Inaugural Dissertation

Housing Markets, the Real Economy and Macroeconomic Policy: A Boundedly Rational Heterogeneous Agents Perspective

vorgelegt von

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Fakultät Sozial- und Wirtschaftswissenschaften

Otto-Friedrich-Universität Bamberg

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SYNOPSIS

The housing market plays a crucial role in economic and social life. The development of house prices has an impact on both the business cycle dynamics and the performance of the financial system. Against this background, the insolvency of Lehman Brothers, but also recently the financial woes of "Evergrande", has shown the dramatic consequences an overheating housing market and, associated therewith, the bubble formation may have on the real economy. Thus it is of utmost importance to gain a better understanding of the complex boom-and-bust behavior of housing markets. This doctoral thesis develops a new housing market model that links the expectation formation and learning behavior of heterogeneous and boundedly rational investors to an elementary housing market. The model is able to produce endogenous house price dynamics with significant bubbles and crashes. With this model framework it is then analyzed how fiscal and monetary policies may affect the housing market's steady state, its stability and out-of-equilibrium behavior. In particular, these policies include public housing construction programs, various interest rate rules by the central bank and different tax policies. The thesis also examines the interactions between the housing market and the real market. These approaches are discussed in four papers which are briefly summarized in the following.

In Paper 1 we explore whether public housing construction programs are able to stabilize the dynamics of housing markets. For this purpose, we use the standard user cost housing market model established by Dieci and Westerhoff that involves a rental and a housing capital market determining key relations between house prices, the housing stock and the rent level. The complex interplay between speculative and real forces can produce realistic housing market dynamics with lasting periods of overvaluation and overbuilding. Housing market investors may switch between an extrapolative and a regressive expectation rule depending on the prevailing market circumstances in order to forecast future house prices. In this model framework we explore the effects of four plausible and well-intended policy measures on the housing market dynamics. Our first intervention strategy implies that public housing construction increases in line with house prices. The second intervention strategy suggests increasing public housing construction in periods in which the housing market is overvalued. With the third strategy, public housing construction increases in periods in which the housing stock is below its fundamental value. The fourth strategy recommends rising public housing construction if house prices increase. Our analysis reveals that these four policy measures may turn out to be a mixed blessing. While they may lower average house prices, neither the amplitude of house price fluctuations nor their volatility can significantly be reduced by these programs. Another undesirable effect is that private housing construction is crowded out by public housing construction through depressed house prices.

In Paper 2 it is examined how the interest rate setting of central banks may affect the stability of housing markets. In contrast to Paper 1, we generalize the stylized housing market model by Dieci and Westerhoff along two important dimensions: First, we introduce a central bank that follows a simple leaning-against-the-wind interest rate rule. Based on this rule, the central bank may not only adjust the base (target) interest rate but also change the interest rate depending on mispricing in the housing market. The latter one means that the central bank increases (decreases) the interest rate if the housing market is overvalued (undervalued) in order to dampen (stimulate) the housing market. And second, we endogenize investors' variance beliefs, i.e. investors may update their variance beliefs. The analytical and numerical analysis reveal that increasing the base (target) interest rate has only limited effects on the housing market. The reason is that high interest rates reduce investors' demand pressure on house prices and, additionally, decrease the fundamental house price which may result in undesirable consequences for the real economy. However, central banks applying the leaning-against-the-wind interest rate rule have a great ability to control housing market fluctuations which significantly improve the stability of housing markets. This can be explained by countering the demand fluctuations driven by investors' destabilizing extrapolative expectations.

The goal of Paper 3 is twofold. First, a novel housing market model is proposed that seeks to explain the complex boom-and-bust behavior of housing markets. Second, this model is used to explore the extent to which policy makers may influence such dynamics by adjusting housing market-related taxes. For this purpose, we include the expectation formation and boundedly rational learning behavior of risk averse housing market investors into a standard user cost housing market setup. Investors choose between extrapolative and regressive expectation rules to forecast future house prices subject to an evolutionary fitness, given by the rules' past profitability. We analytically and numerically show that endogenous boom-and-bust housing market dynamics may arise if investors rely heavily on extrapolative expectations. Moreover, this model framework allows us to study the effect of five different tax policies. First, policy makers may impose a property tax on the value of houses periodically. Second, we propose a tax on rental income which has similar effects as a property tax. Third, policy makers may also levy a tax on owning housing stock. Fourth, policy makers may decide to tax housing construction. And fifth, we suggest a taxation of capital gains on housing. As it turns out, both a property tax and a tax on rental income have a stabilizing effect on housing markets. Unfortunately, it also affects the housing market's steady-state level which could have undesired consequences on the real economy.

Paper 4 demonstrates how a gradually linkage of the housing market and the real market affects the dynamics and stability of these two markets. The basic idea is that such a comprehensive model allows for the simultaneous development of business cycles and house price fluctuations. The user cost housing market model involves a rental and a housing capital market while the real market is represented by an aggregate-demand-aggregate-supply model augmented with a Taylor rule describing the interest setting of the central bank. Expectations in both submodels are formed by boundedly rational and heterogeneous agents choosing between two competing forecasting rules depending on the rules' past accuracy. It is shown that both the housing market model and the real market model are able to produce complex endogenous boom-and-bust dynamics. In addition, we explore how the dynamics changes by merging the housing market model with the real market model in four steps. First, the housing market's interest rate is no longer constant, but develops according to the Taylor rule. Simulations reveal that the house price and the output gap then exhibit pro-cyclical boom-and-bust dynamics over time. Second, the house price distortion is included in the Taylor rule. As a result, the volatility on the housing market can be dampened while the real market remains largely unaffected. In a third step, the house price trend is added to the inflation equation. As it turns out, the distortion in the housing market can be significantly reduced without having considerably effects on the real market. In a fourth step, the aggregate demand equation also depends on the house price distortion. The analysis reveals that the distortion in the housing market can be significantly reduced. However, the volatility of the real market is strongly increased by the adjusted aggregate demand equation.

Each of these four papers is independent from each other and can be read without any prior knowledge of the others. The first paper is jointly written with my Ph.D. supervisor Frank Westerhoff and is published in the *Journal of Economics and Statistics*. The second and third paper are jointly written with Frank Westerhoff and Noemi Schmitt and are published in *Macroeconomic Dynamics* and in the *Journal of Economic Behavior and Organization*. For all papers, each author contributed equally. Thus, my contribution share for Paper 1 is one half and for Paper 2 and Paper 3 one third each. The fourth paper is written by myself and has not been published yet.

CONTENTS

Paper 1

Regulating speculative housing markets via public housing construction programs: Insights from a heterogeneous agent model

Regulating speculative housing markets via public housing construction programs: Insights from a heterogeneous agent model

Carolin Martin and Frank Westerhoff

Abstract

Since the instability of housing markets may be quite harmful for the real economy, we explore whether public housing construction programs may tame housing market fluctuations. As a workhorse, we use a behavioral stock-flow housing market model in which the complex interplay between speculative and real forces triggers reasonable housing market dynamics. Simulations reveal that plausible and well-intended policy measures may turn out to be a mixed blessing. While public housing construction programs may reduce house prices, they seem to be incapable of bringing house prices much closer towards their fundamental values. In addition, these programs tend to drive out private housing constructions.

Keywords: Housing markets, boom-bust dynamics, extrapolative and regressive expectations, heterogeneous agent models, policy experiments, public housing construction programs
JEL classification: D84, R21, R31

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1. Introduction

The collapse of the U.S. housing market in 2006 was at least partially responsible for a global financial crisis that pushed many countries around the world into deep economic recession.[1] Figure 1 depicts the enormous dimension of the U.S. housing bubble. Real house prices in the U.S. almost doubled between 1997 and 2006 - and then the U.S. housing market crashed. Without question, it is thus

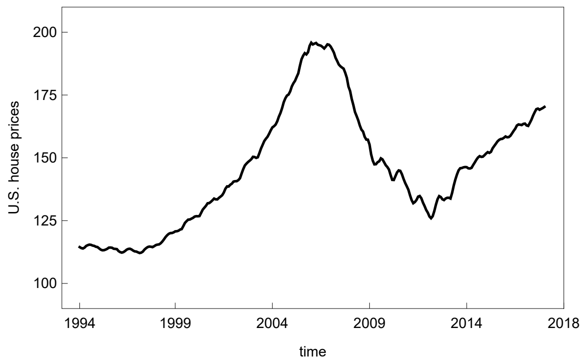

Figure 1: The evolution of Shiller's U.S. house price index from 1994 to 2017, based on 288 monthly observations. The dataset is available at http://www.econ.yale.edu/~shiller/data.htm.

of utmost importance to better understand the complex behavior of housing markets. According to Shiller (2015), the dynamics of housing markets depend to a large extent on market participants' expectations. For instance, a fundamentally justified upswing may turn into a speculative boom if investors' expectations become optimistic. Likewise, a dramatic housing market bust may occur if investors' expectations spontaneously turn pessimistic. Following this lead, a number of behavioral housing market models have been proposed in the recent past (see, e.g. Dieci and Westerhoff 2012, Geanakoplos et al. 2012, Bolt et al. 2014, Kouwenberg and Zwinkels 2014, Eichholtz et al. 2015, Burnside et al. 2016, Diks and Wang 2016, Baptista et al. 2016 and Chai et al. 2017), which explain the intricate dynamics of housing markets via the interactions and the expectation formation behavior of boundedly rational and heterogeneous investors.

In this paper, we seek to go one step further. We use the framework established by Dieci and Westerhoff (2016) as a workhorse to explore whether public housing construction programs are able to stabilize the dynamics of housing markets. Our analysis reveals that well-intended and, at least at first sight, plausible and properly implemented intervention policies may turn out to be a mixed blessing. While these programs may reduce average house prices, they fail to bring house prices much closer towards their fundamental values. Moreover, by depressing house prices, public housing construction drives out private housing construction. Overall, our analysis suggests that neither the amplitude of house price fluctuations nor their volatility can significantly be reduced by these programs, i.e. the boom-bust nature of housing markets seems to be a robust phenomenon.

[1] As is well known, Japan's housing market crash in 1991 also triggered a prolonged economic recession. See Shiller (2015) for many more historical examples.

More precisely, Dieci and Westerhoff (2016) develop a stock-flow housing market model in which speculative forces interact with real forces. The model's basic structure may be summarized as follows. According to the model's rental (flow) market, the rent level decreases with the housing stock. The model's capital (stock) market implies that house prices depend positively on investors' future house price expectations and on the rent level. Investors' expectation formation is crucial for the model's dynamics. In line with empirical evidence (Hommes 2011), investors use extrapolative and regressive expectation rules to forecast future house prices. In particular, more and more investors rely on regressive expectations when the housing market's misalignment increases. Furthermore, house prices depend negatively on the housing stock, which, in turn, depends positively on housing construction and negatively on housing depreciation. Finally, housing construction increases in line with house prices. As it turns out, the dynamics of their model is driven by a two-dimensional nonlinear stochastic map, i.e. house prices and housing stock in period t depend in a nonlinear way on house prices and housing stock in period $t-1$ plus some exogenous shocks.

Dieci and Westerhoff (2016) show that their model can produce realistic housing market dynamics with lasting periods of overvaluation and overbuilding. It is important to note that the underlying parameter setting implies that the model's fundamental steady state is unstable due to a Neimark-Sacker bifurcation, i.e. the dynamics of their stochastic housing market model has a strong endogenous component. In a nutshell, the functioning of this model may be summarized as follows. Suppose that the housing market is slightly overvalued. In such a situation, most investors rely on destabilizing extrapolative expectations. As a result, a bubble may emerge during which an increasingly larger number of housing constructions triggers a lasting and substantial overbuilding process. A major market correction may set in once sufficiently many investors switch to the stabilizing regressive expectation rule. Since the rent level has also become rather low due to the overbuilding process, a high housing stock meets a low housing demand. The market's consequent crash may become quite dramatic. Decreasing house prices turn investors' expectations increasingly pessimistic. Since it takes some time for housing depreciation to correct overbuilding, a lower and lower housing demand is confronted with a still high and only slowly decreasing stock of housing. Eventually, however, the housing market recovers. If house prices are very low, investors return to regressive expectations. In such a situation, they expect increasing house prices and are willing to buy more houses, also because the rent level eventually improves. The lower the stock of housing has become during the downturn, the faster the recovery of the housing market. It is important to note that the model's boom-bust dynamics depends on the combined effect of real and speculative forces, as is the case in real housing markets.

Since their model is able to match some important properties of actual housing markets, it seems suited for conducting a number of policy experiments. In particular, we are interested in the effects of four simple public housing construction programs that policymakers may use to seek to tame housing market dynamics. Our first intervention strategy implies that public housing construction increases in line with house prices. The second intervention strategy recommends increasing public housing construction in periods in which the housing market is overvalued while the third intervention strategy suggests increasing public housing construction in periods in which the housing stock is below its fundamental value. According to the fourth intervention strategy, public housing construction is positive if house prices increase. Although these rules affect the dynamics of housing markets in different ways, they have a number of common effects. First of all, all intervention strategies increase the housing stock. However, a higher housing stock depresses the rent level and, consequently, house prices decrease. Lower house prices reduce the incentive of private constructors to build new houses, yet this crowding out effect does not overcompensate public housing construction, i.e. the total stock of housing increases. In the unregulated housing market, house prices and the housing stock oscillate around their fundamental values. Due to public housing construction programs, the housing stock oscillates on a higher level in the regulated housing market. Similarly, lower house prices imply that house prices tend to fluctuate below their fundamental values. On average, we thus observe an increase in house price distortion, i.e. an increase in the average distance between a house price and its fundamental value, and, ergo, less efficient housing markets. Our analysis also reveals that none of the four intervention strategies manages to reduce the volatility of house prices. Of course, this does not mean that there do not exist other (more sophisticated) public housing construction programs which may achieve to tame housing markets. Moreover, the core predictions of our model should ideally be stress tested against a number of competitor models.

A few additional comments are in order. Our paper is, amongst others, motivated by the current house price hike in Germany. A popular policy remedy frequently reported in the news media is that policy makers simply have to increase the existing housing stock to counter the pressure on house prices. And, in fact, our initial guess was that such programs - even if difficult to implement in reality - may (easily) stabilize housing markets. Yet, quite to our astonishment, our simulations indicate the opposite. In this sense, it is interesting to recall that already Baumol (1961) points out that countercyclical intervention rules may fail to stabilize business cycles.

In general, however, this does not mean that simple feedback rules are unable to stabilize markets. Quite to the contrary: Westerhoff (2008) and, more recently, Franke and Westerhoff (2018) show that policymakers may stabilize financial markets either by trading against the current price trend or by targeting fundamental values. In doing so, they counter the behavior of speculators who rely on extrapolative expectations or they support the behavior of speculators who form regressive expectations, quite similar to the expectation feedback structure within the current housing market model. The stabilizing effects of the interventions in these environments is also surprising since the evolution of financial markets is close to a random walk and thus much more complex than business cycles or house price dynamics. As will become clearer in the sequel, a major reason for the apparent failure of public housing construction programs - with respect to their ability to stabilize housing markets - has to do with a peculiar property of housing markets, namely the durability of the housing stock. Note that a period featuring a larger number of public housing construction increases the current housing stock. Given that depreciation rates of housing markets are quite low, it may take years for overbuilding in a housing market to dissolve. Finally, the effectiveness of public housing construction programs may be better if they could also decrease the existing stock of housing. Since the demolition of housing stock seems to be politically unfeasible, we have abstained from experiments in this direction so far.[2] On the other hand, the effectiveness may also worsen. We assume that public housing construction programs are executed without any significant delays, i.e. they react to the last observable house price. In reality, the initiation of public housing construction programs may take a considerable length of time, which, in turn, can hamper their stabilizing effects even more.

Our paper adds to the literature which deals with the regulation of housing markets. Glaeser et al. (2008) demonstrate that housing bubbles are more likely to occur in places in which housing supply is relatively inelastic. To obtain fewer and shorter bubbles with smaller price increases, policy makers need to make housing supply more elastic, e.g. by providing more building land, as is also argued by Saiz (2010) and Mian and Sufi (2011). Floetotto et al. (2016) study the effect of government interventions that involve aspects such as house buyer tax credits, the introduction of taxes on imputed rents and the removing of tax deductions for mortgage interest payments. Grenadier (1995, 1996) demonstrates that the duration and magnitude of boom-bust cycles in the housing market depends on construction lags and the phenomenon of overbuilding, i.e. the addition of new supply in the face of already high housing stocks. Hence, a policy implication is to help builders to better time their constructions, e.g. by reducing housing market uncertainty. Geanakoplos et al. (2012) and Baptista et al. (2016) study the (dangerous) role of leverage for the emergence of housing market bubbles in a behavioral model with heterogeneous interacting agents.

The remainder of our paper is organized as follows. In Section 2, we first recap the housing market model by Dieci and Westerhoff (2016) and explain its functioning. In Section 3, we then explore the effectiveness of a number of public housing construction programs. In Section 4, we finally conclude our paper and point out some avenues for future research.

2. Boom-and-bust housing market dynamics

To understand the complex boom-and-bust behavior of housing markets, Dieci and Westerhoff (2016) develop a housing market model in which speculative forces interact with real forces. In Section 2.1, we recap their approach. In Section 2.2, we discuss the model's steady state and stability properties while we explore the functioning of their calibrated model in Section 2.3. As we will see, the model is able to produce reasonable housing market dynamics with lasting periods of overvaluation and overbuilding, providing a stage for investigating how certain public housing construction programs may influence the performance of housing markets. In Section 2.4, we introduce a number of summary statistics to measure the performance of housing markets and to evaluate the effects of public housing construction programs.

[2] However, there are some historical examples in which the housing stock has been destroyed to regulate housing markets. For instance, in Eastern Germany a substantial number of buildings (Plattenbauten) have been removed in the late 1990s and early 2000s.

2.1. The basic model setup

Dieci and Westerhoff (2016) propose a stylized stock-flow housing market model, based on Poterba (1984, 1991) and Wheaton (1999), with boundedly rational housing market investors. The housing market consists of two connected markets, namely a rental (or flow) market and a capital (or stock) market. To begin with the rental market, the demand for housing services D_t in each period t is defined as $D_t = k_0 R_t^{-k}$. This isoelastic demand function states that D_t is a decreasing function of the rent level R_t, the price of housing services. Parameter k_0 is a positive parameter and $k > 0$ outlines the constant elasticity of the demand for housing services. The flow of housing services S_t in the same period is proportionally dependent on the initial housing stock H_t, i.e. $S_t = bH_t$, where $b > 0$. Given these two requirements, the market clearing condition for housing services in period t, $D_t = S_t$, leads to the expression $k_0 R_r^{-k} = bH_t$. Accordingly, the rent level depends negatively on the current housing stock, so that

$$R_t = \frac{m_0}{H_t^m}, \tag{1}$$

where $m := \frac{1}{k} > 0$ and $m_0 := (\frac{b}{k_0})^{-\frac{1}{k}} > 0$. Parameter m represents the reciprocal value of the demand elasticity.

Concerning the market for housing capital, investors' demand for housing stock is modeled on the basis of a standard one-period mean-variance framework. Assume that a representative investor is able to spread his wealth between housing capital and an alternative riskless asset over the time horizon from t to $t+1$. From this perspective and given a hypothetical house price level P_t at time t, the investor's end-of-period wealth W_{t+1} is

$$W_{t+1} = (1+r)W_t + H_t^D(P_{t+1} + R_t - (1+r+\delta)P_t). \tag{2}$$

Note that H_t^D denotes the number of housing units held at time t, and all random variables are indexed with $t+1$. Parameter $\delta > 0$ stands for the housing depreciation rate and $r > 0$ is the interest rate. The latter comprises the profit on alternative assets, i.e. the opportunity cost of capital, as well as additional costs of owning a house.[3] According to the real estate literature (Himmelberg et al. 2005), the amount $r + \delta$ can be characterized as the user cost of housing.

The aim for housing market investors is to maximize the certainty equivalent for final wealth that leads to the following mean-variance optimization problem

$$max_{H_t^D}\left[E_t(W_{t+1}) - \frac{\lambda}{2}V_t(W_{t+1})\right]. \tag{3}$$

The functions $E_t(\cdot)$ and $V_t(\cdot)$ represent investors' expectation and variance conditional on final wealth W_{t+1}, and the coefficient of (absolute) risk aversion is indicated by $\lambda > 0$.

For simplicity, investors' beliefs about the variance of the price and payout at the end of the period is assumed to be constant over time, i.e. $V_t(P_{t+1}) = \sigma^2$. In addition, the market expectation of P_{t+1} is formed at the beginning of period t based on observations up to period $t-1$, and is defined

[3] These additional costs are expressed on a proportional basis and include, for instance, insurance and property taxes.

as $P^e_{t,t+1} := E_t(P_{t+1})$. As a result of these considerations, the solution of the above maximization problem is

$$H_t^D = \frac{P^e_{t,t+1} + R_t - (1 + r + \delta)P_t}{\lambda\sigma^2}. \tag{4}$$

Obviously, investors' optimal demand for housing stock is a downward-sloping function of current price P_t and user cost $r + \delta$. Moreover, it depends positively on investors' one-period-ahead price expectations $P^e_{t,t+1}$ as well as on current rent level R_t.

In the following, the total number of investors is set to one. The market clearing condition for the stock of housing

$$H_t^D = H_t \tag{5}$$

implies that

$$P_t = \frac{P^e_{t,t+1} + R_t - H_t\lambda\sigma^2}{1 + r + \delta}, \tag{6}$$

so that P_t outlines the market clearing price. The term $H_t\lambda\sigma^2$ in equation (6) denotes the risk premium and thus $R_t - H_t\lambda\sigma^2$ can be interpreted as the risk-adjusted rent.[4]

The evolution of the housing stock is described as

$$H_t = (1 - \delta)H_{t-1} + I_t^P + I_t^S. \tag{7}$$

I_t^P denotes private housing investments or, in other words, the amount of new private housing constructions in period t, and is defined as

$$I_t^P = q_0 P^q_{t-1}, \tag{8}$$

where $q_0 > 0$. Parameter $q > 0$ describes the constant elasticity of the supply of new housing. Accordingly, the number of new private constructions in period t is an upward-sloping function of the price of the previous period $t - 1$, where $I_t^P > 0$ for $P > 0$. Initially, public housing constructions are set to zero, i.e. $I_t^S = 0$.[5]

In combination with the equation for private housing investment (8) and with the assumption of $I_t^S = 0$, the development of the housing stock in (7) can be rewritten as

$$H_t = (1 - \delta)H_{t-1} + q_0 P^q_{t-1}. \tag{9}$$

A further important component of this model concerns investors' rule-based expectation formation behavior. Based on Dieci and Westerhoff (2016) and consistent with the approaches by Day and Huang (1990), Brock and Hommes (1998), Boswijk et al. (2007) and Westerhoff and Franke (2012), two important expectation rules can be distinguished. First, extrapolative expectations are defined as

[4] Note that (5) implies that the total demand for housing completely stems from the housing demand of investors. See Baptista et al. (2016) for a framework in which the demand for housing depends on renters, first-time buyers, home-movers and buy-to-let investors.

[5] According to (7), there are no differences in the quality of public and private housing constructions. See Baptista et al. (2016) for a framework in which the housing stock displays different quality levels. Moreover, we make the assumption that building land is not restricted in our model, although recent research (Glaeser et al. 2008, Saiz 2010 and Mian and Sufi 2011) point out the relevance of this aspect.

follows:

$$P^e_{t,t+1,E} = P_{t-1} + \gamma(P_{t-1} - P^*_1). \tag{10}$$

As can be seen, investors with extrapolative expectations forecast that the price will continue to move further away from its fundamental value P^*_1, and thus they have a destabilizing effect on prices.[6] Second, investors with regressive expectations act as a stabilizing force because they forecast that price movements will return towards the fundamental value, which can be illustrated by

$$P^e_{t,t+1,R} = P_{t-1} + \theta(P^*_1 - P_{t-1}). \tag{11}$$

Parameters $\gamma > 0$ and $0 \leq \theta \leq 1$ indicate the intensity of investors' reactions to the observed mispricing, i.e. the deviation of the price from its fundamental value.

Investors tend to switch between the extrapolative and the regressive expectation rule depending on the prevailing market circumstances. While investors seek to chase price trends, they also fear bursting bubbles. Investors therefore prefer the regressive expectation rule with increasing misalignments.[7] According to that, the share of investors w_t that has extrapolative expectations can be described by

$$w_t = \frac{1}{1 + V_t(P_{t-1} - P^*_1)^2}. \tag{12}$$

The weighting function (12) has a bell shape, which means that the closer the house price is to its fundamental value, the higher the market impact of the extrapolative rule. In this case, if the price moves away from its fundamental benchmark, most investors expect this gap to become greater and greater, and they seek to profit from it. But with increasing deviation from the fundamental value, the risk of a bursting bubble is assessed as being high by more and more investors. As a consequence, an increasing share of investors turn to the regressive rule. The greater the sensitivity of investors towards the perceived mispricing, expressed by $V_t > 0$, the more quickly such switching occurs.

Note that housing market booms are more pronounced and longer lasting than housing market busts. It is therefore assumed that investors switch more quickly and strongly to the regressive expectation rule if the price is below its fundamental value and simultaneously more strongly distorted. This idea is formalized by

$$V_t = \begin{cases} v_u + c_u(P_t - P^*_1) & P_t \geq P^*_1 \\ v_l - c_l(P_t - P^*_1) & P_t < P^*_1 \end{cases}, \tag{13}$$

where $v_l \geq v_u \geq 0$ and $c_l \geq c_u \geq 0$.

A weighted average of extrapolative and regressive expectations is called the aggregate market expectation $P^e_{t,t+1}$ and is expressed by

$$P^e_{t,t+1} = w_t P^e_{t,t+1,E} + (1 - w_t) P^e_{t,t+1,R} \tag{14}$$

[6]Throughout this paper, the fundamental steady states of house prices and housing stock are indexed with 1 due to the existence of further non-fundamental steady states.

[7]The concept of modeling changes in market sentiment with respect to market circumstances can be traced back to de Grauwe et al. (1993). See Dieci and He (2018) for an excellent survey about the role of expectations in heterogeneous agent models of financial and housing markets.

or

$$P^e_{t,t+1} = w_t(P_{t-1} + \gamma(P_{t-1} - P^*_1)) + (1 - w_t)(P_{t-1} + \theta(P^*_1 - P_{t-1})), \tag{15}$$

respectively, which completes the description of our model.

2.2. Steady states and stability analysis

Combining (1), (6), (9), (12), (13) and (15) reveals that the model dynamics is driven by a two-dimensional nonlinear map

$$P_t = F(P_{t-1}, H_{t-1}) \tag{16}$$

and

$$H_t = G(P_{t-1}, H_{t-1}). \tag{17}$$

Dieci and Westerhoff (2016) define a fundamental steady state (FSS), say (P^*_1, H^*_1), as a steady state at which expectations are realized, i.e. $\overline{P}^e = P^*_1$. Moreover, they show that the FSS is implicitly defined by

$$P^*_1 = \frac{R^*_1 - \lambda\sigma^2 H^*_1}{r + \delta} \tag{18}$$

and

$$H^*_1 = \frac{I^{P*}_1}{\delta}, \tag{19}$$

where $R^*_1 = \frac{m_0}{(H^*_1)^m}$ and $I^{P*}_1 = q_0(P^*_1)^q$. Accordingly, the fundamental price P^*_1 is equal to the discounted value of future (risk-adjusted) rents. Note that (18) gives rise to two no-arbitrage conditions. First, agents are indifferent between investing in the safe asset and in the housing market. Second, agents are indifferent between owning and renting a house. From an economic perspective, the FSS thus has desirable properties.[8] Moreover, the fundamental housing stock depends on the steady-state investment level and the depreciation rate such that housing depreciation is only set off by new housing construction.

The FSS becomes unstable due to a Neimark-Sacker bifurcation if the stability condition

$$\gamma < \frac{r + 2\delta}{1 - \delta} \tag{20}$$

is violated. In such a situation, endogenous quasi-periodic dynamics is set in motion. However, the FSS may also become unstable due to a Pitchfork bifurcation. If the stability condition

$$\gamma < r + \delta + \frac{(\lambda\sigma^2 - R'(H^*))I^{P'}(P^*_1)}{\delta} \tag{21}$$

is violated, the FSS becomes unstable and two new non-fundamental steady states (NFSS), say (P^*_2, H^*_2) and (P^*_3, H^*_3), where $P^*_2 < P^*_1 < P^*_3$ and $H^*_2 < H^*_1 < H^*_3$, are born. The housing market may then get stuck in a permanent bull or permanent bear market. Note that the Pitchfork

[8] Due to the isoelastic nature of the demand and supply curves, the FSS can only be expressed explicitly if investors are risk-neutral. For $\lambda = 0$, the FSS is given by $P^*_1 = (\frac{m_0}{r+\delta})^{\frac{1}{1+mq}}(\frac{\delta}{q_0})^{\frac{m}{1+mq}}$ and $H^*_1 = (\frac{m_0}{r+\delta})^{\frac{q}{1+mq}}(\frac{q_0}{\delta})^{\frac{1}{1+mq}}$.

bifurcation boundary is more binding if $(\lambda\sigma^2 - R'(H_1^*))I^{P'}(P_1^*) < \frac{\delta^2(1+r+\delta)}{1-\delta}$. From an empirical perspective, however, the Neimark-Sacker bifurcation is more relevant than the Pitchfork bifurcation (see Section 2.3). Nevertheless, the Pitchfork bifurcation may play an important role for the model's global behavior.[9] Overall, these results indicate that the stability of housing markets depends on speculators' extrapolative behavior and a number of real factors such as the interest rate and the depreciation rate.

2.3. Unregulated housing market dynamics

Dieci and Westerhoff (2016) explore how the complex interplay between real and speculative forces shapes the dynamics of housing markets. A time period in their calibrated model corresponds to a quarter of a year. Table 1 provides an overview of the model parameters. The real parameters, e.g. the interest rate or the supply elasticity, are based on empirical observations. The remaining model parameters, in particular those that capture agents' expectation formation, are fixed such that the model dynamics mimics a number of important features of actual housing markets. Note that a small amount of exogenous noise is added to the house price equation (shocks are normally distributed with mean zero and standard deviation σ_p). See Dieci and Westerhoff (2016) for more details.

Table 1: Base parameter setting used in the simulations

Interest rate	$r = 0.005$
Depreciation rate	$\delta = 0.005$
Extrapolation parameter	$\gamma = 0.15$
Regression parameter	$\theta = 0.125$
Price volatility (beliefs)	$\sigma = 2$
Absolute risk aversion coefficient	$\lambda = 0.00125$
Additive price noise	$\sigma_p = 2$
Switching parameters	$v_l = v_u = 0.01, c_l = 0.01, c_u = 0$
Demand elasticity	$m = 4, m_0 = 1.5 * 10^8$
Supply elasticity	$q = 4, q_0 = 5 * 10^{-9}$

Note that the calibrated model parameters imply that the FSS is given by $P_1^* = H_1^* = 100$. For completeness, we also mention that the level of private housing construction at the FSS amounts to 0.5 and that all agents form extrapolative expectations. Moreover, $\gamma = 0.15$ implies that the FSS is unstable due to a Neimark-Sacker bifurcation, which occurs at $\gamma^{NS} \approx 0.0151$. Since a Pitchfork bifurcation would require that γ exceeds $\gamma^{PF} = 0.27$, we also know that the FSS is the unique steady state of the calibrated model. In the absence of exogenous noise, the model's house price and housing stock display strong oscillatory fluctuations around their FSS levels.[10]

[9] Theoretically, the model's dynamical system may also give rise to a Flip bifurcation. Instead of a locally stable FSS, we then observe an attracting period-two cycle. Since the stability condition $(\lambda\sigma^2 - R'(H_1^*))I^{P'}(P_1^*) < (2-\delta)(2+r+\delta+\gamma)$ is always fulfilled for realistic parameter values, we can safely neglect this scenario.

[10] According to Shiller (2015), the instability of housing markets depends strongly on market participants' expectations. As pointed out by an anonymous referee, however, the magnitude of housing market bubbles differs across countries and time. A model extension in which the extrapolation parameter γ is time-dependent can account for these observations.

Figure 2 depicts a stochastic simulation run of the model. Since one period corresponds to a quarter of a year, the 200 observations represent a time span of 50 years. The panels show from top to bottom the evolution of house prices (black line) and housing stock (green line), the market share of regressive expectation and private (black line) and public housing constructions (red line), respectively. The gray line featured in the top panel represents the fundamental value of both house prices and housing stock. As can be seen, the housing market is quite volatile and subject to significant bubbles and crashes. Furthermore, there is a mismatch between the fluctuations and turning points of house prices and housing stock. Recall that private housing construction depends on past prices. Hence, the current housing stock may still increase when house prices start to deflate, provided that new housing construction offsets housing depreciation. As it turns out, the level of overbuilding reached during a boom period is, along with investors' price expectations, a crucial factor for the timing and size of housing market crashes.[11]

To be more precise, the functioning of the model - despite being quite intricate due to the complex interplay between speculative and real forces - may be explained as follows. First of all, note that the majority of investors relies on extrapolative expectations when house prices are near their fundamental value. Since extrapolative expectations are destabilizing, we may observe the start of a bubble. In our simulation run, such a development takes place shortly before period 50. As the price runs away from its fundamental value, the market share of regressive expectations increases. This has a stabilizing impact on the dynamics, as can be seen shortly after period 50. However, a real crash typically occurs only in this model if the housing stock also reaches high levels. In fact, shortly after period 50, house prices first recover before they begin to tumble. Recall that a high housing stock implies low rents. Hence, in periods with high house prices and high levels of housing stock, investors predict a price decline, and it is economically uninteresting (low rents) to invest in the housing market. Both effects together depress housing demand which, in turn, pushes house prices downwards. Moreover, once house prices drop below the FSS, investors relying on extrapolative expectations become pessimistic and predict a further price decline. Since the housing stock remains high for a while (the depreciation rate is low), the rent level does not recover. For this reason, the housing market decline continues up to period 100. Now the situation starts to change. Investors switch to regressive expectation and predict an increase in house prices. Eventually, housing demand improves and house prices increase again.

2.4. Performance of the housing market

To evaluate the effects of public housing construction programs and to better understand the functioning of our model, we introduce nine summary statistics. The house price distortion D^P, given by the average absolute relative deviation between house price P_t and its fundamental value P_1^*, is computed

[11] Dieci and Westerhoff (2016) provide a more detailed discussion of the phenomenon of overbuilding and its relevance for housing market crashes. Shiller (2015) presents an interesting overview of the boom-bust behavior of the U.S. housing market and a number of selected cities and metropolitan areas. Similar accounts, also highlighting the relevance of the overbuilding phenomenon, are given by Glaeser et al. (2008), Grenadier (1995, 1996) and DeCoster and Strange (2012).

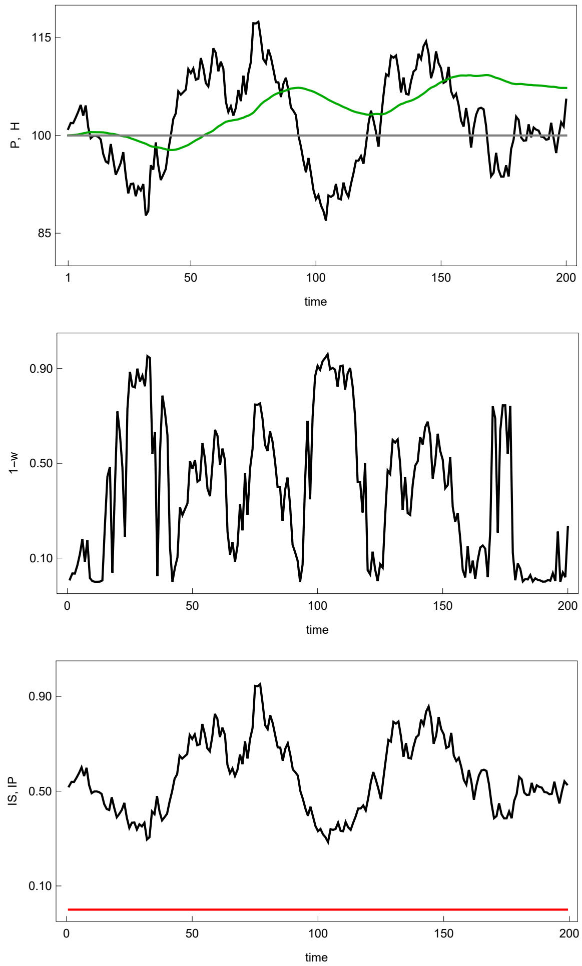

Figure 2: A simulation run under i.i.d. normal additive noise on house prices. The panels show from top to bottom the dynamics of house prices (black line) and housing stock (green line), market impact of regressive expectations, and private (black line) and public housing construction (red line). Parameters as in Table 1.

by

$$D^P = \frac{1}{T}\sum_{t=1}^{T}\frac{|P_t - P_1^*|}{P_1^*}, \tag{22}$$

where T denotes the sample length. Similarly, the housing stock distortion D^H is captured by

$$D^H = \frac{1}{T}\sum_{t=1}^{T}\frac{|H_t - H_1^*|}{H_1^*}, \tag{23}$$

representing the average absolute relative deviation of housing stock H_t from its fundamental value H_1^*. Furthermore, the volatility of house price V is defined as

$$V = \frac{1}{T}\sum_{t=1}^{T}\frac{|P_t - P_{t-1}|}{P_{t-1}}, \tag{24}$$

reflecting the average absolute relative house price change. Average house price $\overline{P}$ and average housing stock $\overline{H}$ can be described by

$$\overline{P} = \frac{1}{T}\sum_{t=1}^{T} P_t \tag{25}$$

and

$$\overline{H} = \frac{1}{T}\sum_{t=1}^{T} H_t, \tag{26}$$

respectively. In addition, average private housing construction $\overline{I^P}$ and average public housing construction $\overline{I^S}$ are characterized by

$$\overline{I^P} = \frac{1}{T} = \sum_{t=1}^{T} I_t^P \tag{27}$$

and

$$\overline{I^S} = \frac{1}{T} = \sum_{t=1}^{T} I_t^S, \tag{28}$$

respectively. To understand the functioning of the model, we also keep track of the average market share of extrapolators

$$\overline{w} = \frac{1}{T}\sum_{t=1}^{T} w_t \tag{29}$$

as well as the average rent level

$$\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t. \tag{30}$$

Computing these nine summary statistics on the basis of $T = 50,000$ observations (after omitting a larger transient period) reveals that the unregulated housing market is not efficient.[12] In particular, $D^P = 0.062$ and $D^H = 0.051$ imply that house prices and the housing stock deviate significantly from their fundamental levels. Note also that house prices are quite volatile ($V = 0.016$). Moreover, the average house price ($\overline{P} = 100.30$) is slightly higher than the fundamental house price ($P_1^* = 100$). This stems back from the fact that the appearance of bull and bear markets is asymmetric, due to speculators' switching between expectation rules. High house prices induce more average private housing construction ($\overline{I^P} = 0.52$) than in the steady state ($I_1^{P^*} = 0.5$). This, in turn, results in a higher average housing stock of $\overline{H} = 104.39$ compared to the fundamental value of $H_1^* = 100$. Due to the high housing stock, the average rent level $\overline{R} = 1.29$ is lower than the steady-state rent level $R_1^* = 1.5$. Initially, average public housing construction $\overline{I^S}$ is set to zero, i.e. $\overline{I^S} = 0$, while the average market share of extrapolators is given by $\overline{w} = 0.56$.

3. Effects of public housing construction programs

So far, we have set public housing construction to zero. Hereafter, we explore the effectiveness of four hypothetical public housing construction programs. In Section 3.1, we present the first intervention strategy, which states that public housing construction depends positively on the level of house prices. In Section 3.2, we consider the second intervention strategy, which implies that public housing con-

[12] Alternatively, one may compute these summary statistics as averages over 250 simulation runs with 200 observations, deleting each time a larger transient period.

struction increases in periods of overvaluation of the housing market. In Section 3.3, we introduce the third intervention strategy, which recommends increasing public housing construction in periods in which the housing stock is below its fundamental value, while the fourth intervention strategy suggests that public housing construction is positive if house prices increase, as described in Section 3.4.

3.1. Dependency on price levels

The first intervention strategy we evaluate implies that public housing construction increases in line with house prices, and can be described by

$$I_t^S = xP_{t-1}, \tag{31}$$

where $x > 0$ is defined as the public intervention parameter. Accordingly, the higher the house prices of the previous period P_{t-1}, the higher the level of public housing construction I_t^S.

To illustrate the performance of this intervention strategy, we depict a simulation run with 200 observations in Figure 3. Since we want to compare the housing market dynamics that arise in the unregulated housing market with those that occur in the regulated market, we use the same design as in Figure 2. The intervention parameter is set to $x = 0.0015$. The first panel shows the evolution of house prices (black line) and housing stock (green line). Apparently, periods of lasting appreciation still alternate with periods of lasting depreciation, i.e. the simulated price dynamics continues to display bubbles and crashes. However, compared to Figure 2, the housing stock oscillates on a higher level and the level of house prices is lower, implying that periods of undervaluation persist longer than periods of overvaluation. For instance, shortly before period $t = 200$, house prices are below the fundamental value in the regulated market and fluctuate very close around P_1^* in the unregulated market. Here again, crashes are particularly pronounced if the housing stock reaches high levels, as can be seen around period $t = 100$. The second panel shows the corresponding market share of regressive expectations which, compared to Figure 2, is lower in periods of overvaluation and higher if house prices are below the fundamental value, due to the overall drop of house prices. This can be seen between periods $t = 50$ and $t = 90$. In the regulated market, house prices fluctuate closer to the fundamental value than in the unregulated market. Therefore the market share of regressive expectations is lower, and most investors follow the destabilizing extrapolative expectation rule.[13] It becomes evident from the bottom panel that private housing construction (black line) fluctuates on a lower level in the regulated housing market. Obviously, public housing construction depresses house prices, driving out private housing construction.

In the following, we use our nine statistics to explain the working of this strategy in more detail. First of all, public housing construction ($\overline{I^S} = 0.14$) increases the housing stock, which is now at the

[13] As will become more clear in the sequel, public housing construction programs affect investors' expectation formation behavior. In particular, all four intervention strategies increase the price distortion in the housing market, prompting investors to put more weight on regressive expectations. Nevertheless, one may argue that investors' learning/adaptation behavior could be more pronounced, e.g. by a change of their expectation rules or their switching behavior. This critical aspect, reminiscent of the famous Lucas critique, is discussed in more detail in Franke and Westerhoff (2018).

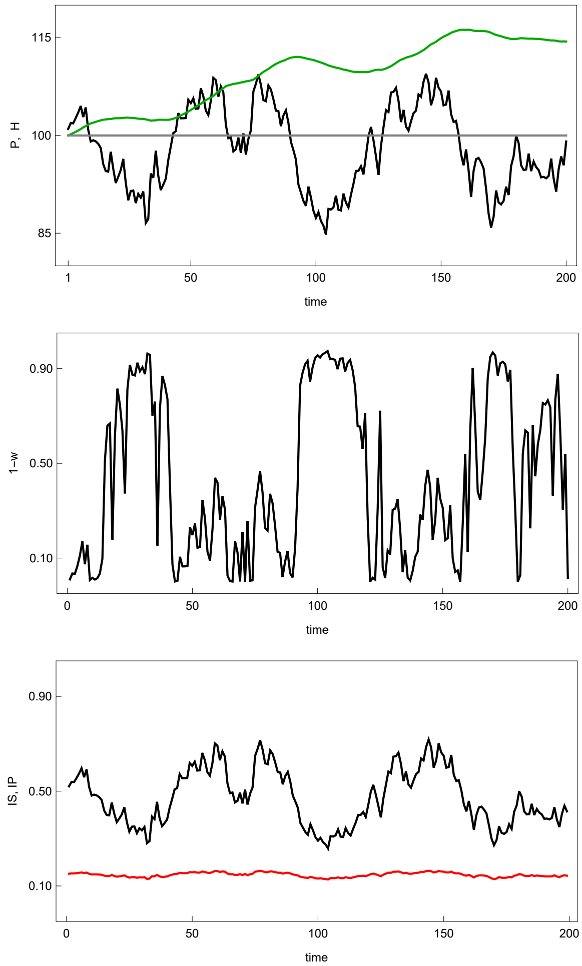

Figure 3: The dynamics of the housing market model, including the first intervention strategy for $x = 0.0015$. The panels show from top to bottom the dynamics of house prices (black line) and housing stock (green line), the market impact of regressive expectations, and private (black line) and public housing construction (red line). The simulation run is based on 200 observations, and parameters are as in Table 1.

average level of $\overline{H} = 109.87$. Consequently, the housing stock distortion increases to $D^H = 0.099$. The higher supply of housing depresses the rent level ($\overline{R} = 1.04$) which, in turn, reduces house prices, meaning that the average house price level drops to a value of $\overline{P} = 94.59$. Hence, on average the house price distortion rises slightly to $D^P = 0.066$ which, in turn, results in a decreased average market share of extrapolators ($\overline{w} = 0.39$). Due to the lower level of house prices, private constructors have less incentives to build new houses and, consequently, private housing construction decreases to an average value of $\overline{I^P} = 0.41$. However, the total stock of housing increases since the crowding out effect does not overcompensate public housing construction. On average, the volatility of house prices increases ($\overline{V} = 0.018$).

In Figure 4, we show how the model performance depends on our first public intervention strategy. The panels illustrate from top left to bottom right the behavior of our nine statistics D^P, D^H, $\overline{P}$, $\overline{H}$, $\overline{I^P}$, $\overline{I^S}$, $\overline{w}$, $\overline{R}$ and V for increasing values of parameter x. As can be seen, the results from Figure 3 are confirmed by Figure 4. Obviously, average public housing construction $\overline{I^S}$ increases in line with parameter x (panel 6). Thus, the stronger the intervention, the higher average housing stock $\overline{H}$ is

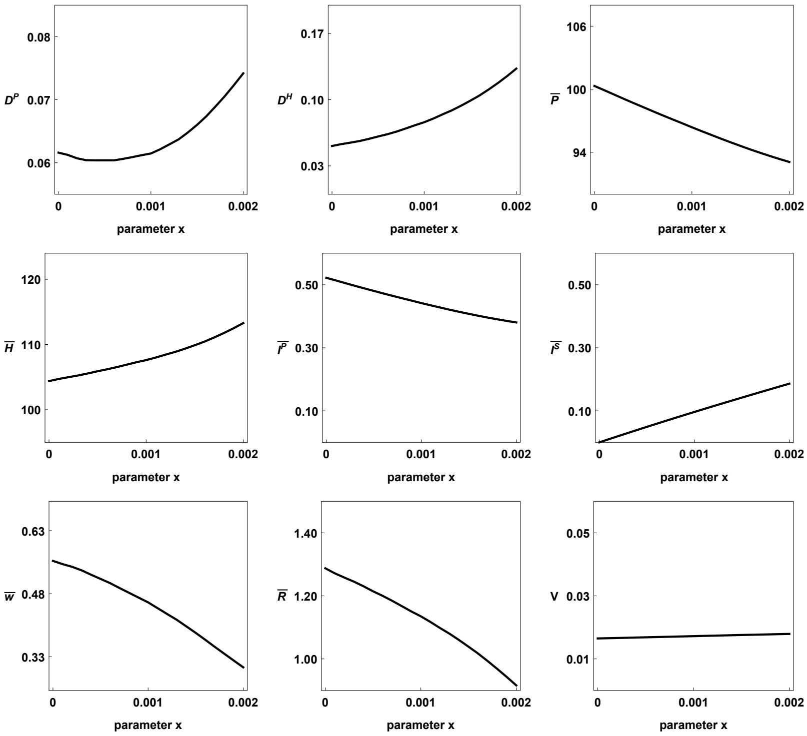

Figure 4: The impact of the first intervention strategy on the performance of the housing market. The panels reveal how the nine statistics D^P, D^H, $\overline{P}$, $\overline{H}$, $\overline{I^P}$, $\overline{I^S}$, $\overline{w}$, $\overline{R}$ and V depend on the intervention parameter x. The computation of the nine statistics is based on 50,000 observations and the base parameters are as in Table 1.

(panel 4). Consequently, both average rent level $\overline{R}$ (panel 8) and average house price $\overline{P}$ (panel 3) decrease with parameter x. Due to the crowding out effect, average private housing construction $\overline{I^P}$ decreases in line with parameter x (panel 5). However, the rise of $\overline{I^S}$ overcompensates the decline of $\overline{I^P}$ so that the average housing stock increases in line with parameter x. As can be seen in the first panel, the house price distortion D^P initially marginally declines in line with parameter x, but as of $x = 0.001$, the price distortion increases sharply. Similarly, the housing stock distortion D^H (panel 2) also grows in line with parameter x. Due to the higher price distortion, the average market share of extrapolators (panel 7) decreases in line with parameter x. To sum up, the first public intervention strategy fails to stabilize the dynamics in the housing market, as can also be seen in the last panel, which shows a slightly growing volatility of house prices as parameter x increases.[14]

3.2. Measures against positive mispricing

According to our second intervention strategy, the level of public housing construction increases in line with the overvaluation of the housing market, i.e. the further house prices move above the fundamental

[14] As a robustness check, we also investigated the dynamics of the housing market for variants of the first intervention strategy. For this purpose, we conditioned the intervention strategy on threshold values of house prices and housing stock. Furthermore, we also explored the effects of nonlinear intervention functions. However, none of these variants led to a significant reduction of the distortion of house prices. Likewise, we increased the supply-response lag of public and private housing construction. Simulations indicate that market efficiency decreases further if housing supply becomes more sluggish.

price, the higher the level of public housing construction. In case of undervaluation, public housing construction is set to zero. This strategy can be characterized by

$$I_t^S = \begin{cases} x(P_{t-1} - P_1^*) & P_{t-1} > P_1^* \\ 0 & P_{t-1} \leq P_1^* \end{cases}. \tag{32}$$

The simulation run depicted in Figure 5 is based on $x = 0.05$. All other parameters used in the simulation are defined as in Table 1. For comparability, the design of Figure 5 is as in Figures 2 and 3. Figure 5 reveals that house prices oscillate around the fundamental value $P_1^* = 100$ (gray line) and

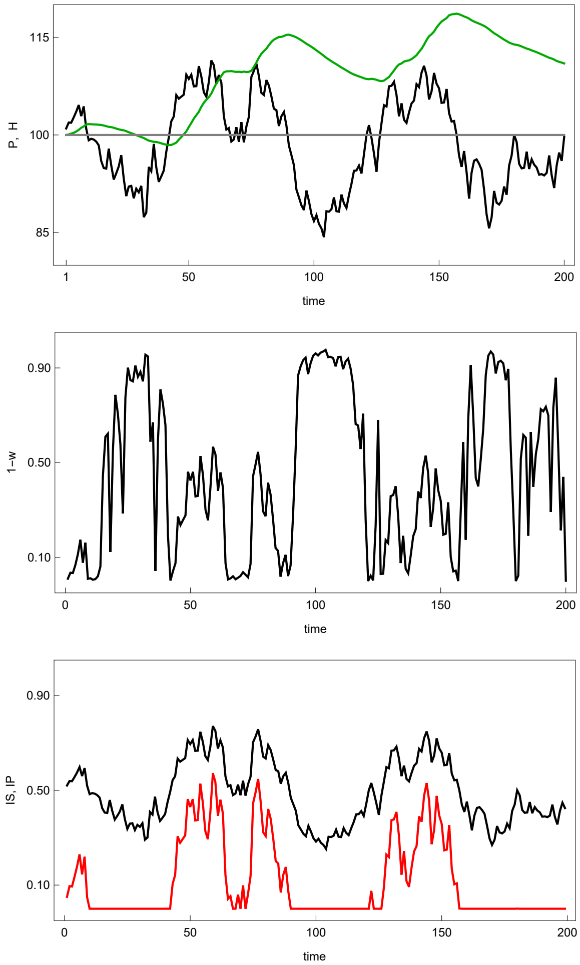

Figure 5: The dynamics of the housing market model, including the second intervention strategy for $x = 0.05$. The panels show from top to bottom the dynamics of house prices (black line) and housing stock (green line), the market impact of regressive expectations, and private (black line) and public housing construction (red line). The simulation run is based on 200 observations, and parameters are as in Table 1.

that significant bubbles and crashes may occur. However, the average house price level is lower than in the unregulated housing market, i.e. house prices fluctuate more frequently below P_1^*. In addition, fluctuations in the housing stock are more pronounced and H_t reaches higher levels than in Figure 2 (above $H = 115$). Therefore, housing market crashes turn out to be stronger, which results in sharp declines in the housing stock. This development can be observed shortly after period 150. After the housing stock rises to over 115, house prices decline sharply and the housing stock falls again. Due to the lower level of house prices, the market share of regressive expectations is lower in periods in which the housing market is overvalued and is higher in periods of undervaluation. This can be seen shortly before period 200. In the regulated market, house prices are below the fundamental value, therefore the market share of regressive expectations is higher than in the unregulated market in which P_t fluctuates very close to P_1^*. Due to the crowding out of private housing construction by public housing construction, I_t^P fluctuates on a lower level. The more the housing market is overvalued, the higher public housing construction is. In periods in which P_t is equal to or below the fundamental value, I_t^S is zero.

The functioning of the second intervention strategy is surprisingly similar to the first intervention strategy. Due to public housing construction ($\overline{I^S} = 0.08$) in periods of overvaluation, the housing stock rises sharply after a short time lag and reaches an average level of $\overline{H} = 107.6$. The distortion of the housing stock also increases to $D^H = 0.081$. This leads to a strong decrease in the average rent level ($\overline{R} = 1.16$), and thus to a sharp drop in house prices ($\overline{P} = 97.16$). As soon as P_t falls below the fundamental value, there is no more public housing construction, i.e. $I_t^S = 0$. Consequently, the housing stock declines with the result that house prices revert towards the fundamental value. Once P_t is higher than the fundamental value, public housing construction and hence H_t increases until the housing market crashes again and the story repeats itself. In fact, this is exactly what we observe between period 50 and 100. Since public housing construction crowds out private housing construction, I_t^P drops to an average value of $\overline{I^P} = 0.46$. Moreover, the average market share of extrapolators falls to $\overline{w} = 0.49$, and the house price distortion decreases slightly to $D^P = 0.061$. Since we observe only a small improvement in D^P but otherwise a deterioration of the other statistics, especially the volatility of house prices increases to $V = 0.017$, the second intervention strategy is also incapable of improving the performance of the housing market.

Figure 6 shows that there is no value of the intervention parameter at which the dynamics of the housing market can be stabilized. Note that the nine statistics depicted in Figure 6 evolve very similarly to those depicted in Figure 4. To be more precise, as the intervention parameter increases, so do average public housing construction, average housing stock and the distortion of the housing stock (panel 6, 4 and 2, respectively). Consequently, the average rent level $\overline{R}$ (panel 8) and the average price level (panel 3) are in constant decline. Up to $x = 0.03$, the price distortion decreases slightly (panel 1). As parameter x increases further, D^P rises strongly, resulting in a decreasing average market share of extrapolators (panel 7). Average private housing construction $\overline{I^P}$ (panel 5) decreases as $\overline{I^S}$ increases, i.e. there is a crowding out effect. Finally, the volatility of house prices rises slightly with increasing parameter x (last panel). In summary, we can say that the second intervention strategy also fails to stabilize the dynamics on the housing market.[15]

[15] We also examined variations of the second intervention strategy. For instance, we replaced the fundamental price by different threshold price levels $\hat{P}$. This means that the further price P_{t-1} moves away from $\hat{P}$ the higher the level of public housing construction is. If $P_{t-1} \leq \hat{P}$, it follows that $I^S = 0$. However, this alternative strategy also failed to stabilize the housing market.

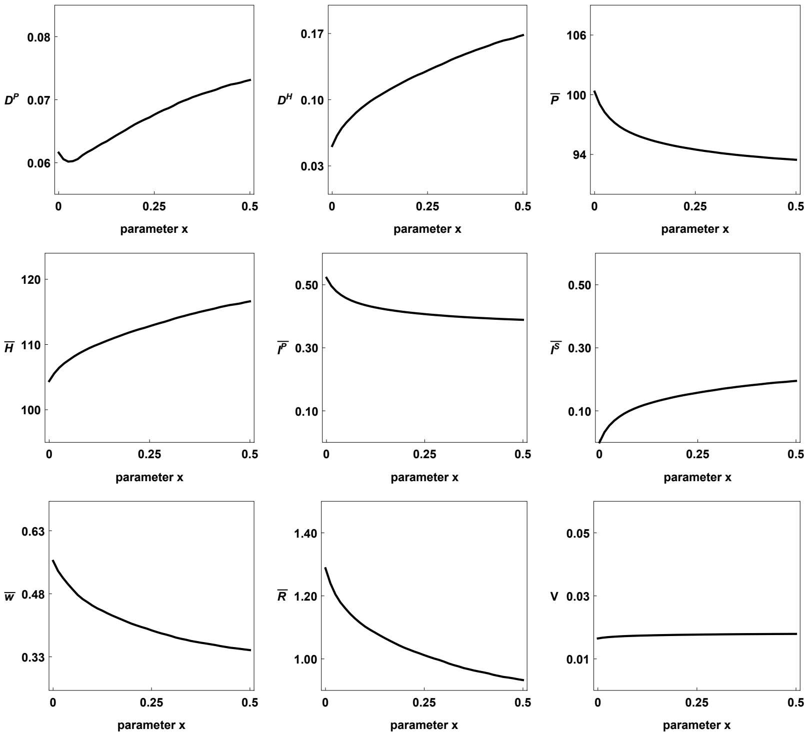

Figure 6: The impact of the second intervention strategy on the performance of the housing market. The panels reveal how the nine statistics D^P, D^H, $\overline{P}$, $\overline{H}$, $\overline{I^P}$, $\overline{I^S}$, $\overline{w}$, $\overline{R}$ and V depend on intervention parameter x. The computation of the nine statistics is based on 50,000 observations, and the base parameters are as in Table 1.

3.3. Countering underbuilding

The third intervention strategy assumes that public housing construction increases in periods in which the housing stock is below its fundamental value, i.e. policymakers seek to counteract housing shortages. However, if the housing stock is equal to or greater than H_1^*, there is no public housing construction, i.e. $I_t^S = 0$. This relationship can be formulated by

$$I_t^S = \begin{cases} x(H_1^* - H_{t-1}) & H_1^* > H_{t-1} \\ 0 & H_1^* \leq H_{t-1} \end{cases}. \tag{33}$$

Figure 7 depicts a simulation run with 200 observations, following the design of Figures 2, 3 and 5. The base parameter setting is as in Table 1, and the intervention parameter is set to $x = 4$. As can be seen, the dynamics only change very little compared with those that occur in the unregulated market. The reason for this is that in our simulation run, the housing stock falls slightly below its fundamental value only between period $t = 25$ and $t = 50$, and thus public public housing construction can only be observed in this period. Consequently, the housing stock is slightly higher and the price level is slightly lower than in Figure 2. As a result, this intervention has no effect on the further evolution of the dynamics. Changes in the development of the market impact of regressive expectations are virtually invisible. The third panel reveals that public housing construction occurs in periods in which the housing stock is below its fundamental value $H_1^* = 100$, which is the case before $t = 50$. Due to

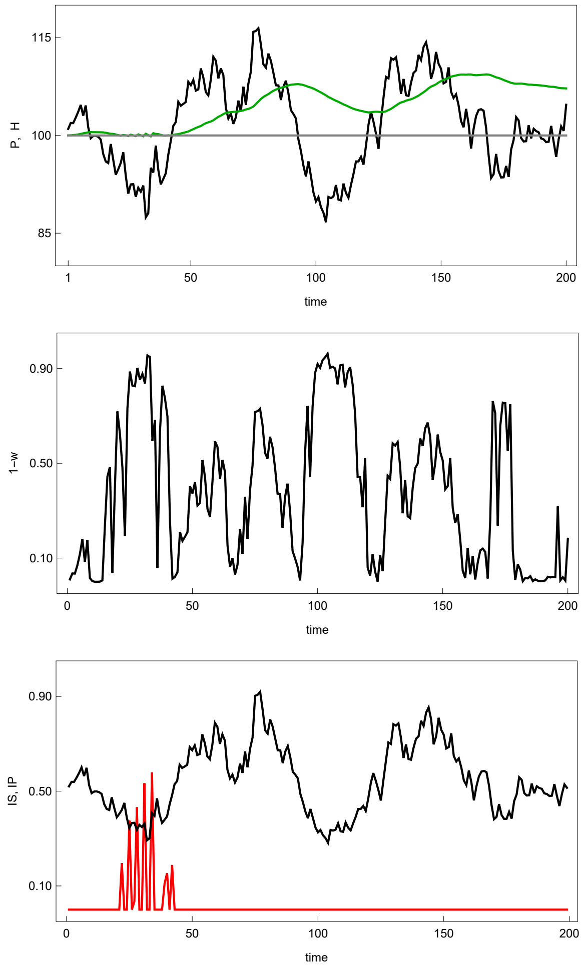

Figure 7: The dynamics of the housing market model, including the third intervention strategy for $x = 4$. The panels show from top to bottom the dynamics of house prices (black line) and housing stock (green line), the market impact of regressive expectations, and private (black line) and public housing construction (red line). The simulation run is based on 200 observations, and parameters are as in Table 1.

the small effect of public intervention, private housing construction changes negligibly.

The third intervention strategy functions according to similar principles as the previous two strategies. Public housing construction in periods in which the housing stock is below its fundamental value leads to an increased average housing stock ($\overline{H} = 104.7$). Since public housing construction is for the underlying intervention parameter rather low ($\overline{I^S} = 0.016$), all other statistics change only marginally. As a result of public intervention, H moves closer to the fundamental value, whereby the distortion of the housing stock decreases to $D^H = 0.047$. Consequently, the average rent level declines ($\overline{R} = 1.27$) as well as the average house price level ($\overline{P} = 99.64$), which, in turn, causes the house price distortion to fall to a value of $D^P = 0.06$. Due to the crowding out effect of public housing construction, the average value of private housing construction falls to $\overline{I^P} = 0.51$. In addition, the average market share of extrapolators is smaller than in Figure 2 ($\overline{w} = 0.55$). The third intervention strategy is also incapable of reducing the volatility of house prices ($V = 0.017$).

Figure 8 demonstrates that there is no value of intervention parameter x that can improve the values of the nine statistics. As parameter x rises, public housing construction increases (panel 6), as

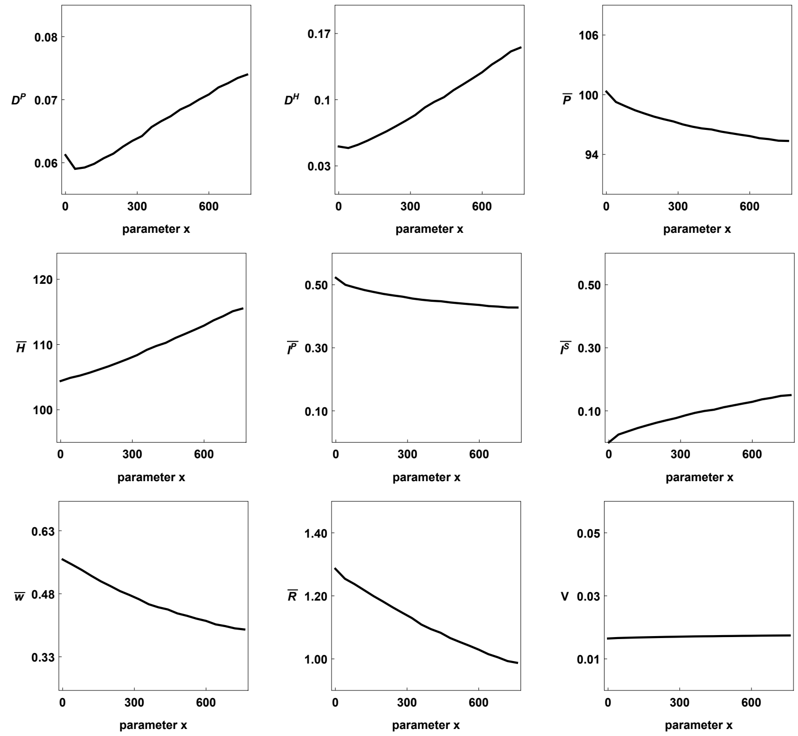

Figure 8: The impact of the third intervention strategy on the performance of the housing market. The panels reveal how the nine statistics D^P, D^H, $\overline{P}$, $\overline{H}$, $\overline{I^P}$, $\overline{I^S}$, $\overline{w}$, $\overline{R}$ and V depend on intervention parameter x. The computation of the nine statistics is based on 500,000 observation runs, and the base parameters are as in Table 1.

does the average housing stock (panel 4). As a result, both the average rent level and the average price level drop as the intervention parameter rises (panels 8 and 3, respectively). After a slight improvement of the two statistics D^P and D^H (panels 1 and 2), both of them rise sharply as parameter x increases further. Due to the increasing price distortion, the average market share of extrapolators falls in line with parameter x (panel 7). Again, the rising level of average public housing construction drives out private housing construction and, consequently, $\overline{I^P}$ declines (panel 5). As the volatility of house prices also increases slightly in line with increasing public intervention (panel 9), the third intervention strategy has no stabilizing effect on the housing market dynamics.[16]

3.4. Anti-trend measures

The fourth strategy proposes that public housing construction increases if house prices increase. In case of falling or constant house prices, there is no public housing construction. This can easily be defined by

$$I_t^S = \begin{cases} x(P_{t-1} - P_{t-2}) & P_{t-1} > P_{t-2} \\ 0 & P_{t-1} \leq P_{t-2} \end{cases}. \tag{34}$$

[16] We also evaluated alternative versions of the third intervention strategy. For instance, we supersede the fundamental housing stock H_1^* by a threshold value of housing stock $\hat{H}$. In this case, public housing construction is positive in periods in which $\hat{H} > H_{t-1}$ and set to zero otherwise. This variation does not result in stable housing market dynamics either.

To illustrate the results of this intervention strategy, we set $x = 0.15$ and show in Figure 9 the dynamics of the consequent housing market. The design is as in Figures 2, 3, 5 and 7, and the base parameters are specified as in Table 1. It can be seen from the top panel of Figure 9 that house prices

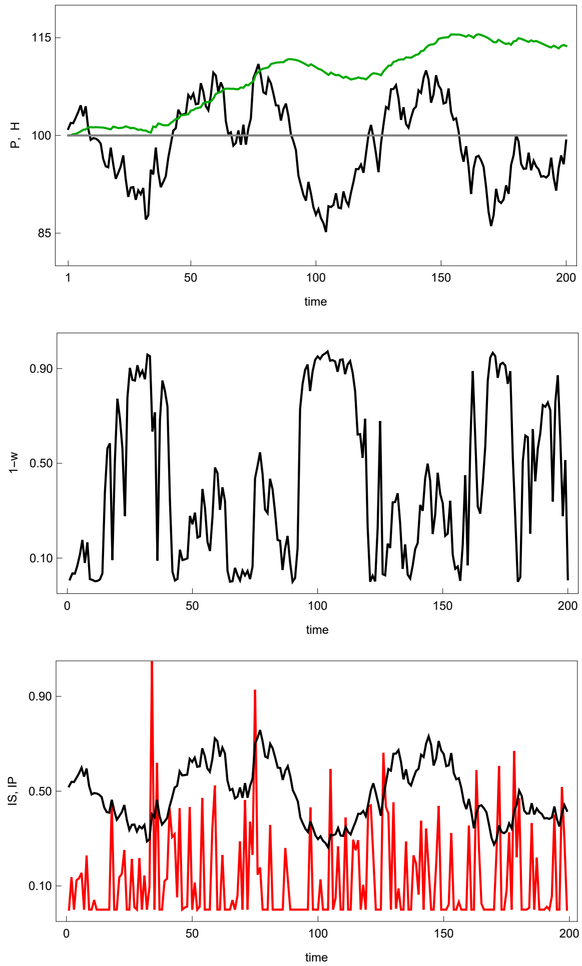

Figure 9: The dynamics of the housing market model, including the fourth intervention strategy for $x = 0.15$. The panels show from top to bottom the dynamics of house prices (black line) and housing stock (green line), the market impact of regressive expectations, and private (black line) and public housing construction (red line). The simulation run is based on 200 observations, and parameters are as in Table 1.

fluctuate on a lower level around the fundamental price $P_1^* = 100$, while the housing stock is always above $H_1^* = 100$, and thus reaches higher values than in Figure 2. Since investors put less weight on extrapolative expectations in undervalued markets, the overall drop of house prices leads to an increase in the market impact of regressive expectations. The bottom panel shows that public housing construction reaches repeatedly high levels and is strongly fluctuating due to the significant house price fluctuations. In contrast, the level of private housing construction decreases to a lower level.

The mode of action of the fourth intervention strategy is again similar to the effects of the previous three strategies. As soon as house prices rise, public housing construction is positive. The stronger prices increase, the higher the level of public housing construction. Due to high price volatility, the average level of public housing construction is quite high ($\overline{I^S} = 0.12$), which, in turn, causes the private housing construction to fall to a value of $\overline{I^P} = 0.42$. As a result, the housing stock grows to a higher level ($\overline{H} = 108.72$). Therefore, the distortion of the housing stock intensifies to more than twice its value in Figure 2, namely $D^H = 0.087$. Consequently, both the average rent level and average house prices decrease to values of $\overline{R} = 1.08$ and $\overline{P} = 95.24$, respectively. The market impact of destabilizing extrapolators decreases ($\overline{w} = 0.42$), but the house price distortion is only marginally lower ($D^P = 0.063$). Moreover, the volatility is higher compared to Figure 2 ($V = 0.017$).

Figure 10 shows that no value of public intervention parameter x diminishes the oscillations of the housing market. An increasing parameter x leads to growing levels of public housing construction

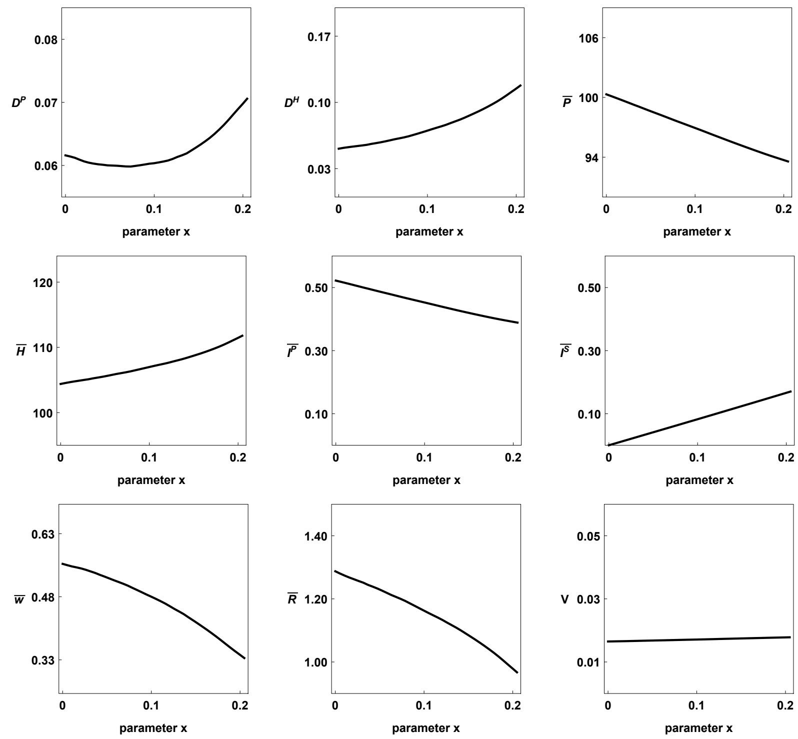

Figure 10: The impact of the fourth intervention strategy on the performance of the housing market. The panels reveal how the nine statistics D^P, D^H, $\overline{P}$, $\overline{H}$, $\overline{I^P}$, $\overline{I^S}$, $\overline{w}$, $\overline{R}$ and V depend on intervention parameter x. The computation of the nine statistics is based on 50,000 observations, and the base parameters are as in Table 1.

(panel 6). However, interventions increase the average housing stock (panel 4), while the average rent and average price level clearly decline (panels 8 and 3, respectively). Higher intervention forces do not bring about a reduction of the housing stock distortion (panel 2), but worsen the situation. It becomes apparent that higher values of parameter x are able to decrease house price distortion in the first place (panel 1), but fail to perform well for values higher than about $x = 0.1$. Panel 9 shows that volatility does not change significantly at all. Due to the increasing house price distortion, the average market impact of extrapolators decreases sharply as parameter x increases (panel 7). Finally, private housing construction is crowded out by public housing construction (panel 5).[17]

[17] Of course, we also discussed variants of the fourth intervention strategy to see if the dynamics of the housing market can be calmed. For instance, we looked not only at the price trend of the last two consecutive periods, but took larger lags into account. But these studies do not lead to any other result either.

4. Conclusions

Housing markets have repeatedly displayed dramatic boom-bust fluctuations in the past. Guided by empirical evidence, the behavioral stock-flow housing market model by Dieci and Westerhoff (2016) explains such oscillations via the interplay between speculative and real forces. In particular, they show that the expectation formation behavior of bounded rational and heterogeneous investors is a crucial factor for the emergence of intricate housing market dynamics. However, the supply side of the housing market, namely price-dependent housing construction in the private sector and the slow depreciation of the existing stock of houses, further complicates these dynamics. Together, these forces can initiate lasting periods of overvaluation and overbuilding, including a mismatch between the fluctuations and turning points of house prices and the housing stock, as is the case in real markets.

Since their model is able to mimic the behavior of actual housing markets at least to some degree, we use it as a workhorse to explore the effectiveness of a number of stabilization policies. Overall, our analysis reveals that plausible and well-intended public housing construction programs fail to tame housing markets. While these programs may reduce average house prices, they do not bring house prices much closer towards their fundamental values. By lowering house prices, public housing construction also crowds out private housing construction, an aspect that most economists would probably regard as undesirable. The main reason for the apparent failure of public housing construction programs has to do with the long-lived durability of the housing stock. During a housing market bubble, private housing construction may create a lasting and substantial overbuilding process. Public housing construction programs that seek to counter a housing market boom amplify the overbuilding process, and thus are at least partially responsible for the consequent housing market bust. Compared to many other intervention policies, say countercyclical governmental expenditure, the effects of public housing construction programs on the existing stock of houses cannot easily be reversed, except if the demolition of houses is considered, which may be quite costly and politically unfeasible.

We conclude our paper by illustrating a number of possible extensions of our work. A first possible extension concerns the underlying housing market model in which investors switch between extrapolative and regressive expectation rules with respect to current market circumstances. A question requiring investigation is whether the simple public housing construction programs we discuss in our paper may appear in a more (or even less) favorable light if investors rely on alternative expectation rules and/or switching motives. For instance, one may study the effects of these programs in a setup in which investors explicitly extrapolate past price changes and/or select expectation rules according to their past performance. Relatedly, one may assume that private housing constructors are able to learn, e.g. by forming expectations that are more sophisticated than those assumed in Dieci and Westerhoff (2016). With respect to the supply side of our model, private constructors may also face larger production lags, different cost functions and land restrictions. Moreover, buildings may have different quality levels and, in particular, public constructors may engage in social building. Within a true agent-based model, one can track the behavior of individual agents and consider, for instance, that they are financially constrained and may become bankrupt. Although we carried out a number of robustness checks, one may consider policymakers applying more complicated intervention policies, depending, for instance, on the housing market's price-rent level or other early warning indicators that may signal the possibility of the onset of a new housing market bubble. Of course, one may also use our setup (or an extended version of it) to explore the effectiveness of alternative policy measures such as subsidizing private housing construction, transaction costs, taxes on rents or rent controls. To sum up, research in this area, despite its obvious relevance, is surprisingly scant so far. We hope that our paper stimulates more work in this important direction.

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Paper 2

Housing markets, expectation formation and interest rates

Housing markets, expectation formation and interest rates

Carolin Martin, Noemi Schmitt and Frank Westerhoff

Abstract

Based on a behavioral stock-flow housing market model in which the expectation formation behavior of boundedly rational and heterogeneous investors may generate endogenous boom-bust cycles, we explore whether central banks can stabilize housing markets via the interest rate. Using a mix of analytical and numerical tools, we find that the ability of central banks to tame housing markets by increasing the base (target) interest rate, thereby softening the demand pressure on house prices, is rather limited. However, central banks can greatly improve the stability of housing markets by dynamically adjusting the interest rate with respect to mispricing in the housing market.

Keywords: Housing markets, heterogeneous expectations, variance beliefs, endogenous boom-bust cycles, interest rates, nonlinear dynamics
JEL classification: D91, E58, R31

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1. Introduction

The past has repeatedly demonstrated that the instability of housing markets may pose serious threats for the real economy. As discussed in Taylor (2009), Glaeser et al. (2013), Shiller (2015) and Piazzesi and Schneider (2016), the enormous boom-bust cycle of the U.S. housing market, which peaked in 2006, initiated one of the most harmful global recessions in history. As a matter of fact, the U.S. housing market boom was caused at least in part by the low interest rate policy adopted by the Federal Reserve System (Fed) in its efforts to combat financial and economic distress in the aftermath of the dot-com bubble. While Himmelberg et al. (2005) conclude that the major decline in interest rates during the early 2000s merely resulted in a (massive) fundamental house price increase, Taylor (2009) critically argues that the Fed's aggressive interest rate adjustments were responsible for the appearance of the U.S. housing market's boom-bust cycle and the consequent financial market turmoil. The intensity of the academic controversy in this line of research over the last couple of years should not be underestimated. Immediately before the crash of the U.S. housing market, Yellen (2005) stated that monetary policy is not the best tool for deflating housing market bubbles, and ventured that economies will be little affected by shrinking housing markets. Ten years later, Glaeser and Nathanson (2015) warn that policymakers should never again be so confident that a housing market crash would not have serious economic consequences. Against this backdrop, the goal of our paper is to explore how the interest rate setting of central banks may affect the stability of housing markets. In particular, we study the conditions under which central banks may prevent – or at least tame – boom-bust cycles in the housing market, and which policies may trigger the opposite effect.

As a workhorse, we use the behavioral stock-flow housing market model by Dieci and Westerhoff (2016). Their model reveals that nonlinear interactions between speculative and real forces can generate significant endogenous fluctuations in the housing market. The speculative forces in this model result from the expectation formation behavior of boundedly rational and heterogeneous investors. Inspired by Day and Huang (1990), de Grauwe et al. (1993) and Brock and Hommes (1998), investors switch between extrapolative and regressive expectation rules to forecast future house prices with respect to current market circumstances.[1] The real forces in this model are due to a standard housing market model (Poterba 1984, 1991, Wheaton 1999) with a rental market and a housing capital market, tying key relations between house prices, the rent level and the housing stock. Based on an empirically motivated parameter setting, the model is able to generate cyclical housing market dynamics with lasting periods of overvaluation and overbuilding, as observed in real markets.

We generalize the model by Dieci and Westerhoff (2016) along two important dimensions. First, we introduce a central bank that follows a simple leaning-against-the-wind interest rate rule (Taylor 2009, Lambertini et al. 2013), consisting of two components. Not only may the central bank autonomously

[1] Deviations from a fully rational behavior are strongly supported by empirical and experimental evidence (Case and Shiller 2003, Case et al. 2012, Hommes 2011). Moreover, Glaeser (2013) and Hommes (2013) point out that simple and plausible rule-governed behavior seems to describe reality better than fully rational behavior.

adjust the base (target) interest rate, it may also decide to automatically change the interest rate with a view to mispricing in the housing market. In the latter case, the central bank increases (decreases) the interest rate if the housing market is overvalued (undervalued) in order to deflate (fuel) the housing market. Second, we endogenize investors' variance beliefs. In the model by Dieci and Westerhoff (2016), investors have constant variance beliefs. Since the central bank's interest rate setting shapes the dynamics of the housing market, we let investors learn (update) their variance beliefs (Gaunersdorfer 2000, Chiarella et al. 2007). While this model feature has interesting implications per se for the model dynamics, since it may amplify housing market crashes, for instance, it also influences the effectiveness of the central bank's interest rate policy.

Our main results may be summarized as follows. The dynamics of our model is driven by a four-dimensional nonlinear map. The model possesses a fundamental steady state in which the price of houses reflects their future risk-adjusted rent payments. However, the fundamental steady state may become unstable due to a Neimark-Sacker bifurcation, i.e. endogenous house price fluctuations arise if investors extrapolate house prices too strongly. Note that such fluctuations are characterized by short-run momentum, long-run mean reversion and excess volatility, important empirical features of actual housing market dynamics (Glaeser 2013). Moreover, the fundamental steady state may also become unstable due to a Pitchfork bifurcation. In such a scenario, two locally stable nonfundamental steady states – surrounding the unstable fundamental steady state – emerge, implying that the housing market is either permanently overvalued or undervalued. The Pitchfork scenario occurs if the housing supply is rather sluggish and if investors use the extrapolative expectation rule too strongly. Finally, there is also the (theoretical) possibility that a Flip bifurcation compromises the stability of the housing market. Interestingly, a certain extrapolative strength of investors is then needed to ensure stability of the fundamental steady state. From an empirical perspective, the Neimark-Sacker bifurcation scenario seems to be the most realistic one. For instance, the calibrated housing market models by Wheaton (1999), Dieci and Westerhoff (2016), Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) as well as the estimated housing market models by Kouwenberg and Zwinkels (2014, 2015) produce endogenous house price oscillations. However, Bolt et al. (2019) and ter Ellen et al. (2020) detect empirical evidence of multiple steady states.

As it turns out, the central bank has a limited ability to increase the parameter domain that guarantees stability of the fundamental steady state by autonomously increasing the base (target) interest rate. Economically, higher interest rates reduce investors' demand pressure on house prices. From a quantitative perspective, the additional gain in the stability-enforcing parameter domain seems to be negligible. Moreover, high interest rates decrease the fundamental house price (and may lead to further adverse effects outside the scope of our model). Simulations also reveal that a decrease in the base (target) interest rate can spark a temporary bubble or even create permanent house price oscillations – a situation reminiscent of the start of the aforementioned U.S. housing market bubble. Central banks should keep this in mind when planning to adjust the interest rate.

Fortunately, the central bank has a great ability to control housing market fluctuations by dynamically adjusting the interest rate with a view to mispricing in the housing market. By increasing (decreasing) the interest rate in periods of overvaluation (undervaluation), such a leaning-against-the-wind policy smooths investors' expectation-driven housing demand. Most importantly, this policy allows the central bank to prevent (or at least reduce) the instability of the housing market arising from the Neimark-Sacker bifurcation, i.e. the housing market remains stable or its oscillations are characterized by a lower amplitude. The stabilizing effect of the central bank's dynamic interest rate setting is also present in the Pitchfork bifurcation scenario, i.e. the central bank has an effective tool to prevent the appearance of nonfundamental steady states. The Flip bifurcation boundary only becomes more relevant if the central bank reacts very aggressively to the housing market's mispricing (though this possibility requires some more extreme parameter constellations).

The remainder of our paper is organized as follows. In Section 2, we survey some related literature. In Section 3, we extend the housing market model by Dieci and Westerhoff (2016). We present our analytical and numerical results in Sections 4 and 5, respectively. In Section 6, we conclude our paper. Proofs and model extensions are presented in the Appendix.

2. Related literature

The important yet intricate relationship between house prices, interest rates and expectations has received increasing academic attention in the recent past. Unfortunately, no clear consensus about their interplay has been reached so far. For instance, Himmelberg et al. (2005) argue that the rapid price growth in the U.S. housing market in the 2000s was primarily caused by fundamental economic factors, especially by low interest rates. The relevance of interest rates for the formation of house prices, already articulated by Poterba (1984, 1991) to explain housing market fluctuations in the 1970s and 1980s, is, more recently, also stressed by Landvoigt (2017). However, Glaeser et al. (2013) conclude that interest rate changes cannot account for more than one-fifth of the U.S. housing market boom. Instead, they conjecture that overly optimistic expectations and mass psychology, as put forward by Case and Shiller (2003), Case et al. (2012) and Piazzesi and Schneider (2009), are major drivers of house price dynamics. Glaeser (2013) and Shiller (2015) sketch a typical boom-bust cycle as follows. While a decrease in interest rates may set in motion a fundamentally justified increase in house prices, the behavior of optimistic momentum investors can transform the initial price increase into a serious boom, resulting, of course, in an inevitable bust at a later stage.

For this reason, the expectation formation behavior of boundedly rational investors is a crucial factor in the housing market model by Dieci and Westerhoff (2016), forming the core of our model, and a number of related housing market models, e.g. by Dieci and Westerhoff (2012), Kouwenberg and Zwinkels (2014), Eichholtz et al. (2015), Burnside et al. (2016), Diks and Wang (2016), Chia et al. (2017), Glaeser and Nathanson (2017), Ascari et al. (2018), Bao and Hommes (2019), Bolt et al. (2019), Schmitt and Westerhoff (2019) and ter Ellen et al. (2020). Overall, these models demonstrate that investors' expectation formation behavior can induce significant endogenous house price oscillations. Although this line of research is still at an early stage, it is worth noting that it is deeply rooted in the heterogeneous agent asset-pricing literature, a rather powerful research strand that convincingly explains the dynamics of financial markets, see, e.g. Day and Huang (1990), de Grauwe et al. (1993), Lux (1995), Brock and Hommes (1998), Farmer and Joshi (2002), Huang and Zheng (2012) and Franke and Westerhoff (2012). Dieci and He (2018) provide an insightful survey.

Returning to the interest rate setting of central banks, Taylor (2009) forcefully states that monetary excess caused the U.S. housing market bubble. It is clear that such a view – stressing a strong relation between interest rates and house prices – has straightforward policy implications. In fact, Taylor (2009) is convinced that a rule-based interest rate policy, moderately adjusting the interest rate with respect to inflation and output (the so-called Taylor principle, going back to Taylor 1993), would have considerably dampened the magnitude of the housing market's boom-bust cycle. In a similar vein, Agnello et al. (2018), exploring the dynamics of housing markets for 20 industrial countries between 1970 and 2012, find that housing market bubbles can be deflated by increasing the interest rate. Therefore, they argue that their work supports the idea that a leaning-against-the-wind monetary policy rule can help to stabilize the housing market. Related to this, Lambertini et al. (2013) show that an interest rate rule that responds to house price growth can foster welfare by reducing the volatility of house prices. In contrast, Iacoviello (2005), also using a model with rational and optimizing agents, concludes that an interest rate response to house prices does not yield significant welfare gains as it fails to improve market stability. Yellen (2005) is even more pessimistic, claiming that monetary policy should not be used to deflate housing market bubbles.

However, it is important to note that a related line of research underlines the role of the supply side for the stability of housing markets. In particular, Glaeser et al. (2008) show that housing market bubbles are more likely to occur in places where housing supply is rather inelastic. They argue that policymakers need to make housing supply more elastic, e.g. by providing more building land or reducing construction costs, to obtain fewer and shorter bubbles with shorter price increases. Similar arguments are offered by Gyouroko et al. (2013), who argue that limitations in building land increase building costs, and by Glaeser and Gyourko (2018), who point out that overly regulated housing markets also exhibit higher building costs. Obviously, elementary laws of demand and supply imply that housing markets will exhibit stronger price reactions to shifts in housing demand, e.g. triggered by changes in interest rates or expectations, when housing supply is inelastic than when it is elastic. Such aspects should not be overlooked when it comes to explaining the dynamics of housing markets.

Our results may help to disentangle the intricate relationship between house prices, interest rates and expectations. On the one hand, our model reveals that interest rates affect the fundamental value of house prices, particularly if interest rates are already low. On the other hand, actual house prices heavily depend on investors' expectation formation behavior. Clearly, investors' expectations can induce endogenous house price fluctuations in which house prices significantly oscillate around their fundamental value, letting any (steady-state) response of the fundamental house price appear rather small. Moreover, a reduction in interest rates may spark a temporary housing market boom or, by increasing investors' demand for housing, permanently compromise the stability of housing markets. Naturally, an increase in the interest rate reduces house prices and enforces more stability, albeit with a rather small effect. The good news is that the central bank can stabilize housing markets by dynamically adjusting the interest rate with a view to mispricing in the housing market. Our analytical and numerical results suggest that, as long as the reaction parameter of the interest rate rule is not too strong, a leaning-against-the-wind interest rate policy will substantially increase the parameter domain that ensures the stability of the housing market or, at least, significantly reduces the amplitude of house price cycles. While the supply side of the housing market influences the duration and magnitude of boom-bust cycles, the central bank can always control these effects by manipulating the demand side of the housing market, which it can do by dynamically adjusting the interest rate. Indeed, it is the demand side of the housing market that is subject to the optimistic/pessimistic expectations of housing market investors.[2]

3. The housing market model

Dieci and Westerhoff (2016) combine a standard stock-flow housing market framework (Poterba 1984, 1991, Wheaton 1999), comprising explicit relations between house prices, the rent level and the housing stock, with a parsimonious approach that captures the expectation formation behavior of boundedly rational and heterogeneous investors (Day and Huang 1990, de Grauwe et al. 1993, Brock and Hommes 1998). According to the stock-flow housing market part of their model, the housing market consists of two interrelated markets: a rental (flow) market and a housing capital (stock) market. For a given housing stock, the demand for housing services determines the rent level in the rental market. House prices depend on investors' demand for housing stock relative to the existing housing stock. Investors' demand for housing stock is a function of their house price expectations, the rent level, the perceived housing market risk and the interest rate, while the housing stock evolves with respect to new housing construction and housing depreciation. The expectation formation part of their model assumes that investors rely on extrapolative and regressive expectation rules to forecast future house prices. In particular, investors increasingly turn to the regressive expectation rule as house prices disconnect from their fundamental values. We extend the model by Dieci and Westerhoff (2016) by introducing a central bank that adjusts the interest rate with a view to mispricing in the housing market. Since the central bank's interest rate setting may affect the (perceived) riskiness of the housing market, we also let investors update their variance beliefs. Technically, this turns the original two-dimensional framework into a four-dimensional model.

Let us start with the rental market. The market clearing condition for housing services implies that the demand for housing services D_t in each period t is equal to the supply (or flow) of housing

[2] Martin and Westerhoff (2019) explore whether public housing construction programs may stabilize housing markets. As it turns out, it is difficult to counter expectation-driven demand changes via supply adjustments, due to the long durability of the housing stock. While the housing stock may grow during a boom to dampen the increase in house prices, the housing stock remains high for a considerable length of time, and may thus worsen the subsequent bust.

services S_t in the same period, i.e.

$$D_t = S_t. \tag{1}$$

The demand for housing services is written as

$$D_t = a - bR_t. \tag{2}$$

Since parameters a and b are positive, (2) indicates that D_t depends negatively on rent level R_t, the price of housing services.[3] The supply of housing services is proportional to the initial stock of housing H_t, and is described as

$$S_t = cH_t, \tag{3}$$

where $c > 0$. By inserting (2) and (3) in (1), rent level R_t is given by a decreasing function of the current housing stock

$$R_t = \alpha - \beta H_t, \tag{4}$$

where $\alpha = \frac{a}{b} > 0$ is a scaling parameter and $\beta = \frac{c}{b} > 0$ represents the sensitivity of the rent level with respect to the housing stock. Of course, α and β have to be such that $R_t \geq 0$.

As regards the capital market, the market clearing condition for housing stock

$$Z_t = H_t \tag{5}$$

indicates that the demand for housing stock Z_t is equal to the supply of housing stock H_t.[4] The development of the housing stock is given by

$$H_t = I_t + (1 - \delta)H_{t-1}, \tag{6}$$

where $0 < \delta < 1$ is the housing depreciation rate and I_t denotes the amount of new housing construction. We assume that houses are built with a one-period production lag. Moreover, home builders are risk neutral and maximize expected profits subject to a quadratic cost function, i.e. $max_{I_t}\{E_{t-1}[P_t]I_t - C_t\}$, where $C_t = \frac{1}{2\gamma}I_t^2$, resulting in $I_t = \gamma E_{t-1}[P_t]$. For simplicity, home builders form naive expectations, i.e. $E_{t-1}[P_t] = P_{t-1}$.[5] As a result, housing construction in period t is given by

$$I_t = \gamma P_{t-1}, \tag{7}$$

implying that

$$H_t = \gamma P_{t-1} + (1 - \delta)H_{t-1}. \tag{8}$$

[3] Dieci and Westerhoff (2016) show that their housing market model produces quite similar dynamics when the linear relation between the demand for housing services and the rent level is replaced by an isoelastic one.

[4] We assume market clearing in the rental and housing capital market, accomplished via an instantaneous adjustment of the rent level and the house price. The disequilibrium asset-pricing approach by Hommes et al. (2005) may serve as an avenue to overcome this assumption. Examples of disequilibrium housing market models include Dieci and Westerhoff (2012, 2013), Geanakoplos et al. (2012), Erlingsson et al. (2014) and Campisi et al. (2018).

[5] Dieci and Westerhoff (2012) study the case in which home builders have perfect foresight expectations, while Campisi et al. (2018) elaborate on the case in which firms rely on a mix of perfect foresight and naive expectations. In Appendix C, we sketch a model framework with larger production lags.

Note that a decrease in the inverse cost parameter $\gamma > 0$ makes the construction of new houses more expensive and the housing stock more sluggish. Since Glaeser et al. (2008) argue that the duration and magnitude of housing bubbles crucially depend on the price-responsiveness of the supply side, parameter γ is a key parameter of our model.

We model investors' demand for housing stock using a standard one-period mean-variance framework. More precisely, investor i faces a wealth allocation problem between housing capital and an alternative riskless asset over the time horizon from period t to $t+1$. For a hypothetical house price level P_t at time t, investor i's end-of-period wealth is given by

$$W_{t+1}^i = (1 + r_t)W_t^i + Z_t^i(P_{t+1} + R_t - (1 + r_t + \delta)P_t), \tag{9}$$

where W_t^i and Z_t^i stand for the wealth and the amount of housing units held by investor i at the beginning of the period. Note that variables indexed with $t+1$ are random. Rent level R_t and interest rate r_t are determined at the beginning of the period. The goal of housing market investors is to maximize the certainty equivalent of final wealth. For investor i, this results in the following mean-variance optimization problem

$$max_{Z_t^i}\left[E_t^i[W_{t+1}^i] - \frac{\lambda^i}{2}V_t^i[W_{t+1}^i]\right], \tag{10}$$

where $E_t^i[W_{t+1}^i]$ and $V_t^i[W_{t+1}^i]$ represent investor i's conditional expectation and variance about his end-of-period wealth, while parameter $\lambda^i > 0$ reflects his (absolute) risk aversion. As is well known, investor i's solution to the above maximization problem yields

$$Z_t^i = \frac{E_t^i[P_{t+1}] + R_t - (1 + r_t + \delta)P_t}{\lambda^i V_t^i[P_{t+1}]}. \tag{11}$$

Obviously, investor i's optimal demand (amount of housing units) increases in line with the expected future house price and the rent level, while it decreases in line with the interest rate, the (current) house price and the perceived housing market risk.[6]

In our paper, we consider the case in which investors' beliefs about future house prices are heterogeneous, while their beliefs about the variance of future house prices are homogeneous, yet time-varying.[7] Let $E_t[P_{t+1}]$ stand for investors' average future house price expectation and $V_t[P_{t+1}]$ for their variance beliefs. Normalizing the mass of investors to one and assuming the same risk aversion for all investors allows us to express investors' total housing demand as

$$Z_t = \frac{E_t[P_{t+1}] + R_t - (1 + r_t + \delta)P_t}{\lambda V_t[P_{t+1}]}. \tag{12}$$

[6]As we will see, investors' housing demand is strictly positive at the steady state and remains positive most of the time when the housing market is out of equilibrium. During larger house price swings, however, investors' housing demand may become negative. Of course, short selling is easier in housing markets than it is in stock markets. The work by Anufriev and Tuinstra (2013), in't Veld (2016) and Dercole and Radi (2020), who study the effects of short-selling constraints within the asset-pricing model by Brock and Hommes (1998), may be useful for studying possible implications of this model aspect in more detail.

[7]For analytical convenience, Brock and Hommes (1998) assume that investors have homogenous and constant variance beliefs, arguing that investors agree more about the variance than about the mean. Gaunersdorfer (2000) and Chiarella et al. (2007, 2013) generalize this assumption by introducing homogenous, yet time-varying variance beliefs. Since the interest rate policy of the central bank affects the underlying data-generating process, we follow their approach, i.e. investors update their variance beliefs.

From the market equilibrium condition (5), we then get

$$P_t = \frac{E_t[P_{t+1}] + R_t - H_t \lambda V_t[P_{t+1}]}{1 + r_t + \delta} + \epsilon_t, \tag{13}$$

where $\epsilon_t \sim N(0, \sigma^2)$ reflects additional exogenous noise affecting the housing market. Apparently, the house price depends positively on investors' house price expectations and the rent level, and negatively on the stock of housing, investors' risk perception and the interest rate.[8]

Hommes (2013) argues that agents are boundedly rational and, when facing complex decision problems, rely on simple yet plausible heuristics. A similar view is offered by Glaeser (2013). To keep the model tractable, investors select only between two expectation rules to forecast future house prices: an extrapolative and a regressive expectation rule. Moreover, expectations formed in period t about the house price in period $t+1$ rely on the last observable house price, namely the house price in period $t-1$. According to the extrapolative expectation rule, investors predict the next period's house price by

$$E_t^E[P_{t+1}] = P_{t-1} + \chi(P_{t-1} - P^*), \tag{14}$$

where $\chi > 0$ denotes the rule's extrapolation strength and P^* stands for the housing market's fundamental price. Hence, the extrapolative expectation rule predicts a continuation of the current boom or bust period in the housing market. In contrast, the regressive expectation rule is based on the assumption that the house price will revert to its fundamental value. This rule is formalized by

$$E_t^R[P_{t+1}] = P_{t-1} + \phi(P^* - P_{t-1}), \tag{15}$$

where $0 < \phi < 1$ stands for the rule's mean-reversion speed. Note that expectation rules (14) and (15) can be traced back to the seminal asset-pricing models by Day and Huang (1990) and Brock and Hommes (1998). Empirical support for these rules is provided by contributions such as Boswijk et al. (2007), Westerhoff and Franke (2012), Hommes and in't Veld (2017), Bolt et al. (2019), Schmitt (2020) and ter Ellen et al. (2020).[9]

Investors' choice of prediction rules depends on current market circumstances. While investors seek to chase price trends, they also fear fundamental price corrections. Assuming that investors prefer the regressive expectation rule with increasing mispricing, the market share of investors that follow the extrapolative expectation rule can be expressed by

$$N_t^E = \frac{1}{1 + \eta(P^* - P_{t-1})^2}. \tag{16}$$

[8]As in Bao and Hommes (2019), Bolt et al. (2019) and Schmitt and Westerhoff (2019), investors update their housing demand in every period by solving a wealth allocation problem between housing capital and an alternative riskless asset. In Appendix C, we propose a model extension in which only a smaller fraction of investors adjust their housing demand in this sense. See Erlingsson et al. (2014) and Ozel et al. (2019) for alternative formulations of investors' housing demand.

[9]Expectation rules (14) and (15), as well as the switching function (16) and the interest rate rule (22), imply that the fundamental house price is common knowledge. As we will see in more detail in Section 4, this is not a too strong assumption since the fundamental house price corresponds to the discounted value of future (risk-adjusted) rents and can thus be identified by investors and the central bank. In Section 5.4.3, however, we consider the case in which all market participants use a moving average of the house price as a proxy for the fundamental house price. Moreover, Schmitt and Westerhoff (2019) and Martin et al. (2020) consider related housing market models in which investors may trend-extrapolate past house price changes.

Near the fundamental value, the market share of the extrapolative expectation rule is relatively high. In such an environment, the bulk of investors regard any price change away from the fundamental value as the start of an exploitable bull or bear market. However, the larger the switching parameter $\eta > 0$, the faster investors switch to the regressive expectation rule as mispricing in the housing market increases. Of course, the market share of the regressive expectation rule is given by $N_t^R = 1 - N_t^E$. The bell-shaped switching function (16) was originally proposed by de Grauwe et al. (1993) to explain the dynamics of foreign exchange markets. See He and Westerhoff (2005), Gaunersdorfer et al. (2008), Dieci and Westerhoff (2012, 2016) and Campisi et al. (2018) for related economic applications and Kilian and Taylor (2003), Gaunersdorfer and Hommes (2007), Menkhoff et al. (2009), Franke and Westerhoff (2012) and Kouwenberg and Zwinkels (2014) for general empirical support of this modeling approach.[10]

Investors' average house price expectations are defined by

$$E_t[P_{t+1}] = N_t^E E_t^E[P_{t+1}] + N_t^R E_t^R[P_{t+1}]. \tag{17}$$

Combining (14)-(17) reveals that

$$E_t[P_{t+1}] = \frac{P_{t-1} + \chi(P_{t-1} - P^*) + P_{t-1}\eta(P^* - P_{t-1})^2 + \phi\eta(P^* - P_{t-1})^3}{1 + \eta(P^* - P_{t-1})^2}, \tag{18}$$

i.e. investors' expectation formation behavior adds a strong nonlinearity to our housing market model.

Following Chiarella et al. (2007, 2013), investors' variance beliefs depend on a fundamental and a speculative component, so that we can write

$$V_t[P_{t+1}] = \Omega + \kappa V_t^S. \tag{19}$$

The fundamental variance component Ω is constant and captures investors' perceived risk associated with owning a house (e.g. damages to the house, not receiving the rent, unforeseen regulations connected with buying and selling houses or other fundamental disturbances).[11] The speculative variance component V_t^S is time-varying and depends on housing market volatility, where parameter $\kappa \geq 0$ measures investors' sensitivity with respect to the latter component. Note that $\kappa = 0$ implies that investors' variance beliefs are constant, as is the case in the original housing market model by Dieci and Westerhoff (2016) and almost all other related heterogeneous agent asset-pricing models (see, for instance, the recent survey by Dieci and He 2018).

We model investors' speculative variance component using a learning rule introduced by Gaunersdorfer (2000), that is

$$V_t^S = \nu V_{t-1}^S + (1 - \nu)(P_{t-1} - U_{t-1})^2 \tag{20}$$

[10] Of course, there are alternative modeling approaches. For instance, Dieci and Westerhoff (2013), Bolt et al. (2019), Schmitt and Westerhoff (2019), ter Ellen et al. (2020) and Martin et al. (2020) study predictor selection frameworks based on the discrete choice approach, using squared prediction errors or realized profits as performance criteria.

[11] Piazzesi and Schneider (2016) point out that the volatility of house prices depends more strongly on idiosyncratic shocks than the volatility of stocks. Since houses are indivisible (they are sold in their entirety and not in small pieces), idiosyncratic shocks to housing are difficult to diversify. Furthermore, real housing markets are usually not very liquid, and are characterized by high transaction costs, aspects that add to the risk of owning a house.

and

$$U_t = \mu U_{t-1} + (1-\mu)P_{t-1}, \tag{21}$$

where $0 < \nu, \mu < 1$ are memory parameters. Accordingly, investors update their speculative variance beliefs by computing a weighted average of their past speculative variance beliefs and the most recent observable squared deviation between the house price and an average house price. Obviously, the average house price is also updated in the form of a weighted average.[12]

Inspired by Taylor (2009), Lambertini et al. (2013) and Agnello et al. (2018), we consider the case in which the central bank sets the interest rate with a view to the fundamental condition of the housing market. More precisely, the central bank tries to stabilize the housing market by using the following interest rate rule

$$r_t = r_0 + \rho\left(\frac{P_{t-1} - P^*}{P^*}\right), \tag{22}$$

where r_0 is the central bank's base (target) interest rate. Furthermore, $\rho \geq 0$ is a parameter that controls how strongly the central bank reacts to mispricing in the housing market. Naturally, $r_t \geq 0$, i.e. the interest rate cannot become negative.[13] Note that (22) suggests increasing (decreasing) the interest rate if the housing market is overvalued (undervalued). In fact, recall from (12) and (13) that higher (lower) interest rates – accomplished by adjusting the base (target) interest rate or reacting to the fundamental condition of the housing market – decrease (increase) investors' demand for housing stock, and therefore depress (elevate) house prices. In the next sections, we use a mix of analytical and numerical tools to explore the extent to which the interest rate rule (22) allows the central bank to control the dynamics of housing markets.

4. Analytical insights

To be able to study the model's deterministic skeleton, we abstract in this section from the exogenous noise component added to the pricing equation (13). By setting $\epsilon_t = 0$ and combining our equations, we can easily express the model by the four-dimensional nonlinear deterministic map

$$S: \begin{cases} P_t = \frac{E_t[P_{t+1}] + \alpha - (\beta + \lambda V_t[P_{t+1}])\gamma P_{t-1} - (\beta + \lambda V_t[P_{t+1}])(1-\delta)H_{t-1}}{1 + r_0 + \rho\left(\frac{P_{t-1} - P^*}{P^*}\right) + \delta} \\ H_t = \gamma P_{t-1} + (1-\delta)H_{t-1} \\ V_t^S = \nu V_{t-1}^S + (1-\nu)(P_{t-1} - U_{t-1})^2 \\ U_t = \mu U_{t-1} + (1-\mu)P_{t-1} \end{cases}, \tag{23}$$

[12] In Appendix B, we study an alternative learning rule proposed by Chiarella et al. (2007, 2013). While their learning rule may affect our model's global behavior, it does not affect the fundamental steady state's stability domain. Further learning rules with fixed memory length are studied by Chiarella and He (2002).

[13] While the interest rate is always positive in the analytical part of our paper (Section 4), it may hit the zero-lower bound when we simulate our model's out-of-equilibrium dynamics (Section 5). In fact, (22) then implies that the model's map is piecewise defined, an aspect that may cause interesting side effects. See Avrutin et al. (2019) for an overview of possible implications of such maps, and tools to explore them, and Schmitt et al. (2017) for economic examples.

where

$$E_t[P_{t+1}] = \frac{P_{t-1} + \chi(P_{t-1} - P^*) + P_{t-1}\eta(P^* - P_{t-1})^2 + \phi\eta(P^* - P_{t-1})^3}{1 + \eta(P^* - P_{t-1})^2}$$

and

$$V_t[P_{t+1}] = \Omega + \kappa V_t^S.$$

As can be seen, the dynamics depends on 14 parameters: α, β, γ, δ, λ, ν, μ, r_0, ρ, χ, η, ϕ, κ and Ω. Nevertheless, we are able to prove the following results (see Appendix A):

Proposition 1. *The dynamical system (23) always gives rise to the fundamental steady state* $FSS = \left(P^*, H^*, V^{S*}, U^*\right) = \left(P^*, \frac{\gamma}{\delta}P^*, 0, P^*\right)$ *with* $P^* = \frac{\alpha\delta}{(r_0+\delta)\delta+(\beta+\lambda\Omega)\gamma}$. *The FSS is locally asymptotically stable if and only if (i)* $\chi > \frac{4+2r_0-2\rho-\delta(\delta+r_0-\rho)-\gamma(\beta+\lambda\Omega)}{\delta-2}$, *(ii)* $\chi < \frac{\gamma(\beta+\lambda\Omega)}{\delta} + \delta + r_0 + \rho$ *and (iii)* $\chi < \frac{2\delta+r_0}{1-\delta} + \rho$, *where a violation of the first, second and third inequality is associated with the emergence of a Flip, Pitchfork and Neimark-Sacker bifurcation, respectively.*

As shown above, the model has a fundamental steady state where the house price equals its fundamental value, i.e. $\overline{P} = P^* = \frac{\alpha\delta}{(r_0+\delta)\delta+(\beta+\lambda\Omega)\gamma}$, while the corresponding values for the housing stock, the speculative variance component and the average house price are given by $H^* = \frac{\gamma}{\delta}P^*$, $V^{S*} = 0$ and $U^* = P^*$, respectively. Note that P^* is independent of any behavioral parameters, such as χ, ϕ or η, and depends only on fundamental parameters. In particular, if the central bank increases the base (target) interest rate r_0, the fundamental house price decreases, which, in turn, implies decreasing values for the housing stock and higher rent levels (and vice versa).

Since the rent level at the fundamental steady state is given by $R^* = \alpha - \beta H^*$, it follows that $P^* = \frac{R^*-\lambda\Omega H^*}{r_0+\delta}$. By defining risk-adjusted rents as $\hat{R}^* = R^* - \lambda\Omega H^*$ (see Dieci and Westerhoff (2016) for more details), the fundamental house price can be expressed as the discounted value of future risk-adjusted rents, i.e. $P^* = \frac{\hat{R}^*}{r_0+\delta}$, where the term $r_0 + \delta$ reflects the user cost of housing. This is a key property of Poterba's (1984, 1991) seminal housing market model. As pointed out by Himmelberg et al. (2005), the nonlinearity in the discounting of risk-adjusted rents can cause sharp fundamental house price changes with respect to interest rate changes. In fact, the sensitivity of the fundamental house price to changes in the interest rate is higher at times when interest rates are already low. In a low interest rate environment, for instance, a given decrease in the interest rate induces a larger increase in house prices than the same decrease in the interest rate would initiate starting from a high interest rate. Of course, the reverse is also true. An increase in interest rates in a low interest rate environment would cause a disproportionally large decline in house prices, especially if risk-adjusted rents remain constant, or adjust only slowly.

To illustrate the stability domain of the fundamental steady state, we plot in Figure 1 the stability conditions (i)-(iii) in (χ,γ)-parameter space. The depiction is stylized and based on $\rho = 0$. The first, second and third stability condition is represented by the red, blue and green line, respectively. Accordingly, the fundamental steady state is locally asymptotically stable for the parameter space that is bounded by the three lines (highlighted in gray). For $0 < \gamma < A1$, an increase in investors' extrapolation behavior may violate the second stability condition, which is associated with a Pitchfork bifurcation, i.e. the fundamental steady state becomes unstable and two additional nonfundamental steady states are created. The housing market then remains permanently either overvalued or undervalued. Note that this scenario may occur if the housing supply is rather sluggish. If $A1 < \gamma < A2$, an increase in χ may cause a Neimark-Sacker bifurcation, and thus the onset of a quasi-periodic motion.[14] If the housing supply reacts more strongly to the past house price, i.e. if $A2 < \gamma < A3$, the local asymptotic stability of the fundamental steady state requires that investors' extrapolation behavior is neither too low (violation of the Flip bifurcation boundary) nor too high (violation of the Neimark-Sacker bifurcation boundary). Hence, there are scenarios where a modest extrapolative behavior of investors is beneficial for the stability of housing markets. Finally, the fundamental steady state is always unstable if $\gamma > A3$, i.e. if the price-responsiveness of the housing stock becomes very large.

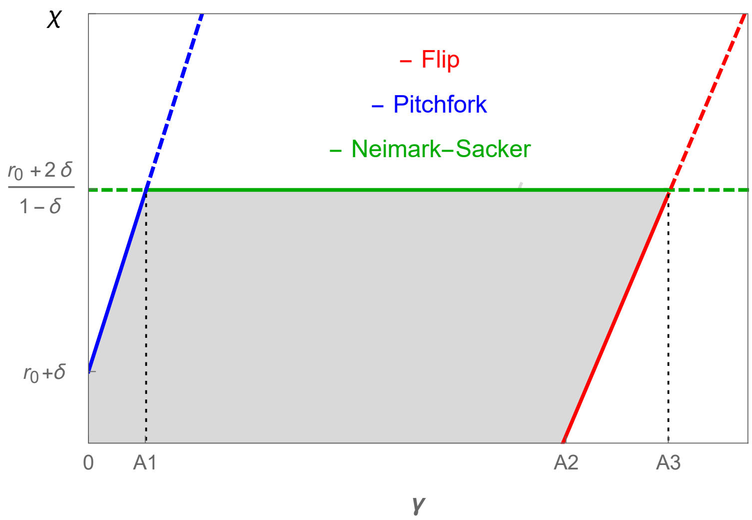

Figure 1: Stability domain of the fundamental steady state in (χ,γ)-parameter space for $\rho = 0$. Since the blue, red and green lines represent the three stability conditions, the parameter space highlighted in gray illustrates the region for which the fundamental steady state is locally asymptotically stable.

The central bank may be able to influence the stability domain of the fundamental steady state by varying r_0 and ρ. If the central bank increases the base (target) interest rate, all three stability conditions become relaxed, and the region for which P^* is locally asymptotically stable becomes larger. From an empirical perspective, however, this effect seems to be rather limited. This can be explained by the following example. For quarterly data, $r_0 = \delta = 0.005$ is a reasonable assumption, implying that $\chi^{NS} \approx 0.015$. Since γ seems to be much larger empirically, say $\gamma^{emp} = 0.15$ (see Section 5 for more details), extremely high (and unrealistic) base (target) interest rates may be needed to stabilize housing markets. In contrast, the stabilizing effect of an increase in the central bank's reaction parameter ρ appears to be much stronger. While an increase in ρ makes the presumably

[14]Several empirical papers indicate that the cyclical nature of housing markets is due to a Neimark-Sacker bifurcation, e.g. Wheaton (1999), Kouwenberg and Zwinkels (2014, 2015), Dieci and Westerhoff (2016) and Glaeser and Nathanson (2017). However, Bolt et al. (2019) and ter Ellen et al. (2020) detect empirical evidence for coexisting attractors due to a Pitchfork bifurcation.

not so important Flip bifurcation boundary more binding, it relaxes the highly relevant Pitchfork and Neimark-Sacker bifurcation boundaries. Given $r_0 = \delta = 0.005$ and $\chi^{emp} = 0.15$, for instance, the central bank needs to set $\rho \approx 0.135$ to ensure stability of the fundamental steady state.

To sum up: investors' extrapolation behavior may destabilize housing markets. In particular, a violation of the Neimark-Sacker bifurcation can set endogenous house price cycles in motion. While the central bank has a limited ability to tame housing markets by increasing the base (target) interest rate, it has a strong potential to stabilize housing markets by following a leaning-against-the-wind interest rate rule. In this sense, our local stability results support the view of Taylor (2009), Agnello et al. (2018) and Lambertini et al. (2013).

5. Numerical insights

Equipped with our analytical insights, we are now ready to explore the model's out-of-equilibrium behavior. In Section 5.1, we first introduce our base parameter setting and explain the functioning of our model. In Section 5.2, we investigate in more detail the extent to which the central bank can stabilize housing markets by adjusting the interest rate with a view to mispricing of the housing market, paying special attention to the Neimark-Sacker bifurcation scenario (Section 5.2.1), the Pitchfork bifurcation scenario (Section 5.2.2) and the Flip bifurcation scenario (Section 5.2.3). In Section 5.3, we discuss how the central bank influences the housing market by adjusting the base (target) interest rate. In Section 5.4, we carry out a number of robustness checks.

5.1. Base parameter setting and functioning of the model

Our base parameter setting, reported in Table 1, closely follows Dieci and Westerhoff (2016). A time period is equivalent to one quarter of a year. The real parameters, such as the base (target) interest rate and the depreciation rate, are grounded on empirical observations. The remaining model parameters, in particular those that include agents' expectation formation, are set such that the model dynamics reflects a number of basic characteristics of real housing markets. In particular, we will see that our model is able to produce boom-bust cycles with short-run momentum, long-run mean reversion and excess volatility, crucial features of actual housing market dynamics (Glaeser and Nathanson 2015, Piazzesi and Schneider 2016). In some of our simulations, a small amount of exogenous noise is added to the house price equation.[15]

The calibrated model parameters imply that the FSS is given by $P^* = E[\overline{P}] = U^* = 100$ and $H^* = 1000$. Since the rent level at the FSS amounts to $R^* = 2$, it follows that the (annual) price-rent ratio is $\frac{P^*}{4R^*} = 12.5$. Furthermore, the steady-state level of investors' variance beliefs and the interest

[15]Future work should try to estimate the parameters of our model. The work by Kouwenberg and Zwinkels (2014, 2015), Hommes and in't Veld (2017), Chia et al. (2017), Bolt et al. (2019), Schmitt (2020) and ter Ellen et al. (2020) may be useful in this respect. However, some parameters may also be identified empirically. For instance, Goldbaum and Mizrach (2008) and Goldbaum and Zwinkels (2014) exploit survey studies to characterize investors' heuristic switching behavior. Experimental work, as done by Anufriev and Tuinstra (2016) and Anufriev et al. (2018), may also be helpful in this respect.

Table 1: Parameter setting used in the simulations (quarterly data)

$\alpha = 62$	scaling parameter	rental market
$\beta = 0.06$	sensitivity of rental market	
$\gamma = 0.05$	sensitivity of home building	supply side of housing market
$\delta = 0.005$	depreciation rate	
$r_0 = 0.005$	base (target) interest rate	central bank
$\rho = 0.1$	reaction parameter of central bank	
$\chi = 0.15$	extrapolative parameter	price expectations
$\phi = 0.125$	regressive parameter	
$\eta = 0.01$	switching intensity	
$\lambda = 0.00025$	risk aversion	risk aversion and variance beliefs
$\Omega = 4$	base fundamental risk	
$\kappa = 0.25$	sensitivity to speculative risk	
$\nu = 0.5$	memory parameter variance	
$\mu = 0.5$	memory parameter mean	
$\sigma = 2$	standard deviation of noise	exogenous shocks

rate is $V^*[P^*] = 4$ and $r^* = 0.005$, respectively. As in Dieci and Westerhoff (2016), the level of exogenous noise (if switched on) corresponds to investors' constant fundamental variance perception, i.e. $\Omega = \sigma^2 = 4$. At the FSS, all agents form extrapolative expectations, i.e. $N^{E^*} = 1$. Note that the Neimark-Sacker condition is violated while the Flip and the Pitchfork conditions hold. Hence, the fundamental steady state is unique but unstable.

To start, Figure 2 shows the functioning of a restricted version of our model. In the depicted simulation run, the parameter of agents' sensitivity to speculative risk κ and the central bank's reaction parameter ρ are set to zero, i.e. investors' variance beliefs are constant and $r_t = r_0$. A small amount of exogenous noise is added to the dynamics ($\sigma = 2$). Note that this setup is close to that of Dieci and Westerhoff (2016), who show that their model can produce reasonable housing market dynamics with lasting periods of overbuilding and overvaluation. The 200 observations represent a time span of 50 years. The panels show, from top left to bottom right, the evolution of house prices, the housing stock, the market share of regressive expectations, the rent level, investors' variance beliefs, and the interest rate, respectively. The gray lines shown in the panels depict the fundamental values. For comparability, we also use this design for Figures 3-5. Obviously, investors' extrapolative behavior causes significant housing market fluctuations. The functioning of the restricted model can be summarized as follows. Initially (at $t = 1$), the housing market is strongly overvalued, which means that the market share of regressive expectations is relatively high. Since regressive expectations have a stabilizing effect, house prices return towards their fundamental value. Moreover, high house prices induce substantial new housing construction, which leads to an expansion of the housing stock and a depression of the rent level. Once house prices drop below their fundamental value, investors with extrapolative expectations become pessimistic. Since their market share is relatively high, prices drop even further until shortly after period $t = 50$. At this point, the situation starts to change. If house prices are very low, more and more investors will switch to the regressive expectation rule and predict an increase in house prices. Since the housing stock is still relatively small, the rent level recovers and the story repeats itself until the next crash occurs.

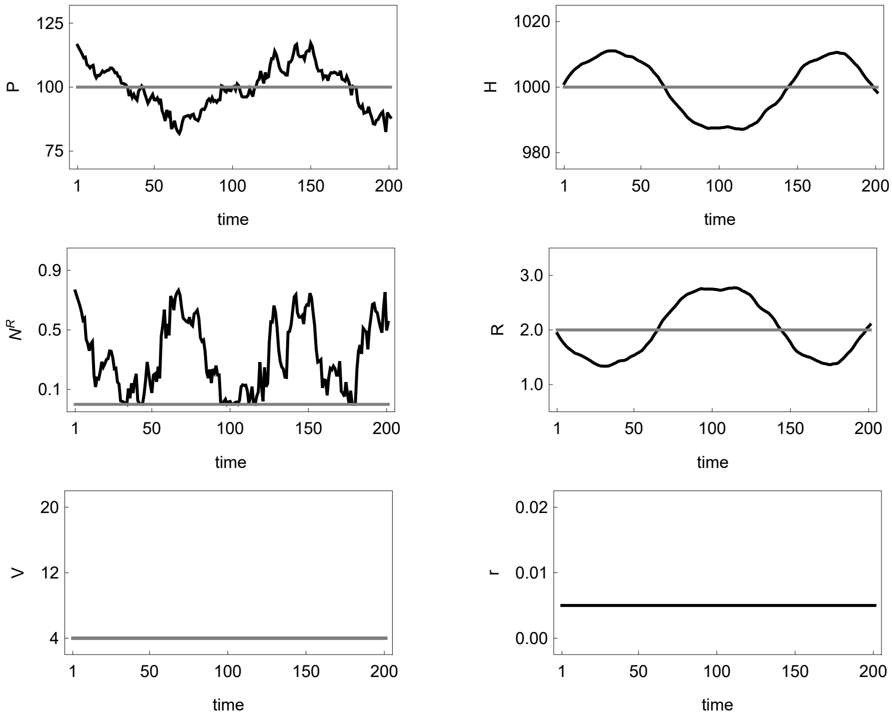

Figure 2: The functioning of the model with constant variance beliefs and a constant interest rate. The panels show, from top left to bottom right, the evolution of house prices, the housing stock, the market share of regressive expectations, the rent level, investors' variance beliefs, and the interest rate, respectively. Base parameter setting, except that $\kappa = 0$ and $\rho = 0$.

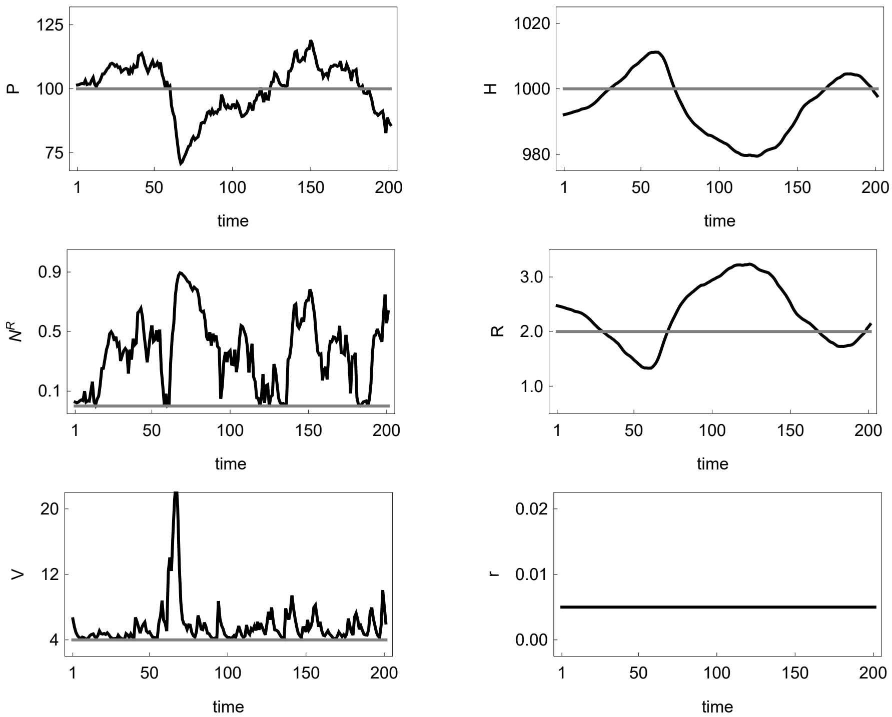

Figure 3: The functioning of the model with endogenous variance beliefs and a constant interest rate. The panels show, from top left to bottom right, the evolution of house prices, the housing stock, the market share of regressive expectations, the rent level, investors' variance beliefs, and the interest rate, respectively. Base parameter setting, except that $\rho = 0$.

To investigate how endogenous variance beliefs change the dynamics of the housing market model, we now set the parameter of agents' sensitivity to speculative risk to $\kappa = 0.25$ instead of $\kappa = 0$. The dynamics of the model with endogenous variance beliefs and constant interest rates (i.e. $\rho = 0$) is illustrated in Figure 3. As can be seen from the bottom left panel, investors' variance beliefs fluctuate slightly above the fundamental value $V^*[P^*] = 4$, except around period $t = 65$, where variance beliefs increase to over $V_t = 20$. According to equations (13) and (19), a rapid drop in house prices leads to an increase in V_t^S, and thus an increase in variance beliefs V_t. In fact, this is exactly what we observe. Around period $t = 65$, house prices fall sharply due to the high level of the housing stock and the low rent level, causing a rapid increase in investors' variance beliefs. Put differently, the sharp drop in house prices makes the housing market appear more risky. Investors then retreat, amplifying the crash, and house prices drop below $P_t = 75$. This gives the model a slightly asymmetric nature, i.e the level of the housing stock decreases and the rent level increases. As house prices rise, investors' uncertainty recedes and their housing demand increases, which further strengthens the upward trend. Glaeser and Nathanson (2015) remark that, while real housing markets are excessively volatile, house prices do not display a constant level of volatility. Instead, house prices experience brief moments of extreme variance that interrupt longer periods of lower variance. Note that our model with endogenous variance beliefs can replicate this empirical property.

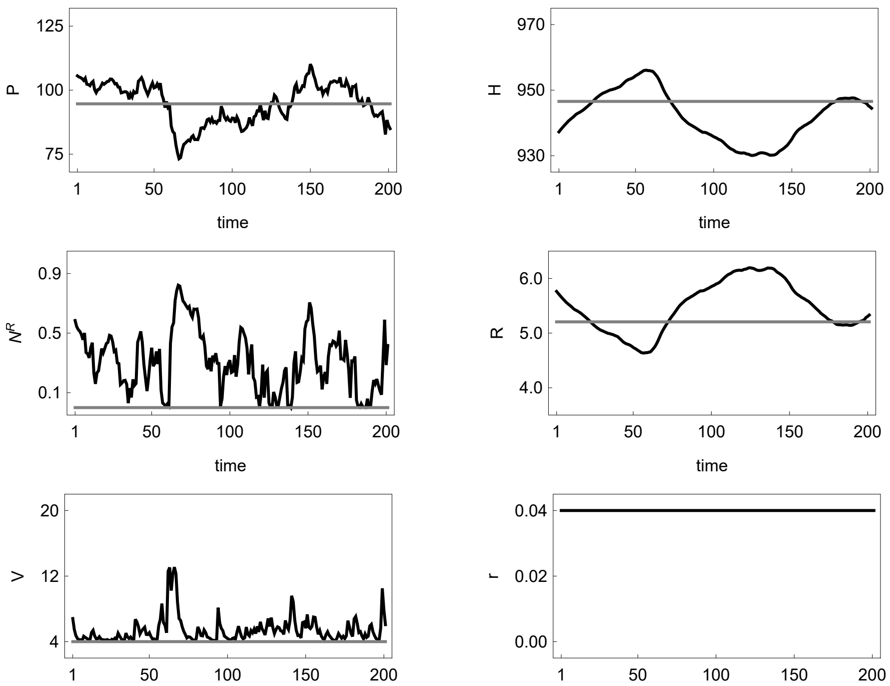

Figure 4: The functioning of the model with endogenous variance beliefs and a constant, yet high interest rate. The panels show, from top left to bottom right, the evolution of house prices, the housing stock, the market share of regressive expectations, the rent level, investors' variance beliefs, and the interest rate, respectively. Base parameter setting, except that $r_0 = 0.04$ and $\rho = 0$.

In Figure 4, we examine the dynamics of the housing market with endogenous variance beliefs and with a constant, yet high interest rate. As can be seen, an increase in the base (target) interest rate has a weakly stabilizing effect on the housing market, since both house prices and the housing stock, as well as the rent level, fluctuate slightly closer around their fundamental values. To be able to visualize these weak effects, here we set the base (target) interest rate to $r_0 = 0.04$, i.e. we increase the annual interest rate from 2% (Figure 3) to 16% (Figure 4). In line with Himmelberg et al. (2005), the fundamental values of house prices and the housing stock decrease to $P^* = 94.66$ and $H^* = 946.57$, respectively, and the fundamental value of the rent level increases to $R^* = 5.21$ (the scaling of H, R and r has been adjusted accordingly). Since a higher interest rate increases the opportunity cost of buying a house, housing demand becomes depressed, pushing house prices down. As a result, housing construction and thus the housing stock decrease, resulting in higher rent levels. While the higher interest rate may slightly stabilize the dynamics of the housing market, decreased house prices and housing stock may result in undesirable consequences that may not be justified by marginally more stable markets.

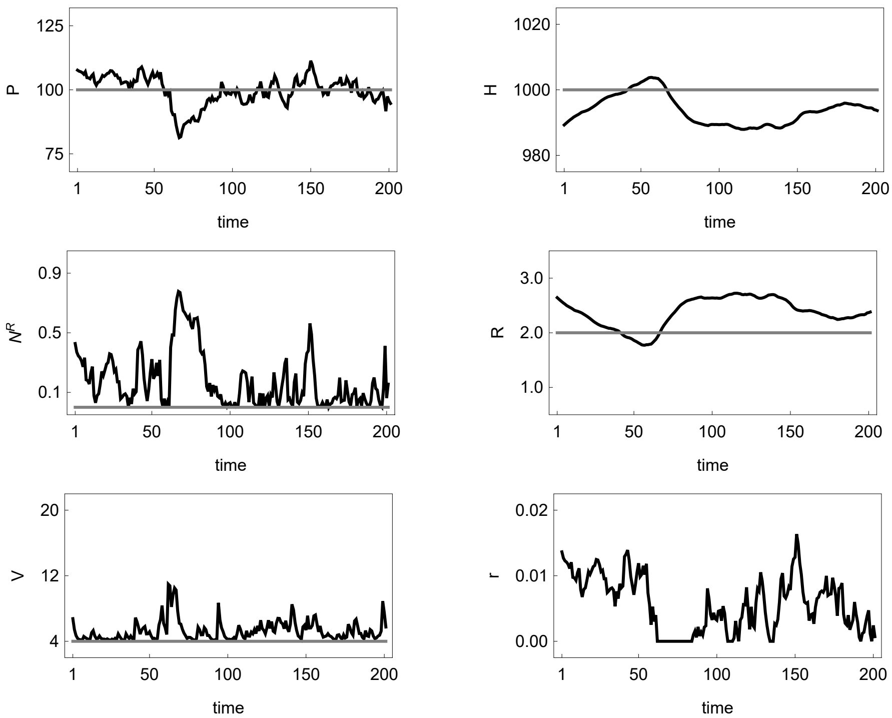

Figure 5: The functioning of the model with endogenous variance beliefs and endogenous interest rates. The panels show, from top left to bottom right, the evolution of house prices, the housing stock, the market share of regressive expectations, the rent level, investors' variance beliefs, and the interest rate, respectively. Base parameter setting.

Finally, we investigate the complete model with endogenous variance beliefs and interest rates. Figure 5 reveals that the dynamic part of the interest rate rule manages to stabilize housing markets. On average, we observe a decrease in house price distortion, i.e. a decrease in the average distance between house prices and the fundamental value, and, hence, a more efficient housing market. Since house prices fluctuate significantly closer around their fundamental value, no strong bubbles or crashes occur. The same can be observed for the housing stock and the rent level, both of which move closer to their fundamental values. A further stabilizing effect of the interest rate rule is that investors' variance beliefs are less extreme. This can be explained as follows. In boom periods, i.e. if $P_t > P^*$, the interest rate is relatively high, leading to a decline in housing demand. This causes house prices to fall. But since the drop in house prices is less extreme, investors' variance beliefs, and thus their demand, remain more balanced. Moreover, new housing construction, the housing supply and the rent level also benefit from more stable house prices. This becomes apparent between periods $t = 1$ and $t = 50$. In the other case, if $P_t < P^*$, the interest rate decreases, which can be observed between periods $t = 50$ and $t = 100$. As housing demand increases, so do house prices. Note that the interest rate fluctuates mainly between 0 and 0.01, which seems to be reasonable. Furthermore, the stability condition is still violated, i.e. the deterministic model still produces endogenous cycles, albeit with a much lower amplitude.

For our further analysis, it is helpful to introduce a policy measure. Let us define the housing market's distortion by

$$distortion = \frac{100}{T}\sum_{t=1}^{T}(\frac{P_t - P^*}{P^*})^2, \tag{24}$$

where T indicates the sample size, set to $T = 50000$ in our experiments. For our base parameter setting, except that $\rho = 0$, the housing market's distortion is given by $distortion = 1.19$. If the central bank uses the leaning-against-the-wind interest rate rule with $\rho = 0.1$, however, the distortion reduces to $distortion = 0.44$. Note that these statistics correspond to the dynamics depicted in Figures 3 and 5. Should the central bank decide to become more aggressive, e.g. by using $\rho = 0.5$, the housing market's mispricing decreases further to a value of $distortion = 0.19$.[16]

5.2. Endogenous interest rate adjustments

In this section, we discuss in more detail how the endogenous component of the central bank's interest rate rule may affect the dynamics of the housing market.

5.2.1. The Neimark-Sacker bifurcation scenario

In Figure 6, we use bifurcation diagrams to relate the house price to the extrapolative parameter χ in the Neimark-Sacker scenario. The parameter setting is as in Table 1, except that $\kappa = 0$, $\rho = 0$, $\sigma = 0$ in the top left panel, $\rho = 0$, $\sigma = 0$ in the top right panel, $\kappa = 0$, $\sigma = 0$ in the center left panel, $\sigma = 0$ in the center right panel and $\kappa = 0$ in the bottom left panel. The top panels of Figure 6 reveal that the stronger investors' extrapolation is, the larger the amplitude of house price fluctuations. Furthermore, the bifurcation route evolves from a stable steady state to quasi-periodic dynamics as χ increases from 0 to 0.4. To be more precise, for small values of χ, the model's fundamental steady state is stable, but becomes unstable when the extrapolative parameter exceeds $\chi = 0.015$, as predicted by our analytical

[16] Of course, one may also consider different policy measures. For instance, the central bank may already be satisfied with the performance of the housing market if the house price remains near its fundamental value. To get a better idea about the significance of bubbles and crashes in the housing market, we remark that the housing market stays outside a price interval of $\pm 10\%$ around the fundamental house price about 39% of the time if the central bank keeps the interest rate constant ($\rho = 0$). This statistic decreases to a value of 11% if the central bank dynamically adjusts the interest rate, using $\rho = 0.1$ (respectively to 4% for $\rho = 0.5$). See Agnello et al. (2015, 2020) for alternative duration measures.

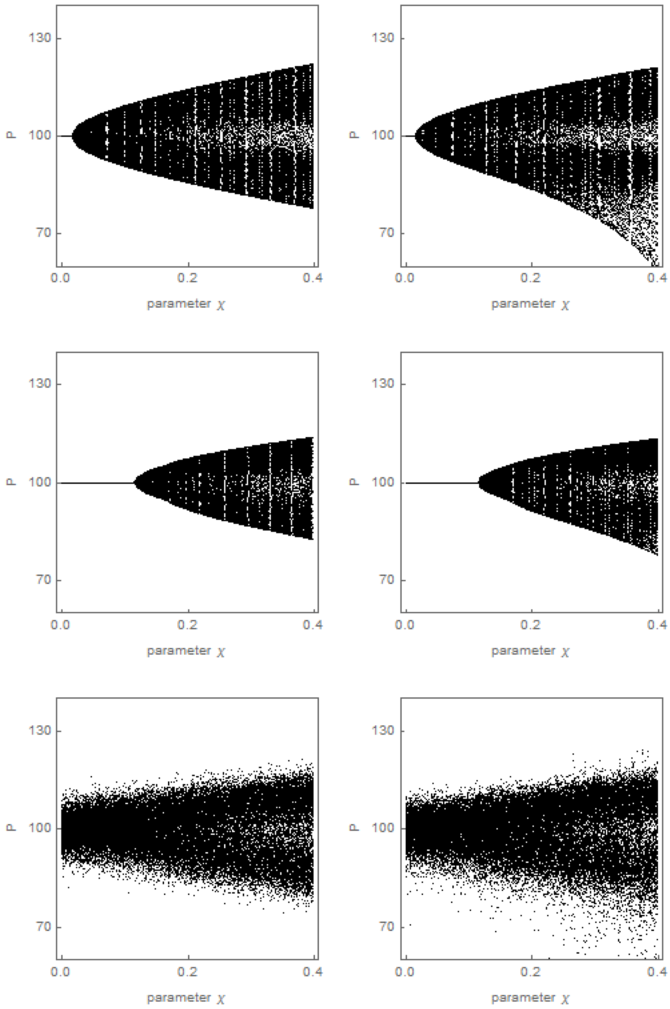

Figure 6: The destabilizing effect of extrapolative expectations in the Neimark-Sacker bifurcation scenario. The panels show bifurcation diagrams for the house price versus extrapolative parameter χ. Base parameter setting, except that $\kappa = 0$, $\rho = 0$, $\sigma = 0$ (top left), $\rho = 0$, $\sigma = 0$ (top right), $\kappa = 0$, $\sigma = 0$ (center left), $\sigma = 0$ (center right), $\kappa = 0$ (bottom left). Moreover, parameter χ is varied between 0 and 0.4.

results. To explore the effect of endogenous variance beliefs, we repeat our simulations from the top left panel in the top right panel, but now with the base setting $\kappa = 0.25$. The corresponding bifurcation route shows that the amplitude of house price swings is biased downwards for high values of parameter χ, an outcome which is due to the crashes induced by variance beliefs.

The stabilizing impact of an endogenous interest rate on house price swings is shown in the two center panels. As can be seen, the amplitude of house price fluctuations can be significantly reduced in both cases, with constant and endogenous variance beliefs. Moreover, the housing market remains stable for larger values of χ, namely up to about $\chi = 0.15$. Again, this observation is in line with our analytical results. Thus the central bank's dynamic interest rate rule has a significant stabilizing effect on the housing market's dynamics by reducing instability, which arises from the Neimark-Sacker bifurcation. The two bottom panels repeat our simulations in a noisy environment. As it turns out, the distorting effect of endogenous variance beliefs is robust with respect to additional exogenous noise

– at least with a view to the amplitude of price fluctuations.

Figure 7 illustrates how the dynamics changes with respect to the central bank's reaction parameter ρ. The extrapolative parameter in the top left panel is set to $\chi = 0.4$, i.e. investors extrapolate house prices quite strongly. Note that if the central bank reacts more aggressively to mispricing in the housing market, the amplitude of house price fluctuations decreases. Moreover, a convergence to the steady state sets in when ρ exceeds 0.385. In the center left panel, we repeat these simulations for the base setting, and again observe a stabilizing effect: the steady state is reached for a lower value of the central bank's reaction parameter, namely for $\rho = 0.13$. In addition, house price fluctuations are significantly more dampened for increasing values of ρ. According to the top right and the center right panel, the stabilizing effect of an increasing parameter ρ holds with respect to exogenous noise, which, in turn, is further supported by the two bottom panels in which we compute the housing market's distortion. Note that both the distortion for $\chi = 0.4$ (bottom left panel) and the distortion for $\chi = 0.15$ (bottom right panel) decrease strongly as parameter ρ increases from 0 to 0.5.

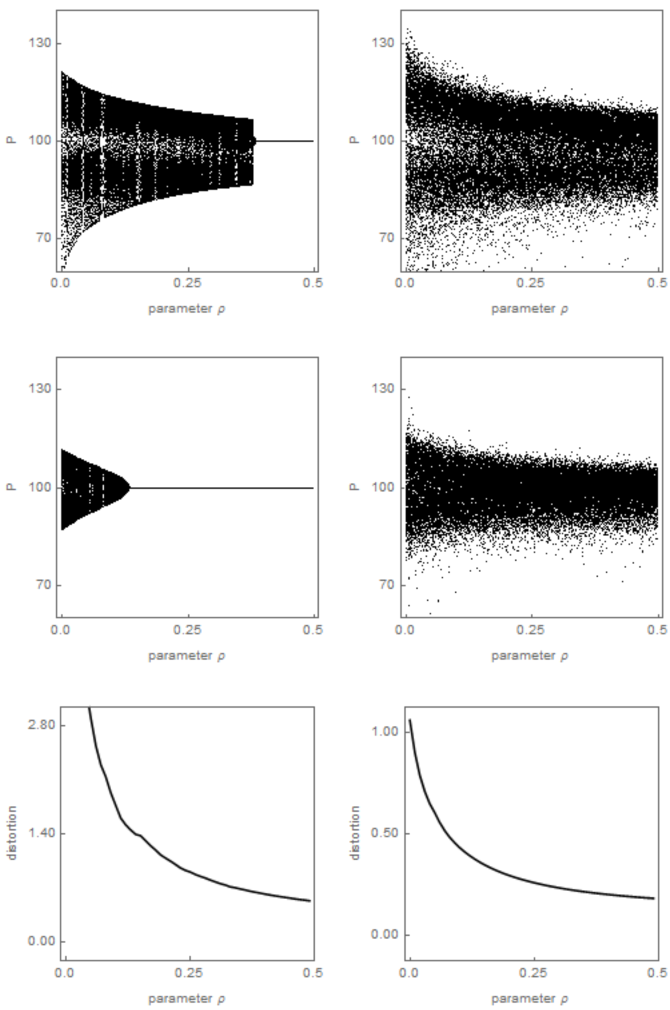

Figure 7: The stabilizing effect of the endogenous part of the interest rate rule in the Neimark-Sacker bifurcation scenario. The first four panels show bifurcation diagrams for the house price versus the central bank's reaction parameter ρ. The bottom two panels show the distortion of the housing market versus the central bank's reaction parameter ρ. Base parameter setting, except that $\chi = 0.4$, $\sigma = 0$ (top left), $\chi = 0.4$ (top right), $\sigma = 0$ (center left), $\chi = 0.4$ (bottom left).

5.2.2. *The Pitchfork bifurcation scenario*

In Figure 8, we analyze the Pitchfork bifurcation scenario. The panels show how extrapolative expectations (top panels) and the endogenous part of the interest rate rule (center and bottom panels) affect the dynamics of our model. Recall that the Pitchfork bifurcation scenario occurs if the housing supply is relatively sluggish. It becomes apparent from the top left panel (base parameter setting, except that $\sigma = 0$, $\beta = 0.0005$ and $\rho = 0$) that if investors use the extrapolative expectation rule too strongly, the steady state becomes unstable and two locally stable nonfundamental steady states emerge, surrounding the unstable fundamental steady state. The housing market then remains permanently overvalued (red line) or undervalued (blue line). Furthermore, mispricing in the housing market increases with the extrapolative parameter χ. The bifurcation route in the top right panel (base parameter setting, except that $\sigma = 0$ and $\beta = 0.0005$) shows that an endogenous interest rate causes the bifurcation to occur for a higher value of the extrapolative parameter, namely for $\chi = 0.128$ instead of $\chi = 0.028$, as can be verified analytically. A comparison of the two top panels also reveals that mispricing in the housing market is – for a given value of χ – lower if the central bank dynamically adjusts the interest rate.

To illustrate these results in more detail, we present three bifurcation diagrams in which we vary the central bank's reaction parameter ρ between 0 and 0.5. In the center left panel (base parameter setting, except that $\sigma = 0$, $\beta = 0.0005$ and $\chi = 0.4$), investors strongly extrapolate prices, and the corresponding bifurcation route undoubtedly reveals that mispricing in the housing market decreases with ρ. As depicted in the center right panel (base parameter setting, except that $\sigma = 1$, $\beta = 0.0005$ and $\chi = 0.4$), this result is robust with respect to noise, but we observe attractor switching (for more details, see Figure 9). This attractor switching occurs more frequently with higher noise, which is illustrated in the bottom left panel (base parameter setting, except that $\sigma = 5$, $\beta = 0.0005$ and $\chi = 0.4$). Here again, the amplitude of house price fluctuations decreases as ρ increases. Further evidence of this result is provided by the bottom right panel (base parameter setting, except that $\sigma = 5$, $\beta = 0.0005$ and $\chi = 0.4$), which reveals that distortion decreases with ρ. Thus, the central bank's dynamic interest rate setting is an effective instrument for preventing the appearance of nonfundamental steady states.

In Figure 9, we analyze coexisting attractors and basins of attraction in the Pitchfork bifurcation scenario in more detail. The top left panel (base parameter setting, except that $\beta = 0.0005$, $\sigma = 1$ and $\rho = 0$) shows a time series for constant interest rates. As can be seen, house price P_t fluctuates around the lower nonfundamental steady state (blue line), and hence the average price is below P^* (gray line). An increase in ρ from 0 to 0.13 brings the nonfundamental steady states closer towards P^*, as depicted in the bottom left panel. As a result, we may observe attractor switching and thus a price correction towards the fundamental price P^*, which is in accordance with the center right panel of Figure 8. The corresponding change in the basins of attraction is visualized in the right panels of Figure 9. In fact, compared to the top right panel (with $\rho = 0$), the basin of the upper nonfundamental steady state (red area) becomes smaller and the basin of the lower nonfundamental steady state (blue area) becomes larger due to the introduction of an endogenous interest rate. In the bottom right panel,

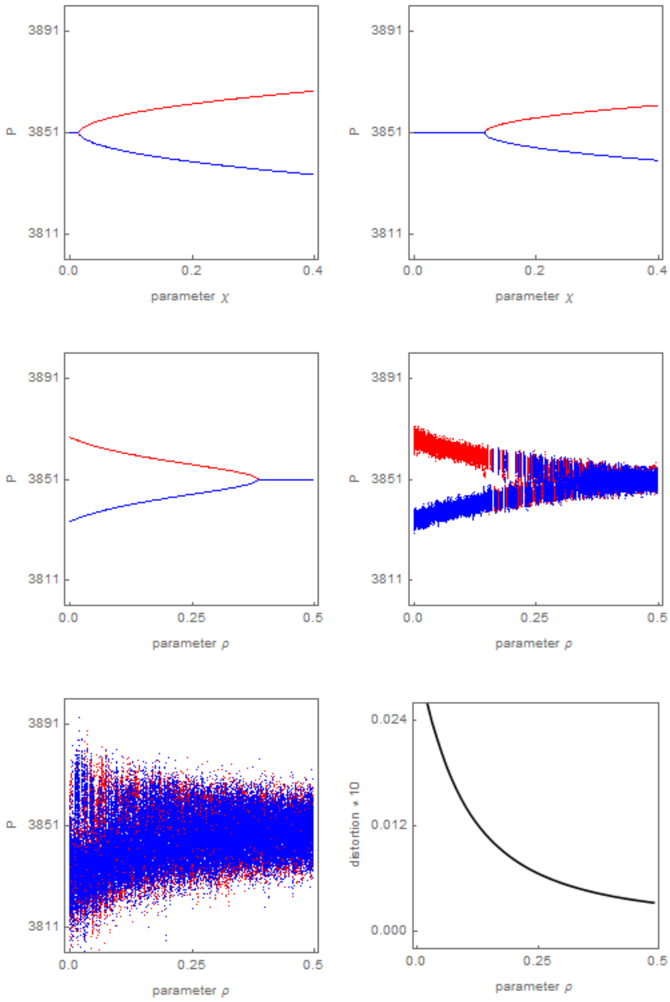

Figure 8: The destabilizing (stabilizing) effect of extrapolative expectations versus the endogenous part of the interest rate rule. The top left panel shows a bifurcation diagram for the house price versus the extrapolative parameter χ for the base parameter setting, except that $\sigma = 0$, $\beta = 0.0005$ and $\rho = 0$. The top right panel shows the same, except that $\sigma = 0$ and $\beta = 0.0005$. The center left panel shows a bifurcation diagram for the house price versus the endogenous part of the interest rate ρ for the base parameter setting, except that $\sigma = 0$, $\beta = 0.0005$ and $\chi = 0.4$. The center right panel shows the same, except that $\sigma = 1$, $\beta = 0.0005$ and $\chi = 0.4$. The bottom left panel shows the same, except that $\sigma = 5$, $\beta = 0.0005$ and $\chi = 0.4$. The bottom right panel shows the corresponding distortion.

however, nonfundamental steady states (red and blue dot) are closer to the borders of their basins of attraction, which explains attractor switching (a nontrivial effect of ρ).

Coexisting attractors may have interesting policy implications. Suppose that the price has converged towards the model's lower nonfundamental steady state in a constant interest rate environment. As long as exogenous shocks are not too large, the system will not leave the steady state's basin of attraction (blue area), and endogenous forces will drive the price back towards its equilibrium value. While endogenous interest rates cause the blue area to increase, nonfundamental steady states move closer to the boundary of their basins of attraction. Policymakers may thus have the opportunity to drive back nonfundamental steady states towards P^* by increasing ρ, reducing mispricing in the housing market.

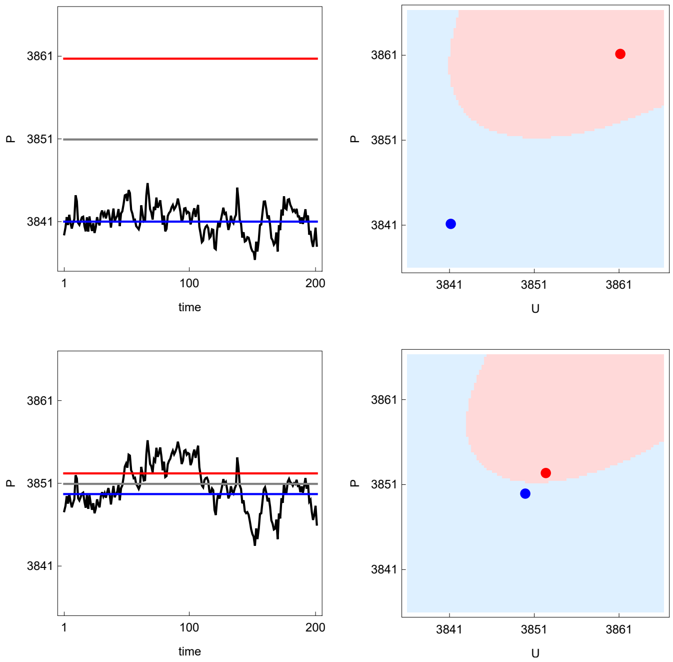

Figure 9: Coexisting attractors and basins of attraction in the Pitchfork bifurcation scenario. The top left panel shows a simulation run of house prices for the base parameter setting, except that $\beta = 0.0005$, $\sigma = 1$ and $\rho = 0$. The nonfundamental and fundamental steady states are given in red, blue and gray. The bottom left panel shows the same for $\rho = 0.13$. The right panels visualize the corresponding basins of attraction for initial conditions of P and H, abstracting from exogenous noise.

5.2.3. The Flip bifurcation scenario

Figure 10 illustrates how extrapolative expectations (top panels) and an endogenous interest rate (center and bottom panels) affect the dynamics of our model in the Flip bifurcation scenario. The left (right) panels show the dynamics of house prices (interest rates). As can be seen in the top panels (parameter setting, except that $\sigma = 0$ and $\rho = 2.5$), the Flip bifurcation value is $\chi = 0.4915$, hence this extrapolative strength is needed to ensure housing market stability. For $\chi < 0.4915$, the steady state is unstable and the system tends to explode until interest rates hit the zero lower bound at which the map becomes piecewise defined (Avrutin et al. 2019). The destabilizing impact of endogenous interest rates is depicted in the second line of panels (base parameter setting, except that $\sigma = 0$ and $\chi = 0.01$). If the central bank reacts very aggressively to mispricing in the housing market (from a value of $\chi = 0.01$), the steady state becomes unstable, and chaotic dynamics emerges. This finding also holds in a noisy environment, as is witnessed in the bottom left panel (base parameter setting, except that $\sigma = 0.2$ and $\chi = 0.01$). The corresponding distortion (bottom right panel) further supports our findings. Clearly, the distortion increases with ρ.

5.3. Exogenous interest rate adjustments

Finally, we investigate the extent to which the autonomous part of the interest rate rule is able to tame housing markets in the Neimark-Sacker bifurcation scenario. The left (right) panels of Figure 11 rely on our base parameter setting, except that $\chi = 0.014$ and $\sigma = 0$ ($\sigma = 0$). In the top left (center left) panel of Figure 11, we increase (decrease) the base (target) interest rate r_0 in period 100

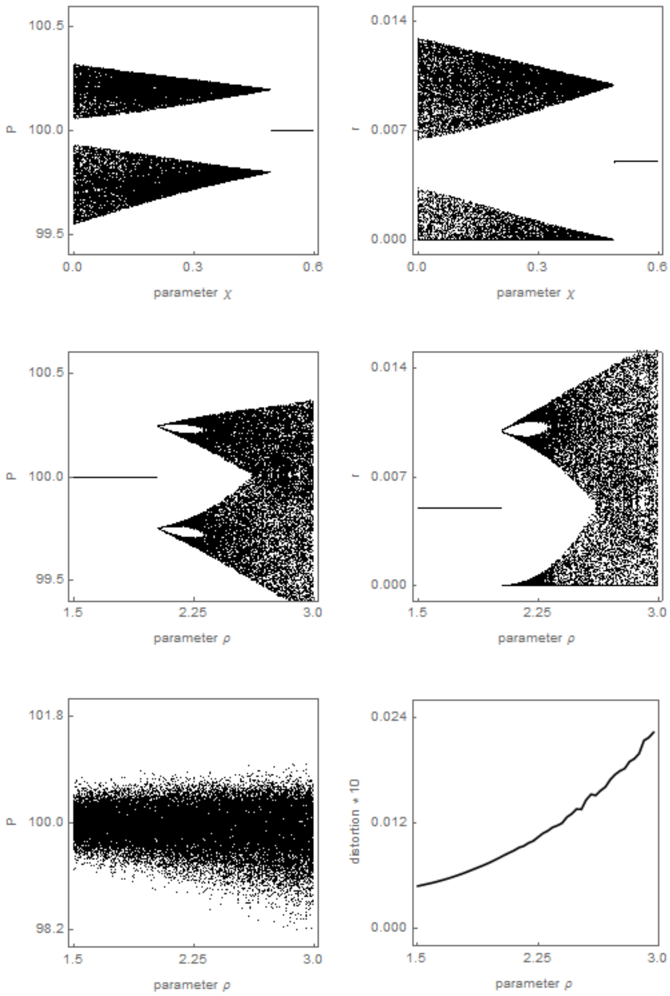

Figure 10: The stabilizing (destabilizing) effect of extrapolative expectation (endogenous interest rates) in the Flip bifurcation scenario. The first line of the panels shows bifurcation diagrams for house prices and the interest rate versus parameter χ for the base parameter setting, except that $\sigma = 0$ and $\rho = 2.5$. The second line of the panels shows bifurcation diagrams for house prices and the interest rate versus parameter ρ for the base parameter setting, except that $\sigma = 0$ and $\chi = 0.01$. The third line of the panels shows bifurcation diagrams for house prices and distortion versus parameter ρ, except that $\sigma = 0.2$ and $\chi = 0.01$.

from 0.005 to 0.010 (0.0005). Before $t = 100$, the house price is equal to the fundamental steady state. The increase in r_0 in $t = 100$ makes the system more stable, but creates an adjustment process with strong house price fluctuations towards a lower steady-state house price level. While our main focus is on endogenous housing dynamics, the relevance of temporary housing dynamics should not be underestimated. For instance, Glaeser and Nathanson (2017) discuss in detail how exogenous shocks may cause temporary fluctuations in a housing market model in which agents form extrapolative expectations. Moreover, Taylor (2009) argues that the Fed's strong interest rate adjustments between 2001 and 2006 have greatly contributed to the instability of the U.S. housing market. Note that a decrease in r_0 can create permanent house price oscillations around an increased steady-state price level. We may observe similar effects of the increase (decrease) in the interest rate in period 500 from 0.005 to 0.05 (0) in the top right (center right) panel. While an increase in r_0 leads to smaller

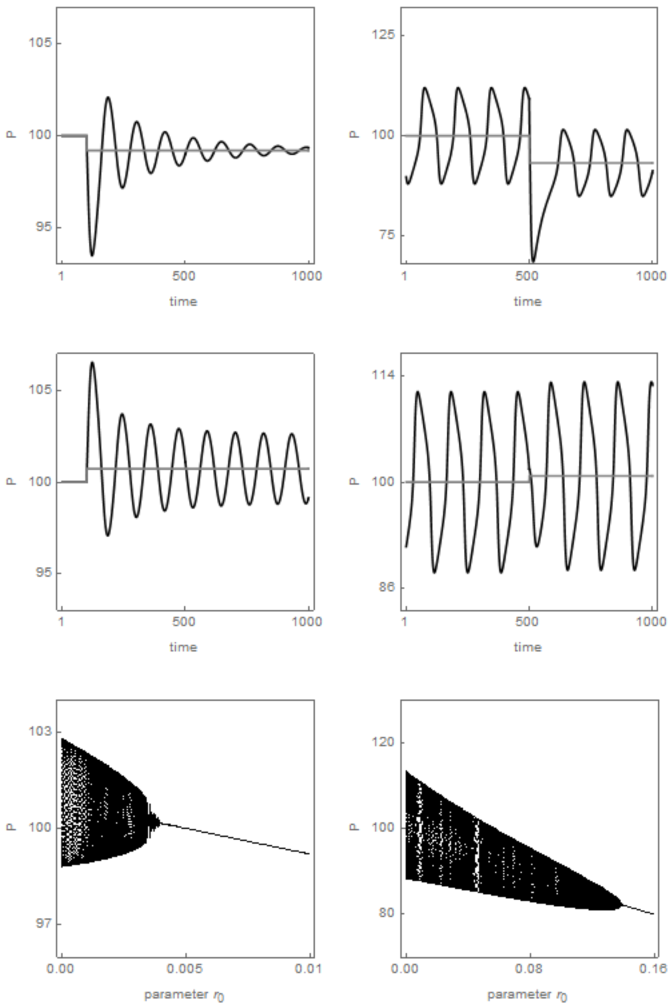

Figure 11: Some effects of the autonomous part of the interest rate rule in the Neimark-Sacker bifurcation scenario. The left panels rely on the base parameter setting, except that $\chi = 0.014$ and $\sigma = 0$. Moreover, in the top left (center left) panel, the interest rate increases (decreases) in period 100 from 0.005 to 0.010 (to 0.0005). The right panels rely on the base parameter setting, except that $\sigma = 0$. In addition, in the top right (center right) panel, the interest rate increases (decreases) in period 100 from 0.005 to 0.050 (to 0).

amplitudes of house price fluctuations around a lower steady-state level, a decrease in r_0 enlarges the amplitude of house price oscillations. Further evidence of these results is provided in the bottom panels, which show the model dynamics for increasing r_0. It can be seen that, with increasing base (target) interest rates, the amplitude of house price fluctuations becomes smaller up to the bifurcation value $r_0 = 0.00393$ (bottom left panel) and $r_0 = 0.13925$ (bottom right panel), respectively. At this point, the quasi-periodic dynamics segues into a stable fundamental steady state. However, the fundamental house price P^* decreases with parameter r_0, which may have further unfavorable effects. All in all, the central bank's ability to reduce the dynamics on housing markets by increasing the base (target) interest rate, weakening the demand pressure on house prices, is rather limited.

5.4. Robustness checks

In this section, we carry out a number of robustness checks.

5.4.1. Base (target) interest rate

So far, we have set the annual base (target) interest rate to 2 percent. As revealed by Figure 5, depicting our base scenario, the annual interest rate then tends to fluctuate between 0 and 4 percent. Since the central bank may opt for a higher or a lower annual base (target) interest rate during specific macroeconomic periods, the question arises as to the extent to which different levels of the base (target) interest rate may affect the effectiveness of the leaning-against-the-wind interest rate rule. In the top left (right) panel of Figure 12, the annual base (target) interest rate is assumed to be 4 percent (1 percent). The black line reveals how the distortion of the housing market reacts to an increase in the central bank's reaction parameter ρ (the gray line recalls the results for our base parameter setting). The depicted simulations underline the robustness of our results, at least from a qualitative perspective. From a quantitative perspective, however, we see that the effectiveness of the central bank's interest rate rule decreases somewhat when the base (target) interest rate is set to a lower value. The main reason for this outcome is that the central bank's interest rate adjustments hit the zero lower bound more frequently when the base (target) interest rate is set to a lower value, losing part of its stabilizing potential in the process. Put differently, the interest rate rule looks more favorable when the base (target) interest rate is set to a higher value, simply because the zero lower bound then affects the central bank's interest rate adjustments less frequently.

5.4.2. The central bank's perception of the fundamental house price

The leaning-against-the-wind interest rate policy may be difficult to implement in reality because the central bank does not generally know the true fundamental house price. As a further robustness check, we therefore assume that the central bank uses a moving average of house prices as a proxy for the fundamental house price, that is

$$\hat{P}_t^* = m\hat{P}_{t-1}^* + (1-m)P_t, \tag{25}$$

where $0 < m < 1$ is a memory parameter. The center left (right) panel of Figure 12 shows how the distortion of the housing market reacts to an increase in the central bank's reaction parameter ρ for our base parameter setting, except that $m = 0.95$ ($m = 0.9$). As can be seen, our results are relatively robust, at least as long as the central bank's memory parameter is sufficiently high. Interestingly, Hennequin and Hommes (2019) report experimental evidence that is consistent with our finding. See also the related study by Bao and Zong (2019). We also simulated scenarios (not depicted) in which the central bank misperceives the fundamental house price, say by overestimating or underestimating its value by 5 or 10 percent. As long as its perception error is not too large, the central bank can stabilize the housing market by following the leaning-against-the-wind interest rate policy.

5.4.3. Perception of the fundamental house price by all market participants

Within our model, investors switch between extrapolative and regressive expectations, subject to the market's actual mispricing. In doing so, we assume that investors are able to compute the true fundamental house price. As a final robustness check, we now assume that all market participants,

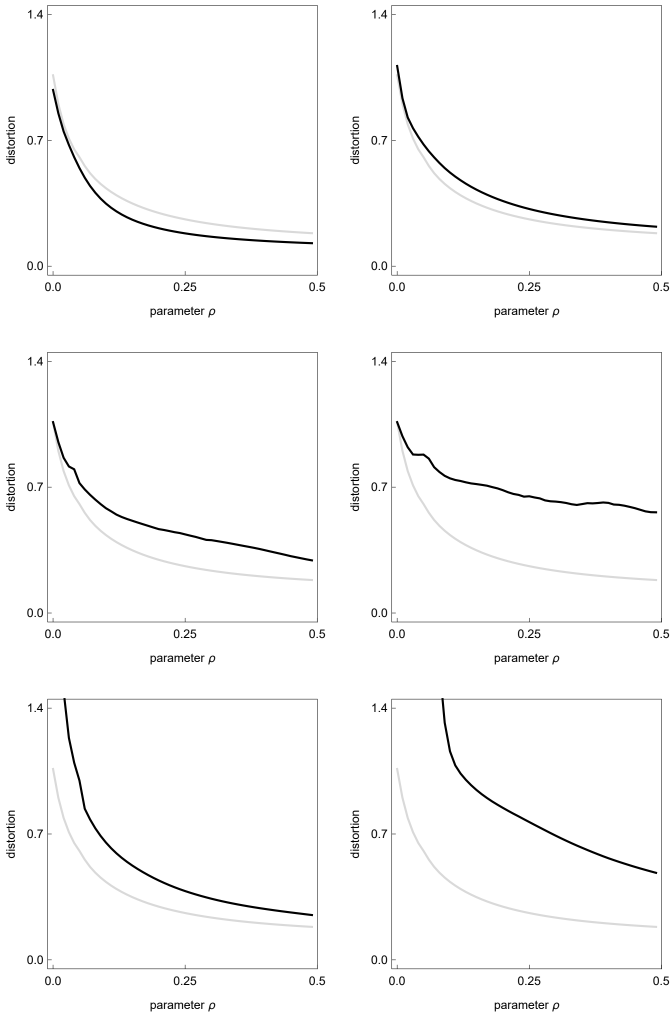

Figure 12: Robustness checks. The top left (right) panel shows the distortion versus parameter ρ for our base parameter setting, except that $r_0 = 0.01$ ($r_0 = 0.0025$). The center left (right) panel shows the distortion versus parameter ρ for our base parameter setting, except that the central bank uses a moving average of the house price as a proxy for the fundamental house price with $m = 0.95$ ($m = 0.9$). The bottom left (right) panel shows the same, except that all market participants use a moving average of the house price as a proxy for the fundamental house price. The gray lines report the distortion for the base parameter setting.

i.e. investors and the central bank, use (25) to compute the fundamental house price.[17] Note that this affects expectation rules (14) and (15), the switching device (16) and the interest rate policy (22). The bottom panels of Figure 12 depict our results for $m = 0.95$ (left) and $m = 0.9$ (right). Once again, our results appear to be quite robust: while the magnitude of housing bubbles tends to increase when market participants use a moving average of the house price as a proxy for the fundamental house price, the central bank is still able to tame the dynamics. We would like to stress that we find these results quite remarkable. Although market participants are unaware of the true fundamental house price, the dynamics does not get out of bounds. The reason why the house price still fluctuates around its true fundamental value has to do with the real side of the housing market. If investors' optimistic

[17] Since it is difficult to determine the fundamental house price in reality, we remark that such a setup may be more suitable when it comes to estimating our model. See also ter Ellen et al. (2020) in this respect.

house price expectations drive the housing market upwards, new housing constructions increase the housing stock, which, in turn, depresses the rent level and makes the housing market less attractive to investors. This mechanism anchors the dynamics of the housing market and keeps investors' perception of the fundamental house price bounded, provided that they do not place too much weight on current house prices.

6. Conclusions

Shiller (2015), Glaeser (2013) and Piazzesi and Schneider (2016) demonstrate that, while the U.S. housing market bubble between 1998 and 2012 may seem extreme, it was hardly unique. In fact, history is replete with dramatic housing market instabilities that have had dire economic consequences. Unfortunately, the economics of housing market bubbles is still in its infancy. According to Glaeser and Nathanson (2015), many important questions remain unresolved, e.g. why did the U.S. boom-bust cycle occur, and what are its policy implications? The goal of our paper is to shed light on the intricate relationship between house prices, expectations and interest rates, also keeping in mind the supply side of the housing market.

For this reason, we generalize the behavioral stock-flow housing market model by Dieci and Westerhoff (2016). Our analysis reveals that interactions between investors' expectations, their variance beliefs and the supply side of housing markets may give rise to substantial boom-bust dynamics. Moreover, we use our setup to explore whether the central bank is able to stabilize the housing market via interest rates. Using a mix of analytical and numerical tools, we find that the central bank has only a limited ability to tame housing markets by increasing the base (target) interest rate. Moreover, any change in the base (target) interest rate causes at least temporary housing market fluctuations. However, we are also able to show that a leaning-against-the-wind interest rate rule, which adjusts the interest rate with a view to mispricing in the housing market, can significantly improve the stability of housing markets. Within our model, and in line with empirical evidence (Case and Shiller 2003 and Case et al. 2012), housing market bubbles are driven by investors' optimistic expectations, an aspect that greatly destabilizes the demand for housing. An interest rate policy that counters these fluctuations in demand may effectively stabilize housing markets.

A few final comments are in order. In this paper, we use a stylized, analytically tractable housing market model to show that the central bank can stabilize the housing market if its interest rate setting takes into account the development of the housing market. Since official central bank mandates are usually geared to control consumer price inflation, policymakers must consider and disentangle the empirically blurry relationship between house prices and consumer prices. Without question, future work should thus study this issue using more elaborate macroeconomic models that explicitly include a housing market, such as the frameworks proposed by Erlingsson et al. (2014), Ozel et al. (2019) and de Grauwe and Macchiarelli (2019). Moreover, these models may also be used to analyze the effectiveness of macroprudential policies, e.g. of loan-to-income or loan-to-value measures. In this respect, we note that Geanakoplos et al. (2012) and Baptista et al. (2016) develop quite realistic agent-based models of the housing market that allow them to investigate the complex effects of leverage and collateral. To conclude, we hope that our paper fosters our understanding of the functioning of housing markets and enables us to design better tools that reduce damage arising from bursting bubbles. More work is undoubtedly needed in this important research direction.

Appendix A

In this appendix, we prove Proposition 1. A steady-state solution $(\overline{P}, \overline{H}, \overline{V}^S, \overline{U})$ of the dynamical system (23) necessarily satisfies the conditions

$$\overline{P} = \frac{E_t[\overline{P}] + \alpha - (\beta + \lambda(\Omega + \kappa\overline{V}^S))\gamma\overline{P} - (\beta + \lambda(\Omega + \kappa\overline{V}^S))(1-\delta)\overline{H}}{1 + r_0 + \rho\left(\frac{\overline{P} - P^*}{P^*}\right) + \delta},$$

$$\overline{H} = \gamma\overline{P} + (1-\delta)\overline{H},$$

$$\overline{V}^S = \nu\overline{V}^S + (1-\nu)(\overline{P} - \overline{U})^2$$

and

$$\overline{U} = \mu\overline{U} + (1-\mu)\overline{P},$$

where $E_t[\overline{P}]$ denotes the price expectations at the steady state. Let us define the fundamental steady state $FSS = (P^*, H^*, V^{S*}, U^*)$ as a steady-state solution to (23) in which $E_t[\overline{P}] = \overline{P}$ and $r_t = r_0$, i.e. $P^* = \overline{P}$. Therefore, expectations are realized at the steady state and the central bank does not change its base (target) interest rate, since prices mirror their fundamental values. It follows that the price at the fundamental steady state is given by

$$P^* = \frac{\alpha\delta}{(r_0 + \delta)\delta + (\beta + \lambda\Omega)\gamma},$$

while $H^* = \frac{\gamma}{\delta}P^*$, $U^* = P^*$ and $V^{S*} = 0$. However, the dynamical system (23) may also give rise to further nonfundamental steady states $(\overline{P}, \overline{H}, \overline{V}^S, \overline{U})$, such that $E_t[\overline{P}] \neq \overline{P}$. While these steady states cannot be expressed analytically, we will numerically encounter them in Section 5.

From the Jacobian of the fundamental steady state, i.e.

$$J(FSS) = \begin{pmatrix} \frac{1-\rho+\chi-\gamma(\beta+\lambda\Omega)}{1+\delta+r_0} & \frac{(\delta-1)(\beta+\lambda\Omega)}{1+\delta+r_0} & -\frac{\alpha\gamma\kappa\lambda\nu}{(1+\delta+r_0)(\delta(\delta+r_0)+\gamma(\beta+\lambda\Omega))} & 0 \\ \gamma & 1-\delta & 0 & 0 \\ 0 & 0 & \nu & 0 \\ 1-\mu & 0 & 0 & \mu \end{pmatrix},$$

we immediately see that two eigenvalues are given by $z_1 = \nu$ and $z_2 = \mu$. Since $0 < \nu, \mu < 1$, we have $|z_{1,2}| < 1$. The two further eigenvalues are the ones of the 2-D block

$$Q = \begin{pmatrix} \frac{1-\rho+\chi-\gamma(\beta+\lambda\Omega)}{1+\delta+r_0} & \frac{(\delta-1)(\beta+\lambda\Omega)}{1+\delta+r_0} \\ \gamma & 1-\delta \end{pmatrix}$$

from which we obtain the characteristic polynomial $P(z) = z^2 - zTr(Q) + Det(Q)$ with $Tr(Q) = \frac{1-\rho+\chi-\gamma(\beta+\lambda\Omega)}{1+\delta+r_0} + 1 - \delta$ and $Det(Q) = \frac{(\delta-1)(\rho-\chi-1)}{1+\delta+r_0}$. Necessary and sufficient conditions (Gandolfo

2009, Medio and Lines 2001) for $z_{3,4}$ to be smaller than one in modulus, which implies local asymptotic stability of the fundamental steady state, are given by $1+Tr(Q)+Det(Q)>0$, $1-Tr(Q)+Det(Q)>0$ and $1-Det(Q)>0$. Rewriting these inequalities in terms of the parameters reveals

$$\chi > \frac{4+2r_0-2\rho-\delta(\delta+r_0-\rho)-\gamma(\beta+\lambda\Omega)}{\delta-2},$$

$$\chi < \frac{\gamma(\beta+\lambda\Omega)}{\delta}+\delta+r_0+\rho,$$

and

$$\chi < \frac{2\delta+r_0}{1-\delta}+\rho.$$

Note that the violation of the first, second and third stability condition is a necessary condition for the emergence of a Flip, Pitchfork and Neimark-Sacker bifurcation. Combined with numerical evidence indicating that such bifurcations occur, they constitute strong evidence (Medio and Lines 2001).

Appendix B

In this appendix, we follow Chiarella et al. (2007, 2013) and express the investors' speculative variance component as

$$V_t^S = mV_{t-1}^S + m(1-m)(P_t-U_{t-1})^2,$$

where

$$U_t = mU_{t-1}+(1-m)P_t.$$

Note that $0<m<1$ also represents a memory parameter. The higher m is, the higher the weight given to past prices. Considering this alternative learning rule yields the following dynamical system

$$S: \begin{cases} P_t = \frac{E_t[P_{t+1}]+\alpha-(\beta+\lambda V_t[P_{t+1}])\gamma P_{t-1}-(\beta+\lambda V_t[P_{t+1}])(1-\delta)H_{t-1}}{1+r_0+\rho\left(\frac{P_{t-1}-P^*}{P^*}\right)+\delta} \\ H_t = \gamma P_{t-1}+(1-\delta)H_{t-1} \\ V_t^S = mV_{t-1}^S + m(1-m)(P_t-U_{t-1})^2 \\ U_t = mU_{t-1}+(1-m)P_t \end{cases},$$

where

$$E_t[P_{t+1}] = \frac{P_{t-1}+\chi(P_{t-1}-P^*)+P_{t-1}\eta(P^*-P_{t-1})^2+\phi\eta(P^*-P_{t-1})^3}{1+\eta(P^*-P_{t-1})^2}$$

and

$$V_t[P_{t+1}] = \Omega+\kappa V_{t-1}^S.$$

Straightforward computations reveal that the fundamental steady state is also given by $FSS = \left(P^*, H^*, V^{S*}, U^*\right) = \left(P^*, \frac{\gamma}{\delta}P^*, 0, P^*\right)$, where $P^* = \frac{\alpha\delta}{(r_0+\delta)\delta+(\beta+\lambda\Omega)\gamma}$. The Jacobian can be written as

$$J(FSS) = \begin{pmatrix} \frac{1-\rho+\chi-\gamma(\beta+\lambda\Omega)}{1+\delta+r_0} & \frac{(\delta-1)(\beta+\lambda\Omega)}{1+\delta+r_0} & -\frac{\alpha\gamma\kappa\lambda}{(1+\delta+r_0)(\delta(\delta+r_0)+\gamma(\beta+\lambda\Omega))} & 0 \\ \gamma & 1-\delta & 0 & 0 \\ 0 & 0 & m & 0 \\ \frac{(m-1)(\beta\gamma+\rho-\chi-1+\gamma\lambda\Omega)}{1+\delta+r_0} & -\frac{(\delta-1)(m-1)(\beta+\lambda\Omega)}{1+\delta+r_0} & \frac{(m-1)\alpha\gamma\kappa\lambda}{(1+\delta+r_0)(\delta(\delta+r_0)+\gamma(\beta+\lambda\Omega))} & m \end{pmatrix},$$

revealing that two eigenvalues are given by $z_{1/2} = m$. Since $0 < m < 1$, the local stability of the fundamental steady state depends only on the two eigenvalues of the remaining 2-D block

$$Q = \begin{pmatrix} \frac{1-\rho+\chi-\gamma(\beta+\lambda\Omega)}{1+\delta+r_0} & \frac{(\delta-1)(\beta+\lambda\Omega)}{1+\delta+r_0} \\ \gamma & 1-\delta \end{pmatrix}.$$

Since it is the same 2-D block as in the other case, we have the same stability conditions for $|z_{3,4}| < 1$.

Appendix C

In this appendix, we briefly sketch three consecutive model extensions. The first extension affects investors' housing demand. In our original model, each investor reconsiders his housing demand in every period. In reality, however, only a (small) fraction of investors will do so. Inspired by the asynchronous updating approach proposed by Hommes et al. (2005), let us define investors' effective housing demand by

$$\tilde{Z}_t = (1-h)\tilde{Z}_{t-1} + hZ_t, \tag{26}$$

where h is the fraction of investors who adjust their housing demand, represented by (12), and $1-h$ is the fraction of investors who adhere to their previous choices. Of course, parameter h is bounded between 0 and 1. From the market equilibrium condition $\tilde{Z}_t = H_t$, we then obtain the house price equation

$$P_t = \frac{E_t[P_{t+1}] + R_t - \frac{H_t-(1-h)\tilde{Z}_{t-1}}{h}\lambda V_t[P_{t+1}]}{1+r_t+\delta} + \epsilon_t. \tag{27}$$

Obviously, we recover our original model by setting $h = 1$. Moreover, the implications of (26) crucially depend on investors' risk aversion. If investors turn risk neutral, i.e. if $\lambda \to 0$, then the frequency with which investors update their housing demand becomes irrelevant. The top left panel of Figure 13 shows a simulation run based on our standard parameter setting, except that the central bank is inactive ($\rho = 0$) and that only 5 percent of investors update their housing demand in the current period ($h = 0.05$). As can be seen, our first model extension yields dynamics that is comparable to that generated by our original model. In fact, the mispricing of the housing market, at *distortion* $= 1.16$, is quite similar to the mispricing observed in our original model (Figure 3, *distortion* $= 1.19$). The top right panel of Figure 13 depicts the dynamics for $h = 0.05$ and $\rho = 0.1$. The central bank is able to stabilize the housing market accordingly by dynamically adjusting the interest rate. To be precise, mispricing decreases to *distortion* $= 0.55$, a slightly higher value than observed for our original model (Figure 5, *distortion* $= 0.44$).

In our original model, housing construction depends on the last observable price, a setup that is consistent with a one-period production lag and naive expectations. To allow for larger production lags, let us express housing construction by

$$I_t = \gamma \frac{1}{\tau} \sum_{i=1}^{\tau} P_{t-i}, \tag{28}$$

where parameter $\tau \geq 1$ indicates home builders' maximal production lag. The center left panel of Figure 13 presents a simulation run based on our standard parameter setting, except that $h = 0.05$, $\rho = 0$ and $\tau = 12$, i.e. housing construction depends on the last 12 observable house prices, representing a time span of three years. Simulated house prices still display bubbles and crashes. Since the housing stock now adjusts with a larger delay, mispricing increases to $distortion = 1.50$. However, the center right panel of Figure 13 reveals that the central bank can still stabilize the housing market by dynamically adjusting the interest rate. For $h = 0.05$, $\rho = 0.1$ and $\tau = 12$, mispricing reduces to $distortion = 0.68$.

As a final model extension, let us assume that housing construction depends negatively on the interest rate. We thus rewrite (28) by

$$I_t = \gamma_1 \frac{1}{\tau} \sum_{i=1}^{\tau} P_{t-i} - \gamma_2 \frac{1}{\tau} \sum_{i=1}^{\tau} r_{t-i} + \gamma_3, \tag{29}$$

where $\gamma_1 = \gamma > 0$, $\gamma_2 > 0$ and $\gamma_3 = \gamma_2 r_0 > 0$. The inclusion of γ_3 simply ensures that the model's steady-state values remain as they are. The bottom left panel of Figure 13 shows the dynamics for $h = 0.05$, $\tau = 12$, $\gamma_2 = 50$ and $\rho = 0$. As long as the central bank keeps the interest rate constant, this model extension will not affect the dynamics (this panel is identical to the center left panel). The bottom right panel of Figure 13 reveals that the central bank is able to tame housing market dynamics if we consider all model extensions. For $h = 0.05$, $\tau = 12$, $\gamma_2 = 50$ and $\rho = 0.1$, distortion decreases to $distortion = 0.80$. The decline in effectiveness of the leaning-against-the-wind interest rate rule may be explained as follows. The central bank increases (decreases) the interest rate when the house price is above (below) its fundamental value to reduce (increase) investors' housing demand. However, the resulting stabilizing effect is partially offset by the fact that there is less (more) housing construction in boom (bust) periods, impeding the stabilizing adjustment of the housing stock.

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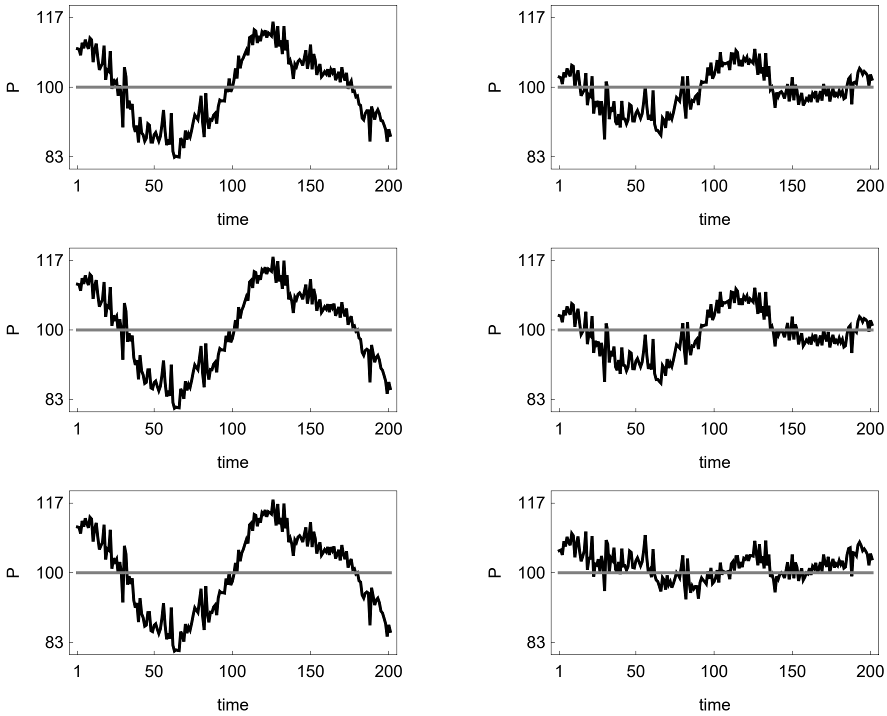

Figure 13: Model extensions. The top panels show the dynamics of the first model extension (base parameter setting and $h = 0.05$). The central panels show the dynamics of the first and second model extension (base parameter setting, $h = 0.05$ and $\tau = 12$). The bottom panels show the dynamics of all three model extensions (base parameter setting, $h = 0.05$, $\tau = 12$ and $\gamma_2 = 50$). Moreover, left (right) panels: $\rho = 0$ ($\rho = 0.1$).

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Paper 3

Heterogeneous expectations, housing bubbles and tax policy

Heterogeneous expectations, housing bubbles and tax policy

Carolin Martin, Noemi Schmitt and Frank Westerhoff

Abstract

We integrate a plausible expectation formation and learning scheme of boundedly rational investors into a standard user cost housing market model, involving a rental and a housing capital market. In particular, investors switch between heterogeneous expectation rules according to an evolutionary fitness measure, given by the rules' past profitability. We analytically show that our housing market model may produce endogenous boom-bust dynamics. Furthermore, we demonstrate that policy makers may use our model as a tool to explore how different tax policies may affect the housing market's steady state, its stability and out-of-equilibrium behavior.

Keywords: Housing markets, bubbles and crashes, heterogeneous expectations, bounded rationality and learning, tax policy, steady state and stability analysis
JEL classification: D84, H24, R31

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1. Introduction

Glaeser (2013), Shiller (2015) and Piazessi and Schneider (2016) stress the fact that history is replete with dramatic housing market bubbles that had serious effects on the real economy. Unfortunately, the reasons for such market turbulence are still not well understood. Against this backdrop, the goal of our paper is twofold. First, we propose a novel model to enhance our understanding of the complex boom-and-bust behavior of housing markets. Second, we use our model to explore the extent to which policy makers can influence such dynamics by adjusting housing market-related taxes.

Our model reveals that endogenous housing market fluctuations may emerge through the interaction of real and behavioral forces. The real forces acting inside our model originate from a standard user cost housing market setup in the spirit of Poterba (1984, 1991), involving a rental and a housing capital market; these forces tie basic relations between house prices, the housing stock and the rent level. The model's behavioral forces are due to the expectation formation behavior of housing market investors who display a boundedly rational learning behavior, as put forward by Brock and Hommes (1997, 1998). Accordingly, investors choose between extrapolative and regressive expectation rules to forecast the future evolution of housing markets, based on the rules' relative past profitability – an assumption that is also in line with empirical observations (Case and Shiller 2003, Hommes 2011, Case et al. 2013, Bao and Hommes 2019). With a view to the omnipresent wilderness-of-bounded-rationality critique, Glaeser (2013) and Hommes (2013) argue that a simple and plausible rule-governed expectation formation scheme describes reality better than a framework with fully rational expectations. Obviously, we follow their line of reasoning in our paper.

Despite the behavioral nature of our model, it possesses a unique (fundamental) steady state, given by the discounted value of future risk-adjusted rents or, in the jargon of the housing market literature, by the relation between risk-adjusted rents and the user cost of housing. Moreover, we analytically derive the conditions under which the housing market's steady state becomes unstable. As it turns out, the steady state's stability domain depends on the housing market's real and behavioral side. For instance, higher interest rates are beneficial for market stability, while the housing market loses its stability and starts to display significant oscillatory fluctuations if investors rely heavily on the extrapolative expectation rule. Policy makers may therefore seek to stabilize the housing market by imposing housing market-related taxes. Using a mix of analytical and numerical tools, our model allows policy makers to clarify how such taxes may affect the housing market's steady state, its stability and out-of-equilibrium behavior.

Our paper is organized as follows. After reviewing some related literature in Section 2, we present our model in Section 3. In Section 4, we study its steady state, stability and out-of-equilibrium behavior, and explore how different tax regimes affect the main properties of our model in Section 5. In Section 6, we conclude our paper.

2. Related literature

In reality, housing market participants face a number of different tax policies. For instance, Barrios et al. (2019), Fatica and Prammer (2018) and Lunde and Whitehead (2016) survey the main features of housing taxation in European countries, covering property (transfer) taxes incurred when buying a house, recurrent property taxes owed by households, (imputed) rent taxation and housing capital gains taxes. Almost all countries in Europe impose a transaction tax on transfer of a primary residential property, with tax rates ranging from one to ten percent. With very few exceptions, European countries also impose a recurrent property tax on owner occupied houses. As pointed out by Barrios et al. (2019), recurrent property taxes represent by large the biggest tax component of the user cost of housing in most European countries. Housing capital gains taxes, in contrast, are less popular in Europe, though Tse and Webb (1999), Shan (2011) and Agarwal et al. (2020) report that this tax is more popular in other regions of the world, e.g. in the United States or in China. For further international evidence on real-world housing taxation, see, e.g., Martin and Hanson (2016) and Norregaard (2013).

To embed our contribution into the theoretical literature, let us briefly discuss some related research in which the effects of expectations and taxes on housing market dynamics were studied. First of all, it is important to note that Poterba (1984) developed his user cost model to explore the extent to which the U.S. housing market boom in the 1970s can be explained by changes in housing market-related taxes. In particular, he demonstrates that a decrease in property taxes led to a reduction of the user cost of housing, which, in turn, was at least partially responsible for the substantial house price increase at that time. Poterba (1984) assumes that housing market investors have perfect foresight, implying that his model exhibits the saddle-path stability property. If the steady state of his model is disturbed, there is a unique path (the so-called "stable arm") along which the system will approach its new steady state. The adjustment path can be summarized as follows. At the time of the shock, the house price overshoots its new steady state since the housing stock is initially fixed. As the housing stock also begins to adjust towards its new steady-state value, the house price monotonically converges towards its new steady state.

While Poterba (1991) underscores the fact that changing tax policies were an important contributory factor in the house price rise in the late 1970s, he also admits that his user cost argument is less able to explain the consequent housing market decline. To better understand the boom-bust behavior of housing markets, Poterba (1991) recommends to take into account the possibility of housing market investors extrapolating past price changes. Weil (1991) and Shiller (1991) strongly agree with this view. For instance, Weil (1991, p. 188) states that *"economists are going to have to bite the bullet and look at models that allow for not-fully rational expectations"*, advocating, amongst others, the modeling of extrapolative expectations. Moreover, Shiller (1991, p. 189) questions both the efficiency of housing markets and investors' forecasting ability, and ultimately stresses that there *"appears to be a purely speculative component of real estate prices"*. In the end, Poterba (1992) concludes that his user cost framework allows a clear-cut analysis of how tax reforms affect the steady-state levels of house prices, the rent level and the stock of housing, but that future work needs to study the dynamics and adjustment processes of (inefficient) housing markets in more detail.[1]

Despite such prominent encouragements, an empirically motivated modeling of market participants' house price expectations within a dynamic context is only slowly taking place in the economic profession. A rare and early exception is Wheaton (1999), who demonstrates that Poterba's (1984, 1991) user cost framework can produce more realistic oscillatory house price dynamics if housing market investors follow a simple rule-of-thumb behavior to predict house prices. Dieci and Westerhoff (2012) present a stylized model of a speculative housing market in which investors switch between extrapolative and regressive expectation rules, subject to the market's mispricing, thereby generating complex house price fluctuations. Dieci and Westerhoff (2013) extend the latter analysis by considering, amongst others, that investors may follow nonlinear expectation rules. Diks and Wang (2016) exploit elements from catastrophe theory to study a housing market model with extrapolators and mean reversion believers. Based on U.S. house price data, Kouwenberg and Zwinkels (2014, 2015) find empirical support for models in which investors switch between heterogeneous expectation rules according to their forecasting ability. Interestingly, the parameter estimates for their nonlinear housing market models suggest that housing markets may indeed be characterized by cyclical dynamics. Similar conclusions can be drawn from the work by Eichholtz et al. (2015) who utilize 350 years of house prices along the Herengracht in Amsterdam. However, the estimation results offered by Bolt et al. (2019) and ter Ellen et al. (2020) suggest that housing markets may be characterized by multiple nonfundamental steady states. Technically speaking, the boom-bust nature of housing markets may therefore also be due to attractor switching phenomena. Relatedly, Bao and Hommes (2019) design an experimental housing market to study how the interplay of real and speculative forces affects the formation of bubbles and crashes. Importantly, all these empirical works indicate that investors' adherence to extrapolative and regressive expectation rules amplifies housing markets' boom-bust behavior. For simplicity, these models abstain from developing an endogenous mechanism for the formation of the rent level, a crucial ingredient of our model.

Another line of research is motivated by Shiller's (2015) observation that mass psychology and investor sentiment are elements that play an important role in the determination of house prices. Burnside et al. (2016) explain the irregular boom-bust behavior of housing markets by a model in which housing market investors' projection of the future (fundamental) state of the housing market is either optimistic or pessimistic; they show that irregular boom-bust house price dynamics may occur due to waves of optimism and pessimism. Piazzesi and Schneider (2009) show that even a small fraction of optimistic housing market investors may be enough to trigger a housing market bubble. Glaeser and Nathanson (2017) propose a powerful framework in which housing market investors are boundedly rational, overconfident and extrapolate past house price changes into the future. Interestingly, their

[1]Further tax-related housing market papers with a similar spirit include Poterba and Sinai (2008) and Himmelberg et al. (2005). Poterba's (1984, 1991) model has been extended in many more directions. See Glaeser and Nathanson (2015) for a recent appraisal.

calibrated model matches key stylized facts of housing markets quite well, thereby underscoring the explanatory power of models that deviate from the assumption of full rationality.

However, our work is related more closely to the papers by Dieci and Westerhoff (2016) and Schmitt and Westerhoff (2019). Dieci and Westerhoff (2016) present a discrete-time generalization of Poterba's (1984, 1991) user cost model in which housing market investors can choose between different expectation rules, subject to market circumstances. Their goal is to explore how the housing market's supply side, in connection with speculative forces, may trigger and shape boom-bust dynamics.[2] Based on the same user cost model, Schmitt and Westerhoff (2019) assume that risk-neutral housing market investors switch between extrapolative and regressive expectation rules with respect to the rules' forecasting accuracy, measured in terms of squared prediction errors. Since investors are risk neutral, the fundamental house price is given by the discounted value of future rent payments. Moreover, they show that endogenous housing market dynamics, characterized by short-run momentum, long-run mean reversion and excess volatility, may only arise if investors rely heavily on extrapolative expectations. Note that this is one of the few contributions in the field where market participants display a boundedly rational learning behavior – an important model ingredient to counter the wilderness-of-bounded-rationality criticism, as advocated in Glaeser (2013) and Hommes (2013).

As already mentioned, the goal of our paper is twofold. First, we develop a novel model to foster our understanding of housing market dynamics. Second, we use our model to explore the extent to which policy makers may influence housing markets by adjusting the tax code. To achieve our goals, we follow Brock and Hommes (1997, 1998) by assuming that housing market investors are risk averse and switch between competing expectation rules subject to the rules' past profitability.[3] In contrast to Schmitt and Westerhoff (2019), the fundamental house price thus corresponds to the discounted value of future risk-adjusted rent payments, an aspect that also influences the model's stability properties and out-of-equilibrium behavior. Due to investors' profit-dependent rule-selection behavior, our model may furthermore serve as a framework to explore the effects of housing market-related taxes. As we will see, policy makers have the opportunity to affect the housing market via the housing market's real and behavioral side. As far as we are aware, such a modeling and, in particular, policy perspective is new in this line of research.

3. The basic model framework

Our model combines the housing market framework by Poterba (1984, 1991) with the heuristic switching approach by Brock and Hommes (1997, 1998). In particular, the housing market consists of two interrelated markets – a rental market and a housing capital market – that fix basic relations between

[2] Martin et al. (2020) use the model by Dieci and Westerhoff (2020) to explore to which extend central banks may stabilize the dynamics of housing markets by adjusting the interest rate.

[3] The heuristic switching approach by Brock and Hommes (1997, 1998) has been used in numerous models and applications. Powerful examples include Droste et al. (2002), de Grauwe and Grimaldi (2006), Boswijk et al. (2007) Anufriev and Hommes (2012) and Schmitt and Westerhoff (2015). Dieci and He (2018) provide a detailed review of this field, and also discuss its connection with the housing market literature.

house prices, the housing stock and the rent level. Investors' demand for housing stock depends on their house price expectations. Motivated by the aforementioned theoretical and empirical literature, we assume that housing market investors select between an extrapolative and a regressive expectation rule to forecast future house prices, depending on the evolutionary fitness of these rules, measured in terms of past realized profits. For ease of exposition, we first abstain from considering housing market-related taxes. These will be introduced in Section 5.

Let us turn to the details of the model. Market clearing in the rental market takes place in every period t, implying that the demand for housing services D_t is equal to the supply of housing services S_t, i.e.

$$D_t = S_t. \tag{1}$$

Demand for housing services is assumed to be linearly decreasing at the current rent level R_t, the price of housing services, and is formalized as

$$D_t = a - bR_t, \tag{2}$$

where a and b are positive parameters. The supply of housing services is proportional to the current housing stock H_t, and can be expressed by

$$S_t = cH_t, \tag{3}$$

where $c > 0$. Combining (1), (2) and (3) reveals that the rent level R_t depends negatively on the existing housing stock, i.e.

$$R_t = \alpha - \beta H_t, \tag{4}$$

where $\alpha = \frac{a}{b} > 0$ is a scaling parameter and $\beta = \frac{c}{b} > 0$ denotes the sensitivity of the rent level with respect to the housing stock.[4] Of course, the model parameters have to ensure that $R_t > 0$.

Housing market investors can invest in a risk-free asset or in housing capital over the time horizon from period t to period $t+1$. The risk-free asset pays a fixed rate of return $r > 0$, while housing generates (imputed) rents R_t, which are fixed at the beginning of the period. By defining P_t as a hypothetical house price level at time t, the wealth of investor i in period $t+1$ is given by

$$W^i_{t+1} = (1+r)W^i_t + Z^i_t(P_{t+1} + R_t - (1+r+\delta)P_t), \tag{5}$$

where W^i_t and Z^i_t represent the wealth and demand for housing stock of investor i at time t. Note that parameter $0 < \delta < 1$ denotes the housing depreciation rate and that variables indexed with $t+1$ are regarded as random.

Housing market investors are assumed to be myopic mean-variance maximizers, implying that their demand for housing stock follows from

$$max_{Z^i_t} \left\{ E^i_t[W^i_{t+1}] - \frac{\lambda^i}{2} V^i_t[W^i_{t+1}] \right\}, \tag{6}$$

[4]For analytical tractability, we model the rental market such that the rent level linearly decreases with the existing stock of houses. See Dieci and Westerhoff (2016) for an isoelastic setup. Of course, extending our partial equilibrium model into a general equilibrium model would be a worthwhile endeavor.

where $E_t^i[W_{t+1}^i]$ and $V_t^i[W_{t+1}^i]$ describe the belief of investor i about the conditional expectation and conditional variance of his wealth in period $t+1$, while parameter λ^i denotes the corresponding (absolute) risk aversion. Solving (6) for Z_t^i then yields

$$Z_t^i = \frac{E_t^i[P_{t+1}] + R_t - (1+r+\delta)P_t}{\lambda^i V_t^i[P_{t+1}]}. \tag{7}$$

Accordingly, investor i's optimal demand for housing stock depends positively on house price expectations and the rent level, and negatively on the interest rate, the depreciation rate, the current house price and the perceived housing market risk.

In the following, we introduce a few simplifying assumptions. First, investors' beliefs about conditional variance of the price are constant for all t and uniform across all investors i, i.e. $V_t^i[P_{t+1}] = \sigma^2 > 0$. Second, all investors have the same risk aversion, i.e. $\lambda^i = \lambda > 0$. Therefore, investors' total housing demand can be expressed as $Z_t = \sum_{i=1}^{N} Z_t^i = \frac{N}{\lambda\sigma^2}\Big(\frac{1}{N}\sum_{i=1}^{N} E_t^i[P_{t+1}] + R_t - (1+r+\delta)P_t\Big)$. Finally, by denoting investors' average house price expectations by $E_t[P_{t+1}] = \frac{1}{N}\sum_{i=1}^{N} E_t^i[P_{t+1}]$, we obtain

$$Z_t = N\frac{E_t[P_{t+1}] + R_t - (1+r+\delta)P_t}{\lambda\sigma^2}. \tag{8}$$

To spare one parameter, let us normalize the mass of investors to $N = 1$.

As equilibrium of demand and supply in the housing capital market is given by

$$Z_t = H_t, \tag{9}$$

the market clearing price P_t can be expressed as

$$P_t = \frac{E_t[P_{t+1}] + \hat{R}_t}{1+r+\delta}, \tag{10}$$

where $\hat{R}_t = R_t - H_t\lambda\sigma^2$. Accordingly, the house price is equal to the discounted value of investors' next period's average house price expectation plus risk-adjusted rent payments; a standard no-arbitrage condition common in models with an asset-pricing nature.

The housing stock evolves as

$$H_t = I_t + (1-\delta)H_{t-1}, \tag{11}$$

where I_t indicates the amount of new housing construction in period t. Note that we assume that houses are built with a one-period production lag. Moreover, home builders are risk neutral, and maximize expected profits, subject to a quadratic cost function, i.e. $max_{I_t}\{E_{t-1}[P_t]I_t - C_t\}$, where $C_t = \frac{1}{2\gamma}I_t^2$. Consequently, new housing construction is given by

$$I_t = \gamma E_{t-1}[P_t], \tag{12}$$

where $\gamma > 0$ is an inverse cost parameter which implies that a lower value of γ generates higher building costs and a more sluggish housing supply. By assuming that home builders form naive expectations, i.e. $E_{t-1}[P_t] = P_{t-1}$, the evolution of the housing stock can be rewritten as

$$H_t = \gamma P_{t-1} + (1-\delta)H_{t-1}. \tag{13}$$

Let us now turn to the expectation formation behavior of housing market investors. Inspired by

Brock and Hommes (1997, 1998), investors select between competing expectation rules to forecast future house prices. In this paper, we concentrate on two representative types of expectation rules: a free extrapolative expectation rule, denoted by $E_t^E[P_{t+1}]$, and a costly regressive expectation rule, i.e. $E_t^R[P_{t+1}]$. Investors' average house price expectations can thus be defined as

$$E_t[P_{t+1}] = N_t^E E_t^E[P_{t+1}] + N_t^R E_t^R[P_{t+1}], \tag{14}$$

where N_t^E and N_t^R stand for the market shares of investors relying on extrapolative and regressive expectations, respectively. Extrapolative expectations presume that house prices move in trends; it can be expressed by

$$E_t^E[P_{t+1}] = P_{t-1} + \chi(P_{t-1} - P_{t-2}). \tag{15}$$

Accordingly, extrapolators pay attention to the most recent price trend, where $\chi \geq 0$ indicates how strongly investors extrapolate past house price trends into the future. For $\chi = 0$, (15) implies naive expectations. In contrast, regressive expectations are formalized as

$$E_t^R[P_{t+1}] = P_{t-1} + \phi(F - P_{t-1}), \tag{16}$$

where F represents the housing market's fundamental value and $0 < \phi \leq 1$ the expected adjustment speed. Thus, investors who follow this rule believe that house prices will return towards their fundamental value over time. We assume that investors have access to the true fundamental value of the housing market, given by $F = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$ (see the appendix for its derivation). The underlying time structure of investors' expectation rules is as follows. At the beginning of period t, investors have to predict the house price for period $t+1$. Since the house price of period t has not yet been determined (it depends on investors' demand, which, in turn, depends on their expectations), their predictions are conditional on information up to period $t-1$. As in Brock and Hommes (1998) and many other related models, investors thus have to predict the house price two-time steps ahead.

In each time step, housing market investors have to determine which expectation rule to follow. This decision depends on the rules' fitness. We assume that the higher the fitness of an expectation rule, the more investors will follow it. As in Brock and Hommes (1997, 1998), and based on Manski and McFadden (1981), we update the market share of investors using the extrapolative and the regressive expectation rule via the multinomial discrete-choice model. Therefore, we obtain

$$N_t^E = \frac{\exp[\nu A_t^E]}{\exp[\nu A_t^E] + \exp[\nu A_t^R]} \tag{17}$$

and

$$N_t^R = \frac{\exp[\nu A_t^R]}{\exp[\nu A_t^E] + \exp[\nu A_t^R]}, \tag{18}$$

where A_t^E and A_t^R denote the fitness of extrapolative and regressive expectations in period t, respectively. Parameter $\nu \geq 0$ measures how sensitively investors choose the most attractive expectation rule. For $\nu = 0$, investors do not observe any fitness differentials between the two expectations rules, and both market shares will be equal to $\frac{1}{2}$. As the intensity of choice parameter ν increases, more and more investors switch to the expectation rule with the higher fitness. For $\nu \to +\infty$, fitness differentials are perfectly observed, and all investors opt for the expectation rule yielding the highest fitness. Since the weights of the two expectation rules add up to 1, the market share of extrapolative (regressive) expectations can also be written as $N_t^E = 1 - N_t^R$ ($N_t^R = 1 - N_t^E$).

The fitness of the two expectation rules in period t depends on realized past profits and can be described by

$$A_t^E = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^E \tag{19}$$

and

$$A_t^R = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^R - c, \tag{20}$$

where

$$Z_t^E = \frac{E_t^E[P_{t+1}] + R_t - (1 + r + \delta)P_t}{\lambda\sigma^2} \tag{21}$$

and

$$Z_t^R = \frac{E_t^R[P_{t+1}] + R_t - (1 + r + \delta)P_t}{\lambda\sigma^2} \tag{22}$$

represent investors' demand for housing stock in period t when forming extrapolative and regressive expectations, respectively. Accordingly, the expectations rules' fitness of period t depends on investors' housing demand of period $t-2$ and investors can determine the fitness values for period t as soon as the house price for period $t-1$ is observable. Note that it may be costly to use the regressive expectation rule since investors have to acquire some kind of knowledge about the economy. In particular, investors have to examine what the housing market's fundamental house price will be, and how quickly the housing market will return to this value. We follow Brock and Hommes (1998) and capture this effort by the information cost parameter $c \geq 0$, which reduces the fitness of regressive expectations. As pointed out by Hommes (2013), realized net profits are a natural candidate for an evolutionary fitness measure since this is what investors seem to care about most in real markets.

4. Implications of our basic model framework

We now explore our basic model framework. In Section 4.1, we first present our main analytical results. In Section 4.2, we then continue with a numerical investigation of our model.

4.1. Analytical insights

In the appendix, we show that the dynamics of our model is driven by a six-dimensional nonlinear map, and prove the following results.

Proposition 1. *The model's unique steady state, implying, amongst others, that* $\overline{P} = F$, *where* $F = \frac{\alpha\delta}{(r+\delta)\delta + (\beta+\lambda\sigma^2)\gamma} = \frac{\overline{R} - \lambda\sigma^2\overline{H}}{r+\delta}$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *loses its local asymptotic stability if either*

$$(i)\ \ \overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} \ < \ \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$$

or

$$(ii)\ \ \overline{N}^R\phi + \frac{\gamma(\beta+\lambda\sigma^2)}{2-\delta} \ < \ 2 + r + \delta + 2\overline{N}^E\chi$$

becomes violated, where $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$*, respectively. Moreover, a violation of the first (second) inequality (while the other inequality holds) is associated with a Neimark-Sacker (Flip) bifurcation, giving rise to cyclical dynamics (a period-two cycle).*

Proposition 1 deserves comment. Let us start with the properties of the model's steady state. Note that $\overline{P}$, $\overline{H}$ and $\overline{R}$ are independent of any behavioral parameters. Since $\overline{P} = F = \frac{\overline{R}-\lambda\sigma^2\overline{H}}{r+\delta} = \frac{\overline{\tilde{R}}}{r+\delta}$, it becomes clear that investors discount future risk-adjusted rent payments to compute the housing market's fundamental value. This is also in line with Poterba (1984, 1991), who defines the term $r+\delta$ as the user cost of housing. Although he considers perfect foresight, our steady state is basically equivalent to the one in his models because our housing market investors make no prediction errors at the steady state. For this reason, we regard the model's unique steady state as a fundamental steady state.

Proposition 1 allows us to draw the following steady-state conclusions. An increase in the interest rate decreases investors' demand for housing stock and, consequently, leads to a reduction of the house price; a lower housing stock; and a higher rent level. Comparable effects are observed if housing market investors become more risk averse and/or perceive a higher housing market risk. If it gets cheaper to build new houses, i.e. if the inverse cost parameter γ increases, house prices as well as the rent level decrease, while the housing stock increases. A higher depreciation rate reduces the stock of housing and, consequently, yields a higher rent level. However, house prices only decrease if $\delta > \sqrt{(\beta+\lambda\sigma^2)\gamma}$. In this case, the effects of an increase in the interest rate and the depreciation rate are qualitatively the same. With respect to the parameters describing the rental market, we can conclude that an increase in the scaling parameter α increases the house price, the housing stock and the rent level, while an increase in the sensitivity parameter β causes the opposite. For completeness, we mention that $\overline{Z}^E = \overline{Z}^R = \overline{H}$, $\overline{A}^E = (\overline{R} - (r+\delta)\overline{P})\overline{H}$ and $\overline{A}^R = (\overline{R} - (r+\delta)\overline{P})\overline{H} - c$. Since $\overline{A}^E - \overline{A}^R = c$, the steady-state fractions of investors relying on extrapolative and regressive expectations, given by $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$, depend only on investors' intensity of choice and on the costs of forming regressive expectations.

Let us now turn to the steady state's stability properties. Note that both stability conditions depend on real and behavioral parameters. Since housing markets display cyclical dynamics, a phenomenon associated with a Neimark-Sacker bifurcation, our main focus is on Proposition 1's first stability condition.[5] First of all, if we assumed naive versus regressive expectations, i.e. $\chi = 0$ and $0 < \phi < 1$,

[5]A Neimark-Sacker bifurcation occurs if the modulus of a pair of complex, conjugate eigenvalues crosses the unit circle, giving rise to periodic or quasi-periodic motion. The contributions by Wheaton (1999), Kouwenberg and Zwinkels (2014), Dieci and Westerhoff (2016), Glaeser and Nathanson (2017) and Bao and Hommes (2019) also focus on scenarios with complex, conjugate eigenvalues, seeking to explain the oscillatory boom-bust behavior of real housing markets, as documented by Glaeser (2013), Shiller (2015) and Piazzesi and Schneider (2016). In contrast, a Flip bifurcation requires that a real eigenvalue passes through -1, causing the emergence of a period-two cycle.

the Neimark-Sacker condition would always be fulfilled. Cyclical housing market dynamics can thus only arise within our model if investors extrapolate past price changes. However, it is also obvious from stability condition (i) that cyclical housing market dynamics becomes less likely if investors expect house prices to return towards their fundamental value more quickly. Furthermore, the Neimark-Sacker condition is also violated when $\overline{N}^E\chi$ moves towards $1+r+\delta$ (see the denominator of the second term on the left-hand side). In this respect, it might be insightful to explore two extreme scenarios. If either information costs c or the intensity of choice parameter ν converge to infinity, then all investors form extrapolative expectations. Hence, stability will be lost at the latest as χ approaches $1+r+\delta$. If c and/or ν converge to zero, only half of investors form extrapolative expectations, and stability will be lost at the latest as χ approaches $2(1+r+\delta)$. In this sense, we can conclude that an increase in c or ν may destabilize the model's steady state. Finally, increasing values of the real parameters β, γ, λ and σ^2 harm the stability of housing markets, while an increase in r has a beneficial effect. Introducing the innocuous assumption that $r+\delta<1$ furthermore reveals that an increase in the depreciation rate also contributes to the stability of housing markets.

From an economic perspective, the violation of the Flip bifurcation boundary, causing a period-two cycle, is of secondary importance. Nevertheless, the second stability condition of Proposition 1 reveals that an increase in parameter ϕ, capturing investors' expected mean reversion speed, may create a period-two cycle, provided that parameters β, γ, λ and σ^2 are sufficiently large. Such a bifurcation becomes more likely if the market share of regressive expectations increases, which is the case if information costs and/or investors' intensity of choice decrease. Finally, we note that an increase in the interest rate, the depreciation rate or investors' extrapolation strength may reverse a Flip bifurcation.

4.2. Numerical insights

Equipped with our analytical insights, we are now ready to explore the model's out-of-equilibrium behavior. Table 1 presents the base parameter setting for our numerical investigations. Since the interest rate and the depreciation rate are given by five percent, one time step in our simulations may roughly be regarded as one year. Accordingly, the production lag in the housing market is also given by about one year, which seems to be a reasonable choice for a model like ours. The remaining parameters are selected such that our model is able to mimic - at least in a qualitative sense - the boom-bust behavior of housing markets, as documented in Glaeser (2013), Shiller (2015) and Piazzesi and Schneider (2016). However, we remark that the behavioral parameters of our model, in particular those affecting investors' expectation formation, are in line with empirical and experimental observations (Case and Shiller 2003, Case et al. 2012, Anufriev and Hommes 2012, Bao and Hommes 2019 and ter Ellen et al. 2020).

Note that the base parameter setting implies that $\overline{P}=F=1$, $\overline{H}=20$, $\overline{R}=0.3$ and $\overline{N}^E\approx 0.731$. Furthermore, the model's steady state is unstable. For instance, the extrapolation parameter, which is given by $\chi=1.1$, is slightly above the Neimark-Sacker threshold $\chi^{NS}_{crit}\approx 1.08$ (while the Flip condition

Table 1: Base parameter setting

$\alpha = 2.3$	scaling parameter	rental market
$\beta = 0.1$	sensitivity of rental market	
$\gamma = 1$	sensitivity of home building	supply side of housing market
$\delta = 0.05$	depreciation rate	
$r = 0.05$	interest rate	
$\chi = 1.1$	extrapolative parameter	price expectations
$\phi = 0.6$	regressive parameter	
$\nu = 1$	intensity of choice	
$\lambda = 6.25$	risk aversion	risk perception
$\sigma^2 = 0.0016$	variance beliefs	
$c = 1$	information cost	fitness

is not violated). And, in fact, the dynamics depicted in Figure 1 displays endogenous boom-bust housing market dynamics. To be precise, the panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The simulation run comprises 30 observations; a longer transient period has been deleted.

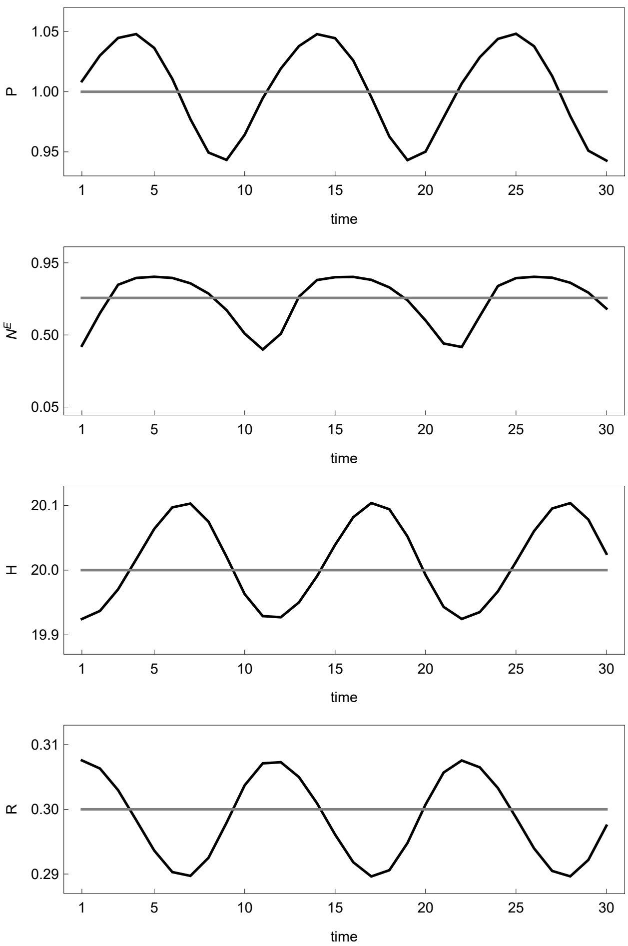

Figure 1: Snapshot of the model dynamics for our base parameter setting. The panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 30 time steps; a longer transient period has been deleted.

The functioning of the model may be explained as follows. Suppose that the house price has just increased above its fundamental value. In such a situation, the extrapolative expectation rule has correctly predicted a further price increase, while regressive expectations have falsely predicted a reversion towards the fundamental value. For this reason, the extrapolative expectation rule is more profitable than the regressive one. As extrapolative expectations now attract more followers, house prices increase further. Eventually, however, the market loses momentum. This could happen for several reasons. First, the market share of extrapolators cannot grow forever. Second, the remaining investors who rely on the regressive expectation rule bet increasingly aggressively on a mean reversion of the housing market. Third, the housing stock has increased due to the construction of new housing during the formation of the bubble. This depresses the rent level and therefore dampens house prices, too. At the bubble's turning point, extrapolative expectations are wrong, while regressive expectations are right. But once the direction of the housing market reverses, both expectation rules correctly anticipate a downturn of the housing market. Moreover, new housing construction – due to house prices that are still relatively high – lets the housing stock grow for a few more periods, pushing the rent level down further. Together, these behavioral and real forces lead to an overshooting in the housing market, i.e. house prices drop below their fundamental value. Then, we once again have a situation in which extrapolative expectations produce more accurate predictions than the regressive expectation rule. However, investors' learning behavior depends on past realized profits, which is why the market share of extrapolators recovers with some delay. In between, the rent level increases again. Since a considerable fraction of investors still uses the regressive expectation rule, prices are pushed upwards, and we see the emergence of the next housing market bubble.

In fact, it is the complex interplay between real and behavioral forces that keeps the dynamics alive. While real forces, in particular the housing stock and rent adjustments, tend to stabilize the housing market, behavioral forces have a double-edged effect. Extrapolative expectations tend to push house prices away from fundamentals; regressive expectations, in turn, exercise mean-reversion pressure. Note that the boom-bust cycle depicted in Figure 1 repeats itself in a more or less regular manner. Figure 2 reveals, however, that our model is also able to produce more irregular dynamics. The simulation run – now for 60 time steps – rests on the base parameter setting, except that $\chi = 1.35$, $\phi = 0.75$ and $\nu = 1.3$. These parameter changes leave the model's fundamental steady state unaffected, although $\overline{N}^E$ increases from 0.731 to 0.786. Of course, the model's instability is still due to a Neimark-Sacker bifurcation. As can be seen, stronger house price cycles result in strong housing stock oscillations, and thus in more volatile rent levels. Needless to say, irregular dynamics may also be observed in the presence of exogenous noise (not depicted), although our model may generate them completely endogenously.

To demonstrate how the dynamics of our model depends on its parameters, we next present a number of bifurcation diagrams in Figure 3. Bifurcation diagrams are a powerful graphical tool to visualize the dynamics of a non-linear dynamical system with respect to its parameters. The so-called bifurcation parameter is discretely varied between selected boundary values. For each parameter value, the model's dynamics is then plotted in the bifurcation diagram, omitting a sufficient large transient

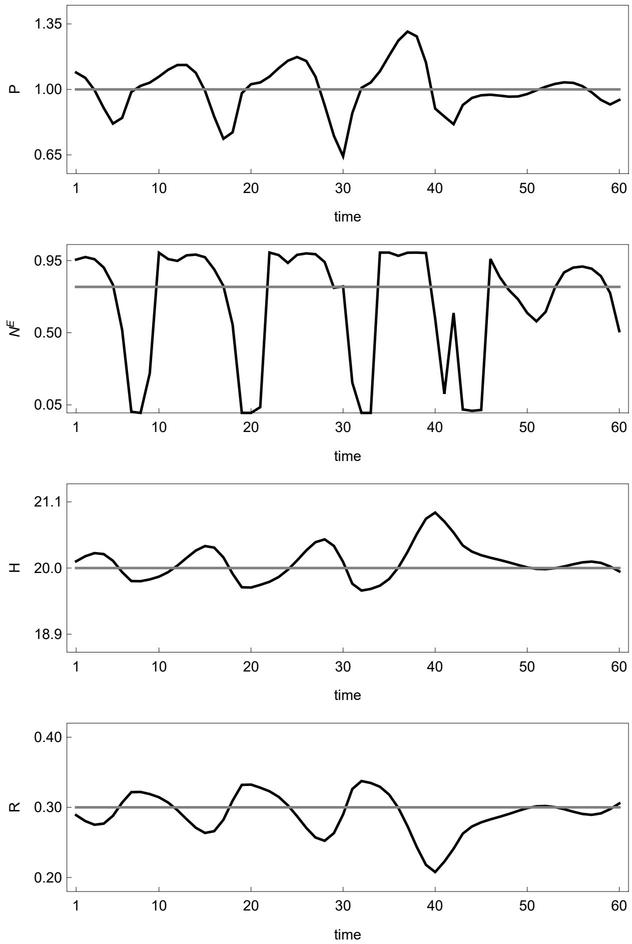

Figure 2: Snapshot of the model dynamics for an alternative parameter setting. The panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 60 time steps; a longer transient period has been deleted. Parameter setting as in Figure 1, except that $\chi = 1.35$, $\phi = 0.75$ and $\nu = 1.3$.

period. Here, we provide examples of how our behavioral parameters χ, ϕ, c and ν may influence house price dynamics. The panels depict, from top left to bottom right, bifurcation diagrams for $1.04 < \chi < 1.16$, $0.52 < \phi < 0.78$, $0.8 < c < 1.2$ and $0.8 < \nu < 1.2$, respectively. As already stated in Proposition 1, these parameters do not affect the housing market's fundamental steady-state price, i.e. $\overline{P} = F = 1$. In the top left panel, the fundamental steady state is initially stable and loses its stability as soon as χ exceeds the critical value $\chi^{NS}_{crit} \approx 1.08$, for which endogenous quasi-periodic dynamics emerges. While the amplitude of house price fluctuations becomes larger if extrapolators react more aggressively to past house price trends, the top right panel shows that a stronger belief in mean reversion reduces their amplitude. In fact, a convergence to the steady state sets in when ϕ surpasses $\phi^{NS}_{crit} \approx 0.674$. Of course, these observations correspond to our analytical results, which are supported further by the bottom two panels. The bifurcation diagram for parameter c shows that the fundamental steady state becomes unstable at $c^{NS}_{crit} \approx 0.953$, after which the amplitude of house price dynamics increases with information costs. The reason for this is that rising information costs increasingly reduce the fitness of the stabilizing regressive expectation rule. Consequently, more and

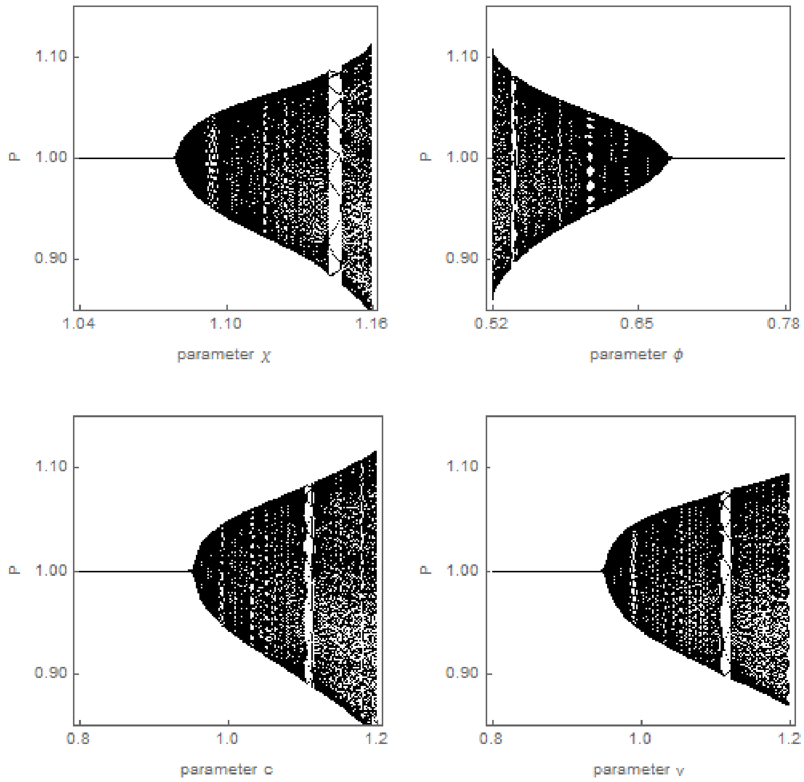

Figure 3: Effects of behavioral parameters on house price dynamics. The panels show, from top left to bottom right, bifurcation diagrams for the extrapolative parameter χ, the regressive parameter ϕ, information costs c and the intensity of choice ν. Parameters are as in our base parameter setting.

more investors switch to extrapolative expectations, which has a destabilizing impact on housing market dynamics. A very similar bifurcation route can be observed in the bottom right panel. As can be seen, at $\nu^{NS}_{crit} \approx 0.953$, the fixed-point dynamics turns into quasi-periodic motion. The destabilizing impact of an increasing intensity of choice can be easily explained. Recall that extrapolative expectations have a higher steady-state fitness than regressive expectations. Since investors react more sensitively to fitness differences as ν increases, more and more of them will opt for extrapolative expectations, which destabilizes the dynamics, and the amplitude of house price fluctuations increases.[6]

In Figure 4, we show how house prices react to an increase in the model's real parameters. The six panels show bifurcation diagrams for $0.02 < r < 0.1$, $0.045 < \delta < 0.065$, $0.9 < \gamma < 1.1$, $0 < \lambda < 12.5$, $0.09 < \beta < 0.11$ and $2.15 < \alpha < 2.45$. It can be seen from the top left panel that an increasing interest rate decreases the amplitude of house price fluctuations. Moreover, the quasi-periodic dynamics converges into a stable fixed point when r exceeds $r^{NS}_{crit} \approx 0.06$. However, the steady-state house price $\overline{P}$ decreases with r. Similar observations are apparent in the top right panel, where the amplitude of house price fluctuations becomes smaller when the depreciation rate increases. At $\delta^{NS}_{crit} \approx 0.058$, the quasi-periodic dynamics segues into our stable fundamental steady state which, in turn, increases

[6]Interestingly, similar effects of the behavioral parameters can be observed in the related asset-pricing and cobweb model by Brock and Hommes (1997, 1998).

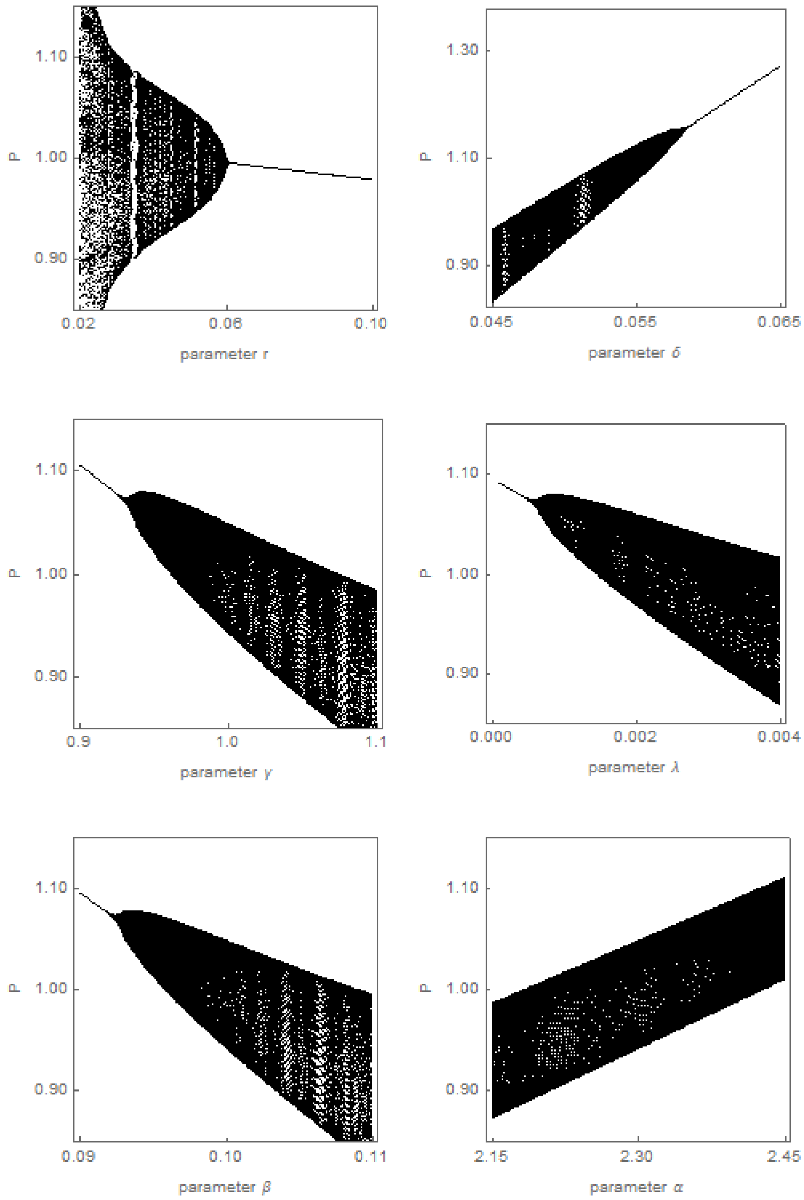

Figure 4: Effects of real parameters on house price dynamics. The panels show, from top left to bottom right, bifurcation diagrams for the interest rate r, the depreciation rate δ, the inverse cost parameter γ, the risk aversion parameter λ, the sensitivity parameter β and the scaling parameter α. Parameters are as in our base parameter setting.

with δ. The destabilizing impact of the inverse cost parameter γ, the risk aversion parameter λ and the rental market's sensitivity parameter β are presented in the two middle panels and the bottom left panel, respectively. Note that their bifurcation routes are very similar. For increasing values of parameters γ, λ and β, the stable steady-state house price decreases and becomes unstable at bifurcation values $\gamma_{crit}^{NS} \approx 0.933$, $\lambda_{crit}^{NS} \approx 1.65$ and $\beta_{crit}^{NS} \approx 0.093$, respectively. At these points, a Neimark-Sacker bifurcation occurs, and the amplitude of house price fluctuations becomes larger as the parameters increase further. Finally, it becomes apparent from the bottom right panel that the scaling parameter α has no effect on the stability of housing market dynamics, as the amplitude of house price oscillations basically remains constant as α increases.

5. The housing market model with taxes

In Sections 5.1 to 5.5, we explore the model's steady state and stability properties as well as out-of-equilibrium effects when various different tax policies are considered. In doing so, we seek to demonstrate that our model provides a useful framework to address the effects of housing market-related taxes. With respect to the steady state's stability domain, we focus on the emergence of cyclical dynamics, that is on the Neimark-Sacker stability condition. Our main results are summarized by Propositions 2 to 5 (their proofs are analogous to the one of Proposition 1, see the appendix) and illustrated in Figure 5.

5.1. Tax on the value of houses

Let us start our analysis with the case of a property tax, imposed periodically (annually) on the value of houses. Note that such a tax affects investor i's wealth equation, which turns into

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta+\tau)P_t), \tag{23}$$

where τ stands for the tax rate. Straightforward computations reveal that investors' total demand then becomes $Z_t = \frac{E_t[P_{t+1}]+R_t-(1+r+\delta+\tau)P_t}{\lambda\sigma^2}$, implying that house prices follow $P_t = \frac{E_t[P_{t+1}]+\hat{R}}{1+r+\delta+\tau}$. Moreover, the fitness functions of the extrapolative and regressive expectation rule now read $A_t^E = (P_{t-1}+R_{t-2}-(1+r+\delta+\tau)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1}+R_{t-2}-(1+r+\delta+\tau)P_{t-2})Z_{t-2}^R - c$, respectively. All other equations remain as before. Proposition 2 summarizes our main analytical results.

Proposition 2. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta+\tau)\delta+(\beta+\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta+\tau-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r+\tau}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs, giving rise to cyclical dynamics.*

A comparison of Propositions 1 and 2 shows that an increase in the property tax rate τ has quite similar effects as an increase in the interest rate r. More precisely, higher tax rates make the housing market less attractive for investors. Therefore, the demand for housing stock decreases, which causes the fundamental house price $\overline{P}$ to fall. Consequently, the fundamental housing stock $\overline{H}$ declines and the fundamental rent level $\overline{R}$ increases.[7] Importantly, higher tax rates may prevent a Neimark-Sacker bifurcation. Note that this also becomes obvious from the bifurcation diagram depicted in the top left panel of Figure 5. Here, we use our base parameter setting, except that τ is varied between 0 and 0.1, and show how the dynamics of the housing market depends on the tax rate. As can be seen, higher property taxes rates initially decrease the amplitude of house price fluctuations, and a convergence to the steady state sets in when τ exceeds $\tau_{crit}^{NS} \approx 0.010$. We can therefore conclude that a property tax

[7] Note that Best and Kleven (2018) and Fritzsche and Vandrei (2019) provide empirical evidence according to which even small increases (decreases) in property taxes may significantly decrease (increase) investors demand for housing stock. Moreover, Barrios et al. (2019) report that property taxes represent a significant component of the user cost of housing in several European countries.

has a stabilizing effect on housing markets, although it also yields lower house prices and, consequently, a lower housing stock and higher rent levels.

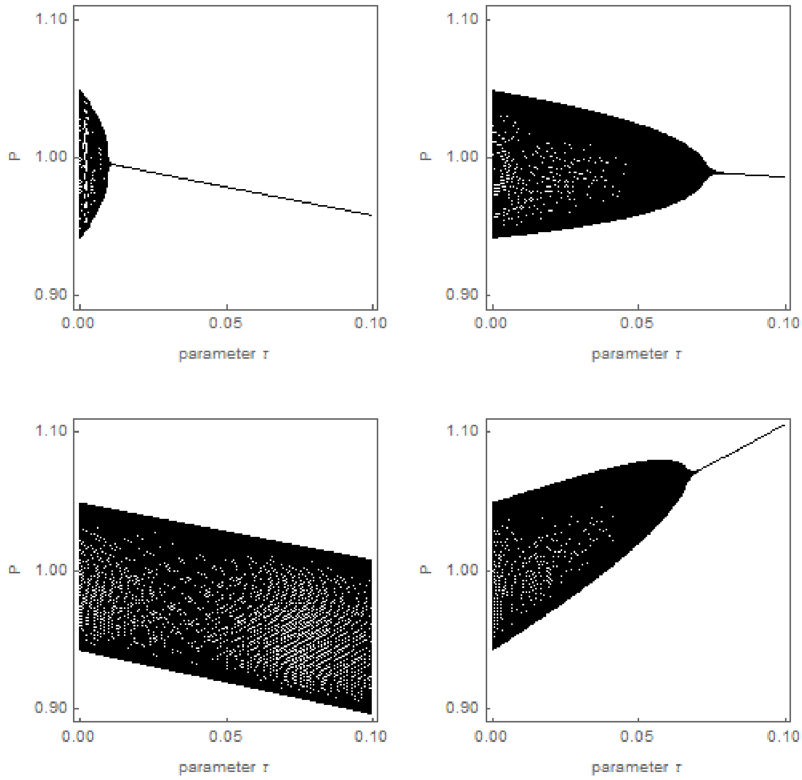

Figure 5: Effects of taxes on the value of houses, rental income, owning housing stock, revenue of housing constructors and capital gains, respectively. Base parameter setting, except that τ is varied as indicated on the axis.

5.2. Tax on rental income

A tax on rental income changes investor i's wealth equation to

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + (1-\tau)R_t - (1+r+\delta)P_t), \tag{24}$$

where τ again denotes the tax rate imposed by policy makers. Accordingly, investors' total demand becomes $Z_t = \frac{E_t[P_{t+1}]+(1-\tau)R_t-(1+r+\delta)P_t}{\lambda\sigma^2}$, and the house price is given by $P_t = \frac{E_t[P_{t+1}]+\hat{R}-\tau R_t}{1+r+\delta}$. The two fitness functions modify to $A_t^E = (P_{t-1} + (1-\tau)R_{t-2} - (1+r+\delta)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1} + (1-\tau)R_{t-2} - (1+r+\delta)P_{t-2})Z_{t-2}^R - c$. Since the other equations do not change, we arrive at the following results.

Proposition 3. *At the model's unique steady state, we have* $\overline{P} = \frac{(1-\tau)\alpha\delta}{(r+\delta)\delta+((1-\tau)\beta+\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma((1-\tau)\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs, giving rise to cyclical dynamics.*

Proposition 3 reveals that a tax on rental income has a similar effect on the model's steady state as a property tax. With an increasing tax rate on rental income, investors have fewer incentives to invest in the housing market. As a consequence, the demand for housing stock is lower and the fundamental house price $\overline{P}$ declines. Therefore, the fundamental housing stock $\overline{H}$ decreases and the fundamental rent level $\overline{R}$ increases. A higher tax on rental income makes a Neimark-Sacker bifurcation also less likely. The bifurcation diagram depicted in the top right panel of Figure 5 shows that increasing values for τ make the amplitude of house price fluctuations smaller up to the point where the threshold value $\tau_{crit}^{NS} \approx 0.073$ is reached. Then, quasi-periodic dynamics turns into fixed-point dynamics.

5.3. Tax on owning housing stock

Policy makers may also consider imposing a tax on owning housing stock. Investor i's wealth equation can then be written as

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - \tau - (1+r+\delta)P_t), \tag{25}$$

where τ is the tax rate.[8] Investors' total demand becomes $Z_t = \frac{E_t[P_{t+1}]+R_t-\tau-(1+r+\delta)P_t}{\lambda\sigma^2}$, house prices are given by $P_t = \frac{E_t[P_{t+1}]+\hat{R}-\tau}{1+r+\delta}$, and the fitness functions turn into $A_t^E = (P_{t-1} + R_{t-2} - \tau - (1+r+\delta)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1} + R_{t-2} - \tau - (1+r+\delta)P_{t-2})Z_{t-2}^R - c$. The following proposition summarizes the main effects of such a tax.

Proposition 4. *At the model's unique steady state, we have* $\overline{P} = \frac{(\alpha-\tau)\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs, giving rise to cyclical dynamics.*

Proposition 4 shows that higher tax rates on owning housing stock also reduces the fundamental house price $\overline{P}$. As a result, the fundamental housing stock $\overline{H}$ decreases and the fundamental rent level $\overline{R}$ increases. However, the Neimark-Sacker stability condition reveals that it is independent of the tax rate τ. Moreover, the bifurcation diagram depicted in the center left panel of Figure 5 indicates that house price oscillations remain basically constant if the tax rate increases. Hence, a tax on owning housing stock merely shifts the dynamics downwards.

5.4. Revenue tax for housing constructors

Alternatively, policy makers may decide to tax housing constructors. For instance, a revenue tax for housing constructors turns their profit maximization problem into

$$max_{I_t}\{(1-\tau)E_{t-1}[P_t]I_t - C_t\}, \tag{26}$$

where τ denotes the tax rate. The optimal supply of new housing is then given by $I_t = (1-\tau)\gamma P_{t-1}$, and the housing stock evolves as $H_t = (1-\delta)H_{t-1} + (1-\tau)\gamma P_{t-1}$. Since all other equations remain unaffected by such a tax, we arrive at the following results.

[8] Note that our modeling of a tax on owning housing stock is reminiscent to a tax on the cadastral value of a property, as, for instance, imposed by a number of EU countries. See Barrios et al. (2019) and references therein for evidence.

Proposition 5. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)(1-\tau)\gamma} = F$, $\overline{H} = \frac{(1-\tau)\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{(1-\tau)\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs, giving rise to cyclical dynamics.*

As stated in Proposition 5, the effects of an increasing revenue tax for housing constructors are qualitatively similar to those of a decrease in the inverse cost parameter γ, i.e. to higher building costs. This can be explained as follows. Higher tax rates make housing construction less profitable for constructors. Since fewer houses are built, the fundamental housing stock $\overline{H}$ declines. Therefore, both the fundamental house price $\overline{P}$ and the fundamental rent level $\overline{R}$ increase. A higher tax rate for housing constructors is beneficial for market stability in the sense that it may counter a Neimark-Sacker bifurcation. In fact, it becomes clear from the bifurcation diagram presented in the center right panel of Figure 5 that the amplitude of house price fluctuations becomes smaller as the tax rate increases. Moreover, the steady state becomes stable if τ exceeds the critical value $\tau_{crit}^{NS} \approx 0.067$.

5.5. Tax on capital gains

Finally, we explore how a taxation of capital gains on housing may affect the dynamics of housing markets. In the presence of a tax on housing capital gains, investor i's end-of-period wealth is determined by

$$W_{t+1}^i = \begin{cases} (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta)P_t) - \tau(P_{t+1} - P_t)Z_t^i & \text{if } P_{t+1} - P_t > 0 \\ (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta)P_t) & \text{if } P_{t+1} - P_t \leq 0 \end{cases}, \tag{27}$$

where $0 \leq \tau < 1$ denotes the tax rate imposed by policy makers on his housing capital gains.[9] Accordingly, investor i's optimal demand for housing stock would be given by

$$Z_t^i = \begin{cases} \frac{(1-\tau)E_t^i[P_{t+1}]+R_t-(1+r+\delta-\tau)P_t}{(1-\tau)^2\lambda V_t^i[P_{t+1}]} & \text{if } P_{t+1} - P_t > 0 \\ \frac{E_t^i[P_{t+1}]+R_t-(1+r+\delta)P_t}{\lambda V_t^i[P_{t+1}]} & \text{if } P_{t+1} - P_t \leq 0 \end{cases}. \tag{28}$$

However, investor i does neither know P_{t+1} nor P_t when he enters the housing market. In the following, we assume that the demand for housing stock of an investor opting for the extrapolative expectation rule is given by

$$Z_t^E = \begin{cases} \frac{(1-\tau)E_t^E[P_{t+1}]+R_t-(1+r+\delta-\tau)P_t}{(1-\tau)^2\lambda\sigma^2} & \text{if } P_{t-1} - P_{t-2} > 0 \\ \frac{E_t^E[P_{t+1}]+R_t-(1+r+\delta)P_t}{\lambda\sigma^2} & \text{if } P_{t-1} - P_{t-2} \leq 0 \end{cases}, \tag{29}$$

while that of an investor selecting the regressive expectation rule is given by

$$Z_t^R = \begin{cases} \frac{(1-\tau)E_t^R[P_{t+1}]+R_t-(1+r+\delta-\tau)P_t}{(1-\tau)^2\lambda\sigma^2} & \text{if } F - P_{t-1} > 0 \\ \frac{E_t^R[P_{t+1}]+R_t-(1+r+\delta)P_t}{\lambda\sigma^2} & \text{if } F - P_{t-1} \leq 0 \end{cases}. \tag{30}$$

[9]Future work may consider the case in which housing capital gains are waived if investors hold their housing stock long enough.

Consistent with his expectation rule, an investor using the extrapolative expectation rule beliefs that he has (not) to pay housing capital gains taxes when house prices have increased (decreased) in the recent past, predicting a continuation of the current upward (downward) price trend. In contrast, an investor applying the regressive expectation rule beliefs that he has (not) to pay housing capital gains taxes when the housing market is currently undervalued (overvalued), projecting a price recovery (decline).[10]

Due to investors' demand schedules, the house price is determined by the following piecewise defined map

$$P_t = \begin{cases} \frac{(1-\tau)(N_t^E E_t^E[P_{t+1}]+N_t^R E_t^R[P_{t+1}])+R_t-(1-\tau)^2\lambda\sigma^2 H_t}{1+r+\delta-\tau} & \text{if } P_{t-1} > P_{t-2} \wedge F > P_{t-1} \\ \frac{(1-\tau)N_t^E E_t^E[P_{t+1}]+N_t^E R_t+(1-\tau)^2 N_t^R(E_t^R[P_{t+1}]+R_t)-(1-\tau)^2\lambda\sigma^2 H_t}{N_t^E(1+r+\delta-\tau)+(1-\tau)^2 N_t^R(1+r+\delta)} & \text{if } P_{t-1} > P_{t-2} \wedge F \leq P_{t-1} \\ \frac{(1-\tau)^2 N_t^E(E_t^E[P_{t+1}]+R_t)+(1-\tau)N_t^R E_t^R[P_{t+1}]+N_t^R R_t-(1-\tau)^2\lambda\sigma^2 H_t}{(1-\tau)^2 N_t^E(1+r+\delta)+N_t^R(1+r+\delta-\tau)} & \text{if } P_{t-1} \leq P_{t-2} \wedge F > P_{t-1} \\ \frac{N_t^E E_t^E[P_{t+1}]+N_t^R E_t^R[P_{t+1}]+R_t-\lambda\sigma^2 H_t}{1+r+\delta} & \text{if } P_{t-1} \leq P_{t-2} \wedge F \leq P_{t-1} \end{cases}. \quad (31)$$

Since the expectation rules' fitness depends on past realized profits, housing capital gains taxes imply that

$$A_t^E = \begin{cases} (P_{t-1}+R_{t-2}-(1+r+\delta)P_{t-2})Z_{t-2}^E - \tau(P_{t-1}-P_{t-2})Z_{t-2}^E & \text{if } P_{t-1}-P_{t-2} > 0 \\ (P_{t-1}+R_{t-2}-(1+r+\delta)P_{t-2})Z_{t-2}^E & \text{if } P_{t-1}-P_{t-2} \leq 0 \end{cases}, \quad (32)$$

and

$$A_t^R = \begin{cases} (P_{t-1}+R_{t-2}-(1+r+\delta)P_{t-2})Z_{t-2}^R - c - \tau(P_{t-1}-P_{t-2})Z_{t-2}^R & \text{if } P_{t-1}-P_{t-2} > 0 \\ (P_{t-1}+R_{t-2}-(1+r+\delta)P_{t-2})Z_{t-2}^R - c & \text{if } P_{t-1}-P_{t-2} \leq 0 \end{cases}, \quad (33)$$

respectively. Clearly, the expectation rules' fitness does not depend on investors' aforementioned belief assumptions – equations (32) and (33) measure their actual realized profits. The model's other equations remain as before.

A few comments are in order. First, all investors expect to pay housing capital gains taxes when the housing market is increasing and undervalued. The upper branch of (31) captures this scenario. Second, no investor expects to pay housing capital gains taxes when the housing market is declining and overvalued; a scenario that the lower branch of (31) comprises. Third, no investor has to pay housing capital gains taxes when the housing market is at rest, independently of whether the house price mirrors its fundamental value or not. Fourth, each of the four branches of map (31) gives rise to a different (virtual) steady state. Recall that a virtual steady state of a branch of a piecewise defined map is a steady state that exists outside the domain for which the branch is defined. Moreover, virtual as well as real steady states may have a significant impact on the dynamics of their underlying branches and, consequently, on the dynamics of the whole system. Fifth, the lower branch of map

[10] Alternative modeling assumptions are possible. For instance, investors may rely on a time-varying weighted average of the upper and lower branch of their demand schedules, where the weights attached to the two branches indicate the strength of their beliefs in rising and falling prices. For simplicity, we assume in (29) and (30) that investors select either the upper or lower branch of their demand schedules.

(31) represents the case in which the housing market is at rest and properly reflects its fundamental value. Since investors neither expect nor have to pay housing capital gains taxes, the steady state of the lower branch of (31) is identical to the one of our original model. We therefore continue to assume that investors compute the housing market's fundamental value by $F = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$.

Since our model with housing capital gains taxes precludes a deeper analytical investigation, we continue our study with numerical experiments. The bottom left panel of Figure 5 reveals that a tax imposed on investor's housing capital gains has a destabilizing impact on the dynamics of housing markets.[11] Although difficult to disentangle, additional simulations (not depicted) suggest that the main reason for this outcome may be summarized as follows. Note first that there are four stages of a housing market cycle: (i) the housing market increases and is overvalued, (ii) the housing market decreases and is overvalued, (iii) the housing market is undervalued and decreases and (iv) the housing market is undervalued and increases, i.e. all four branches of (31) matter. Note furthermore that the differences between the four branches of map (31) increase in line with the tax rate imposed on housing capital gains, as can easily be checked by comparing its upper and lower branch. Since all branches of map (31) are associated with a different (virtual) steady state, their distance and thus the amplitude of the housing market cycles increases with the tax rate. We conclude this section with an important warning message. Well-intended tax policies may not always contribute to a stabilization of housing markets. In particular, the effects of state-dependent tax policies are difficult to anticipate and may yield unintended outcomes.

6. Conclusions

Housing markets regularly display dramatic bubbles. According to Case and Shiller (2003) and Case et al. (2013), such dynamics, which may be quite harmful for the real economy, are due to investors' optimistic house price expectations. However, Glaeser et al. (2008) argue that the real side of housing markets is also relevant for the formation and duration of bubbles. By combining Poterba's (1984, 1991) user cost model and Brock and Hommes' (1997, 1998) heuristic switching approach, we develop a novel housing market model that seeks to take these observations into account.

The real part of our model comprises a rental market and a housing capital market, and determines key relations between the house price, the housing stock and the rent level. The behavioral part of our model consists of housing market investors who switch between competing expectation rules with respect to their past performance, thereby reflecting a boundedly rational learning behavior. Amongst others, our model reveals that endogenous boom-bust housing market dynamics may arise if investors rely heavily on extrapolative expectations. Fortunately, policy makers have the opportunity to stabilize such dynamics by adjusting the tax code. For instance, a property tax or a tax on rental income tames

[11] Although intuitively appealing, at least at first sight, Case (1992) already argued that capital gains taxes might not be effective in curbing speculative housing market bubbles. Amongst others, he reports that Japan had a very high tax on housing capital gains during the 1980s, yet it experienced a dramatic real estate bubble.

the housing market. However, such a tax also affects the housing market's steady-state level, an aspect which should not be overlooked.

Without question, the dynamics of housing markets is driven by a complex interplay between real and behavioral forces, and a complete understanding of the functioning of housing markets is still lacking. However, we hope that our model makes some progress in this direction. Moreover, we would like to stress that our model may serve as a framework to explore how policy makers may affect the steady state, its stability and the out-of-equilibrium behavior of housing markets via adjusting the tax code. Of course, much more work is needed in this exciting and relevant research field.

Appendix

Here, we provide a detailed proof of Proposition 1. Note that the derivation of Propositions 2-7 follows along quite similar lines of reasoning. First of all, we need to express our model in form of a dynamical system. We therefore introduce the auxiliary variables $x_t = P_{t-1}$, $y_t = x_{t-1}$, $z_t = y_{t-1}$ and $k_t = H_{t-1}$. Moreover, it is helpful to use the difference in fractions, given by $m_t = N_t^R - N_t^E = Tanh\big[\frac{\nu}{2}(A_t^R - A_t^E)\big]$. Since $N_t^R + N_t^E = 1$, it follows that $N_t^E = \frac{1-m_t}{2}$ and $N_t^R = \frac{1+m_t}{2}$. The dynamical system of our model can thus be summarized by the following six-dimensional nonlinear map

$$T : \begin{cases} P_t = \frac{E_t[P_{t+1}] + \alpha - \beta(\gamma P_{t-1} + (1-\delta)H_{t-1}) - \lambda\sigma^2(\gamma P_{t-1} + (1-\delta)H_{t-1})}{1+r+\delta} \\ H_t = \gamma P_{t-1} + (1-\delta)H_{t-1} \\ x_t = P_{t-1} \\ k_t = H_{t-1} \\ y_t = x_{t-1} \\ z_t = y_{t-1} \end{cases},$$

where

$$E_t[P_{t+1}] = \frac{1-m_t}{2}\big(P_{t-1} + \chi(P_{t-1} - x_{t-1})\big) + \frac{1+m_t}{2}\big(P_{t-1} + \phi(F - P_{t-1})\big)$$

and

$$m_t = Tanh\left[\frac{\nu}{2}\Big\{\big(P_{t-1} + \alpha - \beta k_{t-1} - (1+r+\delta)x_{t-1}\big)\frac{\phi(F - y_{t-1}) - \chi(y_{t-1} - z_{t-1})}{\lambda\sigma^2} - c\Big\}\right].$$

By imposing the fact that price expectations are realized at the steady state, i.e. $E_t[\overline{P}] = \overline{P}$, implying that $\overline{P} = F$, the model's dynamical system gives rise to the unique steady state $FSS = (\overline{P}, \overline{H}, \overline{x}, \overline{k}, \overline{y}, \overline{z}) = (\overline{P}, \overline{H}, \overline{P}, \overline{H}, \overline{P}, \overline{P})$, where $\overline{P} = F = \frac{\alpha\delta}{\beta\gamma + \delta(r+\delta) + \gamma\lambda\sigma^2}$ and $\overline{H} = \overline{P}\frac{\gamma}{\delta}$. Since prices mirror their fundamental value at the steady state, we call it the fundamental steady state. Furthermore, by using $\overline{R} = \alpha - \beta\overline{H}$ we can also express steady-state prices as $\overline{P} = F = \frac{\overline{R} - \lambda\sigma^2\overline{H}}{r+\delta}$.

To explore the steady state's stability properties, we use the Jacobian matrix, computed at the fundamental steady state, i.e.

$$J(FSS)=\begin{pmatrix} \frac{2-2\gamma(\beta+\gamma\sigma^2)-\phi+\chi-(\phi+\chi)\overline{m}}{2(1+r+\delta)} & \frac{(\delta-1)(\beta+\lambda\sigma^2)}{1+r+\delta} & -\frac{\chi(1-\overline{m})}{2(1+r+\delta)} & 0 & 0 & 0 \\ \gamma & 1-\delta & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

where $\overline{m}=Tanh\left[-\frac{\nu}{2}c\right]$, and derive the characteristic polynomial

$$\kappa^3(\kappa^3-a_1\kappa^2-a_2\kappa-a_3)=0,$$

where $a_1=\frac{-4+2r(\delta-1)+2\delta^2+2\gamma(\beta+\lambda\sigma^2)+\phi+\overline{m}\phi+(\overline{m}-1)\chi}{2(1+r+\delta)}$, $a_2=\frac{(\delta-1)(-2+\phi+\overline{m}\phi)+(\overline{m}-1)(\delta-2)\chi}{2(1+r+\delta)}$ and $a_3=-\frac{(\overline{m}-1)(\delta-1)\chi}{2(1+r+\delta)}$. The fixed point of our model is locally asymptotically stable if and only if all six eigenvalues of the Jacobian matrix are less than one in absolute value. Note that three eigenvalues, say $\kappa_{1,2,3}$, are equal to zero, while the other three eigenvalues, say $\kappa_{4,5,6}$, result from the remaining third-degree characteristic polynomial. For this reason, we follow Lines et al. (2020), who provide a simplified set of conditions to explore a steady state's stability properties for such a problem. In fact, they show that a fixed point of a third-degree characteristic polynomial loses its stability if (I) $1+a_1+a_2+a_3>0$, (II) $1-a_1+a_2-a_3>0$ or (III) $1-a_2+a_1a_3-a_3^2>0$ is violated by a continuous change of a model parameter. Moreover, a violation of (I), (II) or (III), while the other two conditions hold, is associated with a Fold, Flip and Neimark-Sacker bifurcation, respectively. In our case, tedious computations reveal that this results in

$$\text{(I)}\quad \phi(1+\overline{m})\delta>-2\big(\beta\gamma+\delta(r+\delta)+\gamma\lambda\sigma^2\big),$$

$$\text{(II)}\quad \phi(1+\overline{m})<2\Big(r+\frac{\gamma(\beta+\lambda\sigma^2)}{(\delta-2)}+2+\delta+\chi-\overline{m}\chi\Big)$$

and

$$\text{(III)}\quad (1-\overline{m})\chi\delta+\frac{2\gamma(\beta+\lambda\sigma^2)(1-\overline{m})\chi}{2(1+r+\delta)-(1-\overline{m})\chi}<(1+\overline{m})\phi+\frac{2(2\delta+r)}{1-\delta}.$$

Recall that $0\le\phi\le1$, $0<\delta<1$ and $\beta,\gamma,r,\lambda\sigma^2>0$. Also, we have $0\le\overline{m}\le1$ which implies that condition (I) is always satisfied. Finally, we use $\overline{m}=\overline{N}^R-\overline{N}^E$, $\overline{N}^E=\frac{1-\overline{m}}{2}$ and $\overline{N}^R=\frac{1+\overline{m}}{2}$, and rewrite inequalities (II) and (III) as

$$\text{(II')}\quad \overline{N}^R\phi+\frac{\gamma(\beta+\lambda\sigma^2)}{2-\delta}<2+r+\delta+2\chi\overline{N}^E$$

and

$$\text{(III')}\quad \overline{N}^E\chi\delta+\frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi}<\overline{N}^R\phi+\frac{2\delta+r}{1-\delta},$$

which correspond to (ii) and (i) in Proposition 1, respectively. For the derivation of Propositions 2-5, it is helpful to note that the tax parameter τ is always closely related to a real or behavioral model parameter, as pointed out in Section 5. For instance, in the case of a property tax, τ always appears in connection with r.

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Paper 4

Interactions between the housing market and the real economy: A heterogeneous agent approach

Interactions between the housing market and the real economy: A heterogeneous agent approach

Carolin Martin

Abstract

We develop a behavioral macroeconomic model which allows us to analyze interactions between the housing market and the real economy. The user cost housing market model involves a rental and a housing capital market while the real economy is represented by an aggregate-demand-aggregate-supply model complemented by a central bank which influences the interest rate based on a Taylor rule. Our model framework is able to produce complex endogenous boom-and-bust dynamics which arises through the expectation formation behavior of boundedly rational and heterogeneous agents. Moreover, we demonstrate by numerical simulations how a gradually linkage of the housing market and the real market through four steps effects the dynamics and stability of the two markets.

Keywords: Housing markets, real markets, heterogeneous expectations and learning behavior, steady state and stability analysis

JEL classification: D84, E12, R31

1. Introduction

The crash of the U.S. housing market in 2006 and the following global economic and financial crisis have shown how interconnected the markets are within an economy. As a consequence, the developments in the housing market can have serious effects on the real economy. Glaser et al. (2013), Shiller (2015) and Piazzesi and Schneider (2016) emphasize the serious effects huge cyclical movements with strong pronounced bubble formations and volatility in the housing market can have on the real economy. Without doubt, the housing market plays a major role in a country's whole economy. According to Leung (2004) the development of house prices has an effect on business cycle dynamics. He argues that changes in house prices may have significant wealth effects on consumption as well as on investment decisions. Thus, he emphasizes the need to include the housing market in macroeconomic analysis. Recently, Tripathi (2019) also stresses the relevance of macroeconomic determinants to control the house price fluctuations, such as inflation, GDP growth rate and real exchange rate. He shows that these macroeconomic variables have a positive and statistically significant impact on house prices and recommends the policy makers to adjust monetary policies such that house prices could be tamed. Similar arguments are offered by Tsatsaronis and Zhu (2004), who state that the strong and durable relation between inflation and nominal interest rates on the one hand and house prices on the other hand should be considered by the policy of monetary authorities.

Against this background, we pursue two goals in our paper. First, we propose a behavioral macroeconomic model as a working basis to analyze the complex boom-and-bust behavior of housing markets and the real economy. Second, we use our model to explore interactions between the housing market and the real economy in order to present proposals for monetary and policy authorities to tame housing markets. The elementary housing market model is based on Dieci and Westerhoff (2016) and Schmitt and Westerhoff (2019) and reveals that the interacting real and behavioral forces can produce endogenous housing market fluctuations. The model's real forces originate from a standard user cost housing market setup according to Poterba (1984, 1991), involving a rental and a housing capital market which link fundamental relations between house prices, the housing stock and the rent level. The acting behavioral forces come from the expectation formation behavior of boundedly rational and heterogeneous housing market investors in the spirit of Brock and Hommes (1997, 1998). Accordingly, investors switch between extrapolative and regressive expectation rules to forecast future house prices, based on the rules' past accuracy. The real market is represented by a standard aggregate-demand-aggregate-supply model, consisting of the output gap and the inflation rate equation as established by De Grauwe (2010a, b, c, 2012). Furthermore the Taylor rule describes the behavior of the central bank which reacts to deviations of inflation and output gap from their target values. We modify the Taylor rule in the vein of Sbordone et al. (2010). The expectation formation behavior of boundedly rational and heterogeneous agents creates endogenous real market dynamics. As in the housing market model, agents select between extrapolative and fundamentalist expectation rules to forecast the future evolution of the output gap and the inflation rate. This selection mechanism leads the agents to choose the expectation rule which offered the highest performance in the past.

Our main results may be summarized as follows. We show that both the housing market model and the real market model are able to generate complex endogenous boom-and-bust dynamics. Moreover, we investigate the interaction of both markets by means of four steps. First, we investigate the dynamics of the housing market and the real market arising by including the Taylor rule in the housing market. The relevance of a rule-based interest rate policy is emphasized by Taylor (2009). He is convinced that the US housing market bubble, which peaked in 2006, would have been significantly mitigated if the central bank had followed the Taylor rule originating from Taylor (1993). We show that by merging the two submodels using the Taylor rule, the house price and the output gap exhibit pro-cyclical boom-and-bust dynamics over time. Ahearne et al. (2005) also detects an overlap between house price booms and an overheating in real economic activity as well as a house price that precedes the inflation rate.

Second, we examine the effects of an interest rate response to house prices by including the house price distortion in the Taylor rule. In an empirical study, Agnello et al. (2018) show that an increase in the interest rate can dampen housing market bubbles and underline that this evidence supports the idea of a leaning-against-the-wind monetary policy rule to stabilize the housing market. In the same vein, Lambertini et al. (2013) illustrate that an interest rate rule that responds to an increase in house prices and thus reduces house price volatility can stimulate welfare. In fact, our numerical results confirm that an interest rate that reacts to the deviation of the house price from its fundamental value can dampen the housing market's boom-and-bust dynamics and thus brings the house price, the housing stock and the rent level closer to their fundamental values. The dynamics of the real market is not significantly affected by this modification.

In a third step, we investigate how the dynamics of the housing market changes if the house price trend is added to the inflation equation. This approach stems back to the fact that in the euro area the Harmonised Index of Consumer Prices (HICP) only includes rents but excludes owner-occupied housing costs. However, the European Central Bank (ECB) recently proposed the inclusion of owner-occupied housing (OOH) into the HICP as described in Nickel et al. (2021). The importance of owner-occupied housing costs for the inflation rate is already stressed by Cournède (2005). Even if he lists a number of reasons against the inclusion in a price index, he concludes that it is nevertheless worthwhile since - to give a brief explanation - a considerable part of private consumption is spent on owner-occupied housing services and thus also has effects on households' decisions. Consequently, housing costs play an important role when households make economic decisions. Against this background, we show numerically that taking the house price into account in the inflation rate leads to a significant decrease in the distortion of the house price, the housing stock and the rent level without having considerably effects on the real market.

Conclusively, in a fourth step we study the impact of including the house price distortion in the aggregate demand equation on the model dynamics. The relevance of this approach is shown by Case et al. (2005) who explore the relation between housing market wealth and consumption using a crosssectional time-series data set and find empirical evidence that aggregate house prices and aggregate consumption are strongly correlated. They also show that in developed countries the housing market has a stronger effect on consumer spending than the stock market. Similar results are offered by Campbell and Cocco (2005) who estimate the response of homeowners' consumption to house prices using household level data from the UK. In our paper, the numerical analysis reveals that the adjusted aggregate demand equation brings the house price, the housing stock and the rent level much closer to their fundamental values. However, this destabilizes the real market as the magnitude of the real market's boom-and-bust fluctuations is increasing significantly.

The remainder of our paper is organized as follows. In Section 2, we introduce the housing market model by Schmitt and Westerhoff (2019) and the real market model by De Grauwe (2010a, b, c, 2012). We show our analytical and numerical insights of the model in Section 3. In Section 4, we merge the two markets via four steps and explore the effects each has on the model's dynamics. In Section 5, we conclude our paper.

2. Model setup

In the following we present a behavioral model which allows us to analyze interactions between the housing market and the real economy. First, we consider and analyze both markets separately and without interaction, and then we link the two markets via four steps. The total number of agents in both markets is normalized to one, i.e. $N = 1$. In addition, a central bank is able to influence the interest rate directly via monetary policy. The housing market and the real market feature agents with bounded rationality who choose between extrapolative and regressive expectation rules to forecast either the future development of the house price (weighted by $w_t^{P,r}$ and $w_t^{P,e}$) or the future evolution of output gap (weighted by $w_t^{y,f}$, $w_t^{y,e}$) and inflation (weighted by $w_t^{\pi,f}$ and $w_t^{\pi,e}$) depending on which market they are active in. These rules depend on their evolutionary fitness, measured in terms of squared prediction errors. Since agents of both markets tend to exhibit a rather slow and lagged action reaction we use the same time scale for both markets. We consider it to be one quarter of a year. Finally, several stochastic components are added to the model to make its dynamics more realistic. Note, however, this is not needed for the model to produce endogenous dynamics. Let us begin with describing the building blocks of the two markets separately, before we gradually merge them. We first present the elementary housing market model in Section 2.1 and in Section 2.2, we introduce the real market model.

2.1. The housing market model

We model the housing market according to Dieci and Westerhoff (2016) and Schmitt and Westerhoff (2019) and combine a standard user cost housing market setup as proposed by Poterba (1984, 1991) with the heuristic switching approach by Brock and Hommes (1997, 1998). The housing market can be divided into two interrelated markets, namely a rental market and a housing capital market, specifying elementary relation between the house price, the housing stock and the rent level. Let us turn to the details of the model and begin with the rental market. The rent level R_t can be derived from the market clearing condition in the rental market and is defined as

$$R_t = \alpha - \beta H_t, \tag{1}$$

implying a negative dependency of the rent level from the existing housing stock H_t.[1] Parameter $\alpha > 0$ is a scaling parameter and $\beta > 0$ indicates how strongly the rent level reacts to changes in the current housing stock. Note that α and β must be chosen such that $R_t \geq 0$. Concerning the housing capital market, the evolution of the housing stock depends positively on the amount of new housing construction I_t as well as the housing stock of the previous period H_{t-1} less housing depreciation, i.e.

$$H_t = I_t + (1 - \delta)H_{t-1}, \tag{2}$$

where $0 < \delta < 1$ is the housing depreciation rate.[2] By defining housing investments I_t as positively dependent on the past house price P_{t-1}, i.e.

$$I_t = \gamma P_{t-1}, \tag{3}$$

with $\gamma > 0$ representing an inverse cost parameter, the housing stock evolution can be rewritten as

$$H_t = \gamma P_{t-1} + (1 - \delta)H_{t-1}. \tag{4}$$

Thus, a decrease in parameter γ means higher building costs and a more sluggish housing supply. The single house price P_t, derived from a standard intertemporal no-arbitrage condition (see Dieci and Westerhoff (2016) and Martin et al. (2021, 2022) for more details), is paid by all homebuyers and expressed by the discounted value of investors' next period's expected house price $\tilde{E}_t[P_{t+1}]$ plus risk-adjusted rent payments, i.e.

$$P_t = \frac{\tilde{E}_t[P_{t+1}] + R_t}{1 + i + \delta} + \epsilon_t. \tag{5}$$

Parameter $i > 0$ is the risk-free interest rate and the random variable $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ represents additional exogenous noise affecting the housing market. Thus the house price increases with investors' house price expectations and the rent level, while it decreases with the interest rate and the depreciation rate. The tilde on top of the expectation operator points out the bounded rationality of housing market investors' expectations.

[1] According to Dieci and Westerhoff (2016), market clearing in the rental market implies that, in every period t, the demand for housing services, $D_t = a - bR_t$, is equal to the supply of housing services, $S_t = cH_t$, with $a, b, c > 0$. Thus, it follows from $D_t = S_t$ that $R_t = \alpha - \beta H_t$, with $\alpha = \frac{a}{b} > 0$ and $\beta = \frac{c}{b} > 0$.

[2] At this point, it is important to mention some assumptions have to be made concerning housing construction. First of all, houses are built with a one-period production lag. Furthermore, home builders are risk neutral and maximize expected profits $max_{I_t}\{E_{t-1}[P_t]I_t - C_t\}$, subject to the quadratic cost function $C_t = \frac{1}{2\gamma}I_t^2$. Finally, they form naive expectations, i.e. $E_{t-1}[P_t] = P_{t-1}$.

Concerning investors' expectation formation behavior we follow Brock and Hommes (1997, 1998) and define the average house price expectation of heterogeneous and boundedly rational housing market investors as

$$\tilde{E}_t[P_{t+1}] = w_t^{P,r}\tilde{E}_t^r[P_{t+1}] + w_t^{P,e}\tilde{E}_t^e[P_{t+1}]. \tag{6}$$

Similar to Dieci and Westerhoff (2016), investors select between two expectation heuristics to forecast future house prices, namely a regressive expectation rule $\tilde{E}_t^r[P_{t+1}]$ and an extrapolative expectation rule $\tilde{E}_t^e[P_{t+1}]$, weighted by $w_t^{P,r}$ and $w_t^{P,e}$, respectively. Note that housing market investors form their expectations in period t about the house price in period $t+1$ using the last available information set of period $t-1$. Investors relying on the regressive expectation rule, called fundamentalists, predict the house price to move back to its fundamental value F over time, which is formalized by

$$\tilde{E}_t^r[P_{t+1}] = P_{t-1} + \phi(F - P_{t-1}). \tag{7}$$

The reaction parameter $0 < \phi < 1$ indicates how fast the investors expect the house price reverting towards F. This fundamental house price F is assumed to be known by all housing market investors and given by $F = \frac{\alpha\delta}{\beta\gamma+\delta(i+\delta)}$.[3] According to (7), investors assume the price to rise if the last period's house price lies below F, and vice versa.

In contrast, investors forming extrapolative expectations, so-called extrapolators, consider the latest observable house price change. This is described by

$$\tilde{E}_t^e[P_{t+1}] = P_{t-1} + \chi(P_{t-1} - P_{t-2}), \tag{8}$$

stating that investors extrapolate past house price trends into the future. More precisely, we assume that at the beginning of period t, extrapolators observe the development of the house price over the past two periods and form a forecast concerning the house price over the current period. Accordingly, a realized increase in the house price $P_{t-1} - P_{t-2} > 0$ is an indication of a further expected rise in $t+1$. Naturally, the exact opposite is true for a house price decrease $P_{t-1} - P_{t-2} < 0$. The reaction parameter $\chi \geq 0$ measures how aggressively investors react to changes in the house price from period $t-2$ to $t-1$.[4]

Investors' decision on which expectation rule to follow depends on the rules' evolutionary fitness and has to be updated in each time step. We form the fitness functions of the two expectation rules according to Schmitt and Westerhoff (2019, 2022) by using the squared forecasting errors which investors can calculate for each rule and compare them with each other. Thus, the evolutionary fitness

[3] Note that investors make no prediction errors at the steady-state. As illustrated in Section 3.1, F equates to the discounted value of future rents. Therefore investors are able to compute the fundamental house price. See Section 3.1 for more analytical insights and Schmitt and Westerhoff (2019) for more details of the derivation.

[4] If $\chi = 0$, Equation (8) is reduced to $\tilde{E}_t^e[P_{t+1}] = P_{t-1}$, which means naive expectations.

of the regressive expectation rule is given by

$$A_t^r = -g(P_{t-1} - \tilde{E}_{t-2}^r[P_{t-1}])^2 \tag{9}$$

and the evolutionary fitness of the extrapolative expectation rule by

$$A_t^e = -g(P_{t-1} - \tilde{E}_{t-2}^e[P_{t-1}])^2, \tag{10}$$

where $g > 0$ indicates how strong the expectation rules' fitness react to the past forecasting accuracy.[5] Consequently, the smaller the respective forecasting error, the higher is the perceived fitness by the investors.

Inspired by Brock and Hommes (1997, 1998) and in line with Schmitt and Westerhoff (2019) and Martin et al. (2021) we formulate the market share of investors relying on the regressive and the extrapolative expectation rule by using the multinomial discrete-choice approach respectively, i.e.

$$w_t^{P,r} = \frac{\exp[\nu A_t^r]}{\exp[\nu A_t^e] + \exp[\nu A_t^r]} \tag{11}$$

and

$$w_t^{P,e} = \frac{\exp[\nu A_t^e]}{\exp[\nu A_t^e] + \exp[\nu A_t^r]}. \tag{12}$$

Accordingly, the higher the fitness of an expectation rule, the more investors will rely on it. Parameter ν describes the intensity of choice indicating to what degree investors' decision about which expectation strategy to follow is rational given the available information set. Thus, ν is often called rationality parameter. In this context, two extreme cases can be distinguished: For $\nu = 0$, investors are indifferent between the two expectation rules which results in $w_t^{P,r} = w_t^{P,e} = \frac{1}{2}$, such that half of the housing market investors rely on regressive expectations and the other half on extrapolative expectations. As ν increases, more and more investors observe fitness differentials and switch to the expectation rule which provides the higher fitness. In the case of $\nu \to \infty$, all investors immediately choose the expectation strategy with the highest fitness. Two additional properties of (11) and (12) should be highlighted. First, the market shares are bounded between zero and one, i.e. $0 < w_t^{P,r}, w_t^{P,e} < 1$. Second, the sum of the market shares of the two expectation rules is equal to 1, such that the following applies $w_t^{P,r} = 1 - w_t^{P,e}$ and vice versa.

2.2. The real economy model

We continue by specifying the real economy in the form of the behavioral macroeconomic model inspired by De Grauwe (2010a, b, c, 2012). The macroeconomy is described by three equations: an aggregate

[5] Schmitt and Westerhoff (2019, 2022) and Martin et al. (2021) assume that the use of regressive expectations may be costly since, among other things, investors have to obtain knowledge about F. Since the fundamentalist expectation rule in the real market does not include such costs, we also dispense with them in the housing market for the sake of comparability.

demand (AD) equation representing the output gap, an aggregate supply (AS) equation, describing the inflation rate and a Taylor rule (TR), depicting the nominal interest rate. Following De Grauwe (2010a, b, c), the first two equations are micro-founded. This means that they are based on dynamic utility and profit maximization as it is commonly done in Dynamic Stochastic General Equlibrium (DSGE) models. The third equation is taken from Sbordone et al. (2010) and in line with Demary (2017) and De Grauwe and Ji (2020) and describes the behavior of the central bank.

The aggregate demand equation takes the form

$$y_t = a_1 \tilde{E}_t[y_{t+1}] + (1 - a_1)y_{t-1} + a_2(i_t - \tilde{E}_t[\pi_{t+1}]) + \tau_t, \tag{13}$$

where y_t represents the output gap in period t and is an economic measure of the difference between an economy's actual output and its potential output. i_t is the nominal interest rate in period t while π_t describes the inflation rate in period t. The current output gap is specified by a forward-looking component $\tilde{E}_t[y_{t+1}]$ and a lagged one y_{t-1} which are weighted by parameter $0 \leq a_1 < 1$. Moreover, it depends negatively on the real interest rate $i_t - \tilde{E}_t[\pi_{t+1}]$. $a_2 < 0$ is a negative sensitivity parameter. The random variable $\tau_t \sim N(0, \sigma_\tau^2)$ is a white noise disturbance term at time t representing, for instance, an aggregate demand shock affecting the real market. In addition, the tilde above the expectation operator $\tilde{E}_t$ emphasizes that agents who form expectations in t about the real economy in $t+1$ are not perfectly rational, but boundedly rational as is also the case concerning house price expectations. The expectation formation process will be described later on.

The aggregate supply equation - in the form of the so called New Keynesian Philips Curve - is given by

$$\pi_t = b_1 \tilde{E}_t[\pi_{t+1}] + (1 - b_1)\pi_{t-1} + b_2 y_t + \eta_t, \tag{14}$$

where the random variable $\eta_t \sim N(0, \sigma_\eta^2)$ is a white noise disturbance term and can be understood as an aggregate supply shock. The Philips curve is also determined by a forward-looking inflation variable $\tilde{E}_t[\pi_{t+1}]$ and a backward-looking one π_{t-1} which are weighted by parameter $0 \leq b_1 \leq 1$. In addition, $b_2 > 0$ is a positive parameter which measures how flexible the inflation rate reacts to changes in the current output gap. Consequently, a positive output gap implies inflationary pressures, while a negative output gap results in deflationary pressures.

Finally, the central bank sets the interest rate according to

$$i_t = c_3 i_{t-1} + (1 - c_3)(i^* + c_1(\pi_t - \pi^*) + c_2(y_t - y^*)) + \theta_t, \tag{15}$$

which is commonly known as the Taylor rule originally established by Taylor (1993). i^* and y^* are the fundamental values of the nominal interest rate and the output gap, respectively, and π^* is the inflation's fundamental steady state or, more specifically, the central bank's inflation target.[6]

[6]In contrast to De Grauwe (2010a, b, c, 2012) who sets the inflation target equal to zero, we analytically find in

Moreover, the random variable $\theta_t \sim N(0, \sigma_\theta^2)$ is a white noise disturbance term and can be interpreted as a monetary policy shock. Following Demary (2017) and De Grauwe and Ji (2020) and in contrast to De Grauwe (2010a, b, c, 2012) we assume that the central bank smooths the interest rate by adjusting i_t as the weighted average of the lagged interest rate i_{t-1} and the term $(i^* + c_1(\pi_t - \pi^*) + c_2(y_t - y^*))$ where the weighting is given by $0 \leq c_3 < 1$. This interest rate rule implies that, if inflation increases above the inflation target, the central bank raises the interest rate with an intensity given by parameter $c_1 > 0$. Similarly, in case of a positive deviation of the output gap from its fundamental value, the central bank increases the interest rate to dampen investment and consumption. Parameter $c_2 \geq 0$ describes the intensity of the central bank's reaction in this regard. Of course, the opposite is true for an inflation rate and output gap below their fundamental values.

We now specify how heterogeneous and bounded rational agents form their expectations concerning the future output gap $\tilde{E}_t[y_{t+1}]$ as well as the future inflation rate $\tilde{E}_t[\pi_{t+1}]$. For the sake of consistency and comprehensibility, the expectation formation in the real market has the same structure as that of the housing market described in Section 2.1 and is modeled in the same vein of Brock and Hommes (1997, 1998), De Grauwe (2010a, b, c, 2012) and De Grauwe and Ji (2020). Agents again use heuristics and are assumed to select between two competing forecasting rules depending on the rule's fitness: a fundamentalist and an extrapolative expectation rule. Compared to the original De Grauwe model, we use slightly different expectation rules that are more in line with those typically assumed in agent-based financial market models (Brock and Hommes 1997, 1998) and which we already applied in the housing market's house price expectation. We now explain in detail the two rules and the selection mechanism for the expectation formation of the future output gap and afterwards in a short version the expectation formation for future inflation rate due to high comparability.

Agents' weighted average expectations of the output gap are given by

$$\tilde{E}_t[y_{t+1}] = w_t^{y,f} \tilde{E}_t^f[y_{t+1}] + w_t^{y,e} \tilde{E}_t^e[y_{t+1}], \tag{16}$$

where $w_t^{y,f}$ and $w_t^{y,e}$ denote the market shares of agents relying on fundamentalist expectations $\tilde{E}_t^f[y_{t+1}]$ and extrapolative expectations $\tilde{E}_t^e[y_{t+1}]$ concerning future output gap, respectively. Agents applying the fundamentalist forecasting rule expect the output gap to return to its fundamental value y^* if it has deviated from y^* in the previous period. This fundamentalist rule can be expressed by

$$\tilde{E}_t^f[y_{t+1}] = y_{t-1} + f_y(y^* - y_{t-1}), \tag{17}$$

where $0 < f_y < 1$ indicates how fast the fundamentalists assume the output gap to move back to its fundamental value y^*. For the sake of simplicity, all agents in the model have the identical view concerning the fundamental value of the output gap such that we can assume y^* is constant. This is

Section 3.2 that $y^* = 0$ and that $i^* = \pi^*$.

in contrast to De Grauwe (2010a, b, c, 2012) who normalizes the fundamentalist expectation rule to zero.[7] In contrast, extrapolative expectations are based on trend extrapolation and are formalized by

$$\tilde{E}_t^e[y_{t+1}] = y_{t-1} + e_y(y_{t-1} - y_{t-2}). \tag{18}$$

Agents who make use of this rule expect the output gap to further increase from t to $t+1$ if it has already risen from $t-2$ to $t-1$ and vice versa. Hence, $e_y > 0$ is a positive reaction parameter which indicates how strongly and aggressively the extrapolators react to the output gap trend. This also differs from De Grauwe (2010a, b, c, 2012), who uses a naive forecasting rule which states that agents extrapolate the latest observable output gap into the future.[8]

Similarly to the housing market, agents exhibit a learning behavior, which means that they are not limited to one expectation rule but switch between the two forecasting rules in (17) and (18). Inspired by De Grauwe (2010a, b, c, 2012) and Brock and Hommes (1997, 1998) and following the discrete choice approach, this selection mechanism is based on the evolutionary fitness of these rules depending on mean squared forecasting errors that are updated in each period. Thus, agents calculate the forecast performance, or rather the utility of the fundamentalist rule according to

$$U_t^{y,f} = -\sum_{k=1}^{\infty} \omega_k (y_{t-k} - \tilde{E}_{t-k-1}^f[y_{t-k}])^2 \tag{19}$$

and of the extrapolative rule pursuant to

$$U_t^{y,e} = -\sum_{k=1}^{\infty} \omega_k (y_{t-k} - \tilde{E}_{t-k-1}^e[y_{t-k}])^2 \tag{20}$$

and choose the rule that performed best in the past. To include the forgetfulness of agents in the model, parameters ω_k are specified as geometrically declining weights and are defined as $\omega_k = (1-q)q^k$, where $0 \leq q \leq 1$ (see De Grauwe (2010a, b, c, 2012)).[9]

In each period and based on the rules' fitness, agents have to decide which expectation rule to follow. In line with De Grauwe (2010a, b, c, 2012) and as in Brock and Hommes (1997, 1998) the weights of agents relying on the fundamentalist and on the extrapolative expectation rule for future

[7] In his paper, De Grauwe assumes that agents forecast the future output gap by estimating the fundamental steady-state equilibrium value of the output gap. De Grauwe normalizes this steady state value to zero, i.e. there is no gap in the equilibrium. For $f_y = 1$ and $y^* = 0$ the fundamentalist forecasting rule in (17) collapses to the one used in De Grauwe (2010a, b, c, 2012).

[8] In his work, De Grauwe justifies the agents' naivety with the fact that the respective agents do not know the output gap's steady state value. For $e_y = 0$ our extrapolative forecasting rule can be shortened to the naive one used in De Grauwe (2010a, b, c, 2012).

[9] See De Grauwe and Ji (2020) for a short derivation of (19) and (20). Furthermore, parameter $0 < q < 1$ is defined as agents' memory. Accordingly, if $q = 0$ agents have no memory and only remember the last periods' forecast error. In contrast, if $q = 1$ agents have infinite memory and all past forecast errors are equally weighted.

output gap at time t are given by

$$w_t^{y,f} = \frac{exp[\nu U_t^{y,f}]}{exp[\nu U_t^{y,f}] + exp[\nu U_t^{y,e}]} \tag{21}$$

and

$$w_t^{y,e} = \frac{exp[\nu U_t^{y,e}]}{exp[\nu U_t^{y,f}] + exp[\nu U_t^{y,e}]}, \tag{22}$$

respectively. Parameter $\nu \geq 0$ is called intensity of choice which determines the agents' switching sensitivity with which they choose the most attractive expectation rule. We assume that all agents in the economy have the same switching sensitivity and thus the intensity of choice parameter is identical to that of the housing market in Equations (11) and (12) and exhibits the same properties. Consequently, for $\nu = 0$, the weights of both expectation rules are exactly 0.5 since agents cannot differentiate between the fitness of the two rules. With growing parameter ν, the weight of the expectation rule with the higher fitness increases. For $\nu \to +\infty$, all agents choose the expectation rule with the highest fitness since they perfectly realize fitness differences. Note that $0 < w_t^{y,f}, w_t^{y,e} < 1$ and that the weight of the fundamentalist (extrapolative) expectation rule can also be expressed as $w_t^{y,f} = 1 - w_t^{y,e}$ $(w_t^{y,e} = 1 - w_t^{y,f})$.

Agents also have to predict the future inflation rate, using very similar two expectation rules and selection mechanisms as when forming expectations regarding the output gap. Note that expectations for output gap and inflation rate do not intermingle, although the expectation formation process follows identical equations in both cases. It is a concurrent and independent process for both variables. Accordingly, agents' weighted average inflation expectations are defined by

$$\tilde{E}_t[\pi_{t+1}] = w_t^{\pi,f} \tilde{E}_t^f[\pi_{t+1}] + w_t^{\pi,e} \tilde{E}_t^e[\pi_{t+1}], \tag{23}$$

where $w_t^{\pi,f}$ and $w_t^{\pi,e}$ stand for the weights of agents that follow the fundamentalist $\tilde{E}_t^f[\pi_{t+1}]$ and the extrapolative expectation rule $\tilde{E}_t^e[\pi_{t+1}]$, respectively. Thus, agents can decide to make use of the fundamentalist expectation rule which is formulated as

$$\tilde{E}_t^f[\pi_{t+1}] = \pi_{t-1} + f_\pi(\pi^* - \pi_{t-1}), \tag{24}$$

where the central bank's inflation target π^* is assumed to be known by all agents. Furthermore, $0 < f_\pi < 1$ is a reaction parameter that captures the agents' expected adjustment speed of the inflation rate towards its fundamental value π^*. Accordingly, if the inflation rate in $t-1$ is above its fundamental value π^*, the boundedly rational agents expect it to fall again, and vice versa. This is departing from De Grauwe (2010a, b, c, 2012) who assumes that agents use the central bank's inflation target to forecast inflation.[10] Alternatively, agents can choose to follow the extrapolative expectation

[10] De Grauwe argues that agents using this rule trust the central bank's announced inflation target π^* which he assumes to be zero. If this is not the case, agents follow the extrapolative expectation rule. By setting $f_\pi = 1$ Equation (24) is

rule which is given by

$$\tilde{E}_t^{ext}[\pi_{t+1}] = \pi_{t-1} + e_\pi(\pi_{t-1} - \pi_{t-2}), \tag{25}$$

where parameter $e_\pi > 0$ specifies how strongly the respective agents react to changes in the inflation rate. Thus, agents in the model using extrapolative expectations anticipate a further increase (decrease) in the inflation rate if π has already risen (declined) since the period before. Here again, De Grauwe (2010a, b, c, 2012) uses a reduced form of Equation (25) since he states that agents extrapolate the previous observed inflation rate into the future.[11]

The switching mechanism defining which expectation rule concerning inflation forecasting in the real market is dominant in each period is, also for the sake of consistency, formulated in the identical way as in case of output gap expectations above. Hence, the utility of using the fundamentalist and the extrapolative expectation rule are given by the mean squared forecasting errors of the expectation rules and are formulated as

$$U_t^{\pi,f} = -\sum_{k=1}^{\infty} \omega_k(\pi_{t-k} - \tilde{E}_{t-k-1}^f[\pi_{t-k}])^2 \tag{26}$$

and

$$U_t^{\pi,e} = -\sum_{k=1}^{\infty} \omega_k(\pi_{t-k} - \tilde{E}_{t-k-1}^e[\pi_{t-k}])^2, \tag{27}$$

respectively. Here again, parameters ω_k are geometrically declining weights catching the agents' forgetfulness in the model. Concerning the weighting functions, we have

$$w_t^{\pi,f} = \frac{exp[\nu U_t^{\pi,f}]}{exp[\nu U_t^{\pi,f}] + exp[\nu U_t^{\pi,e}]} \tag{28}$$

as the probability of agents relying on the fundamentalist expectation rule and

$$w_t^{\pi,e} = \frac{exp[\nu U_t^{\pi,e}]}{exp[\nu U_t^{\pi,f}] + exp[\nu U_t^{\pi,e}]} \tag{29}$$

determining the probability of agents following extrapolative expectations in period t concerning inflation. The interpretation is very similar to the weighting functions in Equations (21) and (22). The higher the past forecast performance of the fundamentalist expectation rule relative to that one of the extrapolative expectation rule, the more (less) attractive appears the fundamentalist (extrapolative) expectation strategy to the agents and the more (less) agents will follow it. The sensitivity parameter $\nu \geq 0$ controls how fast agents switch to the more attractive expectation rule with increasing utility. Here, the following two properties also applies: $0 < w_t^{\pi,f}, w_t^{\pi,e} < 1$ and $w_t^{\pi,f} = 1 - w_t^{\pi,e}$, and vice versa.

the same as presented in De Grauwe (2010a, b, c, 2012).

[11]For $e_\pi = 0$ the expression breaks down to the one used in De Grauwe (2010a, b, c, 2012).

3. Analytical and numerical insights of our basic model framework

In order to understand the structure and the functioning of our model, we next analyze the model's dynamics, which arises through the agents' interactions and strategy-switching decisions. First of all, we investigate the dynamics of both submodels separately, namely in Section 3.1 of the housing market and in Section 3.2 of the real market.

3.1. Analytical and numerical insights of the housing market model

Table 1 presents the base parameter setting that we use for our housing market simulations. The choice of the model's parameter values is inspired by Martin and Westerhoff (2019) and Schmitt and Westerhoff (2019). The interest rate and the depreciation rate are set to one percent meaning that one time step in our simulations corresponds to one quarter of a year. In addition, some parameters are adjusted such that our model qualitatively reflects important characteristics of housing markets, especially the boom-and-bust dynamics as described by Agnello et al. (2015), Kouwenberg and Zwinkels (2015) and Ascari et al. (2018).

Table 1: Base parameter setting of the housing market model

$\alpha = 12$	scaling parameter	rental market
$\beta = 0.1$	sensitivity of rental market	
$\gamma = 0.01$	sensitivity of home building	supply side of housing market
$\delta = 0.01$	depreciation rate	
$i = 0.01$	interest rate	
$\chi = 2.3$	extrapolative parameter	price expectations
$\phi = 0.9$	regressive parameter	
$\nu = 5$	intensity of choice	
$g = 0.1$	fitness reaction parameter	fitness
$P^* = 100$	fundamental house price	fundamental values
$H^* = 100$	fundamental housing stock	
$R^* = 2$	fundamental rent level	

As shown in Schmitt and Westerhoff (2019), the dynamics of the housing market model is driven by a five-dimensional nonlinear map. Moreover, the model's unique fundamental steady state (FSS) is specified by the fundamental house price $P^* = F = \frac{\alpha\delta}{\beta\gamma+\delta(i+\delta)}$ and the fundamental housing stock $H^* = \frac{\gamma}{\delta}P^*$ (see Schmitt and Westerhoff (2019) for more details on the derivation). As $P^* = F = \frac{R^*}{i+\delta}$, investors discount future rent payments to determine the fundamental house price, more precisely, the steady state at which expectations are realized and investors make no prediction errors.[12] Note that P^* and H^* are only dependent on real parameters. Hence an increase in α leads to a rising P^* while an increase in i, γ and β causes the fundamental house price to fall. A higher depreciation rate only decreases P^* if $\delta > \sqrt{\beta\gamma}$. For completeness, we mention that the fundamental rent level is expressed by $R^* = \alpha - \beta H^*$. Given the parameter setting, the model's FSS is defined at $P^* = 100$, $H^* = 100$,

[12] In line with Poterba (1984, 1991), the term $i+\delta$ is called the user cost of owning a house. As described in Himmelberg et al. (2005) the user cost may also comprise further components as, for instance, a risk premium for owning instead of renting a house, as well as the expected capital gain (or loss).

$R^* = 2$ and $w_t^{P,e^*} = 0.5$, which implies that at the FSS, half of investors follow the extrapolative expectation rule and the other half rely on regressive expectations.

Let us now focus on the steady state's stability properties. Schmitt and Westerhoff (2019) show that the fundamental steady state is locally asymptotically stable if two stability conditions are fulfilled, namely $\phi < \frac{4+2i-\delta(i+\delta)-\beta\gamma+\chi(2-\delta)}{1-0.5\delta}$ and $\chi\delta + \frac{2\chi\beta\gamma}{2(1+i+\delta)-\chi} < \phi + \frac{2(2\delta+i)}{1-\delta}$. Obviously, both stability conditions hinge on behavioral and real parameters. A violation of the first stability condition is associated with a Flip bifurcation, whereas a violation of the second inequality is associated with a Neimark-Sacker bifurcation and endogenous quasi-periodic dynamics is set in motion. Since these cyclical dynamics can be observed in real housing markets, we focus on the second stability condition in the further analysis. As can be seen from the Neimark-Sacker condition, an increase in parameters χ, β and γ may contribute to a destabilization of the FSS and thus to endogenous cycles. In contrast, growing parameters i and ϕ stabilize the system, just like δ under the assumption that $i + \delta < 1$ holds. It is also important to mention that endogenous oscillations only emerge if housing market investors form extrapolative expectations. Thus, in a model with naive versus regressive expectations, i.e. if $\chi = 0$, the Neimark-Sacker condition is always fulfilled.

Given the parameter setting in Table 1 the FSS is unstable since the extrapolation parameter $\chi = 2.3$ is above the threshold value $\chi_{crit}^{NS} \approx 2.04$ at which a Neimark-Sacker bifurcation emerges. And, in fact, our model is able to produce endogenous boom-and-bust housing market dynamics as can be seen in the simulation run displayed in Figure 1. The depicted 100 time steps represent a period of 25 years and the parameter setting is as in Table 1. Furthermore, the panels show from top to bottom the evolution of the house price, the market share of extrapolative expectations, the housing stock and the rent level, respectively. The gray horizontal line in each panel illustrates the respective fundamental value. The functioning of the housing market model can be explained as follows. At the beginning of the simulation (at period $t = 1$), the house price is slightly above its fundamental value. In such a situation, the regressive expectation rule is more profitable than the extrapolative expectation rule since it has properly forecasted a reversion of the house price to its fundamental value and thus, more and more investors follow it. As regressive expectations have a stabilizing impact on the dynamics, the house price falls further towards P^* and even below it. Declining house prices mean less new housing constructions, which leads to a falling housing stock and consequently to a rising rent level with a short time lag. As soon as the house price falls below its fundamental value, the situation starts to change since now extrapolative expectations are right in predicting a further house price decline and therefore attract more followers. The destabilizing behavior of extrapolators leads to a crash. After the crash's turning point the house price moves back towards its fundamental value. Now both expectation rules correctly predict an upward trend of the housing market. But the closer the house price gets to its fundamental value, the more investors switch to the now more profitable regressive expectation rule. Once the house price rises above its fundamental value, investors relying on extrapolative expectations are getting more and more optimistic predicting a further price increase. As a result, the amount of new housing construction increases stimulating the housing stock and depressing the rent level until the housing market bubble bursts and the house price falls towards its fundamental value again.

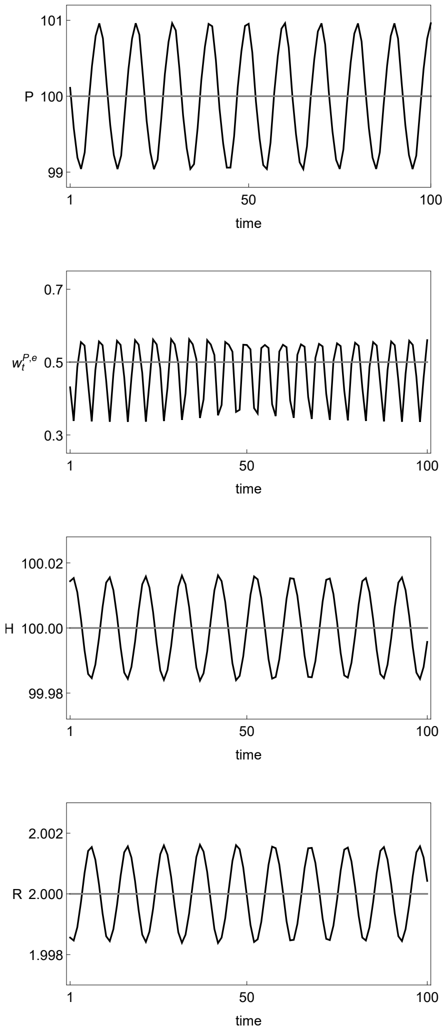

Figure 1: The dynamics of the housing market model for our base parameter setting (Table 1). The panels show, from top to bottom, the evolution of the house price, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 100 time steps and a longer transient period has been cleared.

Next, we examine how the model's dynamics depends on the extrapolation parameter χ for our base parameter setting. For this we present a bifurcation diagram in Figure 2 showing the house price dynamics for $1.9 < \chi < 2.4$. As can be seen, this graphical analysis confirms our analytical results. Accordingly, the fundamental steady state is initially stable. At the bifurcation point $\chi_{crit}^{NS} \approx 2.04$ the system loses its stability and endogenous quasi-periodic dynamics emerges. As parameter χ rises, i.e. the more aggressively the extrapolators react to past house price trends, the larger the amplitude of house price fluctuations.

In Figure 3, we show a stochastic simulation run under i.i.d. normal additive noise on house prices $\epsilon \sim N(0,1)$ resting on the base parameter setting, except that $\chi = 0.35$ and $\phi = 0.125$. The simulation run depicts 200 time steps, representing a period of 50 years. Since the FSS is independent

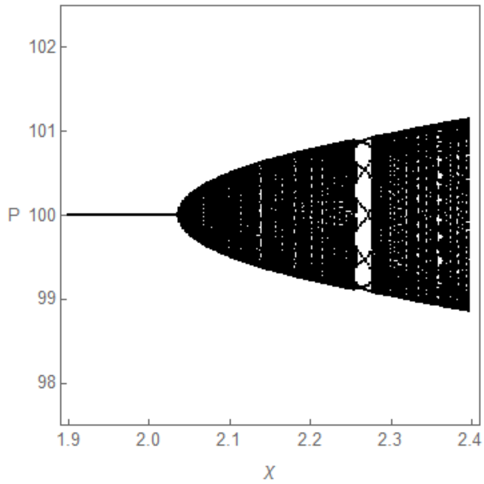

Figure 2: The destabilizing effect of extrapolative expectations on house price dynamics. The panel shows a bifurcation diagram for the house price depending on the extrapolative parameter χ. A Neimark-Sacker bifurcation sets in at $\chi = 2.04$ for the base parameter setting (Table 1).

of any behavioral parameters, it is not influenced by the parameter change. As can be seen, the housing market is very volatile and exhibits significant bubbles and crashes with lasting periods of overvaluation and overbuilding. This boom-and-bust dynamics is maintained through the complex interplay between real and behavioral forces. As this stochastic simulation run produces more realistic housing market dynamics than in Figure 1, we use it for the following analysis.

3.2. Numerical insights of the real market model

Let us now turn to the analysis of the real market model with focus on numerical simulations. The base parameter setting for our simulations is shown in Table 2 which is mostly the same as in De Grauwe (2010a, b, c). Thus one time unit is considered to be one quarter of a year, as is also the case in the housing market model. As already mentioned we assume that all agents in the economy have the same degree of rationality to decide which expectation strategy to follow. For this reason, we take the same value for the intensity of choice parameter as in the housing market, i.e. $\upsilon = 5$. Furthermore, concerning agents' expectation formation we introduce extrapolative and regressive parameters for both the output and inflation expectations to control the trend extrapolation and mean-reversion components, respectively. Taking into account the fact that expectations are realized at the steady state and simultaneously it holds that $\pi_{t-1} = \pi_t = \pi_{t+1} = \pi^*$, we get from Equation (14) that the fundamental output gap is zero, i.e. $y^* = 0$. From Equation (13) it then follows that in the equilibrium, the fundamental values of the inflation rate and the interest rate must be equal, $\pi^* = i^*$, implying that the fundamental real interest rate is zero. Thus, the fundamental values of output gap, inflation rate and nominal interest rate are given by $y^* = 0$, $\pi^* = 0.01$ and $i^* = 0.01$, respectively.

In Figure 4 we show a deterministic simulation run of the real market model for 100 time steps representing a period of 25 years and the parameter setting is as in Table 2. The panels show, from top to bottom, the evolution of the output gap, the inflation rate, the market share of extrapolators

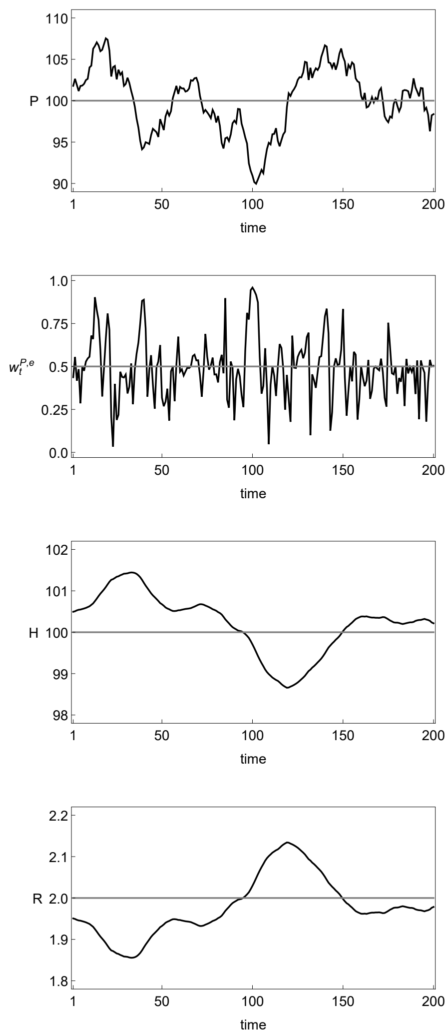

Figure 3: A stochastic simulation run of the housing market model under i.i.d. normal additive noise $\epsilon \sim N(0,1)$ for our base parameter setting (Table 1), except that $\chi = 0.35$ and $\phi = 0.125$. The panels show, from top to bottom, the evolution of the house price, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

concerning the output gap (black line) and inflation (red line) expectations and the nominal interest rate, respectively. As can be seen, the model is able to generate endogenous quasi-periodic boom-and-bust dynamics. Moreover, the output gap precedes the inflation rate. This observation is in line with Claus (2000) who examines whether the output gap indicates inflationary pressures. He argues that output gap does not reliably predict inflation, however can still be a valid tool for the central bank since in two third of the cases there is a relation between a positive (negative) output gap and an increasing (decreasing) inflation in the next quarter.

Next we will look at the dynamics and expectation formation with regard to the output gap. Initially (at $t = 1$) the output gap is still high but decreasing towards its fundamental value which means that the fundamentalist expectation rule is more precise than the regressive expectation rule and thus attracts more and more followers. Due to the stabilizing impact of the fundamentalist rule, the output gap falls further and even below its fundamental value y^*. With a short time lag the

Table 2: Base parameter setting of the real market model

$a_1 = 0.5$	coefficient of expected output	output equation
$a_2 = -0.2$	interest elasticity	
$e_y = 3.5$	extrapolative parameter	
$f_y = 0.9$	regressive parameter	
$b_1 = 0.5$	coefficient of expected inflation	inflation equation
$b_2 = 0.05$	coefficient of output	
$e_\pi = 3.5$	extrapolative parameter	
$f_\pi = 0.9$	regressive parameter	
$c_1 = 1.5$	coefficient of inflation	Taylor rule
$c_2 = 0.5$	coefficient of output	
$c_3 = 0.5$	interest smoothing parameter	
$\nu = 5$	intensity of choice parameter	weighting function
$\omega_1 = 0.25$	geometrically declining weight	
$\omega_2 = 0.125$	geometrically declining weight	
$\omega_3 = 0.0625$	geometrically declining weight	
$y^* = 0$	equilibrium output gap	fundamental values
$\pi^* = 0.01$	central bank's inflation target	
$i^* = 0.01$	equilibrium nominal interest rate	

inflation and the interest rate also decrease towards their fundamental values. Once the output gap drops below y^* the price signals are very low for the fundamentalists, but high for the destabilizing extrapolators, which leads the output gap to fall until the turning point around period $t = 6$. Since the deviation of the output gap from y^* is now relatively large, the fundamentalist expectation rule becomes more profitable again and gains weight which leads the output gap to return to its fundamental value again. As soon as the output gap increases above y^*, extrapolative expectations produce more precise predictions. Consequently, the extrapolators take over the market again, which leads to the next boom around period $t = 12$.

The evolution of the inflation rate and the corresponding expectation formation is very similar to the one of the output gap and can be described in the same way. Note that the function of the inflation rate depends on the output gap positively and linearly. Finally, the nominal interest rate i_t develops as we described in Section 2.2, i.e. the interest rate is positive in periods in which the output gap and inflation rate are above their fundamental values and vice versa.

In addition to the endogenous quasi-periodic boom-and-bust dynamics in Figure 4, we show a stochastic simulation run of the real market model in Figure 5 depicting 200 time steps, that exhibit more irregular but more realistic real market dynamics. In order to achieve this, we add normal additive noise on the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$. The parameter setting is as in Table 2, except that $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The parameter change does not affect the model's fundamental steady state due to its independence of behavioral parameters. The functioning of the stochastic version of the model can be described in the same way as in Figure 4. Thus, extrapolative expectations destabilize the system by pushing the output gap and inflation rate away from their fundamental values, while fundamentalist expectations have a stabilizing effect through their mean-reversion pressure. Depending on the respective shock,

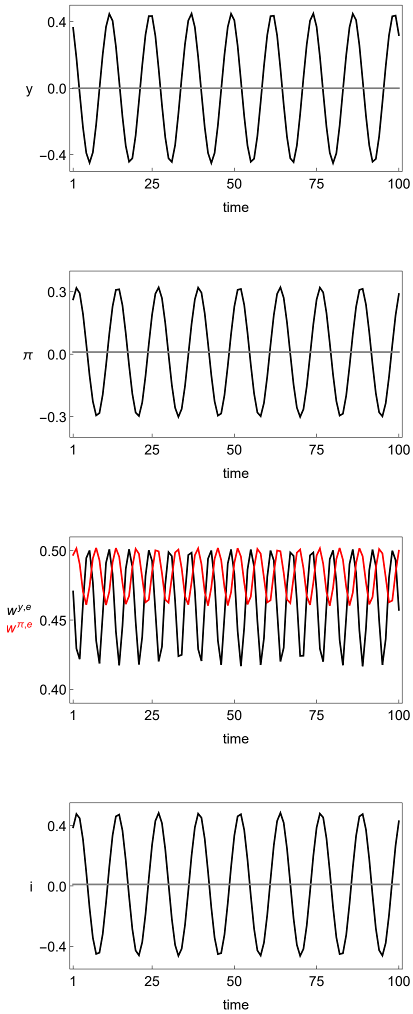

Figure 4: The dynamics of the real market model for our base parameter setting (Table 2). The panels show, from top to bottom, the evolution of the output gap, the inflation rate, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations and the nominal interest rate, respectively. The dynamics is depicted for 100 time steps and a longer transient period has been cleared.

other cycles can be generated that have the same general properties.

Next, we analyze step by step how the dynamics of the real market model depends on the extrapolative parameters of both the output gap and the inflation expectations using a number of bifurcation diagrams in Figure 6. We proceed as follows. First we neglect the inflation expectations and analyze the output gap dynamics for an increasing extrapolation parameter e_y for both a high and a low value of the regressive parameter f_y. Then we do the same concerning the inflation rate, i.e. we neglect the output gap expectations and investigate how the extrapolative parameter e_π affects the inflation dynamics considering two different values of the regressive parameter f_π. In a final step, we analyze the respective dynamics if agents form expectations concerning both the output gap and the inflation rate.

Let us start by considering the output gap dynamics. For this we assume that all agents trust the central bank's inflation target π^* and use it to forecast future inflation, i.e. $\tilde{E}_t[\pi_{t+1}] = \pi^*$. The

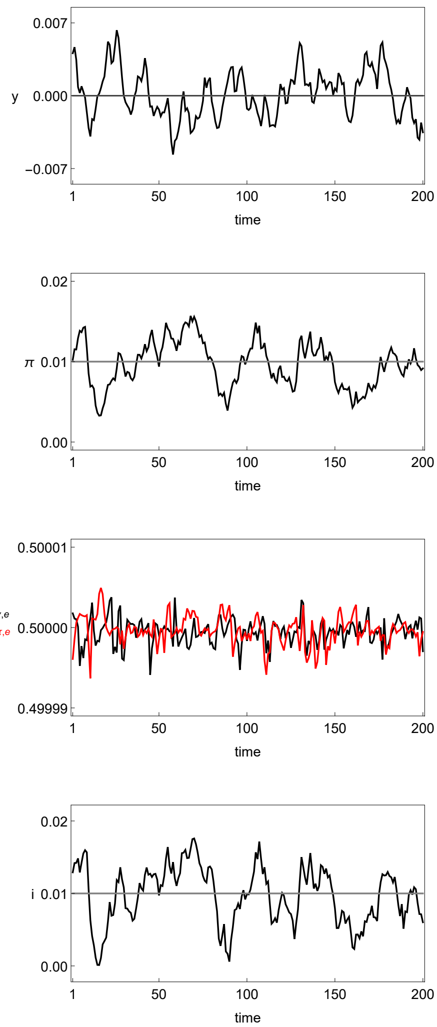

Figure 5: A stochastic simulation run of the real market model under i.i.d. normal additive noise on the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$ for our base parameter setting (Table 2) except that $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The panels show, from top to bottom, the evolution of the output gap, the inflation rate, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations and the nominal interest rate, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

corresponding bifurcation diagrams are illustrated in the two top panels of Figure 6 both of which relate the output gap y to the extrapolative parameter $3.0 < e_y < 5.0$. The parameter setting is used as in Table 2, except that the regressive parameter is set to $f_y = 0.9$ in the top left panel (as in Table 2) and to $f_y = 0.4$ in the top right panel. In both cases the model's fundamental steady state is initially stable, but loses its stability as soon as a critical value is exceeded, from which quasi-periodic dynamics emerges. This point is reached in the top left panel with a higher extrapolative parameter, namely at $e_y^{crit} \approx 4.03$, than in the case of a lower mean reversion speed depicted in the top right panel (at $e_y^{crit} \approx 3.99$). Furthermore, the amplitude of output gap fluctuations increases with the extrapolative parameter regardless of the level of the regressive parameter. However, y oscillates with a smaller amplitude in the top left panel, namely between -0.3 and 0.3, compared to oscillations between -0.5 and 0.5 in the top right panel, if $e_y = 5.0$. Thus, the stabilizing effect of a higher regressive parameter

f_y on the output gap dynamics is clearly confirmed.

The bifurcation diagrams in the two center panels depict how the inflation rate reacts to an increasing extrapolative parameter e_π for two different levels of the regressive parameter f_π using the base parameter setting. More precisely, we show the inflation rate dynamics for $3.0 < e_y < 5.0$ with a high regressive parameter, namely $f_\pi = 0.9$ in the center left panel (as in Table 2) and for $3.0 < e_\pi < 4.2$ with a low regressive parameter, namely $f_\pi = 0.4$ in the center right panel. In addition, we assume that all agents have constant output gap expectations, i.e. $\tilde{E}_t[y_{t+1}] = y^*$. The center left bifurcation diagram shows a bifurcation route that evolves from fixed-point dynamics into quasi-periodic motion at the critical point $e_\pi^{crit} \approx 3.88$. In contrast, this point is reached at a smaller extrapolative parameter in case of a lower mean-reversion speed, namely at $e_\pi^{crit} \approx 3.86$, in the center right panel. Furthermore, the amplitude of inflation fluctuations increases with parameter e_π in both bifurcation diagrams. However, in the center right panel the amplitude of inflation oscillations is nearly twice as large as in the case of a high regressive parameter. Consequently, a stabilizing effect of the regressive parameter can also be demonstrated for the inflation dynamics.

And finally, in the two bottom panels we investigate the model dynamics if heterogeneous agents form expectations concerning output gap and inflation rate. The bottom left panel illustrates a bifurcation diagram that relates the output gap y to the extrapolative parameter $3.0 < e_y < 5.0$ for the base parameter setting. As can be seen, the fundamental steady state becomes unstable at $e_y^{crit} \approx 3.13$ and the stable fixed point converges into quasi-periodic dynamics. Moreover, the amplitude of output gap fluctuations becomes larger when the extrapolative parameter increases. Similar observations can be made in the inflation rate dynamics of the bottom right panel for $3.0 < e_\pi < 4.4$, using the base parameter setting. Thus, the fundamental steady state becomes unstable at $e_\pi^{crit} \approx 3.29$ after which the amplitude of inflation dynamics increases with e_π. As a result, expectations about both the output gap and inflation rate makes the system more unstable since the bifurcation points are reached at a significantly smaller extrapolative parameter compared to the respective panels in the top row.

4. Interactions between the housing market and the real market

So far we have analyzed the dynamics of the housing market and the real market separately. In Section 4.1 we take the first step in order to combine the two markets. First of all we include the interest rate as set by the Taylor rule into the housing market instead of a fixed interest rate as discussed above. The resulting dynamics provides the basis for the further investigations, in which we merge the two markets in three further steps. In Section 4.2 we vary the Taylor rule by also making it dependent on the house price distortion. Next, in Section 4.3 we examine how the inclusion of the house price trend in the inflation equation affects the dynamics. And finally, in Section 4.4 we analyze the effects on the model dynamics if the aggregate demand equation also depends on the house price distortion.

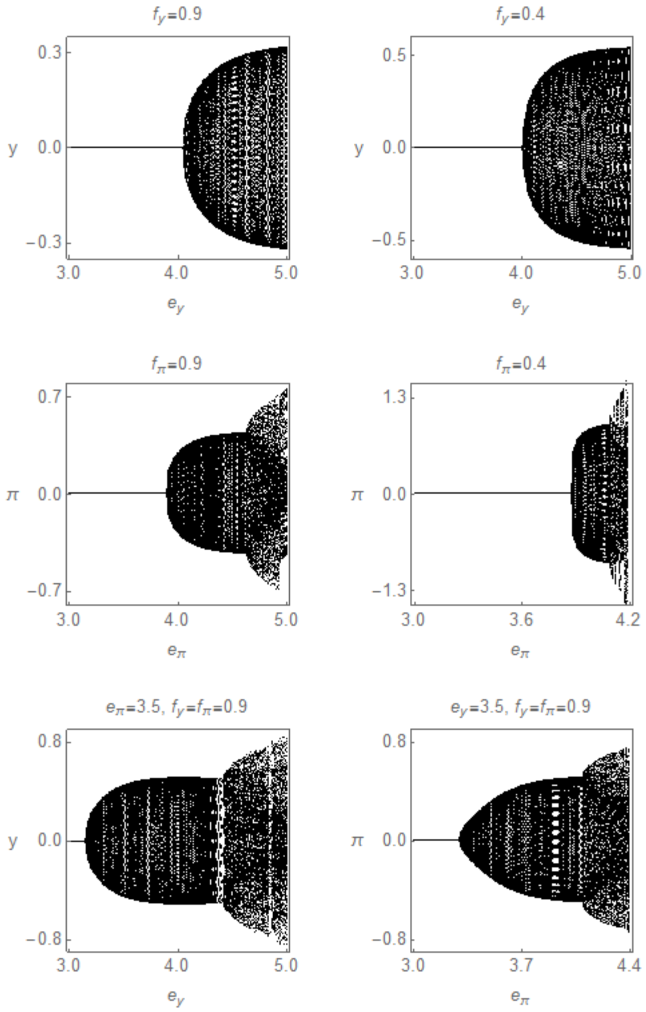

Figure 6: The destabilizing effect of the extrapolative parameters e_y and e_π in the Neimark-Sacker bifurcation scenario. The two top panels show bifurcation diagrams for the output gap versus extrapolative parameter e_y in case of output gap expectations for $f_y = 0.9$ (top left panel) and $f_y = 0.4$ (top right panel). The two center panels display bifurcation diagrams for the inflation rate related to the extrapolative parameter e_π in case of inflation expectations for $f_\pi = 0.9$ (center left panel) and $f_\pi = 0.4$ (center right panel). The bifurcation diagram in the bottom right panel (bottom left panel) relates the output gap (inflation rate) to the extrapolative parameter e_y (e_π) in case of output gap and inflation expectations for the base parameter setting.

4.1. Inclusion of the Taylor rule in the housing market

Initially in Sections 2.1 and 3.1 we assume the interest rate in the housing market constant and to be set by the central bank. We now extend on this assumption by taking into account the interest rate the central bank determines through the Taylor rule in each time step t. Thus including the Taylor rule (15) in the house price equation (5) gives us

$$P_t = \frac{\tilde{E}_t[P_{t+1}] + R_t}{1 + i_t + \delta} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{30}$$

where i_t is the nominal interest rate at time t. The FSS of the housing market is not affected by this change since the interest rate on the housing market in Section 2.1 is the same as the fundamental interest rate on the real market, i.e. $i = i^* = 0.01$. Accordingly the house price rises more sharply in periods in which the interest rate is below its fundamental value than is the case with a constant interest rate and vice versa.

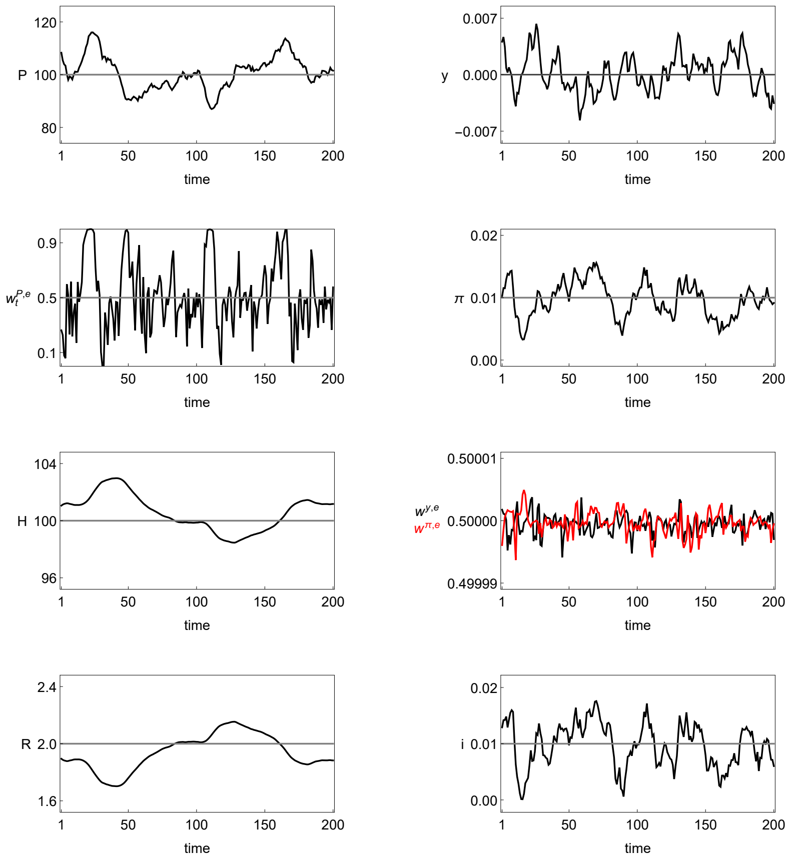

Figure 7: A stochastic simulation run of the housing market and the real market model under i.i.d. normal additive noise on the house price $\epsilon \sim N(0,1)$, the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$ for our base parameter setting (Tables 1 and 2), except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The housing market is connected to the real market via Taylor rule. The panels show, from top left to bottom right, the evolution of the house price, the output gap, the market share of extrapolators concerning house price expectations, the inflation rate, the housing stock, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations, the rent level and the nominal interest rate, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

Figure 7 presents a stochastic simulation run of the dynamics of our merged model, i.e. the housing market is connected to the real market via the Taylor rule. Normal additive noise is added to the house price $\epsilon \sim N(0,1)$, the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$. The 200 time steps represent a period of 50 years and the parameter setting is as in Tables 1 and 2, except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The panels show, from top left to bottom right, the evolution of the house price, the output gap, the market share of extrapolators regarding house price expectations, the inflation rate, the housing stock, the market share of extrapolators regarding output gap (black line) and inflation (red line) expectations, the rent level and the nominal interest rate. The gray lines depict the fundamental values. For comparability we also use this design in Figures 10, 12 and 14. As can be seen, the house price and the output gap both show a similar boom-and-bust development over time. In fact, the two variables exhibit a correlation coefficient of $Corr(P, y) = 0.61$ indicating a positive linear relationship between the house price and the output gap. This is in line with Ahearne et al. (2005) who provide empirical evidence indicating that house prices are pro-cyclical moving with output gaps. They show that house price booms coincide with a strong upturn on the real market, i.e. a positive output gap, just as a downswing of house prices is related to a negative output gap. Furthermore, if we compare Figure 7 with Figure 3, the dependency of the house price on the Taylor rule makes the housing market more unstable. The house price, the housing stock and the rent level fluctuate with larger amplitudes around their fundamental values. If the interest rate falls below its fundamental value, for instance around period $t = 10$ and shortly after $t = 150$, the house price rises more strongly than in case of a constant interest rate $i = 0.01$. The opposite can be observed in periods in which the interest rate increases above its fundamental value, which is the case, for instance, shortly after periods $t = 50$ and $t = 100$. During these time periods the high interest rate leads to more pronounced housing market crashes. Thus the dynamic interest rate intensifies the bubbles and crashes on the housing market. At its peak, the house price in Figure 3 reaches a value of about $P = 108$, while in Figure 7 a value of about $P = 116$, which is, referred to the fundamental value, twice as high. Moreover, a more volatile house price leads to a more distorted housing stock and rent level. One explanation for this development is that the interest rate only depends on y_t and π_t, but does not react in any way to the house price. Thus in Section 4.2 we examine how the dynamics changes if the Taylor rule also takes into account the house price distortion.

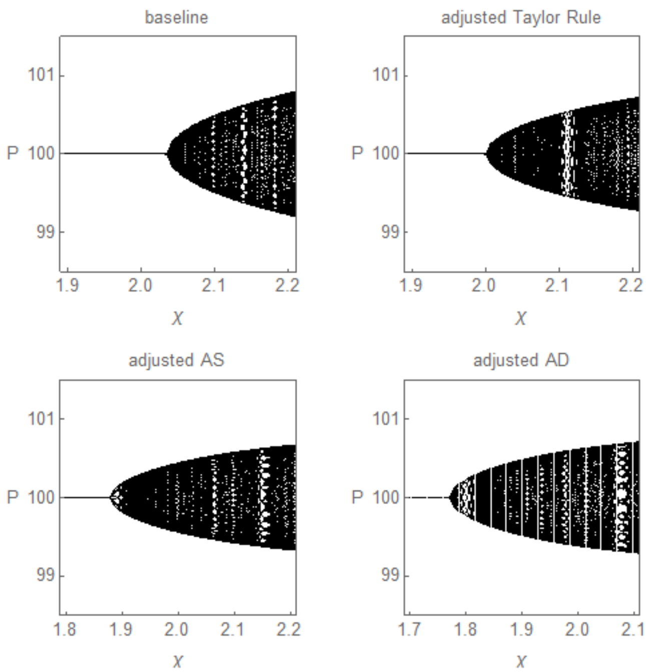

Figure 8: The destabilizing effect of the extrapolative parameter χ on house price dynamics in the Neimark-Sacker bifurcation scenario. The panels show bifurcation diagrams for the house price versus extrapolative parameter χ in case that the housing market is connected to the real market via Taylor rule (top left panel), the Taylor rule depends on the house price distortion (top right panel), the house price trend is added to the inflation equation (bottom left panel) and the house price distortion is included in the aggregate demand equation (bottom right panel). Base parameter setting (Tables 1 and 2), except that $f_y = f_\pi = 0.8$ and $e_y = e_\pi = 1.5$.

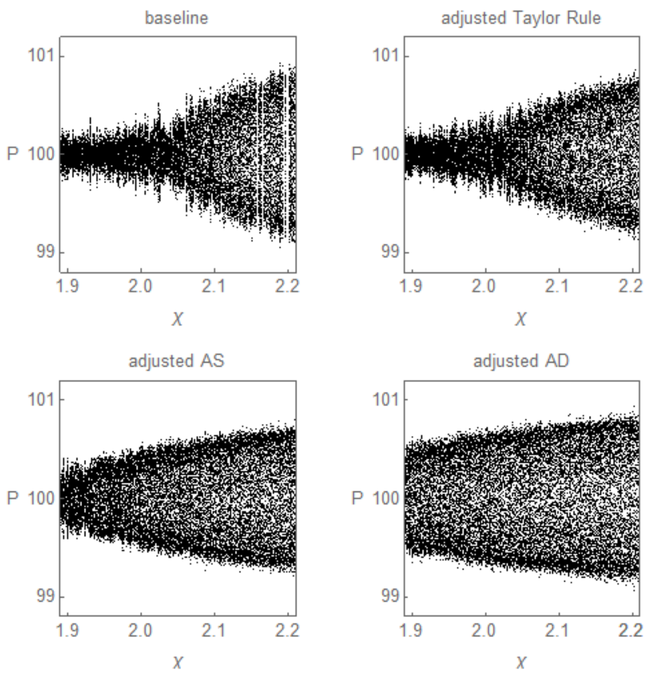

Figure 9: The destabilizing effect of the extrapolative parameter χ on house price dynamics in the Neimark-Sacker bifurcation scenario under i.i.d. normal additive noise on the house price $\epsilon \sim N(0,1)$. The panels show bifurcation diagrams for the house price versus extrapolative parameter χ in case that the housing market is connected to the real market via Taylor rule (top left panel), the Taylor rule depends on the house price distortion (top right panel), the house price trend is added to the inflation equation (bottom left panel) and the house price distortion is included in the aggregate demand equation (bottom right panel). Base parameter setting (Tables 1 and 2), except that $f_y = f_\pi = 0.8$ and $e_y = e_\pi = 1.5$.

Before continuing, we show in Figure 8 how the housing market dynamics depends on the extrapolative expectations. For comparability, the four bifurcation diagrams exhibit the same structure and relate the house price P_t to the extrapolative parameter χ for the base parameter setting except that $f_y = f_\pi = 0.8$ and $e_y = e_\pi = 1.5$. The bifurcation diagram in the top left panel represents the baseline case that the interest rate of the housing market develops according to the Taylor rule. The critical value at which the stable steady state passes into a Neimark-Sacker bifurcation is reached at $\chi^{crit} \approx 2.03$. Moreover, the amplitude of house price fluctuations increases with growing parameter χ. In Figure 9 we repeat our simulations from Figure 8, but now with i.i.d. normal additive noise on the house price $\epsilon \sim N(0, 0.01)$. As it turns out, the destabilizing effect of the extrapolative parameter χ is robust with respect to additional exogenous noise.

4.2. Inclusion of the house price distortion in the Taylor rule

Including the house price distortion in the Taylor rule turns equation (15) into

$$i_t = c_3 i_{t-1} + (1-c_3)(i^* + c_1(\pi_t - \pi^*) + c_2(y_t - y^*) + c_4(p_{t-1} - p^*)) + u_t, \quad (31)$$

where p_{t-1} and p^* denote the logarithmic house price in $t-1$ and the logarithmic fundamental house price, respectively.[13] Parameter $c_4 \geq 0$ indicates how strongly the central bank reacts to the deviation of the house price from its fundamental value. Here, we set $c_4 = 0.05$.[14] Obviously, if the housing market is overvalued, i.e. if $p_{t-1} - p^* > 0$, the central bank will increase the interest rate, while in periods of undervaluation ($p_{t-1} - p^* < 0$) the central bank will decrease the interest rate. According to Equation (30), a higher (lower) interest rate depresses (raises) the house price. In Figure 10, we show a stochastic simulation run depicting the model dynamics in case of the adjusted Taylor rule. In fact, we observe a stabilization of the housing market. The house price, the housing stock as well as the rent level fluctuate significantly closer around their fundamental values. This can be explained as follows. For instance, around period $t = 10$ and shortly after $t = 150$ the central bank reacts to the house price distortion and consequently the interest rate does not fall as sharply as in Figure 7, which leads to a dampened house price and less pronounced bubbles. The opposite can be observed shortly after periods $t = 50$ and $t = 100$. The less rapidly rising interest rate means that there are less pronounced crashes on the housing market. Simultanously, the market share of extrapolators is smaller while the market share of fundamentalists has increased. Moreover, new housing construction, the housing supply and the rent level benefit from a more stable house price which becomes apparent shortly before period $t = 50$ and between periods $t = 100$ and $t = 150$. Furthermore, the adjusted Taylor rule does not have a significant impact on the dynamics of the real market.

Figure 11 illustrates how the model performance depends on the house price distortion in the Taylor rule and as such, on parameter c_4 for the base parameter setting (Tables 1 and 2), except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The panels show from top left to bottom right the behavior of nine statistics for increasing values of parameter $0.0 < c_4 < 0.1$. The nine statistics include the house price distortion $D^P = \frac{1}{T}\sum_{t=1}^{T} \frac{|P_t - P^*|}{P^*}$, the housing stock distortion $D^H = \frac{1}{T}\sum_{t=1}^{T} \frac{|H_t - H^*|}{H^*}$, the volatility of house price $V^P = \frac{1}{T}\sum_{t=1}^{T} \frac{|P_t - P_{t-1}|}{P_{t-1}}$, the average house price $\overline{P} = \frac{1}{T}\sum_{t=1}^{T} P_t$, the average housing stock $\overline{H} = \frac{1}{T}\sum_{t=1}^{T} H_t$, the average rent level $\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$, the average output gap $\overline{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$, the average interest rate $\overline{i} = \frac{1}{T}\sum_{t=1}^{T} i_t$ and the average inflation rate $\overline{\pi} = \frac{1}{T}\sum_{t=1}^{T} \pi_t$. For comparability Figures 13 and 15 have the same design. As can be seen, our previous results from Figure 10 are confirmed. It is evident that the average house price $\overline{P}$ (panel 4) decreases in line with parameter c_4. This incurs a decline in the average interest rate $\overline{i}$ (panel 8). Thus the average housing stock $\overline{H}$ (panel 5) falls with increasing parameter c_4 which leads to an increasing average rent level $\overline{R}$ (panel 6). Since $\overline{P}$ and $\overline{H}$ move towards their fundamental values with increasing c_4, both the house price distortion D^P (panel 1) and the housing stock distortion D^H (panel 2) decrease the stronger

[13] Since all elements in the original Taylor rule are described in percentages (see Taylor 1993), we also express the house price distortion as the percent deviation of the house price P_{t-1} from its fundamental value P^* using the logarithm. More precisely, if for example, $P_{t-1} = 101$ and $P^* = 100$, the central bank increases the interest rate by 0.0005 or 0.05 percentage points, under otherwise identical conditions as in Section 4.1.

[14] Martin et al. (2022) explore whether a central bank may stabilize the dynamics of housing markets by setting different interest rates. Among others, they introduce a dynamic interest rate rule with which the central bank adjust the interest rate with respect to mispricing in the housing market. As it turns out, this dynamic interest rate rule gives the central bank a great ability to control housing market fluctuations.

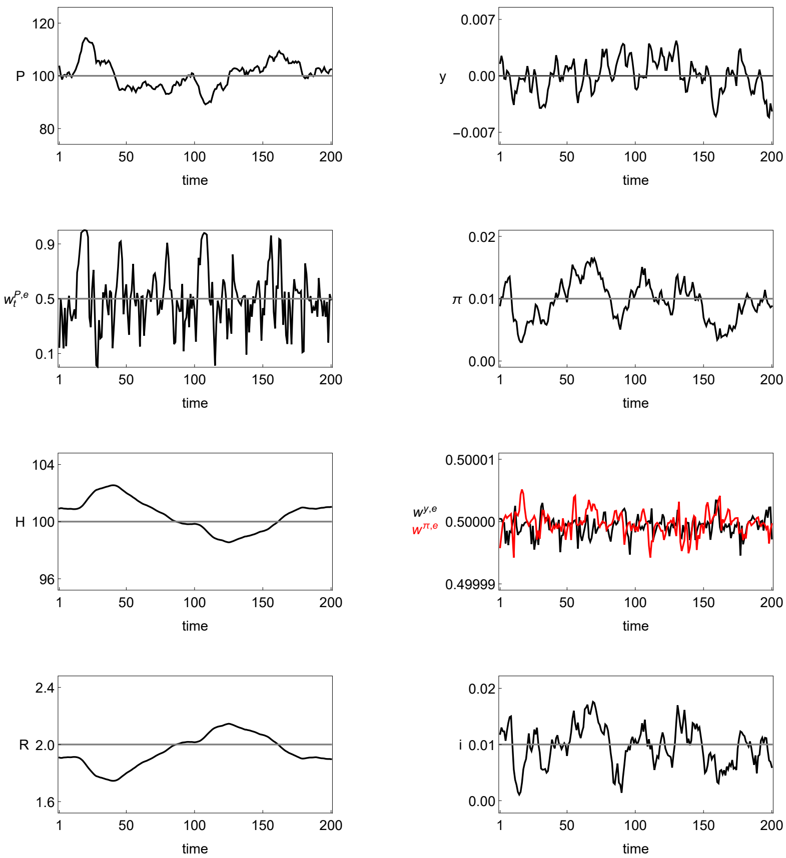

Figure 10: A stochastic simulation run of the housing market and the real market model under i.i.d. normal additive noise on the house price $\epsilon \sim N(0, 1)$, the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$ for our base parameter setting (Tables 1 and 2), except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The Taylor rule depends additionally on the house price distortion ($c_4 = 0.05$). The panels show, from top left to bottom right, the evolution of the house price, the output gap, the market share of extrapolators concerning house price expectations, the inflation rate, the housing stock, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations, the rent level and the nominal interest rate, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

the central bank reacts to the house price distortion. Additionally, an increase in parameter c_4 has a slightly stabilizing effect on the real market. The average inflation rate $\overline{\pi}$ (panel 9) falls with c_4, moving in the direction of its fundamental value. The average output gap $\overline{y}$ (panel 7) initially decreases up to $c_4 = 0.06$ and grows again as c_4 increases further.

Next, we examine in the top right panel of Figure 8 how the dynamics of the house price depends on the extrapolative parameter $1.9 < \chi < 2.2$ using the adjusted Taylor rule. As can be seen, the amplitude of house price fluctuations can be significantly reduced. However, the stable fundamental steady state becomes unstable when the extrapolative parameters already exceeds $\chi \approx 2.0$. To explore the effect of additional exogenous noise, we repeat our simulations in the top right panel of Figure 9 but now with noise on the house price $\epsilon \sim N(0, 1)$. The corresponding bifurcation route is very similar to the one of the top right panel of Figure 8. Therefore, the amplitude of house price fluctuations increases with parameter χ and is smaller than in the baseline scenario of the top left panel.

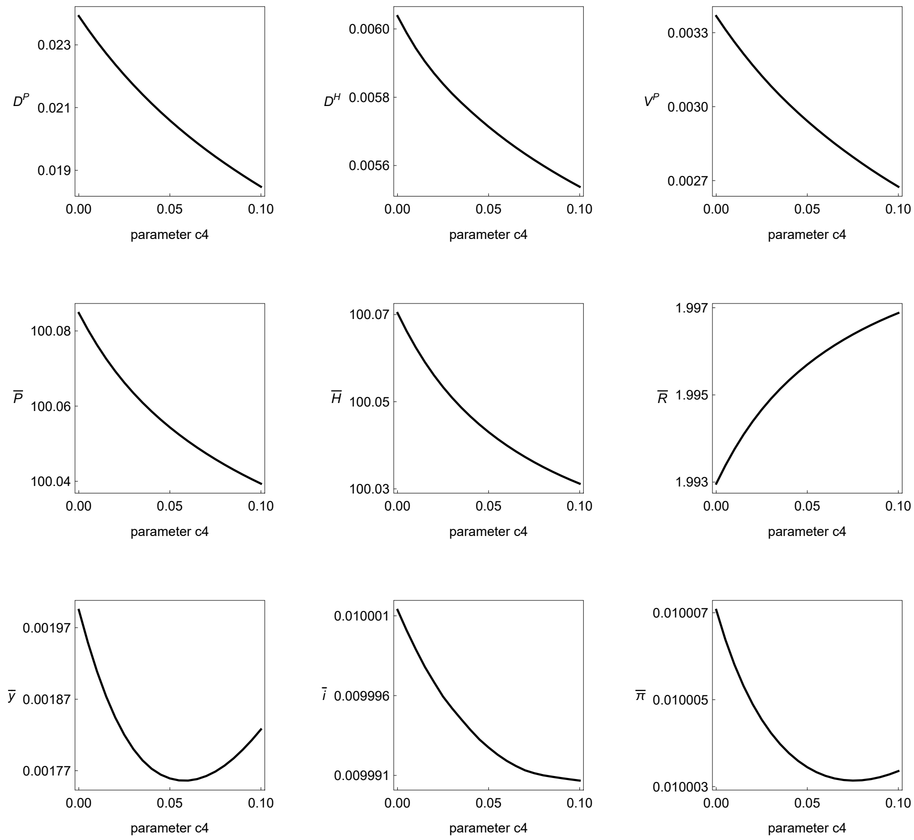

Figure 11: The impact of the house price distortion in the Taylor rule on the performance of the housing and real market. The panels display how the nine statistics D^P, D^H, V^P, $\overline{P}$, $\overline{H}$, $\overline{R}$, $\overline{y}$, $\overline{i}$ and $\overline{\pi}$ depend on the central bank's reaction parameter c_4. The computation is based on 5000 observations and the parameter settings of Tables 1 and 2 are used, except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$.

4.3. Adding the house price trend into the inflation equation

In contrast to the USA, house prices in the euro zone are not yet included in the inflation calculation, except for rents. However, the European Central Bank (ECB) recently proposed to include owner-occupied housing (OOH) into the Harmonised Index of Consumer Prices (HICP). See Nickel et al. (2021) for more details on this project of the ECB. In the following we therefore examine how, in addition to the adjusted Taylor rule, the inclusion of the house price trend in the inflation equation affects the dynamics of the housing and the real market. This turns the inflation equation into

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1} + b_2 y_t + b_3(p_{t-1} - p_{t-2}) + \eta_t, \tag{32}$$

where parameter $b_3 \geq 0$ indicates how sensitively the inflation rate reacts to the house price trend. We define $b_3 = 0.05$. Accordingly, if the house price has increased from period $t-2$ to period $t-1$, i.e. $p_{t-1} - p_{t-2} > 0$, the inflation rate increases, while in the opposite case, i.e. if $p_{t-1} - p_{t-2} < 0$, the inflation rate decreases. Figure 12 reveals that taking the house price trend into account in the inflation equation leads to a significantly more stable housing market. On average we observe a decrease in the distortion of the house price, housing stock and rent level, which means that all three variables move significantly closer to their fundamental values compared to Figure 7 and also to Figure 10. As a result no strong bubbles and crashes occur. Moreover, the market share of extrapolators is smaller which intensifies the stabilizing effect. Regarding the real market a slightly stabilization of the dynamics can be observed since the output gap fluctuates closer around its fundamental value. The adjusted inflation equation results in the following effect. A rising house price leads to a higher inflation rate as can be seen shortly after periods $t = 10$ and $t = 150$. A higher inflation rate, in turn, increases the interest rate, which leads to a falling house price and weakened bubble formation. The opposite can be observed shortly after period $t = 100$. A falling house price means deflationary pressure which in turn leads the central bank to decrease the interest rate. As a result the house price increases.

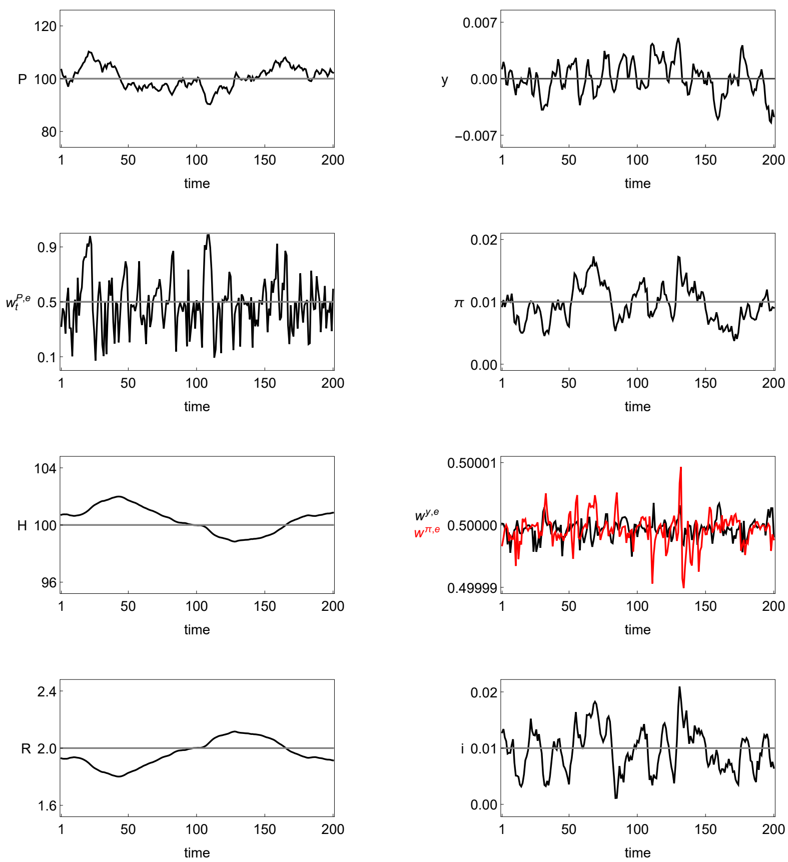

Figure 12: A stochastic simulation run of the housing market and the real market model under i.i.d. normal additive noise on the house price $\epsilon \sim N(0,1)$, the output gap $\tau \sim N(0,0.001)$, the inflation rate $\eta \sim N(0,0.001)$ and the interest rate $\theta \sim N(0,0.001)$ for our base parameter setting (Tables 1 and 2), except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The inflation equation additionally depends on the house price trend ($b_3 = 0.05$). The panels show, from top left to bottom right, the evolution of the house price, the output gap, the market share of extrapolators concerning house price expectations, the inflation rate, the housing stock, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations, the rent level and the nominal interest rate, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

These numerical results are confirmed by Figure 13, which has the same design as Figure 11. As the reaction parameter b_3 increases, so does the average inflation rate $\overline{\pi}$ (panel 9). The average interest rate $\overline{i}$ (panel 8) decrease up to $b_3 = 0.06$ and then rises again, while the average output gap $\overline{y}$ (panel 7) decreases with b_3. As a result the average house price $\overline{P}$ and the average housing stock $\overline{H}$ decline (panels 4 and 5) and the average rent level increases $\overline{R}$ (panel 6). This leads to a falling house price distortion D^P (panel 1) and housing stock distortion D^H (panel 2) as b_3 increases. At least, the volatility of the house price V^P falls with increasing reaction parameter b_3 (panel 3). In summary, the adjusted inflation equation manages to stabilize the housing market and leads to a slightly more stable real market.

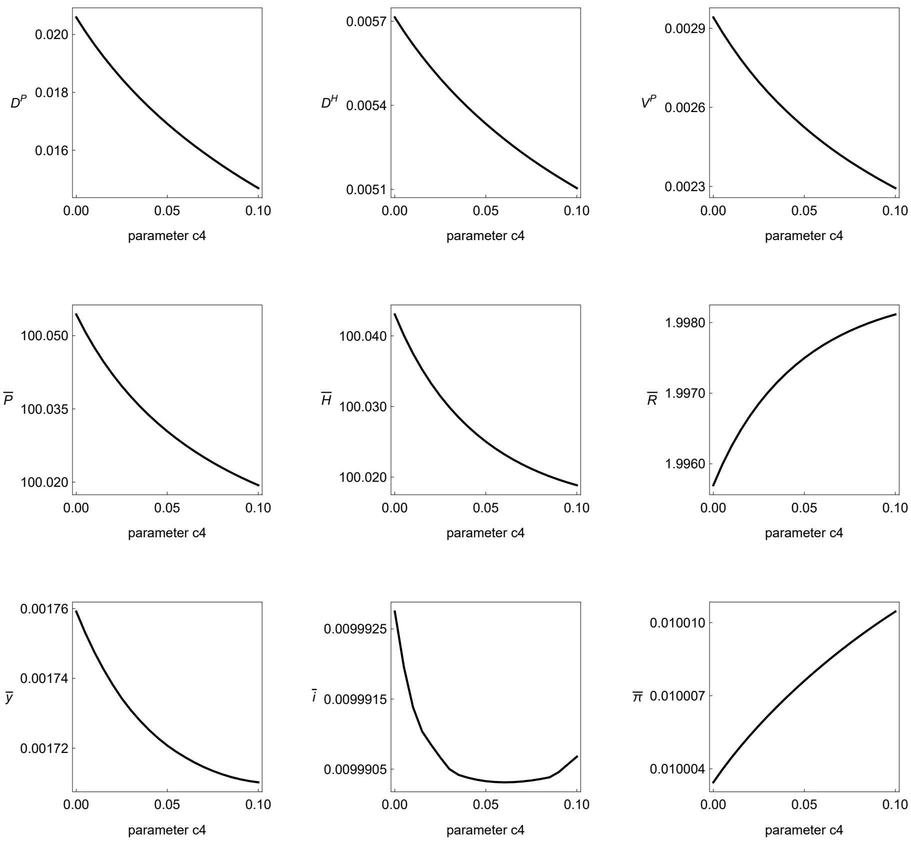

Figure 13: The impact of the house price trend in the inflation equation on the performance of the housing and real market. The panels display how the nine statistics D^P, D^H, V^P, $\overline{P}$, $\overline{H}$, $\overline{R}$, $\overline{y}$, $\overline{\imath}$ and $\overline{\pi}$ depend on the reaction parameter b_3. The computation is based on 4000 observations and the parameter setting of Tables 1 and 2 are used, except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$.

The bottom left panel of Figure 8 illustrates the effect on the house price dynamics with respect to the extrapolative parameter $1.8 < \chi < 2.2$ if the inflation rate also reacts to the house price trend. As can be seen, the amplitude of house price fluctuations can be reduced and is even smaller than in case of the adjusted Taylor rule (top right panel). However, the bifurcation point is reached at a lower value of χ, i.e. the stable steady state turns into quasi-periodic dynamics at $\chi = 1.88$. According to the bottom left panel of Figure 9, the destabilizing effect of an increasing extrapolative parameter χ holds with respect to exogenous noise on the house price $\epsilon \sim N(0, 1)$. Furthermore, the bifurcation route shows higher house price fluctuations between $\chi = 1.9$ and $\chi = 2.0$ which also confirms the results of the bottom left panel from Figure 8.

4.4. Including the house price distortion in the aggregate demand equation

In a final step we examine how the dynamics of the housing and the real market change if the aggregate demand equation additionally depends on the house price distortion, i.e. Equation (13) turns into

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + a_3 (p_{t-1} - p^*) + \epsilon_t \tag{33}$$

Parameter $a_3 \geq 0$ denotes the sensitivity of the aggregate demand with respect to the house price distortion, which is set to $a_3 = 0.05$. Correspondingly, an overvaluation of the house price, i.e. if $p_{t-1} - p^* > 0$, leads to an increase in aggregate demand, while an undervaluation ($p_{t-1} - p^* < 0$) causes y_t to fall.[15] The stochastic simulation run in Figure 14 reveals that the adjusted aggregate demand equation leads to a stabilization of the housing market but destabilizes the real market. As can be seen, the house price, the housing stock and the rent level fluctuate very closely around their fundamental values in small boom-and-bust movements, however without the emergence of strong bubbles and crashes. Furthermore, the market share of extrapolative expectations is observably dropped which also contributes to a more stable market. However, the adjusted aggregate demand equation has a destabilizing effect on the real market as the output gap fluctuates with larger amplitudes around its fundamental value. This effect is strengthened by an increased market share of extrapolative expectations. This can be explained as follows. Around period $t = 20$, for instance, the housing market is overvalued which leads to an increasing output gap and thus to a more articulated output gap bubble. A higher output gap increases the inflation rate both of which raises the interest rate. In addition, a higher interest rate dampens the housing market bubble by decreasing the house price and thus has a stabilizing effect on the housing market. The opposite effect can be observed shortly after period $t = 100$, where the house price falls below its fundamental value which leads to a strong drop in the output gap.

Figure 15 confirms our presented results, which shows the development of the nine statistics for an increasing parameter a_3. As can be seen, the average output gap $\overline{y}$ (panel 7) increases in line with parameter a_3 which leads to a rising average inflation rate $\overline{\pi}$ (panel 9) up to $a_3 = 0.07$ after which it slightly decreases. As a result, the average interest rate $\overline{i}$ rises up to $a_3 = 0.06$ and then falls slightly. The increasing average interest rate leads to a falling average house price $\overline{P}$ (panel 4) and average housing stock $\overline{H}$ (panel 5) and a rising average rent level $\overline{R}$ (panel 6) with increasing parameter a_3. Although this leads to a decrease in both the house price distortion D^P (panel 1) and the housing stock distortion D^H (panel 2), which is a positive effect, the house price volatility V^P (panel 3) cannot be reduced by increasing parameter a_3.

The bottom right panel of Figure 8 illustrates how the house price evolves with increasing extrap-

[15] This seems intuitive and in line with historical experience (see, for example, Iacoviello (2011)). For instance, prior to the subprime crisis in the USA persons without creditworthiness all of the sudden were capable of getting loans to purchase houses. Due to ever-raising house prices creditors were able to increase consumption often using housing as collateral.

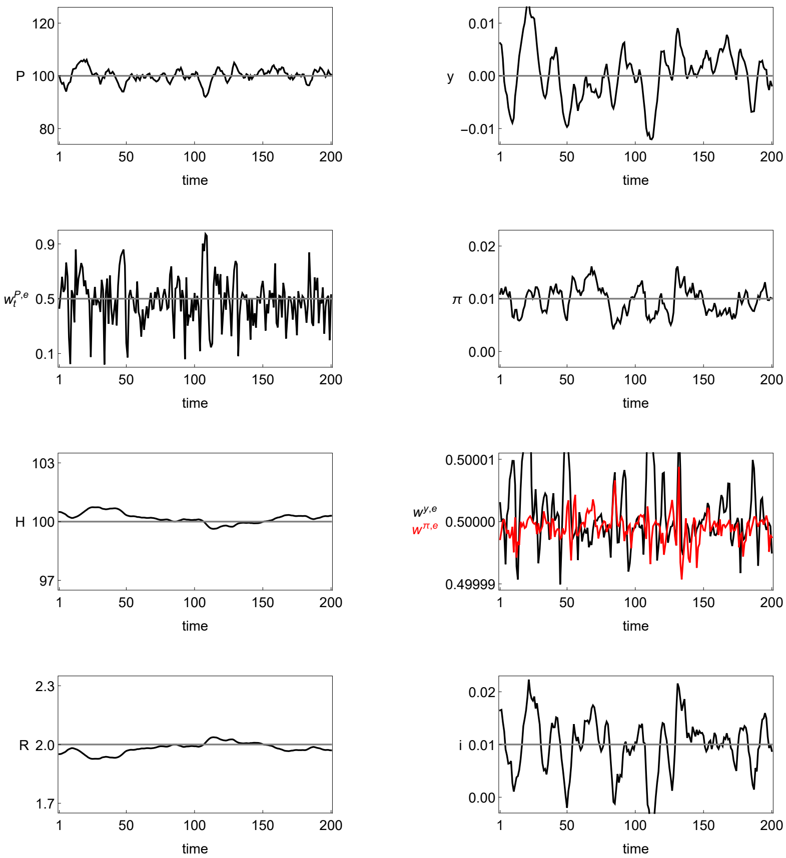

Figure 14: A stochastic simulation run of the housing market and the real market model under i.i.d. normal additive noise on the house price $\epsilon \sim N(0,1)$, the output gap $\tau \sim N(0, 0.001)$, the inflation rate $\eta \sim N(0, 0.001)$ and the interest rate $\theta \sim N(0, 0.001)$ for our base parameter setting (Tables 1 and 2), except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$. The aggregate demand equation additionally depends on the house price distortion ($a_3 = 0.05$). The panels show, from top left to bottom right, the evolution of the house price, the output gap, the market share of extrapolators concerning house price expectations, the inflation rate, the housing stock, the market share of extrapolators concerning the output gap (black line) and inflation (red line) expectations, the rent level and the nominal interest rate, respectively. The dynamics is depicted for 200 time steps and a longer transient period has been cleared.

olative parameter $1.9 < \chi < 2.2$ in case of the adjusted aggregate demand equation. The previous observed stabilizing effect cannot be confirmed. The system loses its stability at a smaller parameter χ, as the bifurcation point is at $\chi = 1.77$. Moreover, the amplitude of house price fluctuations is slightly larger compared to the other three scenarios in Figure 8. Similar results can be observed in the bottom right panel of Figure 9 which repeats the simulation in a noisy environment. As can be seen, the house price exhibits larger fluctuations with increasing χ compared to the other three panels. This observation is most evident between $\chi = 1.9$ and $\chi = 2.0$.

5. Conclusions

The dramatic global economic and financial crisis that followed the collapse of the U.S. housing market in 2006 has intensely demonstrated that developments in the housing market may have serious effects on the real economy. According to Tsatsaronis and Zhu (2004) and Tripathi (2019), interdependencies between the housing market and the real economy should therefore be taken into account. By merging

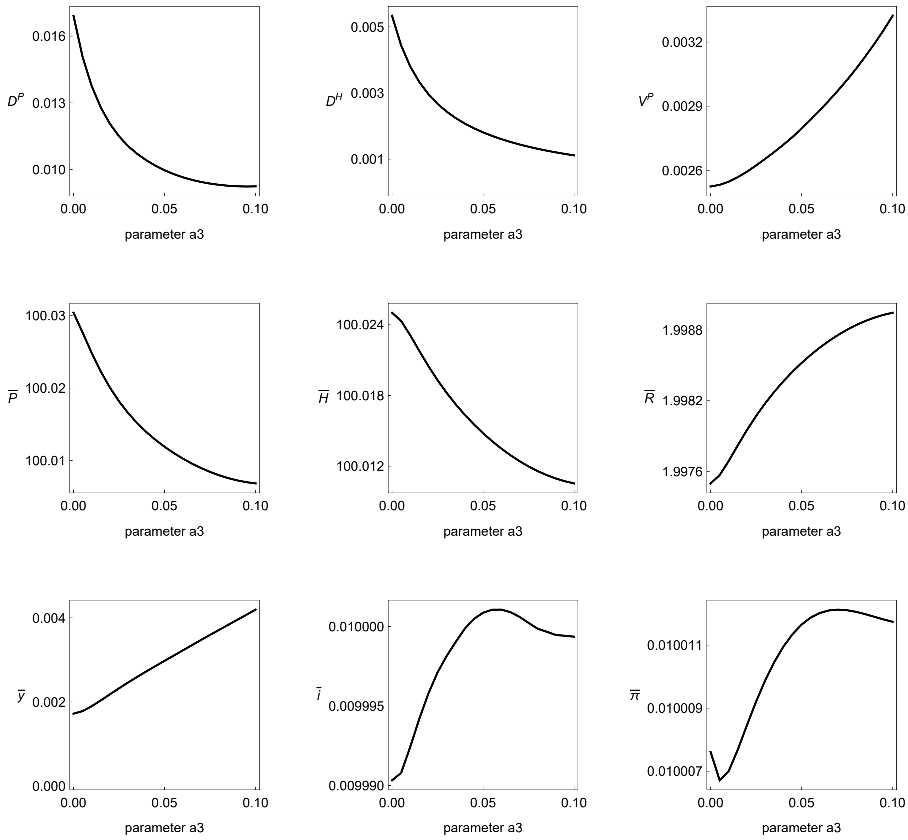

Figure 15: The impact of the house price distortion in the aggregate demand equation on the performance of the housing and real market. The panels display how the nine statistics D^P, D^H, V^P, $\overline{P}$, $\overline{H}$, $\overline{R}$, $\overline{y}$, $\overline{i}$ and $\overline{\pi}$ depend on the inflation's reaction parameter a_3. The computation is based on 4000 observations and the parameter setting of Tables 1 and 2 are used, except that $\chi = 0.35$, $\phi = 0.125$, $e_y = e_\pi = 1.1$ and $f_y = f_\pi = 0.35$.

Schmitt and Westerhoff's (2019) housing market model and De Grauwe's (2010a, b, c, 2012) real market model, we develop a model framework that seeks to take these observations into account.

The housing market model comprises a rental market and a housing capital market determining fundamental relations between the house price, the housing stock and the rent level. The real market is represented by a standard aggregate-demand-aggregate-supply model, consisting of the output gap and the inflation rate, complemented with a Taylor rule describing the behavior of the central bank. Expectations in both submodels are formed by heterogeneous and boundedly rational agents who select between two competing expectation rules depending on the rules' recent forecasting success. We show that this expectation formation behavior means that both the housing market and the real market model are able to generate complex endogenous boom-and-bust dynamics.

Interaction between the housing market and the real market model is examined by four consecutive steps that function as follows. First, we set the interest rate the central bank determines through the Taylor rule in the housing market. Second, we extend the Taylor rule by including the house price distortion. Third, the inflation equation is adjusted by adding the house price trend. Fourth, we include the house price distortion in the aggregate demand equation. We find that a gradually linkage of the two submodels makes the housing market more stable since both the volatility and distortion of the house price, the housing stock and the rent level decrease. However, there is no significant stabilizing effect on the real market. The real market becomes even more unstable by an adjusted aggregate demand equation.

It remains without question, the interactions of the housing market and the real economy is very complex and a complete understanding of these interdependencies is still lacking. However, we hope that our work may offer some new insights in this research field. Moreover, we would like to emphasize that this paper should be seen as an early stage in a broader research agenda and offers a lot of expansion potential. Of course, much more work is needed in this captivating and major research field.

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