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COMPARING MATHEMATICS TEXTBOOKS — AN INSTRUMENT FOR QUANTITATIVE ANALYSIS

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New national curricula and another perspective on mathematics education like for example in the New Math in the 1960th make textbooks change. Thereby, not only the mathematical content differs, even more the presentation and handling of mathematics show great varieties. To identify and explore such differences with statistical methods, a category system for analysing textbooks was developed. By now 30 000 data sets of 14 textbooks are collected. Thereby, first assumptions about differences and changes were made.

INTRODUCTION

Mathematics textbooks vary in many ways. For example, they use different kinds of tasks, reaching from easy technical exercises to such requiring mathematical argumentation or proof. Some textbooks are overloaded with pictures not showing any use for learning mathematics, while other books use images to explain and illustrate mathematics. Great differences can also be seen in the way of presenting mathematics. While some textbooks present a minimal amount of information or contain only a few worked examples, others have long texts, explaining mathematics, or they have special tasks, advising students how to handle a single mathematical problem. To outline such differences, being found in various decades or textbooks of different countries and publishers, an instrument was developed. First of all, this article provides an overview of the research questions and the category system. Afterwards, first hypotheses from an exemplary study are described.

AIMS

Often, the research on mathematics textbooks focuses on very special aspects like the use of materials to show mathematical structure (cf. Dooley, 1961, p. 317) or on aspects “easy to quantify” (translated from Reichmann, 2008, p. 331, „leicht zu quantifizierende Daten“). Thereby, isolated aspects of mathematics textbooks, like the number of images (e.g., Reichmann, 2008; Dooley, 1961) or the use of exercises instead of tasks (Valverde et al., 2004) are regarded. On the other hand, historical studies often use qualitative methods that show very detailed information. But they don’t give a hint on the relevance of a change because differences are only indicated, while the frequency of an aspect is unconsidered.

In consequence of the difficulties shown above, three aims were settled for the following study:

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1. To avoid considering only a few, maybe special, aspects of textbooks some basic categories for analysing textbooks had to be found. Therefore, results from textbook research were used as well as findings about the three main textbook elements (images, explanations and tasks). Additionally, a qualitative study on the base of 30 German textbooks from different decades and school types was done to complete the category system.

2. To decide if differences between books of distinct countries, decades, publishers or school types are significant and to examine what is typical for a decade, the instrument should produce quantitative data. As a result, statistical methods can be used.

3. Finally, not only isolated data should be collected. Instead, coherencies among categories should be considered. So, for example, the correlation between the type of explanations and the mathematical content can be shown.

CATEGORY SYSTEM

Considering these aims, in this study an instrument was developed to measure differences between textbooks by statistical methods. The category system focuses on three main textbook elements: images, explanations and tasks. These can be seen as a conclusion of the more detailed “blocks” by Valverde et al. (2002). To get detailed and adequate statistical characterization of these three elements, from seven up to 17 categories were defined for each element. Every category is specified by properties. Overall, there are around 140 properties that can be analysed individually or related to other properties. To get quantitative data, every textbook element in one textbook is analysed by the categories of the category system. Counting how many times a special property occurs leads to a quantitative set of data that can be handled with statistical methods. Having mainly nominal scaled data chi-square methods are used.

APPLICATIONS

14 textbooks were analysed to show in an exemplary way how the instrument can be used and to get first impressions about changes in German textbooks. These books are published in the years 1935 to 2011. To make changes more clearly, these books can be summarized in five exemplary eras, beginning with the time of the National Socialism in the late 1930s and early 1940s. Because there are no changes in national curricula, the textbooks of the 50s and 60s are considered as one era. They are followed by a special period, the New Math in the late 60s and 70s. Afterwards the 80s and 90s are treated as one era, belonging to the same curriculum. The last era, based on the curriculum of 2004, is resulting from books published between 2004 and 2015.

Data base

To get information, mainly about historical variances, as many as possible variables of the chosen textbooks were kept constant. So, every textbook is published in Bavaria (a traditional federal state in Germany). All books were produced for the 7th grade of the Hauptschule (the lowest school type in the German tripartite school system). In every era at least two textbooks from established textbook publishers were selected (Buchner/Klett, Oldenburg, Ehrenwirth). Wherever possible, the same textbook series (e.g. "Wir rechnen", "Die Welt der Zahl", "Formel") were analysed in different eras.

The textbooks were surveyed completely. That means, not only single chapters of the books were analysed, but every textbook element (image, explanation, task). That leads to a data set of 30 000 textbook elements. Having only a small range of textbooks analysed, no absolute results can be given despite that great number of records. Nevertheless, hypothesis can be shown by the present set of data.

Generic issues

First of all, the collected data can be used for frequency analysis. As a result, it is possible to get information about **number or amount of a special property**. For example, 95 tasks in the three analysed textbooks of the 2000s require mathematical argumentation, which is 3 % of all tasks of this era. 1257 tasks have an extra-mathematical content (in average 419 per textbook or 41 % of all tasks in this era). Looking at the category "type of explanations", three properties are regarded. First, mathematical operations can be explained by *worked examples*. Another way to give information to students is in an explicit, narrative way, here called *narrative explanations*. Finally, information can be reduced on short conclusions of important formulas. This is called *short explanation*. In the textbook "Formel 7" (produced in the year 2004) 61 % of the explanations contain *worked examples*, 33 % *short explanations* and 6 % *narrative explanations*.

To get more detailed insights, a **comparison** of different textbooks is helpful. Comparing the distribution shown above with other publishers' textbooks or another edition of "Formel 7", you can figure out whether this distribution is typical for textbooks of this era. Looking exemplarily at "Mathe aktiv 7" (2005) no significant difference can be established, concerning the "type of explanation". What's more, comparing "Formel 7" (2004) with the following edition, published in 2011, no significant difference can be seen. That's not surprising, knowing that 65 % of the explanations in the newer book are printed in the old one in exactly the same way. This finding should be affirmed by analysing more textbooks from other publishers and other grades. While this category is typical for the publishing period, other aspects of textbooks depend on the respective author or publisher. So it seems, that the number of images used to help the students to orientate in the textbook, depends mainly on the publisher. While in the textbook "Formel 7" (2004) 16 % have this function, in the textbook "Mathe aktiv 7" (2005), there is no such image.

A particular type of comparison is the presentation of distinctions of textbooks published in different eras. Doing that, varieties in textbooks depending on the publishing period can be detected. Looking at the "type of explanation" in different eras, a significant change can be seen:

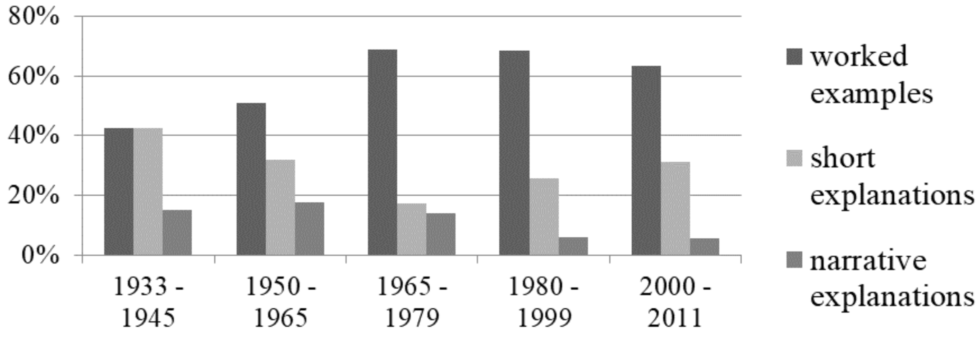

Figure 1. Distribution of the category "type of explanation" in different eras

Another finding, seen from the comparison of different eras' textbooks, is the image type *illustrated diagram*. Occurring mainly in textbooks published in the 50s and 60s, this property can be regarded as typical for this period. Detecting those generic aspects of special eras is another application of the instrument.

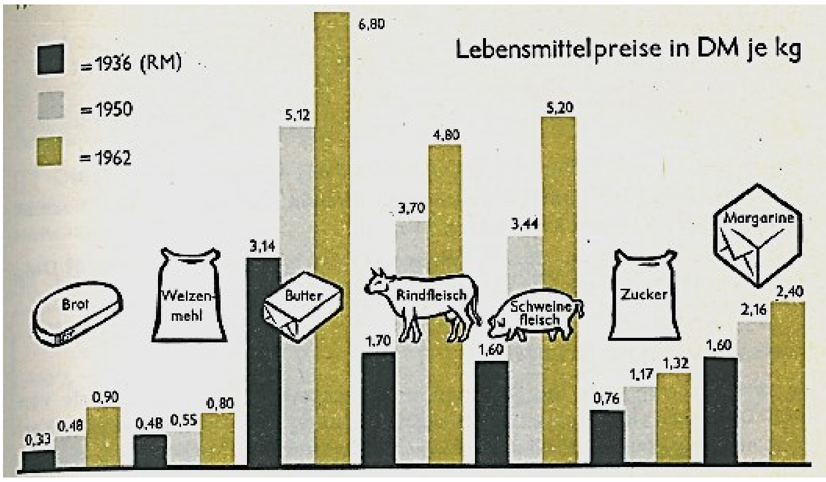

Figure 2. Image with the property *illustrated diagram (*Schlagbauer, 1964, p. 41)

One of the instruments' aims was to enable **measuring coherencies**. That is shown on the example of textbook images. Maybe someone is not only interested in the "types of images" used in a special textbook, but rather in differences in the use of these image types. In general, images are used in different ways. The most important function of images is to *present mathematics* or to *make a text easier to understand*. Other images don't bring any information, but are used to *illustrate* or to *help the students to orientate* in the textbook. Furthermore, images *present information* needed to solve tasks. In the present data a middle coherency between type and function of images can be proved (Cramer's V .39). In particular, images used to *help the students to orientate* in the textbook can be presumed, being of one single type. Therefore, only *drawings* of fantastic or real things are used. Further on, there is a middle coherency between these *drawings* and the use as *illustration* (Cramer's V .44). *Tables* are often used to *present information* needed for tasks (Cramer's V .38), but not as *illustration* (Cramer's V .19). In the table below the number of the main image types and their function can be seen.

	diagrams	geometric drawings	drawings	tables	others
presenting mathematic	0%	4%	2%	1%	5%
make texts easier to understand	1%	7% +	3%	1%	2%
present information for tasks	3%	14%	8% -	20% +	10%
illustration	0%	0% -	10% ++	0% -	2%
others	0%	0%	6%	0%	1%

Table 1. Coherencies between the categories “type of image” and “function of image” (++/-- means there is a middle coherency (Cramer’s V > .40) appearing more/less than expected, +/ - means a weak coherency (Cramer’s V > .20))

Many other coherencies can be analysed by the instrument. Resulting on the present data, a weak coherency can be stated exemplarily between the “type of explanation” and the “mathematics content” (Cramer’s V .24). While *geometric* explanations use *short explanations* mostly (55 % of the geometric explanations), explanations about *numbers and calculation* prefer *worked examples* (74 % of the explanations of this topic).

In the previous applications, the structure of the category system was used to come to new questions. In addition, **further questions** can be surveyed by the help of the instrument. For example, the effect of pedagogical, didactical or mathematical influences on textbooks, like the implementation of national standards or the results of PISA or TIMMS, can be analysed. Therefore, the issues have to be translated into categories or properties analysed in this instrument. This is exemplified for the New Math in the late 1960s. First, basic demands on textbooks had to be found. According to Hayen (1987) one of the demands for modern textbooks in the New Math period was another way of giving information to students, meaning “many examples” (translated from Hayen, 1987, p. 113) and no “reading book for pupils” (Hayen, 1987, p. 115). Translated in the categories of the instrument, the “type of explanation” and the property of tasks *advising students how to handle a single mathematical problem* can be used to find an answer. Solely looking on textbooks of this era will help less. More important is the comparison of New Math books with further editions to see varieties. At first sight, Hayen’s demands seem true for the present data. As shown in figure 1, number and amount of *worked examples* increase in the New Math era. Also, the amount of *narrative examples* decreases. Nevertheless, the actual number of this property rises. To get more differential findings, the two analysed textbooks can be regarded separately. Comparing the distribution of the “types of explanation”, no significant difference between the two books can be proved. A different point of view arises from the comparison of “Gamma 7” (1978) and its previous textbook “Wir rechnen 7” (1964). Contrary to the demands, the number of *narrative explanations* increases. Reasons may be different, but they are not part of the current study.

Special tasks *giving advice how to handle a single mathematical problem* are used in the textbook “Gamma 7”. They present mathematics by concrete activities, guiding students to solve a mathematical problem. Having an amount of 12 % (of all tasks), this finding could support Hayens’ demand for no “reading book”, especially as this property increases in the New Math.

In the same way other characteristics of the New Math can be surveyed. Exemplarily, there is a significant increase of images, using colours to show the mathematical structure and not just make the book more colourful. Rising from 7 % up to 33 % in the New Math and decreasing in the following era, the useful assignment of colours in images can be seen as a property typical for the New Math era.

CONCLUSION

As shown above the instrument is able to show changes in textbooks in different eras. By now, there are only ideas about these changes. To assure those first results, analysing an adequate sample of further textbooks is necessary. Though the instrument produces quantitative data, there are very detailed findings about the way mathematics textbook are structured. This depends on the developed category system, considering a great number of aspects. Some of these categories produce information on their own, or they can be combined to get more detailed information about presentation and handling of images, tasks and explanations. As shown above there are very different ways of using this instrument and many questions about textbooks can be answered.

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