DOI: 10.20378/irb-91725

On Fiscal PONZI Games in a World Economy with Overlapping Generations

Michael Schmid[1]

I. Introduction

Problems surrounding the growth of public debt have been studied anew during the debt-ridden eighties. The survey by Buiter (1985) and the CEPS paper by Blanchard, Buiter, and Dornbusch (1985) offer a wide and balanced view of public deficits and debt (see also Spaventa (1987)). The present paper is confined to what these authors have dubbed 'old-fashioned classical crowding out' or 'full-employment capital accumulation'. The new twist given to the old debate in this article is gained from the assumption of overlapping generations engaged in an 'economic war of ages'[2] and an explicit consideration of national governments competing in a completely integrated international capital market by issuance of public bonds.[3] Secondly, by looking at real capital formation and real government debt simultaneously, I try to avoid a dichotomy frequently found in the literature. There the sustainability of fiscal rules is analysed in a book-keeping manner, i.e., neglecting the effect of fiscal

1 This research was supported by Deutsche Forschungsgemeinschaft. While preparing this paper, I had access to unpublished work by M. Carlberg (1987), T. Ihori (1986), and W. Kitterer (1988) which influenced my own thinking. Persson (1985) is also important for the main topic of this paper. Discussions with M. Carlberg, H. Großmann, J. Michaelis, and H. Schmid are gratefully acknowledged. The difficult diagrams were mastered by C. Schwarz. This paper has been presented at the 1988 EEA-Meeting in Bologna.

2 See TIME, February 22, 1988 for a journalistic general audience perspective and Bierwag-Grove-Khang (1969), Ihori (1978), (1986), Buiter (1979), Atkinson-Stiglitz (1980) for a scientific analysis of public debt in a closed economy modelled along Samuelson-Diamond lines.

3 Debt in open economies and in a world economy populated by overlapping generations is studied in Buiter (1981), Persson (1985), Schmid-Großmann (1986), Schmid (1987), Ihori (1987).

Studies in International Economics and Institutions
F. Gehrels, H. Herberg, H. Schneider, H.-J. Vosgerau (Eds.)
Real Adjustment Processes Under Floating Exchange Rates

rules on the behavior of the private economy. The simple method of coping with this important interrelatedness between public credit and private capital formation is a thorough application of dynamic system analysis, i.e., specifically phase-diagram techniques.

In Part II, the focus is on the closed economy. Section 1 explains private consumer behavior under lifetime taxation and private production and investment behavior which is, by assumption, not affected by fiscal instruments.[4] Government behavior is summarized by a dynamic government budget constraint. The national credit market elucidates government-induced crowding out of private capital formation. Section 2 develops the phase diagram for public (real) debt and private (real) capital formation. The exposition emphasizes bonds finance of public consumption neglecting public investment. More important, I address the possibility of a permanently maintained primary government deficit (p.m.p. deficit) showing that in a closed economy it cannot be sustained under realistic assumptions about private sector behavior. The implications of the non-existence of a steady state with a p.m.p. deficit regime, especially in case of $n-r < 0$, are further investigated in section 3. The law of motion of the system suggests a government debt crisis as the most likely result of a p.m.p. deficit. I discuss next two possibilities of acute crisis management: sudden switch to a p.m.p. surplus regime and partial or complete debt repudiation. Transitory primary deficits and debt removal by a temporary government surplus are also considered as two softer modes of handling a government debt crisis.

Part III attempts to seize upon the relation between government debt and a nation's foreign indebtedness. Relying on previous work *in Schmid (1987)*, section 4 introduces a two-country model of a world economy with different national government behavior. A basic distinction is made between government-induced private behavior which may cause external indebtedness and the direct placement of public debt at an integrated world capital market. Consequently, section 5 first takes up balanced-budget strategies, like tax-financed public consumption or pay-as-you-go social security. Finally, I look into the

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4 See Sinn (1984) on the importance of investment incentives for international capital flows.

possibility of the existence of government PONZI games in a world economy and the stabilization of a government debt crisis in an open economy. From what has been said so far it is obvious that the paper does not address monetization of public debt and public debt and deficits in an unemployed economy among other important problems.

II. The Closed Economy with Public Debt

1. The Basic Model

1.1 Consumer Behavior

Intertemporal household behavior is subjected to lump-sum taxation in a way well known from Atkinson-Stiglitz (1980). The representative consumer of generation t evaluates his lifetime consumption according to[5]

$$u(c_t^1, c_t^2) = \gamma \ln c_t^1 + \delta \ln c_t^2 \qquad \gamma + \delta = 1 \qquad (1)$$

He solves the following problem

$$\max(\gamma \ln c_t^1 + \delta \ln c_t^2)$$

$$c_t^1 = w_1 - \tau^1 - s_t \qquad (2)$$

$$c_t^2 = (1 + r_{t+1}) s_t - \tau^2 \qquad (3)$$

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[5] The Cobb-Douglas functional form has been used by many analysts in the field of intertemporal optimization. It yields simple closed-form decision rules and the loss in generality is not important to the main subject of this paper. However, special effects of this functional form will be mentioned later on. Note further that publicly provided consumption, g_t, which will be introduced later, is not evaluated by the private sector. I am not dealing with optimal government in this paper. See Appendix 1 for a list of symbols.

Alternatively, these budget constraints can be summarized by the lifetime budget constraint.

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} = w_t - \tau^1 - \frac{\tau^2}{1+r_{t+1}} \equiv \hat{w}_t \tag{4}$$

Given wage income, w_t, the lump-sum tax parameters, τ^1, τ^2, and the interest rate for savings of the working period, r_{t+1}, the agent chooses an optimal amount of savings to maximize lifetime utility. The solution of this problem gives optimal consumption demand and optimal savings.

$$c_t^1 = (1-\delta)\hat{w}_t \tag{5}$$

$$c_t^2 = (1+r_{t+1})\,\delta\hat{w}_t \tag{6}$$

$$s_t = \delta\,\hat{w}_t + \frac{\tau^2}{1+r_{t+1}} \quad \text{or} \quad s_t = s(w_t, \tau^1, \tau^2) \tag{7}$$

From (6) and (7) it is obvious that interest and principal of working-period savings cover tax payments and consumption during retirement[6].

$$c_t^2 + \tau^2 = (1 + r_{t+1})(\delta\,\hat{w}_t + \frac{\tau^2}{1+r_{t+1}}) = (1 + r_{t+1})s_t \tag{8}$$

The intertemporal maximization can be visualized in Fig. 1. Three results follow:

(i) Taxation of young consumers' labor income, τ^1, reduces their consumption and savings during the working period. Thus, as shown in A^1, the burden of present taxation is shifted somewhat to the future by lowering retirement consumption as well.

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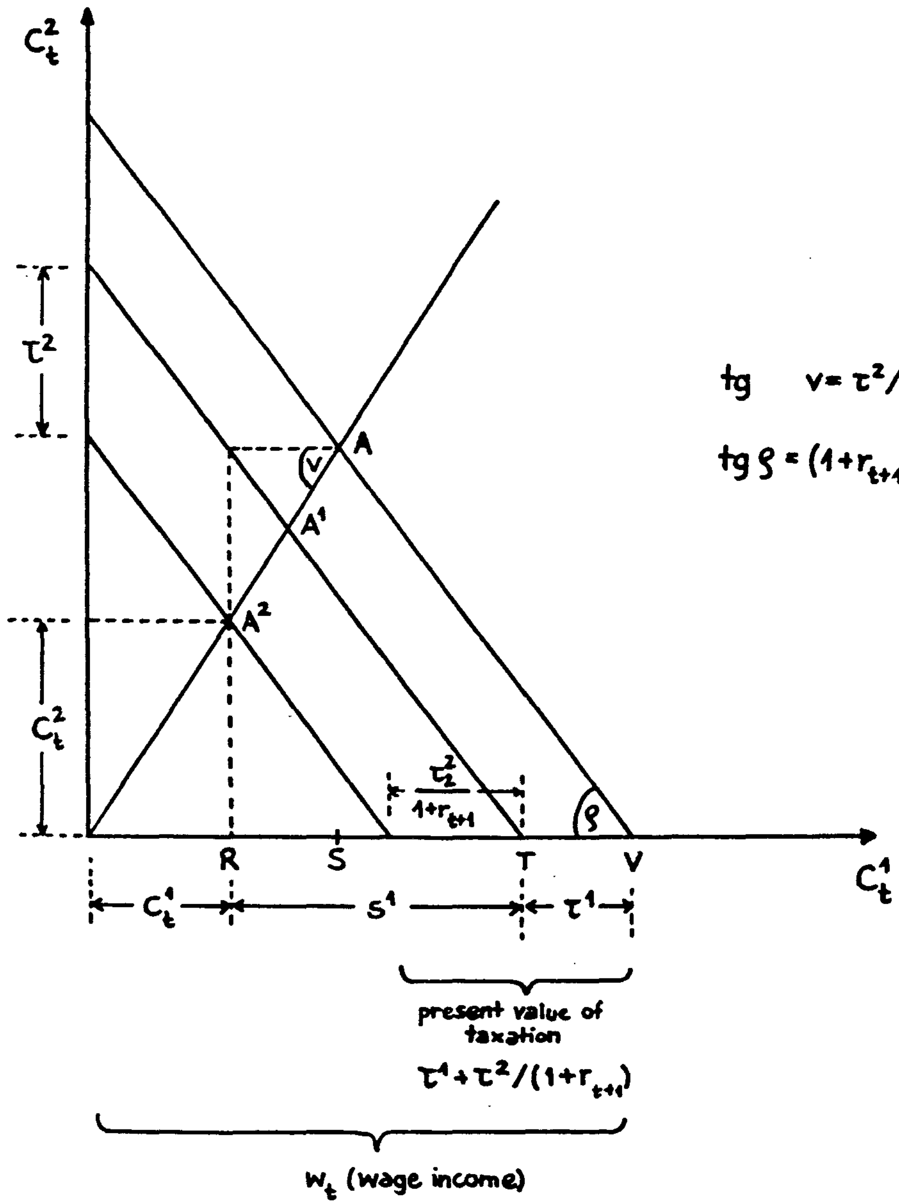
6 Assuming no gifts or bequests in this paper, hence the transfer of wealth between generations is contractual.

Fig. 1: Intertemporal Maximization with Lump-Sum Lifetime Taxation (Cobb-Douglas Case)



(ii) Adding future taxation, τ^2, i.e., taxation during the retirement period, generates an increase in savings. Thus the anticipated future tax burden is shifted to the present by lowering present consumption as well (see point A^2).

(iii) The two conflicting reactions on savings of the young can be combined to obtain a neutrality result: Given a specific amount of the present value of total lifetime taxation, it is always possible to define a tax-burden ratio τ^2/τ^1 such that savings of young people, s, are invariant under lifetime taxation. In Fig. 1, I indicate this burden ratio as tgv $= \tau^2/\tau^1$. Simple geometric reasoning shows that SV = RT.

The members of consecutive generations, L_t, are growing with a constant rate n.

$$L_t = (1+n)L_{t-1} \qquad \text{or} \qquad L_t = L_0(1+n)^t$$

Therefore, it is useful to introduce the wealth variable a_t denoting private wealth per worker, i.e., currently young people[7], at the beginning of period t. Without gifts or bequests, savings of the young members of generation t must equal wealth of the economy at the beginning of period t + 1, A_{t+1}.

$$L_t s_t = A_{t+1} \qquad \text{or} \qquad s_t = (1+n)a_{t+1} \tag{9}$$

Making use of the wealth variable, it is possible to express consumption of generation t−1 during period t, i.e., consumption of currently old people.[8]

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7 Obviously, the model distinguishes L_t, the working population of period t, from $L_t + L_{t-1}$, the consuming population of period t. We will refer to the working population when we use per-capita notation.

8 Apart from their size, different generations are populated by identical members.

$$\frac{L_{t-1}}{L_t} c_{t-1}^2 = \frac{1}{1+n} c_{t-1}^2 = (1+r_t)\, a_t - \frac{\tau^2}{1+n} \tag{10}$$

Obviously, in period t, old people live and pay taxes by spending principal and interest of their working-period savings.

In an economy with overlapping generations, old people dissave their wealth, a_t, while young people save. Consolidated savings of the household sector, $\tilde{s}$, equals aggregate national savings.

$$\tilde{s}_t = s_t - a_t = (1+n)\, a_{t+1} - a_t \tag{11}$$

Only in a growing economy, net savings are positive and can be used to finance investment, government deficits or a current account surplus.

In a closed economy, savings of young people represent the supply side of a credit market. Next I turn to the demand side which consists of private investment demand and government's demand for credit.

1.2 Production and Investment

Labor and a (non-depreciating) real capital stock - carried over from period t−1 - are combined within the production sector of the economy to produce output of period t. There is a well-behaved neoclassical production technology, thus output per capita, x_t, is a function of the capital-labor ratio, k_t:

$$x_t = f(k_t)$$

The allocation of factors is perfectly competitive, hence the usual condition for factor rewards holds

$$w_t = f(k_t) - k_t f_k(k_t) \quad \text{and} \quad r_t = f_k(k_t) \tag{12}$$

From these conditions, we obtain more information for further use.

$$\frac{dw_t}{dk_t} = -k_t f_{kk} > 0 \qquad \frac{dr_t}{dk_t} = f_{kk} < 0 \qquad \frac{dw_t}{dr_t} = -k_t < 0 \qquad (13)$$

The investment in period t is carried out by firms whose managers maximize the net value of the firm by choosing the optimal capital stock, k_{t+1}, given the current stock, k_t. They operate with a two-period planning horizon: during the (first) period t, the financing of k_{t+1} is obtained by offering securities to the young generation which will have to be redeemed during the second period when the members of this generation t will have grown old and will then be wishing to dissave. Assuming that some cash flow is paid, the firms' optimization problem is

$$\max_{k_{t+1}} \left(-k_{t+1} + \frac{x_{t+1} - w_{t+1}}{1+r_{t+1}} + \frac{k_{t+1}}{1+r_{t+1}}\right)$$

with respect to

$$x_{t+1} = f(k_{t+1})$$

From this optimization follows that the optimal capital stock, k_{t+1}, carried over to the next period, has been attained when the interest rate equals the marginal productivity of the future capital stock.

$$r_{t+1} = f_k(k_{t+1}) \qquad (14)$$

By definition, per-capita investment, $i_t = (K_{t+1} - K_t)/L_t$, can be written

$$i_t = (1 + n)\, k_{t+1} - k_t$$

Hence, investment is a declining function of the interest rate. Note that securities backing up the real capital stock are one store of value in this economy.

By redeeming securities from the old generation and reissuing them to the young generation at the same point in time, a transfer of ownership of the immortal physical capital stock between generations has been achieved.

1.3 Government

The other store of value consists of government bonds , B_t, which are assumed perfect substitutes of ownership titles of the private capital stock, i.e., they are one-period bonds paying the same rate of return. The government is considered an immortal agent. In period t, taxes are collected from young and old people, interest and principal, $(1+r_t)B_t$, are paid to the holders of government debt, i.e., to the members of the old generation, and a new emission of government bonds, B_{t+1}, is sold to members of the young generation who carry them over to period t+1. The government also finances public consumption, G_t. The role of the government is summarized by the following dynamic government budget constraint.

$$B_{t+1} = (1+r_t)B_t + G_t - L_t \tau_t^1 - L_{t-1} \tau_t^2 \tag{15}$$

Defining $g_t = G_t/L_t$ and $b_t = B_t/L_t$ permits (15) to be written in per-capita terms.

$$\underbrace{(1+n)\,b_{t+1} - b_t}_{\substack{\text{government deficit}\\ \text{(new emission of}\\ \text{government debt)}}} = \underbrace{r_t b_t}_{\substack{\text{interest}\\ \text{payment}}} + \underbrace{g - \tau^1 - \frac{1}{1+n}\,\tau^2}_{\substack{\text{primary government}\\ \text{deficit}}} \tag{16}$$

Note that a primary government deficit (surplus) implies a new emission of government debt larger (smaller) than debt service on government bonds. Note further that the primary deficit or surplus will be considered permanent; hence I shall omit the time subscript.

1.4 Momentary Equilibrium of the Capital Market

Equilibrium in the capital market can be derived by starting with the familiar statement of goods-market equilibrium.

$$f(k_t) = c_t^1 + \frac{1}{1+n} c_{t+1}^2 + g + i_t \; ; \quad i_t \equiv (1+n)k_{t+1} - k_t \qquad (17)$$

It is well known that the goods-market equilibrium can be rewritten as a savings-investment equilibrium. More precisely, total savings of consumers must finance private investment and the government deficit.[9]

$$[(1+n)a_{t+1} - a_t] = [(1+n)\,k_{t+1} - k_t] + [(1+n)\,b_{t+1} - b_t] \qquad (18)$$

In period t, the variables a_t, k_t, and b_t are given by history, and wealth of households equals the sum of real capital stock and government debt.

$$a_t = k_t + b_t \qquad (19)$$

Thus equilibrium in the capital market requires that the savings of young people must be sufficiently large to finance the debt issues of private firms and the government. Using (9) and (19), we rewrite (18)

$$s_t = (1+n)\,[k_{t+1} + b_{t+1}]$$

9 Equation (18) can be derived from (17) by using (2), (10), and (16).

Given k_t, τ^1, τ^2, we know s_t from (7) and (12). Given additionally b_t and g, we know b_{t+1} from (16). Note that from (14) k_{t+1} is a function of r_{t+1}. Thus a capital-market equilibrium is defined by an interest rate r_{t+1} such that demand and supply of loanable funds are equalized.

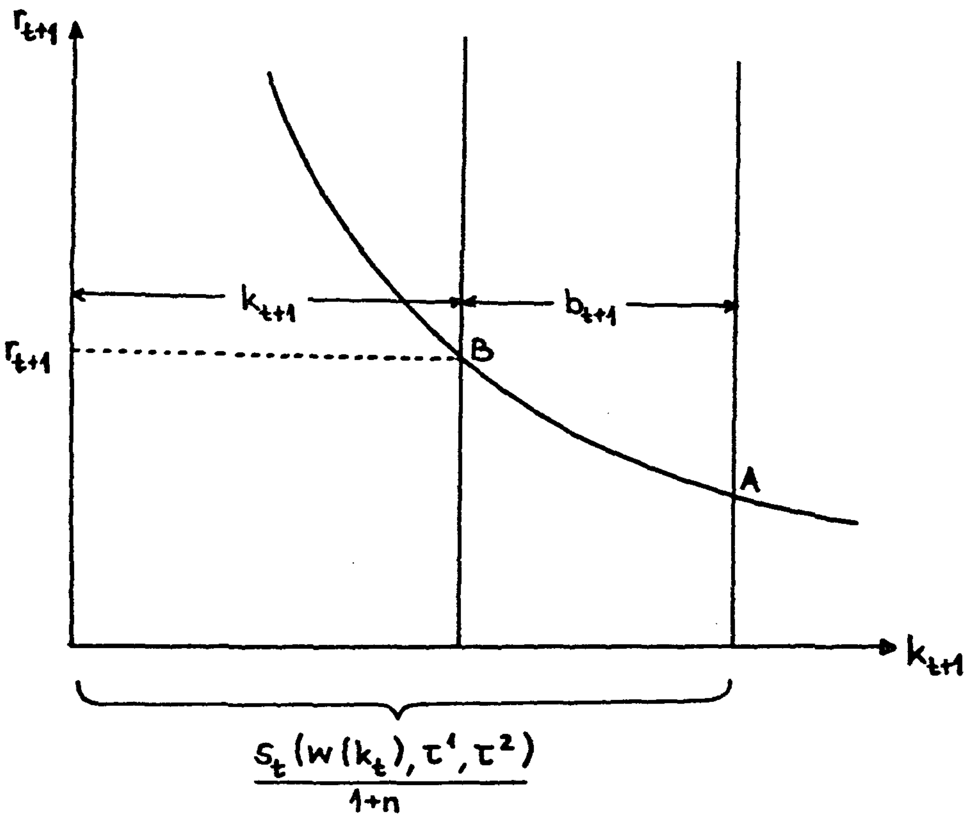
$$s_t(w(k_t), \tau^1, \tau^2) = (1+n)[k_{t+1} + b_{t+1}] \qquad (20)$$

The capital-market equilibrium is illustrated in Fig. 2. Two observations are important: (i) Only investment demand of firms is interest-elastic while the demand for credit of the government is completely inelastic. The supply of credit of young savers is inelastic by assumption.[10] (ii) The government demand for credit crowds out investment demand by driving up the interest rate from point A to point B. Thus, given the historic values of k_t and b_t, k_{t+1} will be the lower the bigger the primary deficit in (16).

The capital-market equlibrium (20) and the government budget constraint (16) provide two non-linear difference equations in k and b to study the adjustment process towards a steady-state solution. I shall analyse the existence of steady state in the next section and return to stability problems of steady states later.

10 This is due to the Cobb-Douglas form (1). But nothing of importance would be changed if savings were a function of the interest rate.

Fig. 2: Credit Market Equilibrium



2. Steady-State Equilibrium

2.1 Equations and the Geometry of the Model

A steady-state equilibrium is determined by the following set of equations[11].

$$w = f(k) - kf_k(k)$$

$$r = f_k(k)$$

$$(1+n)a = s(w, r, \tau^1, \tau^2) \tag{21}$$

$$(n-r)b = g - \tau^1 - \frac{1}{1+n}\tau^2$$

$$a = k + b$$

Given the fiscal parameters g, τ^1, τ^2, equations (21) can be reduced to two equations in the basic variables k, b.

$$\begin{aligned} &(1+n)(k+b) = s(w(k), r, \tau^1, \tau^2) \\ & \qquad\qquad\qquad\qquad\qquad\qquad \text{with } r = f_k(k) \\ &(n-r)b = g - \tau^1 - \frac{1}{1+n}\tau^2 \end{aligned} \tag{22}$$

Before studying the graph of the equation system (22), it is useful to depict the steady state of an overlapping-generations model in a modified Solow-growth diagram. Therefore, I present the steady-state expressions for the other variables of the model[12].

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11 Time subscripts are suppressed when I refer to steady states. It is important to realize that the partial derivative s_r reflects only the discount factor of future taxation in (7). Thus $s_r = 0$ if $\tau^2 = 0$.

12 Note that $s = (1+n)(k+b)$ is a definition of wealth. An equilibrium condition is obtained only if the behavioral function $s = s(\cdot)$ is used.

$$c^1 = w - \tau^1 - (1+n)(k+b) \qquad \frac{c^2 + \tau^2}{(1+n)} = (1+r)(k+b)$$

$$s = (1+n)(k+b) \qquad \tilde{s} = na = n(k+b) = \frac{n}{1+n} s \qquad (23)$$

$$s = s(w(k), \tau^1, \tau^2) \qquad i = nk$$

$$g - \tau^1 - \frac{1}{1+n}\tau^2 \gtrless 0 \quad \text{primary deficit (surplus)}$$

$nb > 0$ steady-state government deficit (new emission of government debt)

Setting $b = g = \tau^1 = \tau^2 = 0$, equation (23) can be illustrated in Fig. 3. This is the standard overlapping-generations model without government activity[13]. Fig. 3 shows the use of GNP and the functional distribution of national income as well. The most important curve is the savings line, s(w(k)). Due to our Cobb-Douglas specification, this line is obtained by applying the savings parameter $0 < \delta < 1$ to the wage line, w(k). Apparently by variation of δ, I can generate any steady-state equilibrium. Notably, I assume in Fig. 3 that with $\bar{\delta}$ a steady-state equilibrium can be established at $\bar{k}$, the golden-rule capital-labor ratio. From a theoretical point of view, δ is a parameter of the intertemporal utility function[14], hence we could expect steady states $k \gtrless \bar{k}$.

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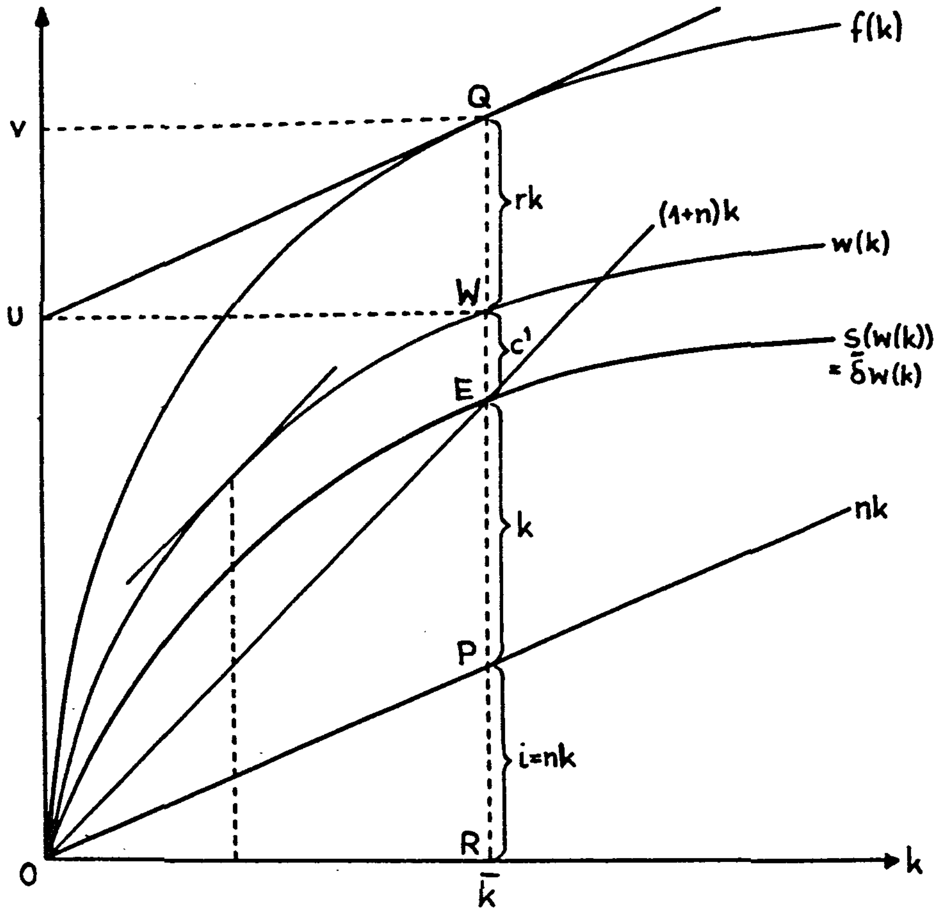
13 In fact, Fig. 3 employs Cobb-Douglas functional forms for utility and technology. The modified Solow diagram in Fig. 3 has been used by Kitterer (1988) to demonstrate geometrically other properties of the overlapping-generations model.

14 δ determines the rate of time preference, $(1-\delta)/\delta$.

Fig. 3: The Solow-Diagram with Overlapping Generations



It is possible, however, to demonstrate that a steady state $k < \bar{k}$ is more likely if in accordance with empirical observations I assume a national savings rate smaller than the capital-income share in GNP, i.e., $\sigma < \alpha$. This implies, a steady state $k < \bar{k}$ is more likely if $n-r < 0$[15].

In Fig. 4, the standard overlapping-generations model is modified by fiscal variables[16]. To simplify I disregard taxation of old people, thus $\tau^2 = 0$[17] in (23). To begin with I deal with the case of a balanced budget in Fig. 4. If $g - \tau^1 = 0$ and $b = 0$ for $n-r < 0$ savings of young people must be taken from disposable wage income, $w - \tau^1$. Hence the savings line in Fig. 3 shifts downwards. This is shown in Fig. 4 where the balanced-budget capital-market equilibrium is given by point E^{bb} at the capital-labor ratio $k^{bb} < \bar{k}$. If the government decides to raise loans, $b > 0$, it generates income from new bonds issue, $nb > 0$, on the one hand, but it has to make an interest payment, $rb > 0$, on the other hand.

If $n-r < 0$ the net cash flow is negative, and from the government-budget constraint it is obvious that either taxes, τ^1, must be increased or public consumption, g, must be reduced. In Fig. 4, the latter is assumed and therefore E' is the new equilibrium point[18]. Point E' would be shifted even farther to the left had we increased taxes. There is a simultaneity between b, k and a fiscal strategy given by the instrument variables g, τ^1.

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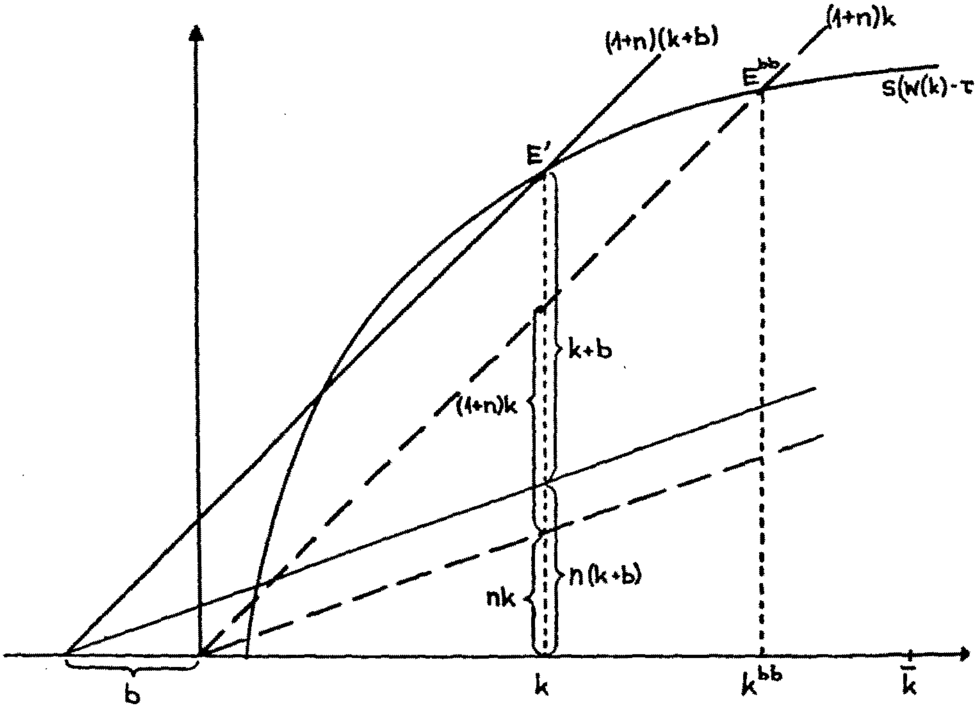
15 See Appendix 2 for a justification.

16 This diagram was developed in Phelps and Shell (1969) for a Solow growth model with government debt. The first application to overlapping generations seems to be in Kitterer (1988). Kitterer does not use the $s = s(w - \tau^1, \tau^2)$ function, however.

17 In section 5, I shall consider $\tau^2 > 0$ in a social-security context.

18 Note that the demand for credit line, $(1+n)k$, is shifted parallel in an upward direction to $(1+n)(k+b)$ if the government enters the capital market, i.e., $b > 0$. Hence private investment is reduced and a lower capital-labor ratio being the result of this debt issue.

Fig. 4: The Solow-Diagram with Public Debt



2.2 The Phase Diagram of Public Debt and Private Capital Formation

It is useful to discuss this simultaneity by looking at the phase diagram of real capital formation and government debt[19]. The phase diagram in Fig. 5 is constructed for a debtor government ($b > 0$). Assuming $\tau^2 = 0$, we repeat the equation system (22) for convenience[20].

cc line: $$s(w(k) - \tau^1) = (1+n)(k+b) \qquad (24.1)$$

bc line: $$(n-r)b = g - \tau^1 \qquad (24.2)$$

The cc line represents a capital-market equilibrium. The shape of cc can be explained by Fig. 4. Given the tax parameter τ^1, the savings curve $s(w(k)-\tau^1)$ has a specific position. The cc line is determined by the savings curve and the $(1+n)(k+b)$ line. For each $0 < b < b^{max}$ there exists a pair of lower k and upper k such that the capital market is in equilibrium[21]. The shape of the cc line can be discussed more rigorously by differentiation of (24.1).

$$(1+n)db = -[(1+n) + \delta k f_{kk}]dk - \delta d\tau^1$$

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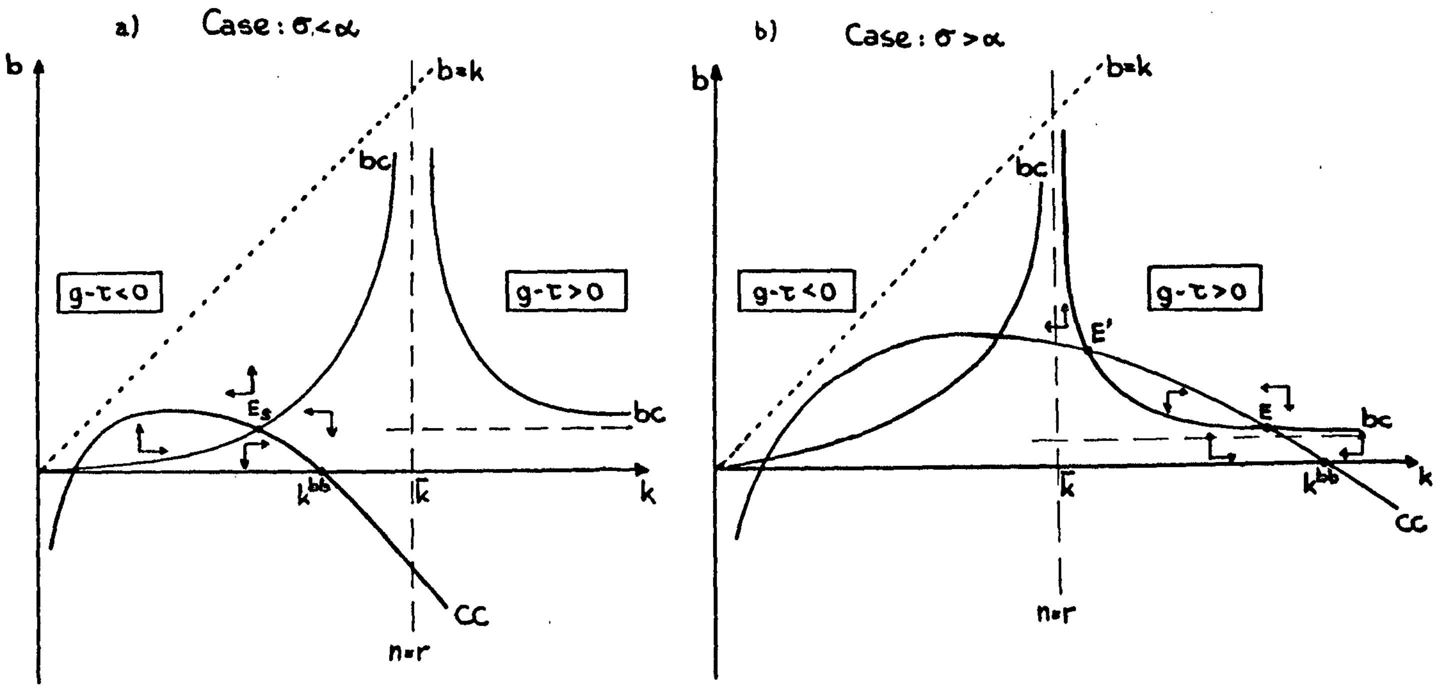
19 A phase-diagram analysis of growth and government debt has been presented by Phelps and Shell (1969) in a Solow growth context. Not using phase-diagram techniques, Carlberg (1988) studies debt and growth in a Solow growth context. Masson (1985) and Ihori (1986) provide the first attempts at phase-diagram analysis of growth and government debt with overlapping generations.

20 I attempt a phase-diagram analysis in k,b space, while Ihori (1986) and Masson (1985), apart from other differences, work in r,b space.

21 Note that with proportional wage-income taxation, the cc line starts at the origin. With lump-sum taxation, the cc line originates at a positive k. Note further that the cc curve shifts downwards if taxes τ^1 are increased.

Fig. 5: Phase Diagram
Debtor Government: b > 0



cc line: $s(w(k) - \tau^1) = (1+n)(k+b)$

bc line: $(n-r)b = g - \tau^1$

From Cobb-Douglas technology it is well known that $kf_{kk} = -ßf_k$. Therefore,

$$(1+n)db = -[(1+n) - \delta ß f_k]dk - \delta d\tau^1 \quad \text{and} \tag{25}$$

$$\frac{db}{dk} \gtrless 0 \quad \text{if and only if} \quad (1+n) - \delta ß f_k \lesseqgtr 0 \quad \text{or } n - \sigma r \lesseqgtr 0 \tag{26}$$

Furthermore, $\bar{k}$, the golden-rule k, is shown in Fig. 5a by the dashed vertical line, n = r. It is important to recall from Appendix 2 that a steady state with b = 0 can a priori imply $k \gtrless \bar{k}$. Hence the intersection point of the cc line with the k axis could be a priori to the left or to the right of $\bar{k}$ depending on the size of national savings, σ. Thus Fig. 5a (5b) depicts a relatively low (high) national savings rate $\sigma < \alpha$ ($\sigma > \alpha$). Above I have also argued that balanced-budget steady states must have lower values for k than steady states without fiscal parameters. Therefore, if I want to show a balance-budget steady state I will have to make the distinction between $k^{bb} < \bar{k}$ (see Fig. 5a) and $k^{bb} > \bar{k}$ (see Fig. 5b).

Next I discuss the government-budget constraint, i.e., the slope of the bc line. Even if I concentrate on government indebtedness only (b > 0) the bc line has two parts which can be associated with the alternate fiscal regimes of primary deficit or primary surplus. On the right-hand section, i.e., to the right of the r=n asymptote (n−r > 0), primary deficits, $g-\tau^1 > 0$, are captured, and the left-hand section (n−r < 0) covers a primary-surplus regime. Obviously, the two regimes are exclusive cases[22]. Closer analysis reveals that the left-hand section of the bc line must start in the origin while the right-hand section must have a horizontal asymptote. This asymptote's distance from the k axis is directly related to the size of the primary deficit, $(g-\tau^1)/n$, and increases (decreases) if and only if the primary deficit grows (shrinks). Differentiating (24.2) confirms these observations.

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22 The bc line must be properly interpreted: If the government decides on a specific primary-budget regime only one of the two sections of the bc line will appear. Only if the government switches regimes the other section will become operational. Recall that here I am concerned with a permanently maintained primary fiscal deficit or surplus.

$$(n-f_k)db = bf_{kk}dk + dg - d\tau^1 \tag{27}$$

Cobb-Douglas technology simplifies further.

$$(n-f_k)db = -(\beta f_k/k)\,dk + dg - d\tau^1 \tag{28}$$

$$\frac{db}{dk} \gtrless 0 \quad \text{if and only if} \quad n-r \lesseqgtr 0$$

Summarizing, public consumption affects only the bc line, while taxation of young people affects both the cc and the bc line. An increase in the primary deficit (surplus) shifts the bc line in an upward direction.

2.3 Steady-State Comparative Statics and Dynamics

Starting from a balanced-budget position in Fig. 5b, two simple experiments are possible within the regime of primary deficits.

(i) Holding the level of public consumption constant, I substitute tax finance for bonds finance by lowering taxes.

(ii) Keeping taxes constant, I raise public consumption financed by government bonds.

From my discussion of the phase diagram, the result of issuing bonds is an unambiguously decreasing capital-labor ratio in Fig. 5b. The decline of the capital-labor ratio is less pronounced in case (i) because the capital market is less tight due to the tax cut[23]. The most remarkable observation, however, is the disappearance of the steady state in Fig. 5b if the primary deficit exceeds a critical value $(g-\tau^1)^{max}$. From Fig. 5a, it can be seen immediately that in the

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23 In case (i), both the cc and bc line shift upwards, yet the cc line moves less strongly.

more likely case of low savings, $\sigma < \alpha$, a strategy of primary deficits is never feasible, because a steady state cannot exist.[24] Hence with the help of phase-diagram techniques, I confirm an important theorem in Carlberg (1983, 1988) for a Solow growth model.

Theorem: A permanently maintained primary deficit is never sustainable in the long run if an empirically relevant savings rate, $\sigma < \alpha$, is assumed. If the savings rate is relatively high, $\sigma > \alpha$, and the primary deficit remains below a relatively low critical value primary deficits are sustainable. This, however, can be considered a purely theoretical possibility only.[25]

Nevertheless, I can ask what would happen if the government embarked upon a strategy of primary deficits[26]? To answer this question it is necessary to analyse the dynamics of the phase diagram. The dynamics of debt and real-capital formation is governed by the system of two non-linear difference equations (16), (20) repeated here for convenience.

$$(1+n)b_{t+1} - b_t = r_t b_t + g_t - \tau^1$$

$$(1+n)\,[k_{t+1} + b_{t+1}] = \delta\,[w_t - \tau^1] \tag{29}$$

$$r_t = f_k(k_t) \qquad \text{and} \qquad w_t = f(k_t) - f_k(k_t)k_t$$

Appendix 3 explains how the directional arrows shown in Fig. 5 can be obtained from a qualitative stability analysis. A regime of primary government deficits kept sufficiently low in the case of $\sigma > \alpha$ generates two

24 It follows from (24.2) that for $(n-r) < 0$ and $g-\tau^1 > 0$ only $b < 0$ yields a steady state. This, however, violates the debtor assumption. Put geometrically, for $g-\tau^1 > 0$ there exists an intersection of cc and bc with negative b in the $n-r < 0$ domain which is not shown in Fig. 5a.

25 Michaelis (1989), chapt. 5, has shown the validity of the Carlberg theorem for overlapping-generations models. A similar view is reported in Ihori (1986), p. 25.

26 This question was raised by Carlberg (1983) and posed again in Carlberg (1988).

steady-state equilibria in Fig. 5b: E, a stable node, and E', an equilibrium with saddle-point stability. As discussed above, in the case of $\sigma < \alpha$ there exists no steady state. Within a regime of primary government surplus, the only stable equilibrium is the saddle point E_S. Recalling that in an empirically relevant case, $\sigma < \alpha$, a balanced budget steady state k^{bb} is located within the $n-r < 0$ region I observe in Fig. 5a that the most likely equilibrium is the saddle point E_S.

Theorem: In a growth equilibrium with government debt under the realistic assumption of an undercapitalized economy, $n-r < 0$, there exists a steady state if and only if the government runs a primary budget *surplus*. The steady state equilibrium must be a saddle point.

3. Government Debt Crisis and Crisis Management

This information is highly useful in an evaluation of the dynamic economic adjustment to a permanently maintained primary (p.m.p.) government deficit.

Theorem: Starting from a balanced-budget equilibrium, a p.m.p. budget deficit can be sustained and forces the economy on a stable adjustment path only in the unrealistic case of an overcapitalized economy, $\sigma > \alpha$, and only if the p.m.p. deficit is sufficiently small. In the realistic case of an undercapitalized economy, $\sigma < \alpha$, or in the case of $\sigma > \alpha$ with a too large p.m.p. deficit, there exists no steady state and the adjustment is unstable.

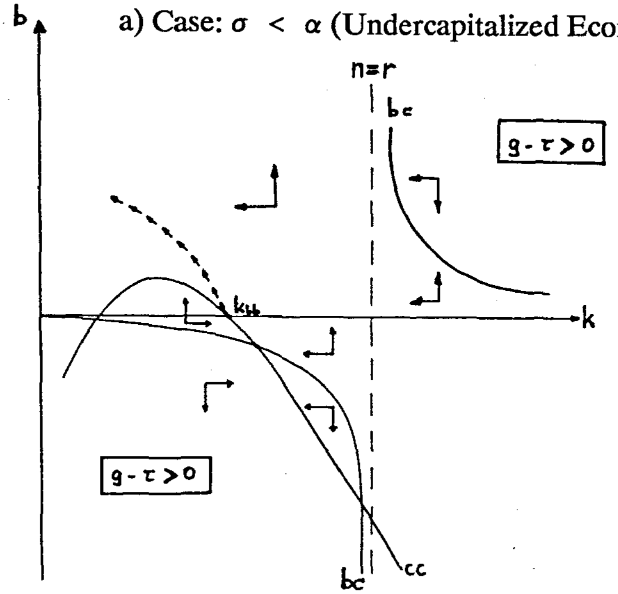
The two cases of unstable adjustment are shown in Fig. 6a and Fig. 6b where the law of motion for k and b can be seen from the directional arrows. I refer to this pattern of adjustment as a government debt crisis[27].

---

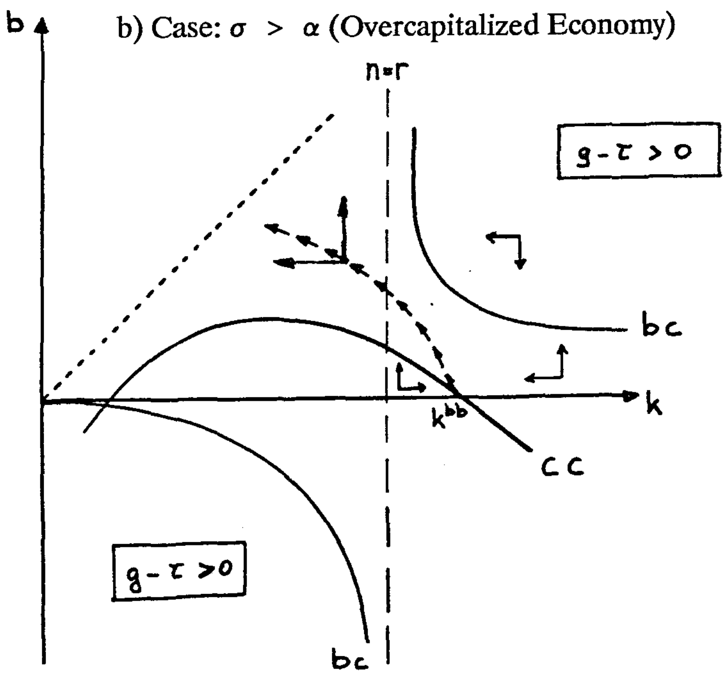
[27] By calculating a numerical example, Carlberg (1988) generates exactly this type of "dynamic disequilibrium".

Fig. 6: **Government Debt Crisis**
Primary Fiscal Deficit: $\sigma < \alpha$ and $n - r < 0$
Starting Point: Balanced Budget k_{bb}

a) Case: $\sigma < \alpha$ (Undercapitalized Economy)



b) Case: $\sigma > \alpha$ (Overcapitalized Economy)



The reader should realize that it is the general performance of an economy with $k_t \rightarrow 0$ and $b_t \rightarrow \infty$, particularly the slow strangulation of private capital formation, which is being critical here. The debt service is always maintained during adjustment. Hence no debt repudiation takes place although it becomes the more likely the more time passes. I do not explicitly analyse the distributional problems involved since debt service must effectively be paid by 'young' workers and accrues to 'grey' capitalists.

3.1 Stabilization of a Government Debt Crisis

Completely aside from the possibility of debt repudiation, the problem I want to pose now is very simple: How can this critical development be stabilized? Fig. 7a suggests a solution. Note from Fig. 5a that there exists a saddle-point stable equilibrium E_S within the regime of a p.m.p. budget *surplus*. Thus in Fig. 7a, I show the stable saddle path sp. The economy starting in k^{bb} because of a p.m.p. deficit of some assumed size (see Fig. 7b) must move towards the saddle path. If the p.m.p. deficit is switched to a p.m.p. surplus the moment the economy will reach the sp path, it can move along the saddle path towards the final equilibrium E_S. I have shown the time profiles of the p.m.p. budget and the achieved stabilization of the variables k_t and b_t after period t_s in Fig. 7b[28].

The reader feeling it might be difficult to switch fiscal regimes just at the right moment should realize that what matters really for re-entry is less the timing but the increasing rigor of the necessary austerity program the government must demand from the young. The longer the crisis lasts the more biting will be the tax burden and the more disruptive will be the distribution problem of nearly confiscatory taxation.

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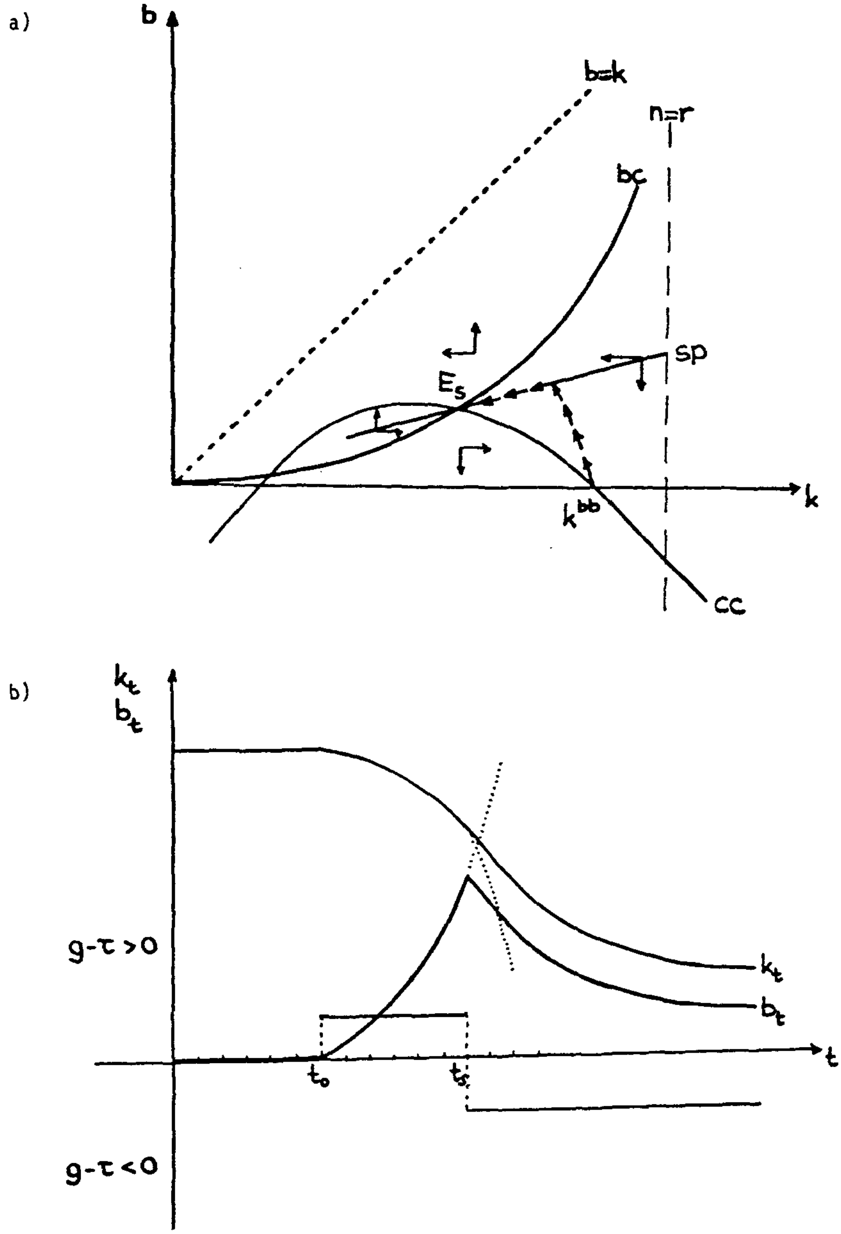
[28] For the sake of convenience I have smoothed the dynamic paths of k_t and b_t which are in discrete time.

Fig. 7: Stabilization of a Government Debt Crisis



Thus in Fig. 7a, there seems to exist a maximum bearable p.m.p. surplus. Switching from a p.m.p. deficit to a p.m.p. surplus smaller than this maximum surplus always stabilizes the public debt as long as the crisis path has not yet reached the saddle path belonging to the maximum bearable p.m.p. surplus. All these stabilization programs do not result in a positive steady-state debt, they rather yield crash landings for public debt, i.e., $b = 0$. It follows from this discussion: p.m.p. deficits are most likely not sustainable and must be stabilized sooner or later by a p.m.p. surplus of some size depending on the duration the crisis was allowed to persist. Considering the most realistic case of an undercapitalized economy, i.e., $n - r < 0$, I conclude that the only permanently sustainable fiscal regime is a primary government surplus with a public debt which does not impose an excessive tax load.

3.2 Debt Demolition by Debt Repudiation

If, however, the accumulation of public debt has lasted too long the tax burden to service the debt could become unbearable. In terms of Fig. 7 this might happen if there exists no intersection point of bc and cc in the $n - r < 0$ region because a too high tax load has shifted the bc line (cc line) too far in an upward (downward) direction. Or even before this will occur, an existing steady state might imply an unbearable tax load at an intolerably low capital intensity. Writing off the government debt then appears as a last resort if accompanied by a drastic tax reduction. If the size of the write-off and the tax cut are carefully coordinated the economy can be placed on a saddle-point path leading to a higher k and a lower b. As an extreme case the complete write-off accompanied by a balanced primary budget would allow the economy to reach the capital ratio k_{bb}. This is obvious since the private economy freed from the public debt burden would start a vigorous accumulation.The historical experience of public debt repudiation shows that the implied breach of contract has been accepted by the public at least after the trauma of wars. Of course, there are severe distributional problems involved in public debt repudiation which have been mastered by a scheme of 'Lastenausgleich' in German economic history.

3.3 Transitory Budget Deficits

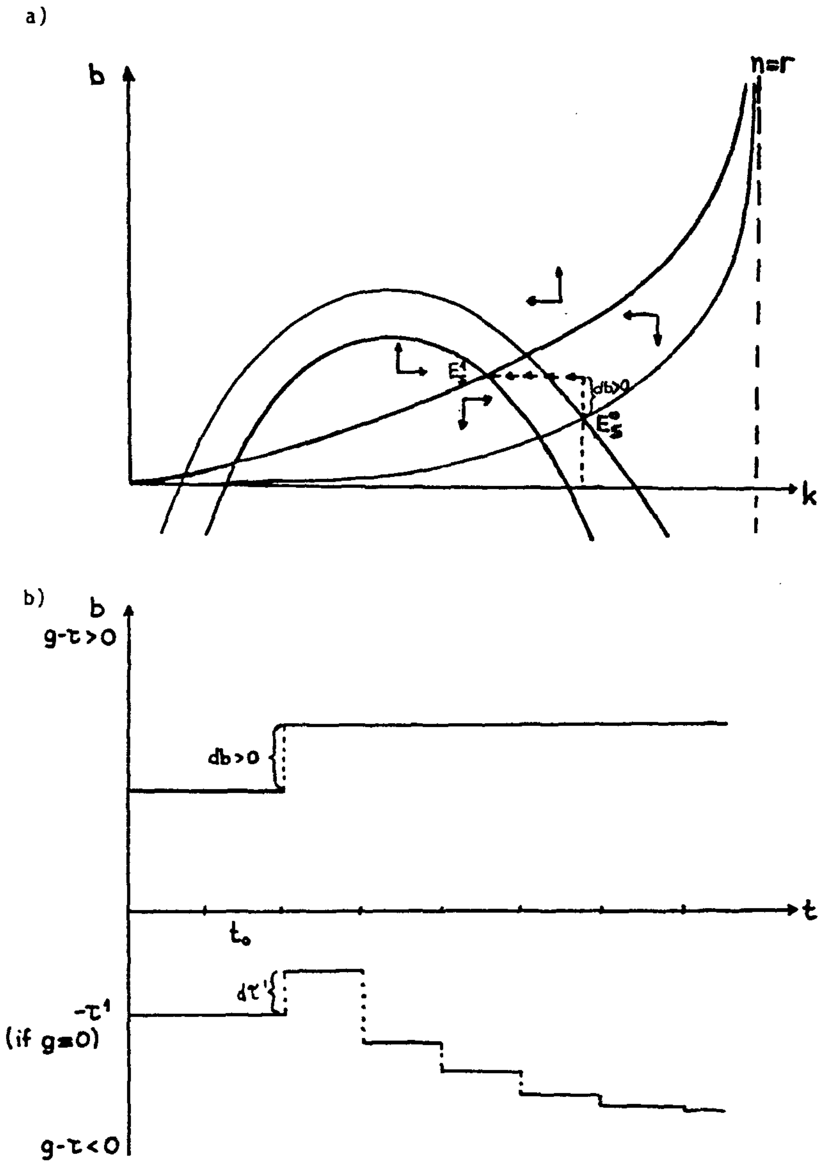
Up to now, I described a sort of a front-loaded government behavior distributing public consumption first and worrying about the consequences later. Persson (1985) models a more debt-conscious government behavior. He assumes a temporary government deficit followed by a tax policy aimed at a stabilization of a new constant level of government debt. The Persson experiment can be readily captured by the phase-diagram technique in Fig. 8. Realistically, Persson starts from a steady state, as E_s^o in Fig. 8a, in the undercapitalized region $n-r < 0$. In this situation, debt service exceeds revenues from new bonds issuance and a primary surplus is necessary to balance the government budget constraint[29]. At time t_0, a temporary tax cut, $d\tau^1 < 0$, will be effective for only one period. Thus the stock of bonds must increase by $db = - d\tau^1 > 0$ in Fig. 8a and Fig. 8b. From now on, this higher level of government bonds is to be stabilized by endogenous taxation. As seen from Fig. 8b, only a boost in taxation to a higher than the initial level with a series of consecutive tax hikes of diminishing increase steers the economy towards the new steady state E_s^1 in Fig. 8a. Compared with the initial state, this steady state is characterized by a higher tax, i.e., primary surplus, and a lower capital-labor ratio. This thoughtfully designed experiment teaches two things: first, it provides a soft landing instead of a crash in the unstable saddle-point region. Secondly, by reversing the direction of the experiment, I demonstrate how the government can remove public debt from the economy in a less dramatic way as discussed above: a *temporary* increase of the primary surplus, e.g., an initial period of high taxation, may be used to repurchase part of the public debt.

To stabilize the lower debt level a series of later tax reductions is possible while the economy increases the capital labor ratio and the interest rate and debt service are declining. In the long run the economy can enjoy a higher capital labor ratio and lower taxation.

---

29 Persson has no public consumption, therefore $g = 0$. Nevertheless, this is to be considered a primary surplus. Kitterer (1988) conducts a similar experiment; he concentrates, however, on steady states and less on dynamics.

Fig. 8: Transitory Budget Deficits



III. Fiscal Activity in a World Economy

1. The Two-Country Model

In the world economy I assume an integrated capital market, i.e., $r_t^* = r_t$. The sizes of national labor forces and labor growth rates are assumed identical, i.e., $L_t = L_t^*$ and $n = n^*$. With respect to intertemporal taste, technology and government behavior, there may be differences in both countries. If tastes and technologies are identical across countries[30] I speak of identical (private) economies; if, in addition, government parameters are identical in both countries we get the same pattern as above, the only difference now being that a world economy faces a world government. To treat a meaningful and not too complicated case in what follows later I shall assume therefore identical economies but non-identical governments, i.e., primary government budgets. The only tax instruments are a lump-sum taxation of domestic and foreign young people; hence there is neither taxation of production or investment nor taxation of any border-crossing economic activities. Also, a discussion of e.g. economic aid by intergovernmental transfers is excluded.

Following the presentation in Schmid (1987), I define net wealth for the domestic and foreign economy.

$$a_t = k_t + b_t + z_t \qquad a_t^* = k_t^* + b_t^* - z_t \tag{30}$$

$z_t \gtrless 0$ denotes the net foreign asset position of the domestic country. If the domestic country is a creditor country domestic national income, y_t, exceeds the value of the domestic product, $f(k_t)$, by the interest payments due to government debt and foreign investment.

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30 Buiter (1981) introduced the two-country case with identical technologies, Schmid (1987) analysed differences in taste and technologies, showing that Buiter's external debt criterion is neither necessary nor sufficient.

$$y_t = f(k_t) + r_t(b_t + z_t) \qquad y_t^* = f^*(k_t^*) + r_t(b_t^* - z_t) \qquad (31)$$

$$y_t = w_t + r_t a_t \qquad y_t^* = w_t^* + r_t a_t^*$$

Domestic or foreign government bonds and ownership titles to domestic or foreign firms are perfect substitutes. Hence, z_t must not be specified explicitly as a bond issued by the foreign government or an investment in foreign firms. Only the net asset position matters. From the world goods market constraint

$$[f(k_t) - c_t - \frac{1}{1+n} c_{t-1} - i_t - g_t] + [f^*(k_t^*) - c_t^{*1} - \frac{1}{1+n} c_t^{*2} - i_t^* - g_t^*] = 0$$

I can derive a well-known saving-investment statement properly modified by government deficits. In case of identical technologies, $f(k)$ and $f^*(k^*)$ would be identical and thus leading to $i_t = i_t^*$.

$$\{[(s_t - a_t) - i_t] - [g + r_t b_t - \tau^1]\} + \{[(s_t^* - a_t^*) - i_t^*] - [g^* + r_t b^* - \tau^{*1}]\} = 0 \qquad (32)$$

Obviously, the terms in { } brackets denote the national current accounts which must sum up to zero. The equilibrium condition for the world credit market (34) is a consequence of (32) taking into account the two national government budget constraints

$$(1+n)\, b_{t+1} - b_t = r_t b_t + g - \tau^1 \qquad (33)$$

$$(1+n)\, b_{t+1}^* - b_t^* = r_t b_t^* + g^* - \tau^{*1}$$

and the wealth definitions (30).

$$s_t + s_t^* = (1+n)\,[k_{t+1} + b_{t+1} + k_{t+1}^* + b_{t+1}^*] \tag{34}$$

$$s_t = s_t\,(w\,(k_t) - \tau^1)$$

$$s_t^* = s_t^*\,(w^*(k_t^*) - \tau^{*1})$$

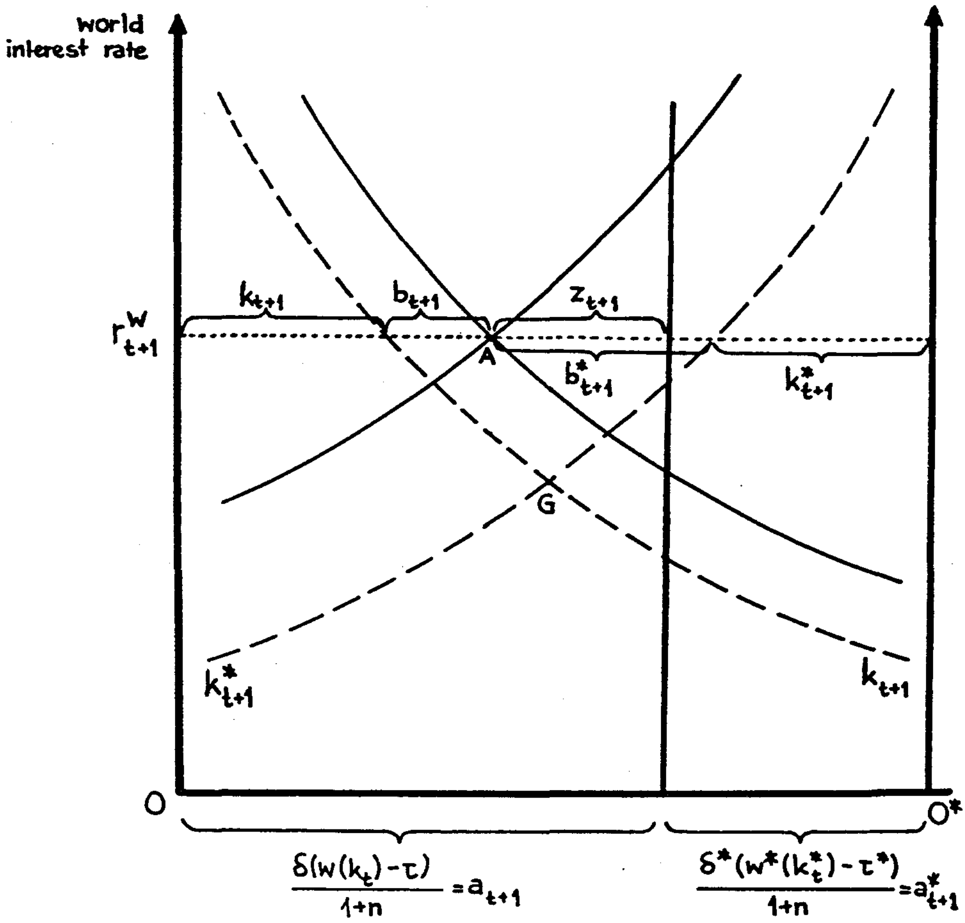
The world capital-market equilibrium is illustrated in Fig. 9 where national excess demands for credit are equalized at an equilibrium interest rate, r_{t+1}. Note that, given historic values for k_t, k_t^*, b_t, b_t^* and the permanent fiscal parameters τ^1, τ^{*1}, g, g*, the credit market determines the foreign asset position, z_{t+1}, and real capital stocks given government bonds issuance[31].

It has been shown in Buiter (1981) and Schmid (1987) that without fiscal activity the dynamic adjustment of the world economy is governed by the difference equation (34) only. Modelling government behavior enforces two more difference equations in (33). Thus the dynamics would be governed by a system of three non-linear difference equations with time-constant disturbance terms these being the primary government budget terms. Here I am not interested in further analysis of the system dynamics, I rather look more deeply into the problem of the existence of steady states and their comparative statistics.

---

31 The demand for credit of the government and the supply of credit of young people is assumed interest inelastic. The assumption of identical economies shifts the middle axis to point G if and only if national lump-sum taxes are equal, $\tau^1 = \tau^{*1}$.

Fig. 9: The World Credit Market



A steady state of the world economy is given by the following set of equations (see Schmid-Großmann (1986) and Schmid (1987)):

$$[a - b - k] + [a^* - b^* - k^*] = 0 \tag{35.1}$$

$$s(w(k) - \tau^1) = (1+n)\,a$$

$$s^*(w^*(k^*) - \tau^{*1}) = (1+n)\,a^* \tag{35.2}$$

$$r = f_k(k) \qquad w = f(k) - f_k(k)\,k$$

$$r^* = f_k^*(k^*) \qquad w^* = f^*(k^*) - f_k^*(k^*)\,k^* \tag{35.3}$$

Perfect international capital mobility yields

$$r = r^* \tag{35.4}$$

The government budget constraints are ($b, b^* > 0$ public debt)

$$(n - r)b = g - \tau^1$$

$$(n - r)b^* = g^* - \tau^{*1} \tag{35.5}$$

The (net) foreign asset positions are ($z > 0$ domestic creditor position)

$$z = a - b - k \qquad z^* = -z = a^* - b^* - k^* \tag{35.6}$$

The current account is ($q > 0$ domestic surplus)

$$q = n[a - k - b] = nz \tag{35.7}$$

and the trade account is ($h > 0$ domestic trade surplus)

$$h = q - rz = (n - r)\,z \tag{35.8}$$

2. Fiscal Strategies in a World Economy

The steady-state version (35) of the dynamic system (33), (34) is rather general, but fortunately it can be stripped down for further analysis. Employing the assumptions of identical private economic behavior and Cobb-Douglas forms for taste and technology, yields

identical technology:

$$f(k) = f^*(k^*) \quad ; \quad w^* = w = \beta f(k) \quad ; \quad rk = \alpha f(k) \qquad (36)$$

identical savings before tax:

$$s(\cdot) \equiv \delta \, [\beta f(k) - \tau^1] \quad ; \quad s^*(\cdot) \equiv \delta \, [\beta f(k) - \tau^{*1}]$$

This is useful because now the net foreign asset position is not a reflex of national differences in private tastes and technology as in Schmid (1987). On the contrary, external debt must be completely dependent on fiscal activity, i.e., (i) the imbalance of the government budget and (ii) the private reaction to income taxation. This way, I am able to separate conveniently fiscally related foreign debt from external indebtedness caused by private economic behavior. Obviously, if worldwide government behavior is completely harmonized, i.e., identical governments, there will be no fiscally related foreign indebtedness. Therefore, I need asymmetric government behavior and, as an extreme case, it is sufficient to consider $b > 0$ and $b^* = 0$ later. In the rest of the paper, I will study balanced budget strategies first and government debt in open economies later.

2.1 Balanced Budget Strategies: Foreign Indebtedness without Public Debt

A balanced budget is implemented by setting $b = b^* = 0$ and

$$g = \tau^1 \;;\quad g^* = \tau^{*1} \tag{37}$$

Note that the size of the balanced budget may differ across countries. Using (37) and (36) in (35), a very simple condition for credit-market equilibrium appears.

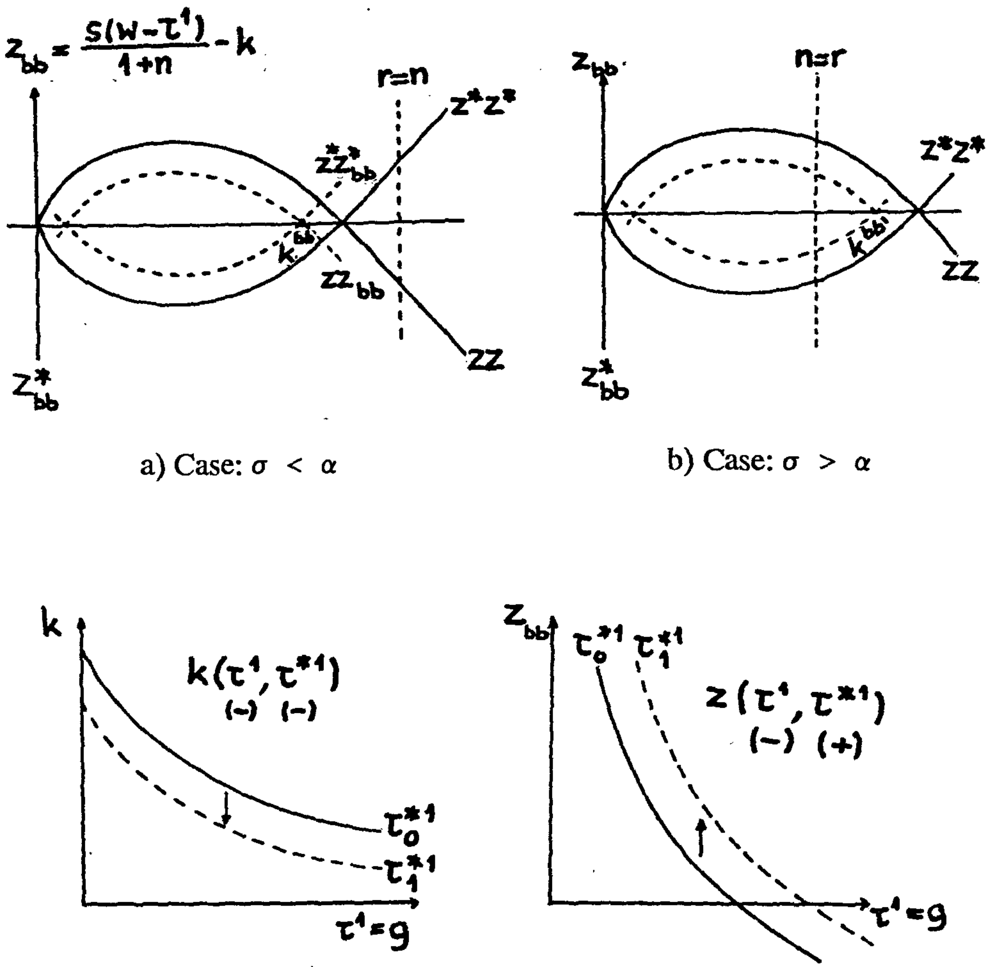
$$\underbrace{\left\{\frac{\delta[\beta f(k) - g]}{(1+n)} - k\right\}}_{z_{bb}} + \underbrace{\left\{\frac{\delta[\beta f(k) - g^*]}{(1+n)} - k\right\}}_{z^*_{bb}} = 0 \tag{38}$$

The steady-state equilibrium (38) can be illustrated in Fig. 10 where I use a geometric apparatus developed for a world economy without government activity in Schmid-Großmann (1986) and Schmid (1987). Setting $g = g^* = 0$ in (38), the two terms in { } brackets represent free of government the national excess supply for credit, i.e., the net foreign asset position, $z = z(k)$ and $z^* = z^*(k)$. The $z(k)$ and $z^*(k)$ functions are shown as the zz and z^*z^* loci in Fig. 10[32]. It follows from the assumption of the world economy's private sectors being perfect replicas that the intersection point of the zz and z^*z^* locus, i.e., the international steady state, is located right on the k axis. Adding a government sector with a balanced budget yields the zz_{bb} and $z^*z^*_{bb}$ lines. This contraction is an expression of the fiscal drag on private savings of young people.

---

32 Fig. 10a and Fig. 10b show two possible cases. Assuming a relatively high (low) savings rate, the international steady state must be located to the right (left) of the $n=r$ line, i.e., the overcapitalized (undercapitalized) region. For proof, see Schmid-Großmann (1986).

Fig. 10: Balanced Budget: b = b* = 0
(Tax Financed Public Consumption)



a) Case: $\sigma < \alpha$

b) Case: $\sigma > \alpha$

c)

d)

If both nations run a balanced budget of the same size, $g = g^* > 0$, the new steady state k_{bb} must show a lower capital-labor ratio and there is no external indebtedness. If the balanced-budget size differs across nations the government will cause external debt. The dependence of k and z on the fiscal instruments, $\tau^1 = g$ and $\tau^{*1} = g^*$, is summarized in Fig. 10c and 10d proving the following

Theorem: Two identical nations with different balanced-budget sizes will always have external indebtedness for the country affording the larger balanced-budget size.

Another interesting balanced-budget strategy is the case of a pay-as-you-go social security system[33]. To demonstrate the balanced-budget nature of a pay-as-you-go social security system recall the general government-budget constraint (16) for the special case of a steady state without public debt and public consumption. The following constraint must hold

$$\tau^2 = -(1+n)\tau^1 \qquad \tau^1 > 0 \qquad (39)$$

The pay-as-you-go social-security system asks the young population to contribute $L_t\tau^1$ during their working period and pays this amount in benefits to the currently old population, i.e., $L_t\tau^1 = -L_{t+1}\tau^2$. Note that the social-security tax amounts to $\tau^1 > 0$ and the benefits are to be considered a negative tax, $\tau^2 < 0$, i.e., a transfer. Using (39) in (7), yields

$$-\frac{\delta(1+r) + (1-\delta)(1+n)}{1+r}\tau^1 < 0$$

From the logic of the intertemporal maximization, it is clear that any social-security tax reduces the savings of currently young people, and the expected

33 See e.g. Auerbach-Kotlikoff (1987), chapt. 10, for a discussion of social security for a closed economy with overlapping generations.

transfer during retirement reduces the propensity to save even further[34]. Thus, savings of young people shrink unambiguously and decisively. The case of social security in an open economy therefore is very similar to the case of pure tax finance of public consumption. The zz locus contracts towards the k axis and I can formulate the following

Theorem: Assume two identical nations with pay-as-you-go social security systems. Then the country asking the bigger social-security tax must show external indebtedness.

2.2 Fiscal PONZI Games in a World Economy: Public Debt and Foreign Indebtedness

As experienced recently by Latin American debtor countries a rise in fiscal deficits and debt may cause a critical amount of external indebtedness. This subsection explores the sustainability of permanently maintained primary fiscal deficits in a world economy. This is important, since I consider the world debt crisis in analogy to section 3 as a temporary attempt by several countries at maintaining unsustainable debt profiles.

The steady-state conditions are now captured by a system of three equations with the variables k, b, b*.

$$\underbrace{\left\{\frac{\delta[\beta f(k) - \tau^1]}{1+n} - k - b\right\}}_{z_b} + \underbrace{\left\{\frac{\delta[\beta f(k) - \tau^{*1}]}{1+n} - k - b^*\right\}}_{z_b^*} = 0 \qquad (40.1)$$

$$(n - r)b = g - \tau^1 \qquad (40.2)$$

$$(n - r)b^* = g^* - \tau^1 \qquad (40.3)$$

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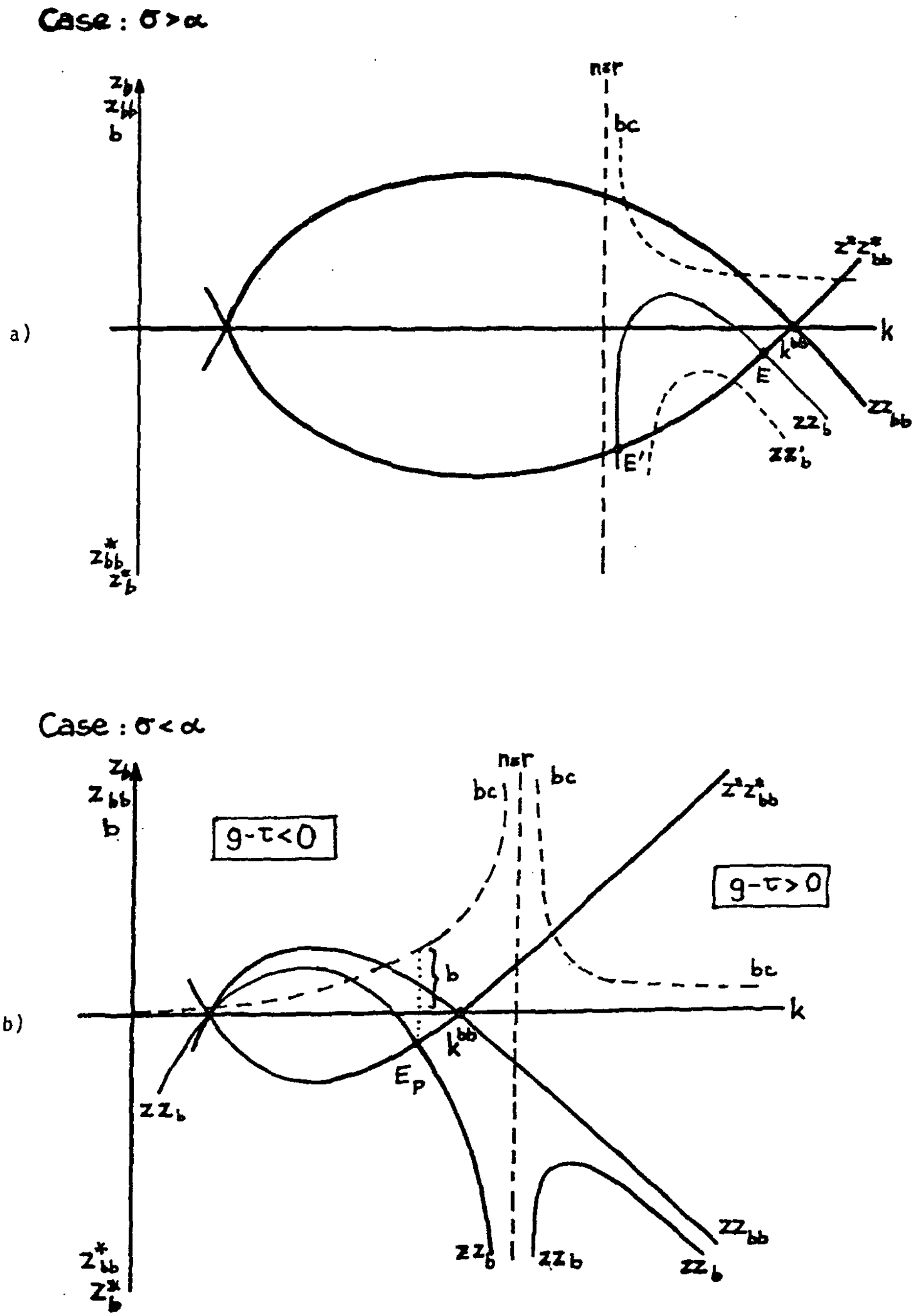
34 This effect is valid independently of the present value of the social-security scheme,

$$pw = -\frac{n\text{-}r}{1+r}\,\tau^1,$$ which is ambiguous.

Fig. 11: Government Debt in an Open Economy



Without loss of generality, I further assume that government behavior in the foreign country is restricted to a balanced budget, i.e., $b^* = 0$ and $g^* = \tau^{*1}$, while the domestic government starting from a balanced budget of equal size is allowed to raise loans in the world credit market.

Before the domestic government issues bonds, the steady state of the world economy is located at k_{bb} in Fig. 11 which may be situated to the right (see Fig. 11a) or to the left (see Fig. 11b) of the golden-rule $\bar{k}$ depending on the size of the (identical) national savings ratios. Suppose the domestic government raises public consumption beyond the original level and finances this increase by government bonds[35], the resulting primary government deficit appears as the dashed bc locus in the $n-r > 0$ region of Fig. 11a. In an open economy, government bonds can be sold to foreigners and, in fact, are placed abroad whenever supply of public bonds exceeds the net demand of the private sector. On the other hand, the foreign asset position of a country must be positive if the private sector's (net) demand for bonds is larger than the public bonds supply. Thus in (40.1), we subtract b from z_{bb} to obtain z_b. In Fig. 11a, the zz_b locus is obtained by subtracting the bc locus from the zz_{bb} locus. As a result, we observe (two) steady-state equilibria, E and E', where the domestic p.m.p. deficit is financed (partly) by external debt. Concentrating on the stable equilibrium E, two conclusions can be drawn: (i) The steady-state capital-labor ratio is higher in a world economy than it would be in a closed economy due to the savings of young foreigners. (ii) In a world economy, a single nation can finance p.m.p. deficits of a much larger size than possible in a closed economy (see the dashed zz_b' locus in Fig. 11a). However, there again is a critical size of the p.m.p. deficit, given the savings capacity of the foreign country, i.e., the $z^*z^*_{bb}$ line. Furthermore, if the foreign government would run a p.m.p. deficit, too, the existence of a steady state would be threatened very quickly. To summarize I state the following

Theorem: Assuming the empirically not relevant high savings case, $\sigma > \alpha$, p.m.p. deficits certainly are not more likely to be possible in an open economy

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[35] Note that a bonds-financed increase in public consumption is the simplest case since it leaves the z_{bb} locus unchanged. A switch from tax to bonds finance would shift the z_{bb} locus in an upward direction.

when more than one government runs them. However, there seems to exist the possibility of one single (small) nation running a p.m.p. deficit in a large world economy[36].

Note that this statement is conditioned on an overcapitalized world economy and I have to analyse the more realistic case of an undercapitalized world economy. Fig. 11b clearly demonstrates that now a steady state cannot exist in case of a domestic p.m.p. deficit of any size. Interestingly enough, there exists an international steady state in case of a p.m.p. surplus with positive government debt, $b > 0$, and a positive external debt for the domestic country, $z < 0$. The equilibrium E_p is shown within the realistic $n-r < 0$ region in Fig. 11 b[37, 38]. Since public interest payments to serve the debt held by domestic and foreign residents exceeds revenues of new bonds, a p.m.p. surplus is necessary to balance the overall budget.

2.3 Transitory Government Deficits

The debt from transitory deficits could be stabilized in an open economy as reported in Persson (1985). There exists an initial public debt b_0 in the $n-r < 0$ domain of Fig. 12. The steady state E_p illustrates that this debt is held by foreigners (z_0^*) and domestic residents. Suppose the domestic government runs a (one-period) transitory deficit by lowering the tax necessary to maintain steady state E_p. Suppose furthermore that $db > 0$ is the resulting increase in domestic debt which is to be stabilized from now on. Obviously, taxation must become endogenous because only a series of consecutive tax hikes guarantees a constant b while the economy decumulates real capital along the arrowed path in Fig. 12.

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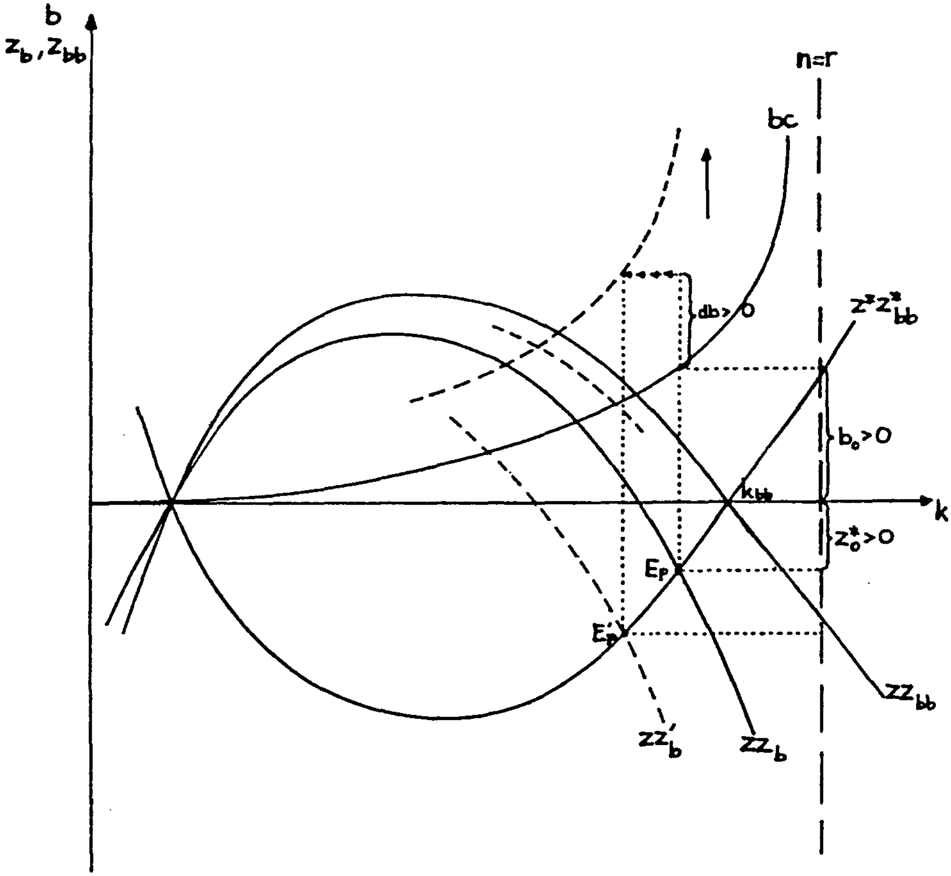
36 The theorem in Carlberg (1983) is still valid in a world economy, but only in a much weaker form.

37 This steady state is in the background of Persson (1985).

38 The zz_b locus is obtained by subtracting the upward-sloping bc line from zz_{bb}.

Fig. 12: Transitory Deficits in a World Economy



In the new steady state E_p' external debt has risen since part of the additional domestic government debt was credited by foreigners. A larger domestic primary budget surplus is necessary to service the national debt at a higher world interest rate. Foreigners enjoy an increase in their foreign investment income while the domestic current account worsens.

By reversing the direction of this experiment, I demonstrate how the government could remove public debt from the economy in a silent way: A temporary increase of the primary surplus, e.g., an initial period of higher taxation, can be used to repurchase part of the public debt. To stabilize the lower debt level a series of later tax reductions are possible while the economy vigorously increases the capital-labor ratio with interest rate and debt service declining. The current account would improve.

IV. Conclusions

This paper offers a unifying dynamic system approach to real government debt and real capital formation in a world economy. The sustainability of permanently maintained primary fiscal deficits is investigated in an open economy. In particular if national governments compete in the issuance of debt at an international capital market sustainable debt profiles appear only as a theoretical (i.e., not empirically valid) curiosity. Within the more realistic regime of an undercapitalized world economy a nation can only run a permanently maintained primary surplus. Starting from a p.m.p. surplus the paper also demonstrates the viability of temporary deficits implying an increase in taxation later to stabilize the fiscal debt. By reversing the argument this shows that the right way to reduce government debt in a non-traumatic manner is to run a higher temporary surplus via higher taxation. Using this extra-surplus to buy back fiscal debt the economy may reduce taxation later while enjoying vigorous capital accumulation towards a higher capital-labor ratio. Furthermore, the paper shows that ceteris paribus a relatively high social security load and a relatively high size of a balanced budget causes external indebtedness via consumption oriented current account deficits. It is left for further research to see what happens if government borrows for public investment instead of public consumption.

V. Appendices

Appendix 1: List of Symbols

Variables:

c_t^1	working-period consumption of a member of generation t
c_t^2	retirement-period consumption of a member of generation t
τ^1, τ^2	lump-sum tax during working and retirement period
r_{t+1}	interest rate for savings during period t and carried over to period t+1
s_t	savings of young people
$\bar{s}_t$	national savings
w_t	wage income
$\hat{w}_t$	present value of lifetime income net after taxes
L_t	number of members, i.e., size, of generation t
k_t	capital-labor ratio
$x_t = f(k_t)$	output-labor ratio; f(·)production function
y_t	national income including interest on government debt and net foreign investment income
$i_t = (1+n)k_{t+1} - k_t$	per-capita investment
b_t	government bonds
g	public consumption

$a_t = k_t + b_t$	private wealth at the beginning of period t
$(1+n)b_{t+1} - b_t$	current overall government deficit
$g - \tau^1 - \tau^2$	primary government deficit

Parameters:

α	capital-income share
ß	labor-income share
δ	savings ratio of young people; $(1-\delta)/\delta$ time preference
σ	national savings ratio
v	tax burden ratio
n	growth rate of labor

Appendix 2

Assume the technology is given by $f(k) = k^{\alpha}$ together with linear logarithmic utility (1). Then α represents the share of capital income and $ß = 1 - \alpha$ is the wage share of the economy. Now steady state is given by

$$\delta ß f(k) = (1+n)k$$

This can be rewritten

$$\delta ß r = (1+n)\alpha$$

From the definition of the national savings ratio, $\sigma \equiv \tilde{s}/f(k)$, we obtain from (23)

$$f(k)\sigma = \frac{n}{1+n}s = \frac{n\delta w}{1+n} \qquad \text{or} \qquad \sigma = \frac{n\delta ß}{1+n}$$

Using the definition of σ in the above steady-state condition, yields

$$\frac{n\delta \beta}{1+n} r = n\alpha \qquad \text{or} \qquad \sigma r = n\alpha$$

$\sigma \leq \alpha$ implies $r \geq n$. Thus $\sigma \leq \alpha$ implies $k \leq \bar{k}$.

Appendix 3

System (29) can be rewritten

$$b_{t+1} - b_t = \frac{(r_t - n)}{1+n} b_t + \frac{g - \tau^1}{1+n}$$

$$k_{t+1} - k_t = \frac{\delta[\beta f(k_t) - \tau^1] - (1+n)k_t - (1+r_t)b_t - (g - \tau^1)}{1+n}$$

and the following set of partial derivatives can be derived

$$\begin{bmatrix} \dfrac{(f_k - n)}{1+n} \lesseqgtr 0 & \dfrac{bf_{kk}}{1+n} < 0 \\ \\ -\dfrac{1+f_k}{1+n} < 0 & \dfrac{\delta\beta f_k - (1+n) - bf_{kk}}{1+n} \gtrless 0 \end{bmatrix}$$

Stability of the process of real capital accumulation requires $-\dfrac{f_{kk}(\delta k + b)}{1+n} < 1$.

Assuming this stability condition, I obtain the law of motion indicated by arrows in Fig. 5.

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