT = 3                          #the replenishment lead time
A = 502655                      #the total market area
Lambda = 200000                 #demand per time unit for A
d = Lambda/A                    #demand per time unit per unit area
v = 0.5                         #delivery time perception slope parameter
u = 1.5                         #delivery time perception scale parameter
bt = 0.7                        #transport time slope parameter
ttransport = 0.02               #transport time scale parameter
twarehouse = 0                  #constant fulfillment time per order
bc = 0.7                        #transport cost slope parameter
ctransport = 0.02               #transport cost scale parameter
cwarehouse = 0                  #constant fulfillment cost per item
R = 25                          #reorder costs per replenishment
h = 0.2               #inventory holding costs for one product unit per time unit
Optimize(LT, A, d, v, u, bt, ttransport, twarehouse, bc, ctransport,
cwarehouse, R, h)
#yBisectionSearch(LT, A, d, v, u, bt, ttransport, twarehouse, bc, ctransport,
cwarehouse, R, h)
```

UNIVERSITY OF BAMBERG
CHAIR OF OPERATIONS MANAGEMENT & LOGISTICS

This PhD thesis is about logistics and its importance in B2C e-commerce retailing of physical goods (e-tailing). Logistics encompasses a wide range of activities in e-tailing, including the purchase of goods, their storage, as well as the packing and shipping of orders. In many cases, these logistical processes are at the core of an e-tailer‘s value chain. Consequently, inexpensive, fast, and reliable logistics are also frequently emphasized by e-tailers as part of their marketing.

A prominent example of this is the Amazon Prime subscription (membership-based free fast shipping) and the Fulfillment by Amazon service, which third-party sellers on the Amazon marketplace can use to tap into Amazon‘s logistics capabilities (i.e., logistics outsourcing). At the same time, there are strong economies of scale effects in logistics, and thus also in e-tailing. Accordingly, it can be argued that competition in the B2C e-commerce market is strongly influenced by logistical scale.

This PhD thesis explores this interrelated set of topics through a combination of exploratory and confirmatory, qualitative and quantitative, empirical and mathematical research. The perspectives of customers as well as e-tailers, marketplace operators and third-party sellers are considered. Outsourcing to logistics service providers, transaction costs and possible regulation of the B2C e-commerce market are also discussed.