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Social sampling, self-anchoring and redistribution

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ABSTRACT

We examine the effect that agents anchor their perceptions of societal mean incomes on their own incomes. As we show analytically and verify by means of numerical simulation, this self-anchoring implies that even if the sampling of agents is unbiased and fully random, including their own income into their estimate of the average can lead to substantial misperceptions, especially, if the sampling ratio is small. Within a Meltzer–Richard framework, we then demonstrate that self-anchoring leads to a decrease in implemented redistribution.

1. Introduction

It is now a well-established fact that belief-formation in many domains is strongly based on social samples, i.e., based on a sample of people we interact with and extrapolate to the whole population (Galesic et al., 2012). While the effects of biased selections of social samples was already extensively studied (Schulz et al., 2022), we examine another issue in this letter: Even if the sample selection is unbiased, agents will know their own situation, leading to an anchoring of individual beliefs to their own income. We show analytically that for any distribution with a mean income above the median income, the majority of agents will underestimate the societal mean income, and explicate the implications for the implemented redistribution.

2. Perception model

We assume that agents form their beliefs about societal mean income based on a random sample of the overall population and include their own income into this assessment. The effect of this self-anchoring can lead to substantial bias, even for fully random sampling.

Perceptions p of agent i with income y_i based on a random sample of size d of the set of incomes Ω are thus given by

$$p_i = \frac{y_i + \sum_{j\in\Omega\setminus y_i}^{d}\{y_j\}}{d+1}, \tag{1}$$

where $\{y_j\}$ are realisations of sampling randomly from the income distribution Ω with equal probability.

By taking expectations, we can simplify the expression as

$$\mathbb{E}[p_i] = \mathbb{E}[\frac{y_i + \sum_{j\in\Omega\setminus y_i}^{d}\{y_j\}}{d+1}] \tag{2}$$

$$= \frac{y_i + d(N\bar{y} - y_i)/(N-1)}{d+1}, \tag{3}$$

with $\bar{y}$ as the population mean income.

3. Results

In expectation, this implies that a majority of agents will underestimate the population mean income in the empirically relevant cases.

Theorem 1. *For any income distribution where a majority of agents has an income below the mean income, this majority will underestimate the mean societal income $\bar{y}$ in expectations.*

Proof.

(1) Agents with own income below the mean income will underestimate the mean income in expectations, i.e., $\mathbb{E}[p_i] = \frac{y_i+d(N\bar{y}-y_i)/(N-1)}{d+1} \begin{cases} < \bar{y} \text{ for } y_i < \bar{y} \\ = \bar{y} \text{ for } y_i = \bar{y} \\ > \bar{y} \text{ for } y_i > \bar{y}. \end{cases}$

(2) We consider distributions with $y^m < \bar{y}$, i.e., the median income is below the mean income.

The proof follows immediately due to the fact that perceptions are monotonously rising in income in expectations, i.e., $\frac{\partial \mathbb{E}[p_i]}{\partial y_i} = \frac{1-\frac{d}{N-1}}{d+1} > 0$ for $d < N-1$. By (1), perceptions will increase and thus be lower than the population mean income. By (2), the majority of agents will have incomes below the population mean and thus, by (1) and (2),

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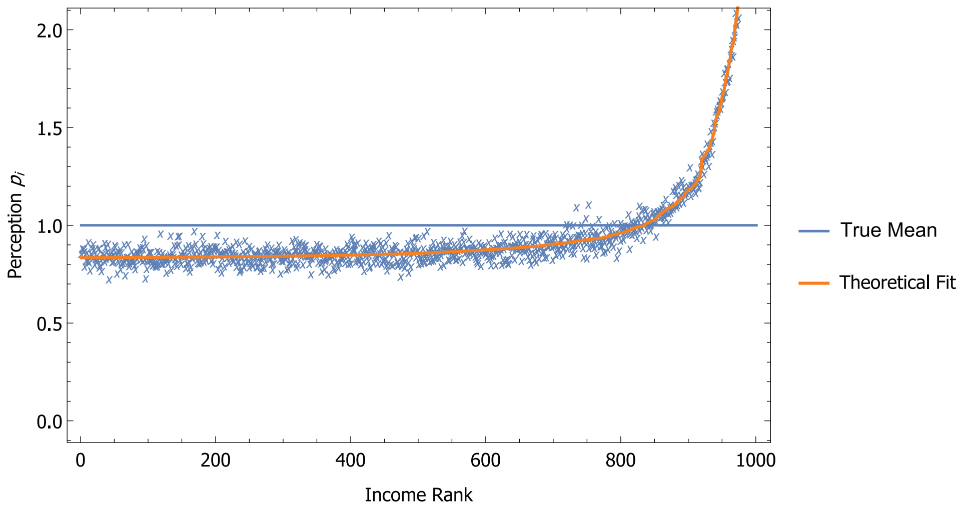

Fig. 1. Perceptions per income rank for intial lognormal distribution with mean 1 and $\sigma = 1$ for $N = 1,000$ agents and $d = 5$. Averaged over 1,000 simulation runs.

the majority of agents underestimate the population mean income in expectations. □

Thus, perceptions tend to be biased downwards by self-anchoring for distributions with a majority of incomes below the mean which is the empirically relevant case for virtually all empirical income distributions (see e.g. Kuhn et al., 2020, for the US case). This result is verified by numerical simulation, as shown in Fig. 1. Here, incomes are initialised following a lognormal distribution with $N = 1,000$, $d = 5$, mean of 1, dispersion σ also 1 and averaged over 1,000 simulation runs. The theoretical predictions by Eq. (1) shows an excellent fit to the simulation outcomes. Indeed, given the skewness of the lognormal for this parametrisation, the majority of agents underestimates the true societal mean, as shown for the more general case in Proposition 1.

So far, we only analysed the case of a static income distribution. Yet, as the canonical paper by Meltzer and Richard (1981) (MR) shows, demand for redistribution is increasing in the mean-to-median ratio. As a counteracting effect of self-anchoring, the amount of agents underestimating the population mean income in expectation is also increasing in the mean-to-median ratio, however, for general conditions. We illustrate this result for the analytically convenient case of a lognormal distribution. Yet, it holds generally for every positively skewed distribution for which the p-quantile is increasing in the mean-to-median ratio which is the case for virtually all parametric unimodal distributions with positive skew.

Theorem 2. *For a lognormal income distribution, the share of agents underestimating the population mean income is increasing in the mean-to-median ratio.*

Proof.

(1) The p-quantile for a lognormal distribution is given by $F^{-1}(\bar{y}) = \frac{1}{2}\mathrm{erfc}\left(-\frac{\sigma}{2\sqrt{2}}\right)$ which is by $dF^{-1}(\bar{y})/d\sigma = e^{-\frac{\sigma^2}{8}}/2\sqrt{2\pi} > 0$ for $\sigma > 0$ a monotonously increasing function of the dispersion parameter σ and independent of the location parameter μ.

(2) The mean-to-median ratio $r = e^{\frac{\sigma^2}{2}}$ for a lognormal distribution is also a monotonously increasing function of the dispersion parameter σ and independent of the location parameter μ.

By (1) any increase in the mean-to-median ratio r implies that the dispersion parameter σ has increased which also implies by (2) that the p-quantile of the mean observation $F^{-1}(\bar{y})$ has increased and therefore the share of agents with income below the mean. By Theorem 1, all agents with income below the mean will underestimate the population mean income in expectations. Thus, an increase in the mean-to-median ratio will increase the share of agents who underestimate the population mean income. □

It follows that while for correct perceptions, increasing the mean-to-median ratio increases the demand for redistribution as in Meltzer and Richard (1981), it also increases the amount of agents underestimating the societal mean income and thus of a potential transfer. To see the quantitative effect of these two channels, consider a stylised version of the model by Meltzer and Richard (1981) that can be microfounded by assuming an iso-elastic utility function for the labour-leisure trade-off (Bastani and Lundberg, 2017). We assume k as the elasticity parameter of post-tax income with respect to the net-of-tax rate $(1-t)$ indicating the marginal disutility of labour and a flat-tax system with revenue being redistributed in equal shares. This implies that the size of the transfer T for an agent i for a tax rate $t \in (0,1]$ is

$$T_i(t; y_i, k, \bar{y}) = t(1-t)^k \cdot \bar{y} - t \cdot y_i. \tag{4}$$

As shown above, self-anchoring biases typically lead to a situation where $\mathbb{E}[p_i] \neq \bar{y}$ even for fully random samples. This holds, in particular, for the decisive agent with median income who, for the assumption of $\bar{y} = 1$ has income of $e^{-\sigma^2/2}$. As Lopez-Velasco (2020) shows for the same functional form, voters' preferences over tax rates are single-peaked and thus, the median voter theorem applies. The implemented tax rate t^* will thus be such as to maximise the transfer for the agent with median income, i.e.,

$$t^* = \arg\max_t \quad T(t; y^m, k, \bar{y}). \tag{5}$$

For tractability, we assume that the marginal disutility of labour $k = 1$. By the FOC, this directly leads to the implemented tax rate t^*

$$t^* = \frac{\bar{y} - y^m}{2\bar{y}}. \tag{6}$$

Without loss of generality, we continue to assume $\bar{y} = 1$. This implies that the implemented tax rate without self-anchoring is given as

$$t^* = \frac{1 - e^{-\sigma^2/2}}{2}, \tag{7}$$

which unanimously increases in the dispersion parameter σ. Thus, we recover the original intuition of Meltzer and Richard (1981) that an increase in pre-tax inequality should increase redistribution. With self-anchoring, the agent with median income bases their estimate on a random sample of agents and their own denoted by p_m. In expectation, the implemented tax rate $\hat{t}^*$ for self-anchoring thus reads

$$\mathbb{E}[\hat{t}^*] = \mathbb{E}[\frac{p_m - e^{-\sigma^2/2}}{2}] \tag{8}$$

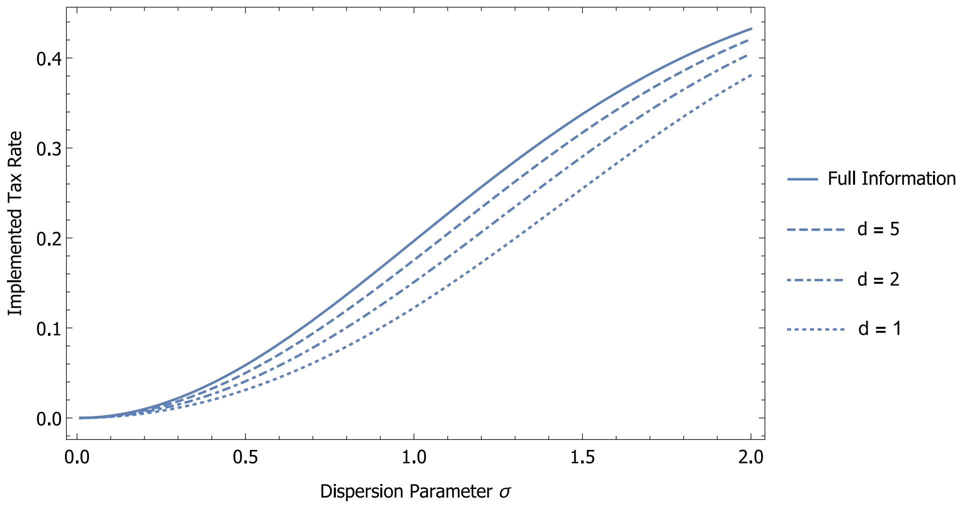

Fig. 2. Implemented tax rates for the full information (MR) benchmark and for several levels of self-anchoring with varying d and for $N = 1,000$. Incomes are assumed to follow a lognormal distribution with varying dispersion parameter $\sigma \in (0, 2]$.

$$= \frac{\mathbb{E}[p_m] - e^{-\sigma^2/2}}{2} \tag{9}$$

$$= \frac{d N e^{-\sigma^2}\left(e^{\frac{\sigma^2}{2}} - 1\right)\left(d\left(N e^{\frac{\sigma^2}{2}} - 1\right) + N - 1\right)}{(d+1)^2 (N-1)^2 \left(1 - e^{-\frac{\sigma^2}{2}}\right)} \tag{10}$$

As is easily verified, $\mathbb{E}[\hat{t}^*] \leq t^*$ for $d\langle N, N\rangle 1$ and $\sigma > 0$, since $\mathbb{E}[p_m] < \bar{y}$. Thus, self-anchoring biases implemented redistribution downwards. To see the effects play out quantitatively, we plot $\mathbb{E}[\hat{t}^*]$ for different d and σ for a fixed $N = 1,000$ (see Fig. 2). Self-anchoring in expectation thus leads to a downward bias in implemented redistribution that can be sizeable for realistic numbers of d based on the cognitive limits imposed by Dunbar's number (Dunbar, 1992). For lower k compared to $k = 1$ we impose here for tractability, this effect can be even larger.

4. Conclusion

This letter demonstrates that self-anchoring will for otherwise unbiased perception and the empirically observed income distributions lead to substantial misperceptions of the average societal income in expectation. The number of agents underestimating the societal mean income will increase in the mean-to-median ratio. Self-anchoring therefore directly counteracts the mechanism proposed by Meltzer and Richard (1981) that an increase in the mean-to-median ratio of the pre-tax distribution will increase redistribution. Self-anchoring might thus partially explain the empirically dampened response of redistributive taxation to increased market inequality by relaxing an apparently innocuous assumption of the MR framework. Indeed, as we show for the example for a lognormal income distribution, implemented redistribution will be lower than for the MR benchmark.

Data availability

No data was used for the research described in the article.

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