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Modeling the Performance of ARQ Error Control in an LTE Transmission System

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Abstract. We consider the transmission of protocol data units in an LTE eNodeB in downlink direction and focus on the radio link control (RLC) and hybrid Automatic-Repeat-Request functionality at layer 2 of the transmission system. We model the associated window flow control of RLC frames in terms of an open queueing network with two stations and describe RLC frames waiting for acceptance by the flow control window by repeated objects collected in an orbit in front of this multi-server queueing network. We describe the correlated arrivals of frames by a general Markovian arrival process and the transfer of data transport blocks along the orthogonal frequency-division multiplexing channels by state-dependent service processes and varying channel capacities. We derive a versatile finite Markovian queueing model and show that its steady-state distribution can be computed in terms of a level-independent QBD process. Then we determine the basic performance metrics of the system in terms of the latter steady-state distribution.

Keywords: LTE performance analysis · Window flow control
Hybrid ARQ · MAP · Level-independent QBD

1 Introduction

At present mobile cloud services and multimedia applications generate the majority of current traffic load in the Internet and demand a fast evolution of the underlying mobile networks from Long Term Evolution (LTE) and LTE-Advanced to dynamically evolving 5G technologies. Considering the protocol stack of LTE transport systems, many improvements established in recent years have concerned layer 1 and 2 functionalities. They are related to improved transmission techniques such as hybrid Automatic-Repeat-Request (ARQ), advanced adaptive space-time coding methods that are dependent on channel states as well as multiple-input/multiple-output (MIMO) and beamforming techniques.

The paper is devoted to the transmission of radio link control (RLC) protocol data units (PDUs) in an LTE base station called eNodeB in downlink direction.

It focuses on the RLC error control and hybrid ARQ functionality as basic control method at layer 2 (cf. [3,15]). We model the flow control scheme of radio link control frames that is governed by an ARQ selective repeat (SR) policy in terms of an open queueing network (QNW) with two multi-server stations. We describe those frames waiting for acceptance by the flow control window by retrying objects that are collected in an orbit in front of this multi-server queueing network. The correlated arrivals of RLC frames are modelled by a general Markovian arrival process (MAP) which can be mapped easily to real data (cf. [2,7]). The transport of radio data blocks along the orthogonal frequency-division multiplexing (OFDM) channels is described by state-dependent service processes with varying channel capacities. In this regard the sketched model extends our past modeling approaches that either did not map the correlated arrival process to an adequate point process or considered a simpler closed queueing network of the flow control process (cf. [12]). In this regard our open queueing network and its underlying continuous-time Markov chain (CTMC) also differ substantially from the discrete-time models of ARQ and hybrid ARQ systems developed in recent years by Zorzi et al. and others (cf. [1,14] and references therein). There the main focus was given by the performance of control algorithms in lower layers of a wireless network whereas our approach studies the intertwining between the RLC error control scheme at layer 2 of an LTE transmission system and a correlated packet arrival process, e.g. a TCP flow arising from advanced cloud and multimedia applications. It allows us to understand the impact of the layer 2 functionality on the quality-of-service (QoS) and quality-of-experience (QoE) performance of mobile cloud applications in a better way (cf. [5]).

We derive a versatile finite Markovian queueing model and show that its steady-state distribution can be computed in terms of a quasi birth-and-death (QBD) process. Then we determine the basic performance metrics of the system in terms of the latter steady-state distribution taking into account its matrix-geometric closed form.

Looking at the requirements of tactile Internet based on software-defined 5G wireless networks in the near future, we may argue that gaining insight on the performance of basic layer 2 techniques such as ARQ selective repeat (ARQ-SR) and hybrid ARQ error control of RLC frames is a fundamental step towards a self-optimizing resource assignment and an adaptive orchestration methodology. Analytic models constitute a first simple step in that direction to optimize delay-throughput and QoE performance of such challenging architectures. In this regard the presented modeling approach may provide a first analytic guideline how to reach these challenging goals.

The paper is organized as follows. First we present a brief description of the RLC error control and hybrid ARQ scheme and its modeling by a queueing network. Regarding the error control in the considered LTE transmission system, we derive a Markovian model on a finite state space and compute its steady-state vector in Sect. 3. In Sect. 4 we determine the basic performance metrics of the model. Finally we discuss some conclusions of our modeling approach.

2 Modeling ARQ Error Control of RLC Frames on Logical Channels in an LTE eNodeB

In the following we regard the protocol stack of an LTE network at layers 1 to 3 and the transmission of the corresponding protocol data units that are arising from some advanced application on top of the IP network layer, see Fig. 1 (cf. [3]). The latter are associated with an LTE bearer service, first segmented and encapsulated into PDUs of the protocol data convergence protocol (PDCP) by some compression and encryption transformations and then disassembled and converted into radio link layer control PDUs. The objects of these RLC streams are multiplexed into MAC PDUs. In our paper we focus on the transmission of RLC frames as part of MAC PDUs in an LTE base station called eNodeB in downlink direction. We study the logical protection of transported RLC frames at layer 2 of the transmission system at the LTE air interface (cf. [3,15]).

2.1 Error Control of RLC Frames in an LTE Transmission System

In an LTE eNodeB IP packets are transformed during the complex downlink processing steps such that MAC PDUs are formed from this payload (SDU) as shown in Fig. 1 (cf. [3]). The latter comprise the potentially segmented IP payload in terms of RLC frames that are transferred along a logical channel, namely a data traffic channel (DTCH). These RLC PDUs are then mapped to transport channels on the MAC layer, namely the downlink shared channels (DL-SCHs) and to one or multiple physical downlink shared channels (PDSCHs) at the physical layer in terms of transport blocks, see Figs. 1 and 2 (cf. [3]). Thereafter, the generated physical layer PDUs are transmitted along the latter channels by digital signal streams towards a mobile client.

In an LTE eNodeB hybrid ARQ schemes such as a parallel version of the simple stop-and-wait policy (ARQ-SW) combined with smart adaptive channel coding are applied at the LTE air interface to the MAC PDUs (cf. [3], [10, Chap. 3.4.4], [11]). During the preparation of those transport blocks on physical channels further coding elements such as forward error correction (FEC) code words are attached to the original PDUs at layers 2 and 1 and appropriately encoded by space-time coding techniques.

Following Zorzi's line of research [1,14], we study here the effective ARQ selective repeat policy that can be used at the RLC sublayer to retransmit RLC PDUs. It is applied in the acknowledged mode to a data flow traversing a logical channel. It complements the hybrid ARQ functionality with its advanced adaptive FEC techniques that is used at the physical and MAC layers for the associated PDUs (see Fig. 2, cf. [3]). We investigate the latter ARQ window flow control protocol at the sender side by analytical means based on an open queueing network. More precisely, we model the associated transformations from the logical RLC channels to the transport and physical channels and the associated transport process of RLC PDUs by a queueing network with three stations. This derived LTE transmission model describing the flow controlled transport of RLC frames consists of two building blocks, namely, an inner block comprising a multi-server queueing network with two substations and an outer substation with K

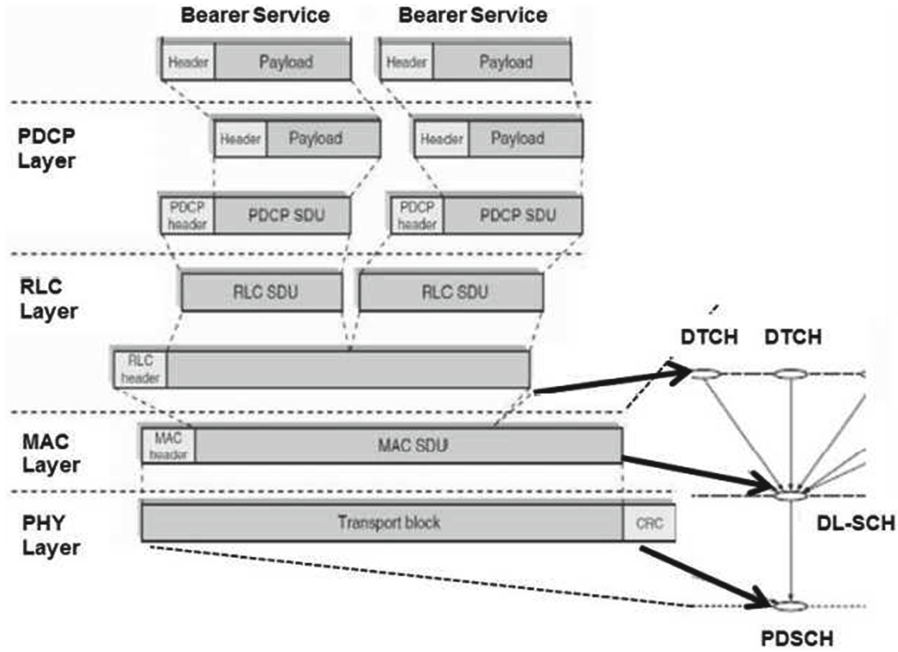

Fig. 1. Data processing in a protocol stack of an LTE eNodeB.

places and a single server with exponentially distributed service times. This is, of course, a first rough abstraction of many complicated processing steps applied to RLC PDUs by the LTE eNodeB stack that cannot be captured in full detail by any queueing model. We assume that the flow control window of the selective repeat scheme which is applied to aggregated streams of RLC frames comprises a finite number M of PDUs. They are served by J_1 parallel servers with exponentially distributed service rates μ_1 that describe a set of corresponding downlink shared transport channels (DL-SCHs) at the MAC layer. Completed frames are forwarded to the second substation consisting of a queue served by J_2 service stations. The latter describe the mapping of the RLC frames embedded into MAC frames from transport channels onto the physical downlink shared OFDM channels assigned by the radio resource control (RRC) protocol (cf. [3]). The employed hybrid ARQ scheme induces a variable FEC part on each MAC PDU which is encoded by symbol sequences corresponding to the associated transport blocks on the multiple-input multiple-output (MIMO) channels of the LTE air interface based on spatial multiplexing techniques. Thus, we model these complex structures by random service times with state-dependent rates $\mu_{2,s}$. The physical channel capacity is described in this simplistic abstraction by a modulating Markovian channel environment J_C with $S \in \mathbb{N}$ states and an irreducible generator matrix $Q_C \in \mathbb{R}^{S\times S}$, e.g. by means of a Gilbert-Elliott error model or its various extensions, in accordance with previous studies (cf. [1,11,14,16,19]).

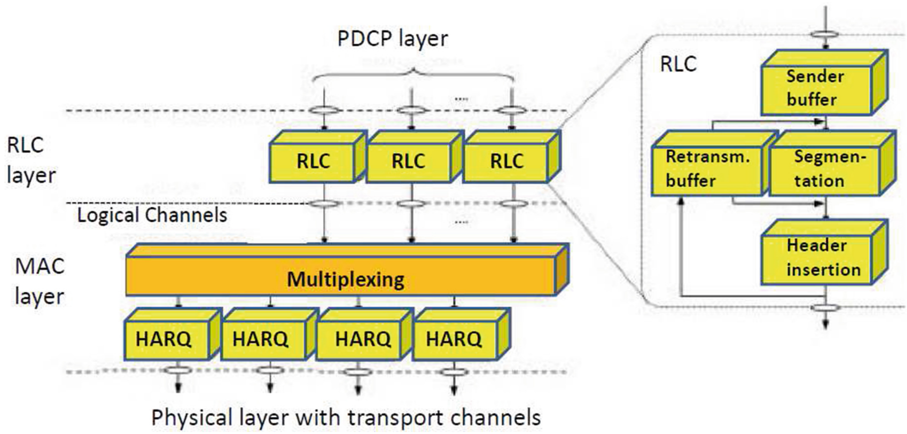

Fig. 2. RLC frame processing along downlink channels of an LTE eNodeB.

2.2 Derivation of a Queueing Model for ARQ-SR Error Control

We consider a stream of certain application packets arriving at a considered LTE eNodeB. They are entering its protocol stack at layer 2 after some processing steps as discussed previously. As these RLC PDUs of a tagged stream may be multiplexed into a single MAC PDU during the mapping of a logical downlink traffic channel onto a transport channel in the LTE transport system, we may model the transfer of the corresponding transport blocks of this tagged and other RLC streams included in the MAC PDU on the associated physical channels by a service period of related mean length $1/\bar{\mu}_2$. We assume that the service distribution of the PDU transmission follows an exponential distribution and that the service times of all frames are independent.

As the radio channel is subject to signal impairments, a transmission may fail with certain probabilities q_i that depend on the state $i \in \{1, \ldots, S\}$ of the channel environment J_C (cf. [8,11]). We further adopt this dependency on the channel states i for the service periods of a PDU. Thus, we assume that the exponential service rates are also state-dependent objects and described in state i by the service rate $\mu_{2,i} > 0$. We suppose that a failed transport of a tagged PDU occurs with probability $q_i > 0$. It models the aggregated negative acknowledgments of all those erroneous transport blocks associated with the transferred PDUs that are received by the multiple stop-and-wait mechanisms of the hybrid ARQ scheme in the MAC layer. A successful service completion occurs with probability $1 - q_i$. It means that the rate of a successful transport service in state i is given by $\mu_{2,i}(1 - q_i) > 0$. Then the frame forever leaves the inner subsystem of the QNW describing the successful frame transmission along the LTE air interface. Hence, a PDU slot becomes available in the flow control window. Furthermore, the overall mean service rate is given by $\bar{\mu}_2 = \sum_{i=1}^{S} \eta_i \cdot \mu_{2,i}$ where η denotes the unique steady-state distribution of J_C, i.e. $\eta^t \cdot Q_C = 0, \eta^t \cdot e = 1$. The incoming RLC frames of a specific stream of packets and their correlated interarrival times are modelled by an ordinary Markovian arrival process (MAP) controlled by a finite Markovian environment J_A with $E \in \mathbb{N}$ states $d \in \{1, \ldots, E\}$ and a generator matrix $Q_A \in \mathbb{R}^{E \times E}$ (cf. [2,7]). It is independent of the modulating environment J_C of the channel. We can assume that this arrival process is the result of the superposition of independent MAP arrival streams of those processes that describe the integration of packets into a single MAC PDU. Further, we assume that only single arrivals occur with the nonnegative rate matrix $0 \leq D_1 \in \mathbb{R}^{E \times E}$. Here and in the following we apply the ordering relations $0 \leq v, 0 < v$ to vectors or matrices v with an element-by-element meaning. It is paraphrasing the nonnegativity or positivity of all individual elements. Internal phase shifts on E without any arrivals are described by the regular Metzler-Leontief matrix $D_0 \in \mathbb{R}^{E \times E}$, i.e. $-D_0$ is an invertible M-matrix with positive diagonal elements $-D_0 \cdot e = D_1 \cdot e > 0$ where $e \in \mathbb{R}^E$ is the vector of all ones. Then the generator of the random environment J_A is determined by $Q_A = D_0 + D_1 \in \mathbb{R}^{E \times E}$. We assume that Q_A is irreducible. The mean number of arrivals is determined by $\lambda = \xi \cdot D_1 \cdot e$ where $0 < \xi \in \mathbb{R}^E$ denotes the steady-state vector of the modulating environment J_A, i.e. $\xi^t \cdot Q_A = 0, \xi^t \cdot e = 1$ (cf. [7]).

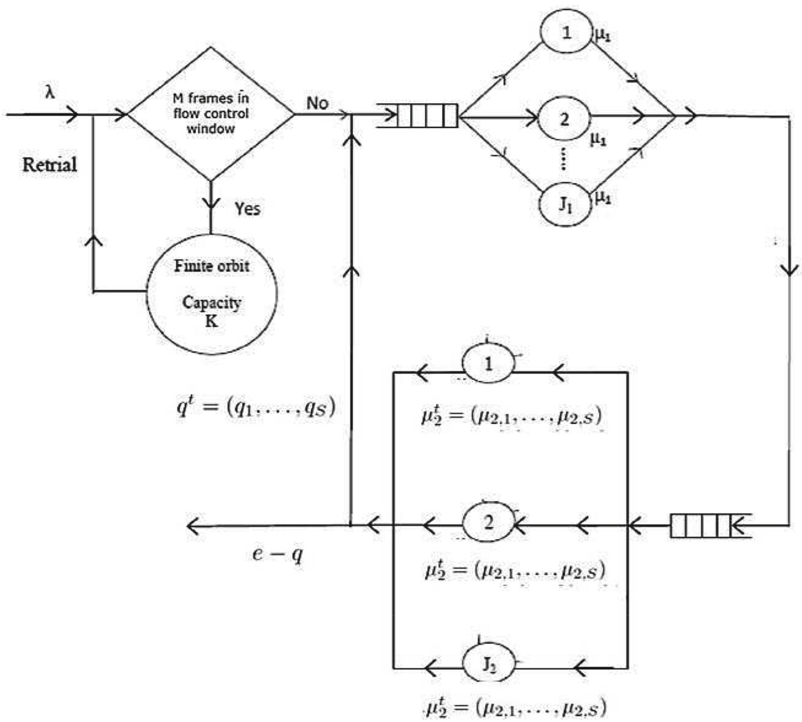

Fig. 3. Queueing network model of the error control for RLC PDUs that is based on an ARQ-SR controlled frame transport along a logical downlink traffic channel at an LTE eNodeB.

If an arriving RLC frame comes in and there is an idle slot in the flow control window of size M it can enter the queueing network of the RLC transport system. If all slots are occupied it is entering an orbit queue and waits for an empty slot in the ARQ flow control window. The status is periodically inspected. This process is modelled by an orbit with $K \in \mathbb{N}$ places including a single server with exponentially distributed independent service times with common mean $1/\nu$. The service time is modelled as a multiple of the basic transmission slot, i.e. the length of the transmission time interval (TTI) of the LTE transport system corresponding to one millisecond, and approximates the feedback delay of the hybrid ARQ status reports along the transport channels (cf. [3,15]).

The capacity of the overall queueing network with its two inner stations and the outer orbit covers $\tau = M + K \in \mathbb{N}$ RLC PDUs. If all slots are occupied incoming frames are lost in our model. The overall queueing system of the RLC error control is depicted in Fig. 3.

3 A Markovian Model of the ARQ-SR Error Control in an LTE Transmission System

3.1 Structure of the Markov Chain

Following the investigation of a related multi-server queueing network with retrials by Kumar et al. [9], we can describe the state $X(t)$ of the derived queueing model of ARQ-SR error control in LTE at time $t \geq 0$ by a vector process

$$X(t) = (O(t), R(t), P(t), J(t)) \in \Sigma,\ t \geq 0,$$

on the finite state space $\Sigma \subset \mathbb{N}_0^5$. Here $O(t) = n \in \{0, 1, \ldots, K\}$ denotes the RLC frames waiting in the orbit station for admittance to the flow control window $W(t) = R(t) + P(t)$ of size $M > 0$. Without loss of generality we assume $M \geq J_1 + J_2 \geq 3$. $R(t) = l \in \{0, 1, \ldots, M\}$ represents the number of frames in the first substation of the inner model with J_1 servers that describes the PDU processing on the transport channels of the LTE transmission system. $P(t) = k = m - l \in \{0, 1, \ldots, M\}$ denotes the number of frames in the second substation of the LTE transmission system that describes the RLC PDU transport along the physical LTE channels. We impose here the condition $0 \leq R(t) = l \leq m \leq M$. Then $W(t) = R(t) + P(t) = m \in \{0, 1, \ldots, M\}$ frames are in the queueing network modeling the RLC PDU transport along the air interface at each time t. It is limited by the maximal number of frames $M \in \mathbb{N}$ in the flow control window, i.e. $W(t) = m$ represents the state of the flow control window at time $t > 0$.

In the following let $I_S, I_E, I_{SE}, I_{M+1-l}$ denote identity matrices of sizes S, E, SE, and $M + 1 - l$, respectively.

$J(t) = (J_C(t), J_A(t))$ denotes the overall state of the modulating environment comprising the state $J_C(t)$ of the channel impairment model and the modulator $J_A(t)$ of the arrival process. It is governed by the common generator matrix $Q_J = Q_C \oplus Q_A = Q_C \otimes I_E + I_S \otimes Q_A$ where $\oplus$ denotes the Kronecker sum defined in terms of the Kronecker product $\otimes$ of the individual generator matrices of J_C and J_A, respectively.

The five-dimensional vector process $X(t) = (O(t), R(t), P(t), J(t)) = x \in \Sigma, t \geq 0$, is a continuous-time Markov chain on a finite state space $\Sigma \subset \{0, \ldots, K\} \times \{0, \ldots, M\}^2 \times \{1, \ldots, S\} \times \{1, \ldots, E\}$ since all components are determined by a state-dependent set of independent memoryless distributions of sojourn times in the states $x \in \Sigma$.

As the restriction $0 \leq W(t) = R(t) + P(t) = m \leq M$ holds, we realize that for given $R(t) = l \in \{0, 1, \ldots, M\}$ we get state-dependent subsets $S_l = \{0, 1, \ldots, n_l - 1\}$ of size $n_l = M - l + 1 \in [1, M + 1] \subset \mathbb{N}$ as realizations of $P(t)$. These varying dimensions govern the structure of the underlying generator matrix Q of the CTMC $X(t)$. Then we realize that $W(t) = (R(t), P(t)) = (l, m - l)$ varies in a set of $w = \sum_{l=0}^{M} n_l = \frac{(M+1)(M+2)}{2}$ states and, hence, the state space Σ has the finite size $\kappa = \frac{1}{2}(M + 1)(M + 2)(K + 1)SE$.

The associated generator matrix $Q \in \mathbb{R}^{\kappa \times \kappa}$ of the CTMC $X(t)$ has a block tridiagonal structure:

$$Q = \begin{pmatrix} Q_{0,0} & Q_{0,1} & \mathcal{O} & \mathcal{O} & \ldots & \ldots & \mathcal{O} \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & \mathcal{O} & \ddots & \ddots & \vdots \\ \mathcal{O} & Q_{2,1} & Q_{2,2} & Q_{2,3} & \ddots & \ddots & \vdots \\ \mathcal{O} & \mathcal{O} & Q_{3,2} & Q_{3,3} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathcal{O} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & Q_{K-1,K} \\ \mathcal{O} & \ldots & \ldots & \ldots & \mathcal{O} & Q_{K,K-1} & Q_{K,K} \end{pmatrix} \tag{1}$$

All blocks $Q_{i,j}, 0 \leq i, j \leq K$, have the same size $\kappa' = \frac{1}{2}(M + 1)(M + 2)SE$ and state-dependent internal structures. Here and subsequently, $\mathcal{O}$ denotes block matrices with all zeros whose dimension is determined by those block matrices of the surrounding context. $X(t)$ represents a quasi-birth-and-death (QBD) process where $O(t) = n \in \{0, \ldots, K\}$, is the level. $(R(t), P(t), J(t)) \in \Sigma_n$ is the phase process where Σ_n stems from the projection of Σ for fixed n.

The diagonal block matrices $Q_{i,i}, 0 \leq i \leq K$, represent again block tridiagonal matrices of size κ'. We first consider

$$Q_{0,0} = \begin{pmatrix} {C^{(0)}}_{0,0} & {A^{(0)}}_{0,1} & \mathcal{O} & \mathcal{O} & \ldots & \mathcal{O} \\ {B^{(0)}}_{1,0} & {C^{(0)}}_{1,1} & {A^{(0)}}_{1,2} & \mathcal{O} & \ddots & \vdots \\ \mathcal{O} & {B^{(0)}}_{2,1} & {C^{(0)}}_{2,2} & {A^{(0)}}_{2,3} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \mathcal{O} \\ \vdots & \ddots & \ddots & \ddots & \ddots & {A^{(0)}}_{M-1,M} \\ \mathcal{O} & \ldots & \ldots & \mathcal{O} & {B^{(0)}}_{M,M-1} & {C^{(0)}}_{M,M} \end{pmatrix} \tag{2}$$

with $n = 0$ elements in the orbit. Then $R(t) = l \in \{0, \ldots, M\}$ varies the number l of frames in the first substation of the LTE transmission system.

The upper diagonal blocks $A^{(0)}{}_{l,l+1}, 0 \leq l \leq M-1$, are $n_l SE \times n_{l+1} SE = (M-l+1)SE \times (M-l)SE$ matrices and correspond to RLC PDU arrivals according the internal MAP structure J_A with rate matrix $0 < D_1 \in \mathbb{R}^{E \times E}$, i.e.

$$A^{(0)}{}_{l,l+1} = \begin{pmatrix} M^{(0)}{}_{0,0} & \mathcal{O} & \mathcal{O} & \dots & \mathcal{O} \\ M^{(0)}{}_{1,0} & M^{(0)}{}_{1,1} & \mathcal{O} & \ddots & \vdots \\ \mathcal{O} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \mathcal{O} \\ \mathcal{O} & \dots & \mathcal{O} & M^{(0)}{}_{M-l-1,M-l-2} & M^{(0)}{}_{M-l-1,M-l-1} \\ \mathcal{O} & \dots & \mathcal{O} & \mathcal{O} & M^{(0)}{}_{M-l,M-l-1} \end{pmatrix} \quad (3)$$

where we get for $k \in \{0, \dots, M-l-1\}$:

$$M^{(0)}{}_{k,k} = I_S \otimes D_1 \in \mathbb{R}^{SE \times SE} \quad (4)$$

The lower diagonal blocks are given for $s_k = \min(k, J_2)$, $k \in \{0, \dots, M-l\}$, by matrices

$$M^{(0)}{}_{k,k-1} = s_k \cdot Z_S(q, \mu_2) \otimes I_E \in \mathbb{R}^{SE \times SE} \quad (5)$$

with the diagonal matrix

$$Z_S(q, \mu_2) = \begin{pmatrix} q_1\mu_{2,1} & 0 & \dots & 0 \\ 0 & q_2\mu_{2,2} & \ddots & \vdots \\ \vdots & \ddots & q_{S-1}\mu_{2,S-1} & 0 \\ 0 & \dots & 0 & q_S\mu_{2,S} \end{pmatrix} \in \mathbb{R}^{S \times S}. \quad (6)$$

It represents the departure rates $\mu_2^t = (\mu_{2,1}, \dots, \mu_{2,S})$ of the second substation of the LTE transmission system that are modulated by the state s of the channel environment J_C and its state-dependent feedback probabilities $q^t = (q_1, \dots, q_S)$ of an unsuccessful frame transmission.

The lower diagonal blocks $B^{(0)}{}_{l,l-1}$, $l \in \{1, \dots, M\}$, describe the departure events $R(t) = l$ to $R(t) = l-1$. They correspond to the occurrence of a service completion of the first station among the $r_l = \min(l, J_1)$ active servers with rates $r_l\mu_1$. Then these rates are taken into account by the upper diagonal blocks of this block matrix

$$\begin{aligned} B^{(0)}{}_{l,l-1} &= \begin{pmatrix} \mathcal{O} & r_l\mu_1 \otimes J_{0,1} & \mathcal{O} & \dots & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & r_l\mu_1 \otimes J_{1,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \dots & \mathcal{O} & r_l\mu_1 \otimes J_{M-l,M-l+1} \end{pmatrix} \\ &= \left(\mathcal{O}, r_l\mu_1 I_{M+1-l} \otimes I_S \otimes I_E \right) \in \mathbb{R}^{n_l SE \times n_{l-1} SE} \end{aligned} \quad (7)$$

where $J_{k,k+1} = I_{SE} = I_S \otimes I_E, 0 \leq k \leq M-l$.

The diagonal block matrices ${C^{(0)}}_{l,l} \in \mathbb{R}^{n_l SE \times n_l SE}, 0 \le l \le M$, describe the changes within the state variable $P(t) = k \in \{0, \dots, M-l\}$ given $R(t) = l$ and reflect a tridiagonal structure of the local QBD behavior

$$
{C^{(0)}}_{l,l} = \begin{pmatrix}
{G^{(0,l)}}_{0,0} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} \\
{F^{(l)}}_{1,0} & {G^{(0,l)}}_{1,1} & \mathcal{O} & \ddots & \vdots \\
\mathcal{O} & {F^{(l)}}_{2,1} & {G^{(0,l)}}_{2,2} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \mathcal{O} \\
\mathcal{O} & \cdots & \mathcal{O} & {F^{(l)}}_{M-l,M-l-1} & {G^{(0,l)}}_{M-l,M-l}
\end{pmatrix}. \tag{8}
$$

The lower diagonal blocks ${F^{(l)}}_{k,k-1}, k = 1, \dots, M-l$, correspond to departures $P(t) = k$ to $P(t) = k-1$ from the second LTE station after a successful frame transmission with probability $1 - q_s$ which is modulated by the state s of the channel environment J_C. Thus, it is given by

$$
\begin{aligned}
{F^{(l)}}_{k,k-1} &= \begin{pmatrix}
(1-q_1)s_k\mu_{2,1} & 0 & \cdots & 0 \\
0 & (1-q_2)s_k\mu_{2,2} & \ddots & \vdots \\
\vdots & \ddots & (1-q_{S-1})s_k\mu_{2,S-1} & 0 \\
0 & \cdots & 0 & (1-q_S)s_k\mu_{2,S}
\end{pmatrix} \\
&\quad \otimes I_E \\
&= s_k \cdot Z(e-q, \mu_2) \otimes I_E \in \mathbb{R}^{SE \times SE}
\end{aligned} \tag{9}
$$

with $s_k = \min(k, J_2)$ active servers in the second substation. Here $e \in \mathbb{R}^S$ is the vector of all ones.

The diagonal block matrices ${G^{(0,l)}}_{k,k}, 0 \le k \le M-l-1 = n_l - 2$, of an idle orbit with $n = 0$ comprise the rates $S_0 = Q_C \in \mathbb{R}^{S \times S}$ of the channel environment $J_C(t)$, the rates $D_0 \in \mathbb{R}^{E \times E}$ of the modulating MAP arrival process $J_A(t)$ among those states of J_A without incoming frames, and a compensating diagonal vector $\Delta_{n,l,k}$. The latter is such that $Qe = 0$ is satisfied where we disregard the diagonal elements $-D_1 e = D_0 e$ of D_0 and S_0. Then we may represent ${G^{(0,l)}}_{k,k}, 0 \le k \le M-l-1$, in the form

$$
{G^{(0,l)}}_{k,k} = S_0 \oplus D_0 + \mathrm{diag}(\Delta_{0,l,k}). \tag{10}
$$

There is no arrival for $P(t) = k = M-l$ since an incoming frame is blocked and sent to the orbit if $O(t) = n < K$ holds. Incrementing $O(t)$ it is taken into accounted in terms of $Q_{n,n+1}$.

Apart of the variability of the diagonal compensation $(\Delta_{n,l,k})$ all matrices $Q_{n,n}$, $0 \le n < K$, have the same structure. Only if the orbit is full in the last state $O(t) = K$ and the flow control window is also fully occupied, i.e. $W(t) = R(t) + P(t) = M$, in the blocking states $B = \{\chi = (l, M-l) \mid 0 \le l \le M\}$, an incoming frame is blocked from the system and lost. This event is reflected by the modified structure

$$
{G^{(K,l)}}_{M-l,M-l} = S_0 \oplus (D_0 + D1) + \mathrm{diag}(\Delta_{K,l,M-l}) \tag{11}
$$

of the diagonal blocks in those states χ with a corresponding compensating diagonal vector $(\Delta_{K,l,M-l})$.

The upper diagonal block matrices $Q_{i,i+1}, 0 \leq i \leq K-1$, of Q cover the arrivals of the MAP process to the orbit. It is accounted for by the state variable $O(t) = n < K$ which is incremented to $O(t) = n+1$. It is starting with the idle orbit $n = 0$ and ends with the last empty slot $n = K-1$ of the orbit. It is driven by a full flow control window with the state $W(t) = R(t) + P(t) = M$ of the inner queueing system in the overall network. Only in this state the MAP arrival is redirected to the orbit and the residual states generate no increment of $O(t) = n < K$. These events are represented by the block matrix

$$Q_{n,n+1} = \begin{pmatrix} H_0 & \mathcal{O} & \dots & \mathcal{O} \\ \mathcal{O} & H_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathcal{O} \\ \mathcal{O} & \dots & \mathcal{O} & H_M \end{pmatrix}, \; 0 \leq n \leq K-1. \tag{12}$$

It is described by a set of block diagonal matrices H_l along the diagonal

$$H_l = \begin{pmatrix} 0 \otimes I_{M-l} \otimes I_S \otimes I_E & \mathcal{O} \\ \mathcal{O} & I_S \otimes D_1 \end{pmatrix} \in \mathbb{R}^{n_l SE \times n_l SE}, \; 0 \leq l \leq M-1 \tag{13}$$

$$H_M = \begin{pmatrix} I_S \otimes D_1 \end{pmatrix} \in \mathbb{R}^{SE \times SE}. \tag{14}$$

If $R(t) = l \in \{0, \dots, M\}$ holds then only in the last state $0 \leq P(t) = M-l \leq M$, i.e. the last state $W(t) = R(t) + P(t) = M$ of the flow control window, this redirection to the orbit occurs.

The lower diagonal block matrices $Q_{n,n-1}, 1 \leq n \leq K$, represent departures from the orbit in state $O(t) = n$ and arrivals to the first inner station of the LTE transmission system. They are accounted by state variable $R(t) = l < M$ which is incremented by one arrival to $R(t) = l+1 \leq M$ with rates ν of the exponentially distributed flow-control inspection interval. If there are $W(t) = R(t) + P(t) = l + k = m < M$ frames in the inner system in the case $O(t) = n \leq K$, such an arrival is possible, otherwise we get again a redirection to the orbit in state $W(t) = R(t) + P(t) = M$. Then the lower diagonal block matrix $Q_{n,n-1}, 1 \leq n \leq K$ is characterized by an upper block tridiagonal matrix

$$Q_{n,n-1} = \begin{pmatrix} \mathcal{O} & I_{0,1} & \mathcal{O} & \dots\dots & & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & I_{1,2} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \mathcal{O} \\ \vdots & \ddots & \ddots & \ddots & \mathcal{O} & I_{M-1,M} \\ \mathcal{O} & \dots & \dots & \dots & \mathcal{O} & 0 \otimes I_S \otimes I_E \end{pmatrix} \tag{15}$$

which is identical for all $n \in \{1, \ldots, K\}$. The matrices $I_{l,l+1}, 0 \leq l \leq M-1$, represent the arrival to the flow control window $W(t) = R(t) + P(t) = l + k$ with the exception of the final state $R(t) = l, P(t) = M - l$. They are given by

$$I_{l,l+1} = \begin{pmatrix} \nu \cdot I_{n_{l+1}} \\ \zeta^t \end{pmatrix} \otimes I_S \otimes I_E \in \mathbb{R}^{n_l SE \times n_{l+1} SE} \tag{16}$$

with identity matrices $I_{n_{l+1}}, I_S, I_E$ of size $n_{l+1} = M - l, S, E$, respectively, and a row vector $\zeta^t = (0, \ldots, 0)$ with $M - l$ zeros.

3.2 Computation of the Steady-State Distribution

The Markovian queueing network $\{X(t), t \geq 0\}$ modeling the ARQ-SR error control scheme in the LTE transmission system with a MAP arrival process has a QBD structure on a finite state space Σ. Imposing the condition on the irreducibility of the generator matrix $Q_J = Q_C \oplus Q_A$ of the modulating environment $J(t) = (J_C(t), J(_A(t))$, we may conclude that the CTMC $X(t)$ has an irreducible generator matrix Q with a block tridiagonal structure and a single, zero eigenvalue $\rho(Q) = 0$. Thus, the corresponding steady-state vector $\Pi \in \mathbb{R}^{\kappa \times \kappa}$ is the unique solution of the following balance equations with the zero vector $0 \in \mathbb{R}^{\kappa}$ as right hand side and the additional normalization condition:

$$\Pi^t \cdot Q = 0 \qquad e^t \cdot \Pi = 1 \tag{17}$$

The QBD structure of Q on the finite state space implies that there exists a superposition of two matrix-geometric terms that constitute the steady-state vector Π (cf. [7,9,13,17,18]). To describe its structure, we partition the steady-state vector according to the levels $O(t) = n$ of the CTMC $X(t)$ as

$$\Pi^t = (\pi_0^t, \pi_1^t, \ldots, \pi_K^t).$$

According to the phase process $(R(t), P(t), J(t)) = (R(t), P(t), J_C(t), J_A(t)) = (l, k, s, d)$ and a given fixed $n \in \{0, \ldots, K\}$ each component vector π_n on a level $O(t) = n$ is further decomposed into $\pi_n^t = (\pi_{(n,l,k,s,t)} \mid \forall (l, k, s, t) \in \Sigma_n)$.

Let us define the block matrices $A_2 = Q_{n,n-1}, n \in \{1, \ldots, K\}$, $A_1 = Q_{n,n}, n \in \{1, \ldots, K-1\}$, and $A_0 = Q_{n,n+1}, n \in \{1, \ldots, K-1\}$ which are independent of the level n. We define the matrix polynomial

$$\begin{aligned} \mathcal{A}(z) &= z^2 \cdot A_2 + z \cdot A_1 + A_0, \quad z \in \mathbb{C} \\ \mathcal{A}(1) &= A_2 + A_1 + A_0 \end{aligned} \tag{18}$$

and assume that $z \in \mathbb{C}$ exists such that $\det(\mathcal{A}(z)) \neq 0$.

Then we conclude that the steady-state vector Π can be represented by a sum of two scaled marix-geometric solutions in the following way (cf. [7]):

$$\pi_n^t = \begin{cases} \pi_0^t & , \quad n = 0 \\ \pi_1^t \cdot R^{n-1} + \pi_K^t \cdot \Phi^{K-n} & , \quad 1 \leq n \leq K \end{cases} \tag{19}$$

The basic matrices R, Φ are determined as minimal nonnegative solutions of the fundamental matrix-geometric equations:

$$\mathcal{O} = R^2 \cdot A_2 + R \cdot A_1 + A_0 \tag{20}$$

$$\mathcal{O} = A_2 + \Phi \cdot A_1 + \Phi^2 \cdot A_0 \tag{21}$$

Then a normalized variant of the steady-state vector $\Pi = (\pi_n)_n$ that satisfies

$$\pi_0^t \cdot e + \left(\pi_1^t \cdot \sum_{n=0}^{K-1} R^n + \pi_K^t \cdot \sum_{n=0}^{K-1} \Phi^n \right) \cdot e = 1 \tag{22}$$

fulfills the balance equations and normalization condition (17).

Hajek [4] has shown that there exists a related nonnegative matrix $G(R)$ satisfying the relation $0 = A_2 + A_1 \cdot G(R) + A_0 \cdot G^2(R)$ as its minimal nonnegative solution. The nonnegative solution matrices $R, G(R)$ are related in terms of the following relations which are used to compute the final solution Π (cf. [7]):

$$W(R) = A_1 + A_0 \cdot G(R) = A_1 + R \cdot A_2$$
$$R = A_0 \cdot [-W(R)]^{-1}, \quad G(R) = [-W(R)]^{-1} \cdot A_2$$

The missing initial terms $x^t = (\pi_0^t, \pi_1^t, \pi_K^t)$ of the matrix-geometric representation (19) of the steady-state vector Π can be computed as normalized solution $x \neq 0$, $x^t \cdot \widehat{A} = 0, x^t \cdot e = 1$, of the boundary system:

$$\widehat{A} = \begin{pmatrix} \widehat{A}_{0,0} & \widehat{A}_{0,1} & \mathcal{O} \\ \widehat{A}_{1,0} & \widehat{A}_{1,1} & \widehat{A}_{1,K} \\ \widehat{A}_{K,0} & \widehat{A}_{K,1} & \widehat{A}_{K,K} \end{pmatrix} \tag{23}$$

$$\widehat{A}_{0,0} = Q_{0,0}, \quad \widehat{A}_{1,0} = Q_{1,0}, \quad \widehat{A}_{K,0} = \Phi^{K-1} \cdot Q_{1,0}$$
$$\widehat{A}_{0,1} = Q_{0,1}, \quad \widehat{A}_{1,1} = W(R), \quad \widehat{A}_{K,1} = \Phi^{K} \cdot Q_{1,2}$$
$$\widehat{A}_{1,K} = R^{K-2} \cdot [Q_{1,2} + R \cdot Q_{K,K}], \quad \widehat{A}_{K,K} = \Phi \cdot Q_{1,2} + Q_{K,K}$$

4 Performance Metrics of the ARQ-SR Error Control Model

The basic performance parameters of the ARQ-SR error control system in steady state can be computed in terms of the matric-geometric solution vector $\Pi = (\pi_n)_n$. They comprise the following probabilistic metrics and basic mean values:

- the probability that there are $O = n \in \{0, \ldots, K\}$ frames waiting for transmission in the orbit queue: $p_n = \pi_n^t \cdot e$
- the probability that the orbit is idle: $p_0 = \pi_0^t \cdot e$
- the utilization probability that at least one RLC frame is waiting in the orbit: $p_U = 1 - p_0 = 1 - \pi_0^t \cdot e$

– the blocking probability of the overall system when there are $O = K$ frames in the orbit and $W = R + P = M$ in the flow control window:

$$p_{Loss} = \sum_{l=0}^{M} \sum_{s=1}^{S} \sum_{d=1}^{E} \pi_{(K,l,M-l,s,d)}$$

– the probability that there are $W = R + P = M$ in the flow control window and an arriving frame is either going to the orbit or rejected:

$$p_W = \sum_{n=0}^{K} \sum_{k=0}^{M} \sum_{s=1}^{S} \sum_{d=1}^{E} \pi_{(n,k,M-k,s,d)}$$

– the throughput of the overall flow control model based on the MAP input:

$$T = (1 - p_{Loss}) \cdot \lambda = \xi \cdot D_1 \cdot e \cdot (1 - p_{Loss})$$

– the mean number of frames waiting for flow control admittance in the orbit:

$$\mathbb{E}(O) = \sum_{n=0}^{K} n \cdot \pi_n^t \cdot e = \sum_{n=1}^{K} \left(n \cdot \pi_1^t \cdot R^{n-1} \cdot e + n \cdot \pi_K^t \cdot \Phi^{K-n} \cdot e \right)$$

– the mean number of frames in the LTE MAC processing queue:

$$\mathbb{E}(R) = \sum_{l=1}^{M} l \cdot \left(\sum_{n=0}^{K} \sum_{k=0}^{M-l} \sum_{s=1}^{S} \sum_{d=1}^{E} \pi_{(n,l,k,s,d)} \right)$$

– the mean number of frames in the LTE transmission queue:

$$\mathbb{E}(P) = \sum_{k=1}^{M} k \cdot \left(\sum_{n=0}^{K} \sum_{l=0}^{M-k} \sum_{s=1}^{S} \sum_{d=1}^{E} \pi_{(n,l,k,s,d)} \right)$$

– the mean number of frames in both the LTE MAC processing and transmission queues of the flow control window when there are $W = R + P = m \in \{0, \ldots, M\}$ frames processed:

$$\mathbb{E}(W_{FCW}) = \sum_{m=1}^{M} m \cdot \left(\sum_{l=0}^{m} \sum_{n=0}^{K} \sum_{s=1}^{S} \sum_{d=1}^{E} \pi_{(n,l,m-l,s,d)} \right)$$

– the mean sojourn time of a random frame in the flow control window until a successful transmission:

$$\mathbb{E}(S_{FCW}) = \frac{\mathbb{E}(W_{FCW})}{(1 - p_{Loss}) \cdot \lambda} = \frac{\mathbb{E}(W_{FCW})}{(1 - p_{Loss}) \cdot \xi \cdot D_1 \cdot e}$$

– the mean sojourn time of a random frame in the orbit:

$$\mathbb{E}(S_O) = \frac{\mathbb{E}(O)}{(1 - p_{Loss}) \cdot \lambda} = \frac{\mathbb{E}(O)}{(1 - p_{Loss}) \cdot \xi \cdot D_1 \cdot e}$$

5 Conclusion

The paper is devoted to the transmission of RLC PDUs in downlink direction at an LTE eNodeB. It focuses on the modeling of the ARQ error control functionality at layer 2 of the LTE protocol stack, namely the ARQ selective repeat policy complimented by a hybrid ARQ scheme (cf. [3,15]). We have modelled the flow control of radio link control frames on an LTE air interface in terms of a simple queueing network with three stations. Regarding the error control, we have described those RLC frames encapsulated by MAC PDUs that are waiting for the acceptance by the flow control window in terms of repeated objects. The latter are collected in an orbit in front of a queueing network with two multi-server stations that models the transport and physical channels of an LTE eNodeB. We have described the correlated arrivals of RLC frames by a general Markovian arrival process and the transfer of these PDUs by transport blocks along the OFDM channels by state-dependent service processes and varying channel capacities.

We have derived a basic finite Markovian queueing model as major outcome of our LTE modeling approach and stated the associated generator matrix with its block tridiagonal structure. It is shown that its steady-state distribution can be computed in terms of a level-independent QBD process. The latter is realized as a superposition of two matrix-geometric terms. Then we have determined the basic performance metrics of the ARQ error control model in terms of the latter steady-state distribution.

Our future investigations will concern further theoretical studies regarding the accuracy of the ARQ error control model of an LTE air interface and the derivation of optimal design parameters with respect to the flow control system. It will also be necessary to estimate the important parameters of our queueing model and to compare its plausibility based on channel simulations or emulations of the LTE air interface. (cf. [6]). Furthermore, extensive comparisons with existing hybrid ARQ models, e.g. by Zorzi et al. [1,14], will be required to assess the quality and benefits of the new modeling approach.

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